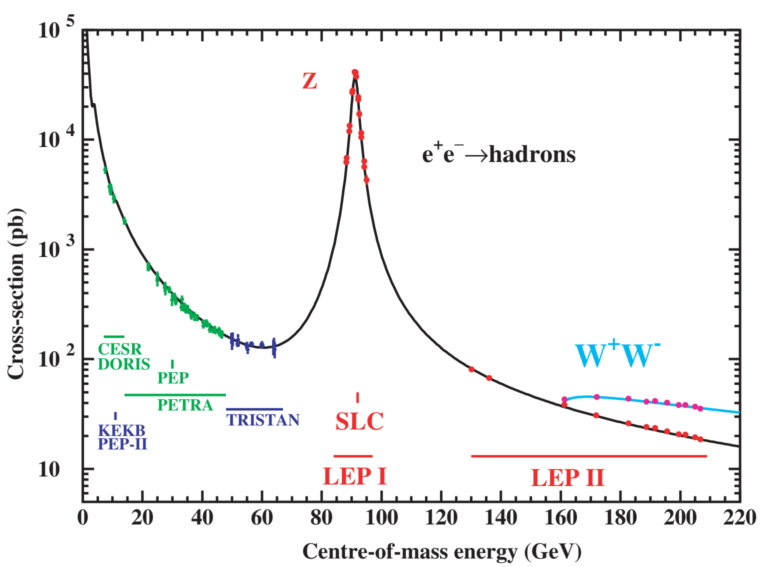
Status and prospects of the electroweak SM fit after the Higgs discovery with Gfitter

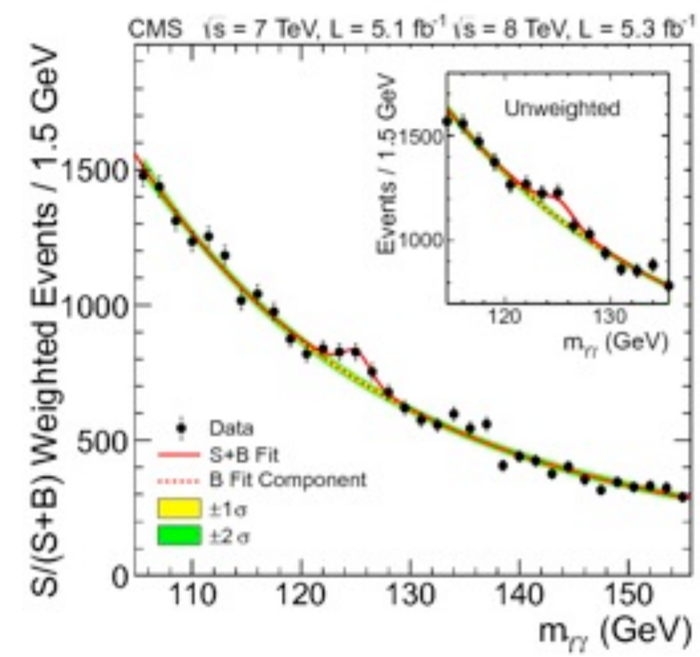


Roman Kogler

for the Gfitter group

LHC Run1 Aftermath

Bad Honnef, Oct 1, 2013

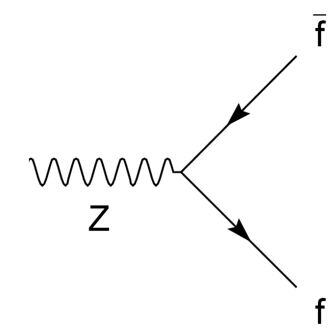


The Gfitter group: M. Baak (CERN), J. Cuth (Univ. of Mainz) J. Haller (Univ. of Hamburg), A. Hoecker (CERN), R. K. (Univ. of Hamburg), K. Mönig (DESY), M. Schott (Univ. of Mainz) J. Stelzer (Univ. of Michigan)

Predictive Power of the SM

Tree level relations for Z → f $\bar{f}$

$$g^{(0)}_{V,f} \equiv g^{(0)}_{L,f} + g^{(0)}_{R,f} = I^f_3 - 2Q^f \sin^2\theta_W$$
$$g^{(0)}_{A,f} \equiv g^{(0)}_{L,f} - g^{(0)}_{R,f} = I^f_3$$

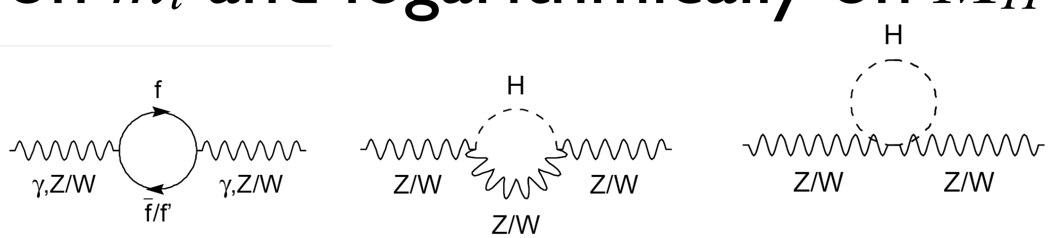


$$i\bar{f}\gamma^\mu\left(g_{V,f} - g_{A,f}\gamma_5\right) f Z_\mu$$

- Unification connects the electromagnetic and the weak couplings
- M_W can be expressed in terms of M_Z and G_F

Radiative corrections

- Parametrisation through electroweak form factors ρ, κ, Δr
- Effective couplings at the Z-pole
- ρ, κ, Δr depend nearly quadratically on m_t and logarithmically on M_H



$$\sin^2\theta^f_{\mathrm{eff}} = \kappa^f_Z \sin^2\theta_W$$

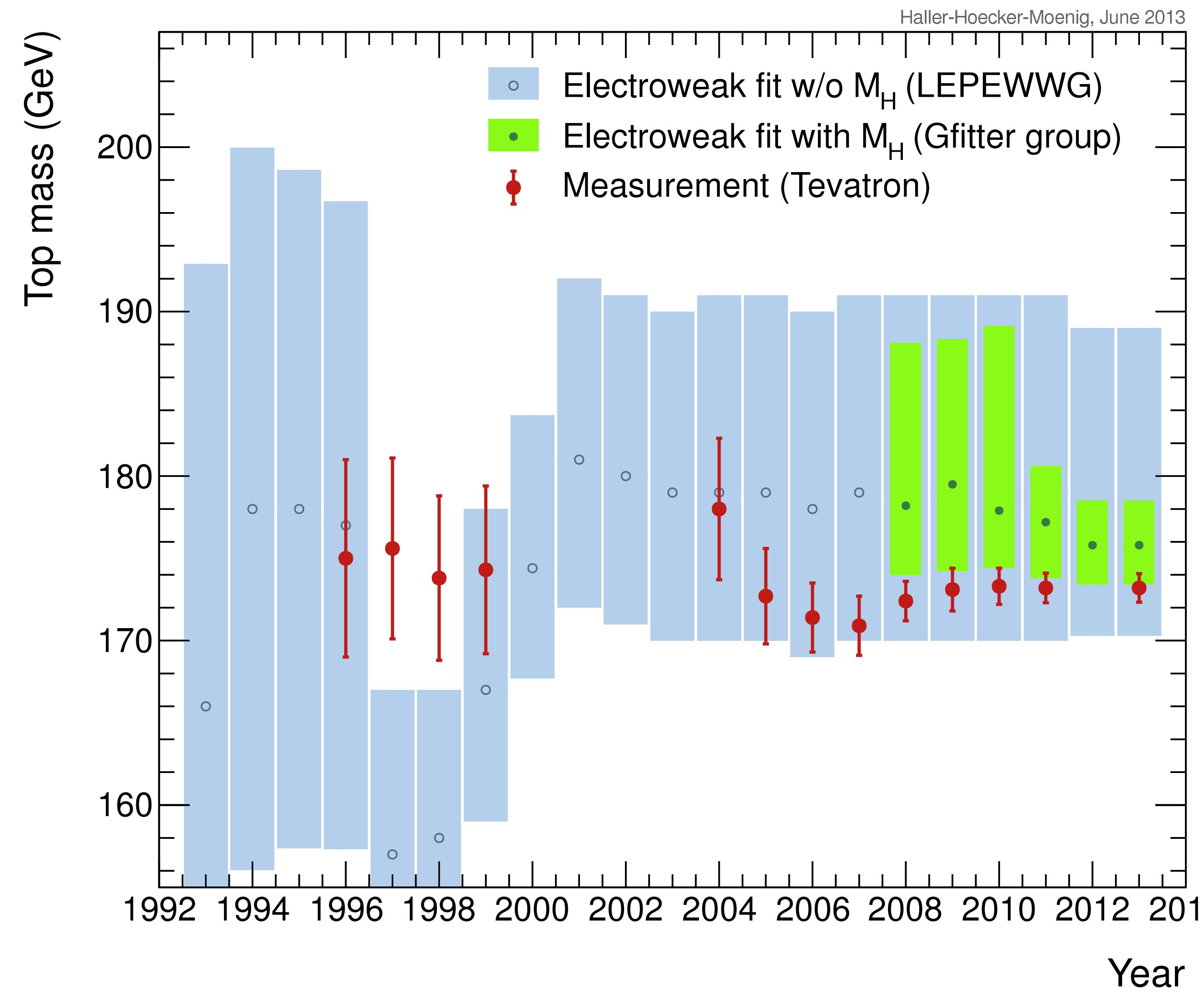
$$g_{V,f} = \sqrt{\rho^f_Z}\left(I^f_3 - 2Q^f\sin^2\theta^f_{\mathrm{eff}}\right)$$
$$g_{A,f} = \sqrt{\rho^f_Z}\, I^f_3$$

$$M^2_W = \frac{M^2_Z}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1+\Delta r)}{G_F M^2_Z}}\right)$$

Electroweak Fits

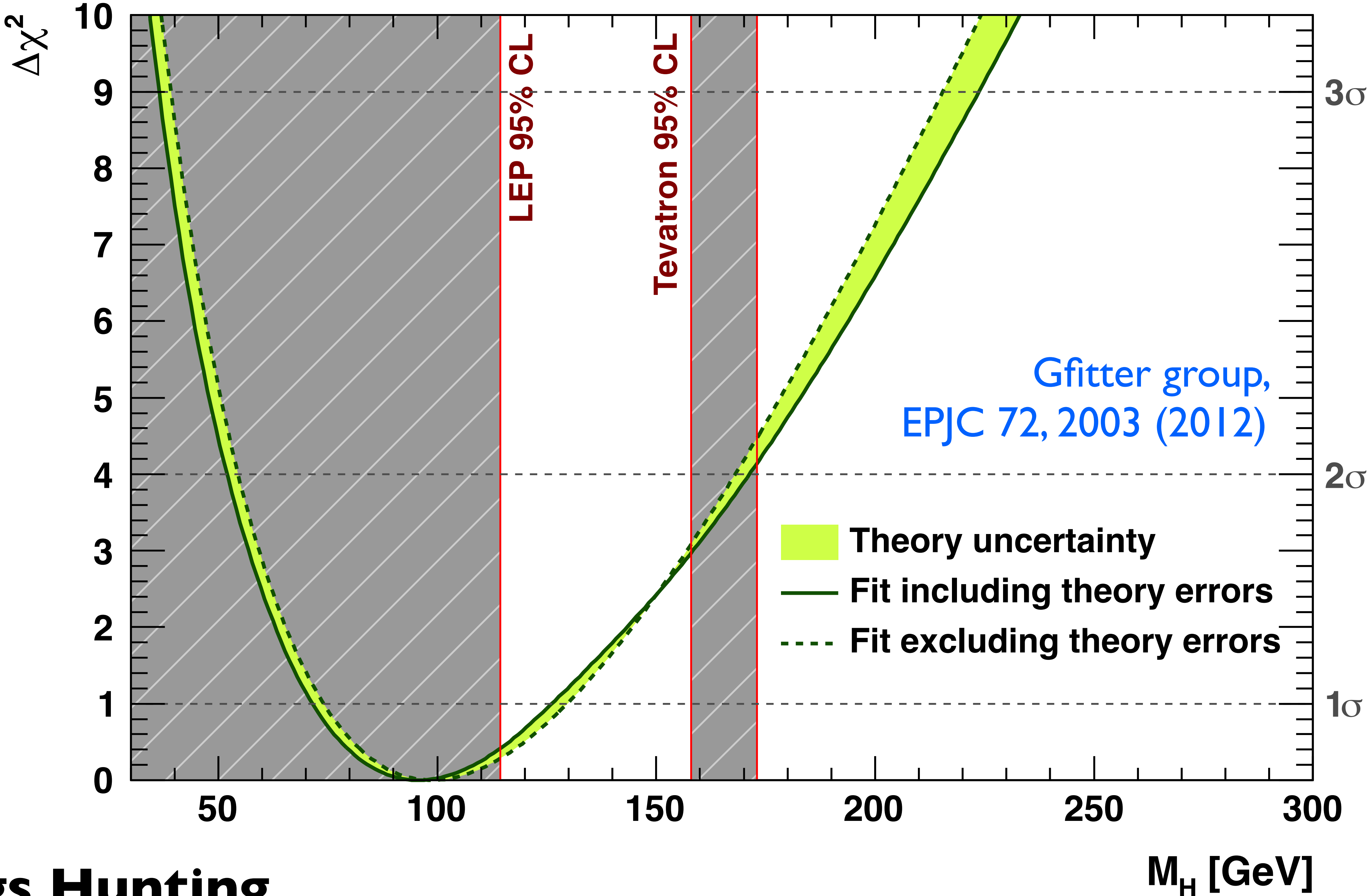
A long tradition

- Huge amount of pioneering work to precisely understand loop corrections
- Observables known at least in two-loop order, sometimes higher orders available
- Precision measurements crucial, after the LEP/SLC era results from Tevatron and LHC become available



- Top mass predictions from loop effects available since ~1990
  - LEPEWWG fits since 1993
- The EW fit has always been able to predict the top mass correctly

The Last Missing Piece



Higgs Hunting

- Indirect determination (2011): $M_H = 96\,^{+31}_{-24}$ GeV
- Exclusion limits incorporated in EW fits: $M_H = 120\,^{+12}_{-5}$ GeV

Outline

1. Status of the Global EW Fit

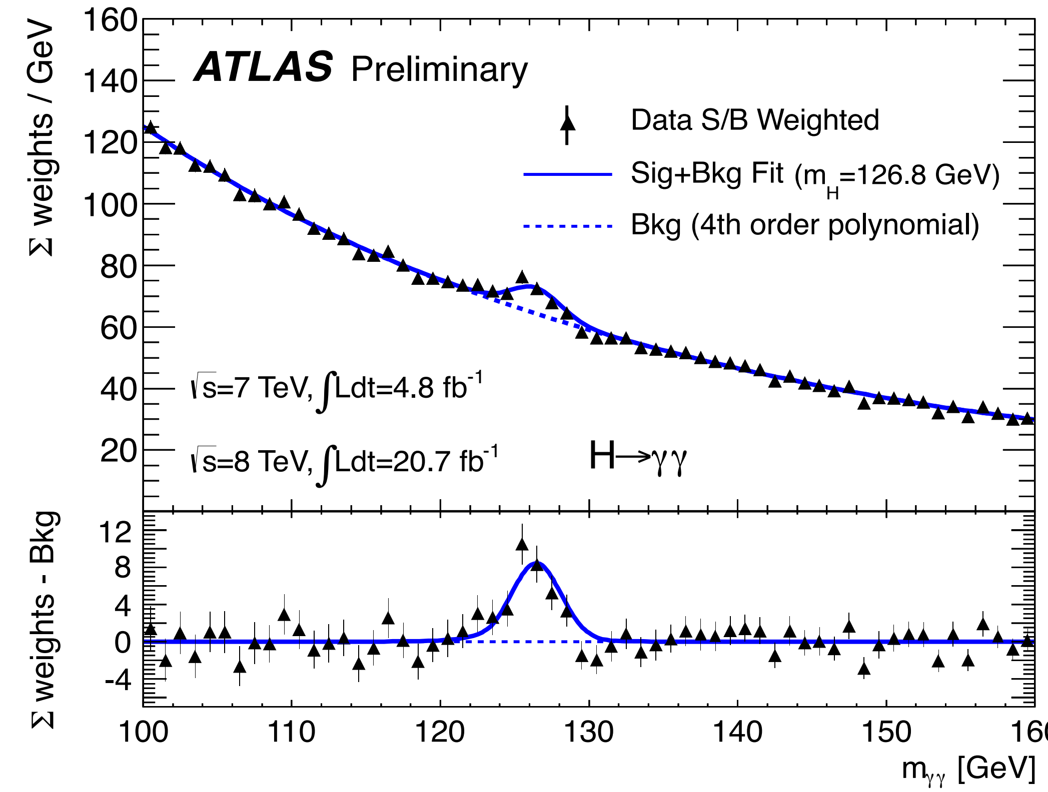
2. BSM and Higgs Coupling

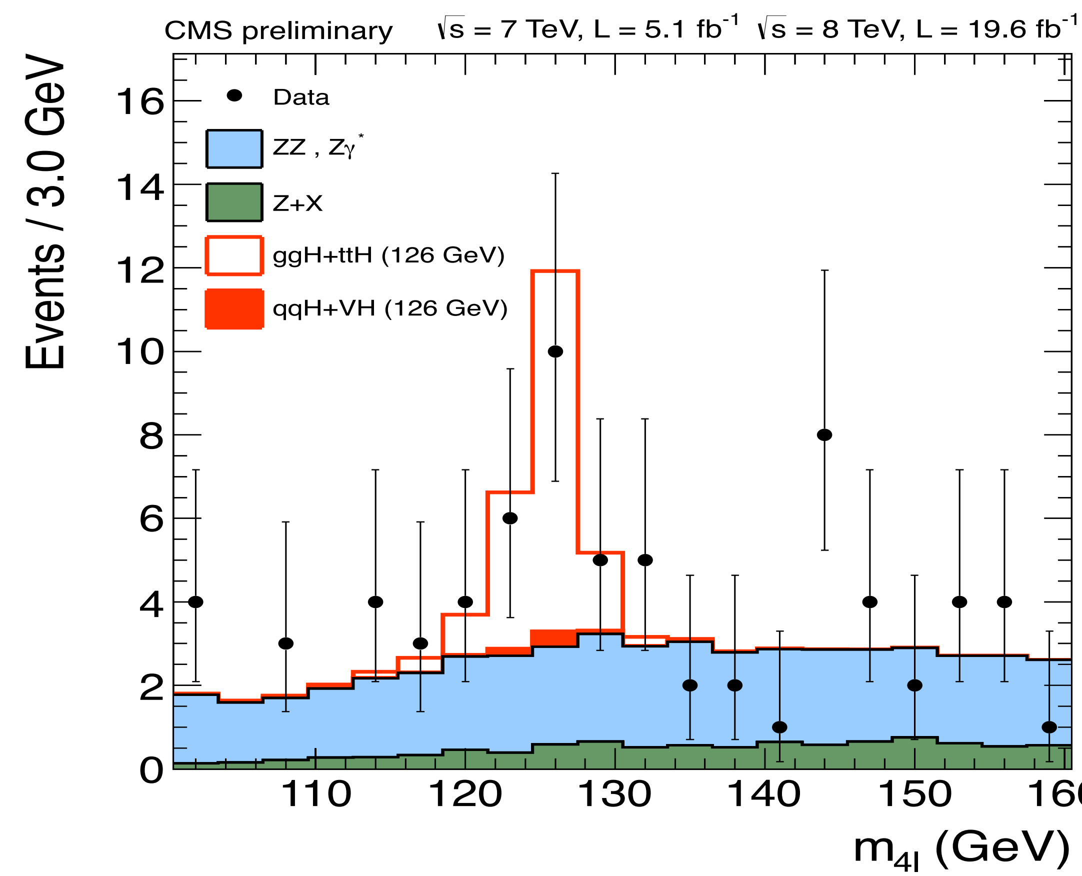
3. Prospects of the EW Fit

The SM Fit with the Higgs

The discovery of a new boson

- Cross section, production rate times branching ratios, spin, parity sofar compatible with predictions for the SM Higgs
- Assume that the boson is the SM Higgs
- EW fit: M_H = 125.7 ± 0.4 GeV
  ATLAS: M_H = 126.0 ± 0.4 ± 0.4 GeV
  CMS: M_H =125.3 ± 0.4 ± 0.5 GeV
  [arXiv:1207.7214, arXiv:1207.7235]
- Change between fully uncorrelated and fully correlated systematic uncertainties minor:
  δM_H : 0.4 → 0.5 GeV





The SM Fit with Gfitter

Unique situation

- For first time SM is fully over-constrained.
- electroweak observables can be unambiguously predicted at loop level
- Powerful predictions of key observables now possible, much better than without M_H

Calculations used

- M_W mass of the W boson [M. Awramik et al., Phys. Rev. D69, 053006 (2004)]
- Γ_Z, Γ_W partial and total widths of the Z and W [Cho et. al, arXiv:1104.1769]
- $\sin^2\theta^l_{\mathrm{eff}}$ effective weak mixing angle [M. Awramik et al., JHEP 11, 048 (2006), M. Awramik et al., Nucl.Phys.B813:174-187 (2009)]
- Γ_{had} QCD Adler functions at N3LO [P.A. Baikov et al., Phys.Rev.Lett. 108, 222003 (2012)]
- R_b partial width of $Z \rightarrow b\bar{b}$ [Freitas et al., JHEP08, 050 (2012), Erratum. 1305 (2013) 074] **NEW!**

Updated Calculation of R^0_b

- R^0_b = partial decay width of $Z \to b\bar{b}$ to $Z \to q\bar{q}$
- We use calculation with full EW 2-loop corrections of $Z \to b\bar{b}$
- A. Freitas et al., JHEP 1208 (2012) 050, Erratum ibid. 1305 (2013) 074

Recently a mistake was found in this calculation

- **Old**: Two-loop corrections to R^0_b comparable to experimental uncertainty (6.6×10^{-4})
  - Moved theoretical prediction by 1.5σ
  - Much more than the originally estimated theory uncertainty!
- **New**: bug in calculation of R^0_b has been corrected
  - sizeable reduction of the size of the two-loop correction
- All results shown here and on Gfitter homepage use the corrected R^0_b calculation

Experimental Input

Observables:

- Z-pole observables: LEP/SLD results
  [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
- M_W and Γ_W: LEP/Tevatron
  [arXiv:1204:0042, arXiv:1302.3415]
- m_t : Tevatron [arXiv:1305:3929]
- $\Delta\alpha_{had}^{(5)}(M_Z)$ [M. Davier et al., EPJC 71, 1515 (2011)]
- $\overline{m}_c$, $\overline{m}_b$: world averages
  [PDG, J. Phys. G33, 1 (2006)]
- M_H : LHC [arXiv:1207.7214 , arXiv:1207.7235]

7 (+2) free fit parameters:

- M_Z, M_H, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, $\overline{m}_c$, $\overline{m}_b$, m_t
- nuisance parameters for theoretical uncertainties
  δM_W (4 MeV), $\delta\sin^2\theta^l_{eff}$ ($4.7 \cdot 10^{-5}$)

M_H [GeV]°	$125.7^{+0.4}_{-0.4}$	LHC
M_W [GeV]	80.385 ± 0.015	Tevatron
Γ_W [GeV]	2.085 ± 0.042	
M_Z [GeV]	91.1875 ± 0.0021	LEP
Γ_Z [GeV]	2.4952 ± 0.0023	
σ^0_{had} [nb]	41.540 ± 0.037	
R^0_ℓ	20.767 ± 0.025	
$A^{0,\ell}_{FB}$	0.0171 ± 0.0010	
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	SLC
$\sin^2\theta^\ell_{eff}(Q_{FB})$	0.2324 ± 0.0012	
A_c	0.670 ± 0.027	SLC
A_b	0.923 ± 0.020	
$A^{0,c}_{FB}$	0.0707 ± 0.0035	LEP
$A^{0,b}_{FB}$	0.0992 ± 0.0016	
R^0_c	0.1721 ± 0.0030	
R^0_b	0.21629 ± 0.00066	
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.20 ± 0.87	Tevatron
$\Delta\alpha^{(5)}_{had}(M^2_Z)$ $^{(\dagger\triangle)}$	2757 ± 10	

SM Fit Results

[The Gfitter Group, EPJC 72, 2205 (2012)]

Fit comes in three flavours

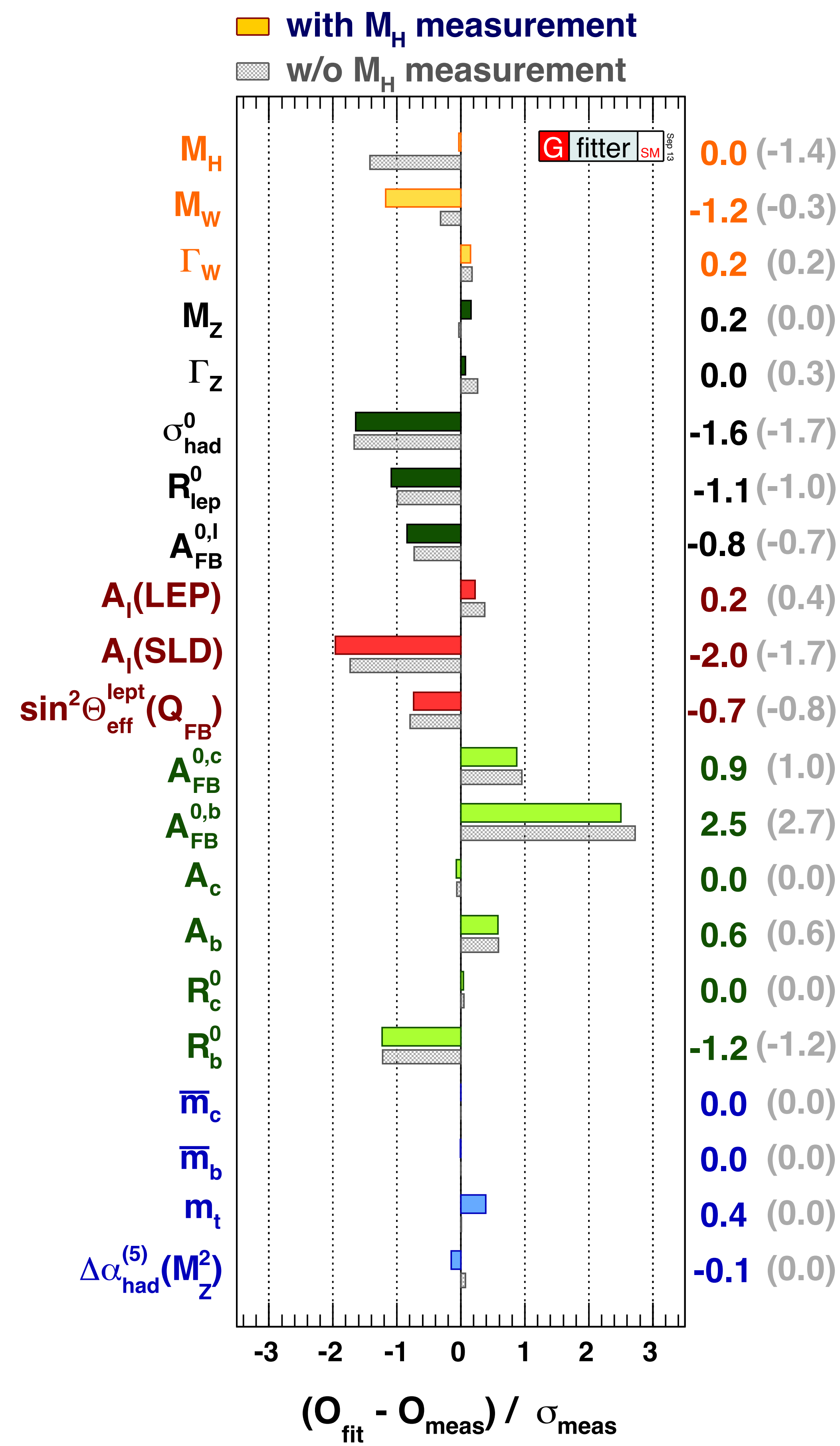
- left: full fit incl. MH
- middle: full fit w/o MH
- right: fit w/o observable in given row

Parameter	Input value	Free in fit	Fit Result	Fit without M_H measurements	Fit without exp. input in line
M_H [GeV]$^\circ$	$125.7^{+0.4}_{-0.4}$	yes	$125.7^{+0.4}_{-0.4}$	94.7^{+25}_{-22}	94.7^{+25}_{-22}
M_W [GeV]	80.385 ± 0.015	–	$80.367^{+0.006}_{-0.007}$	$80.367^{+0.006}_{-0.007}$	80.360 ± 0.011
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1878 ± 0.0021	91.1978 ± 0.0114
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4954 ± 0.0014	2.4954 ± 0.0014	2.4950 ± 0.0017
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.479 ± 0.014	41.479 ± 0.014	41.471 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.740 ± 0.017	20.740 ± 0.017	20.715 ± 0.026
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	$0.01626^{+0.0001}_{-0.0002}$	$0.01626^{+0.0001}_{-0.0002}$	0.01624 ± 0.0002
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1472 ± 0.0007	0.1472 ± 0.0007	–
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	$0.23149^{+0.00010}_{-0.00008}$	$0.23149^{+0.00010}_{-0.00008}$	0.23150 ± 0.00009
A_c	0.670 ± 0.027	–	$0.6679^{+0.00034}_{-0.00028}$	$0.6679^{+0.00034}_{-0.00028}$	0.6680 ± 0.00031
A_b	0.923 ± 0.020	–	$0.93464^{+0.00005}_{-0.00007}$	$0.93464^{+0.00005}_{-0.00007}$	0.93463 ± 0.00006
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	0.0738 ± 0.0004	0.0738 ± 0.0004	0.0737 ± 0.0004
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	0.1032 ± 0.0005	0.1032 ± 0.0005	0.1034 ± 0.0003
R^0_c	0.1721 ± 0.0030	–	0.17223 ± 0.00006	0.17223 ± 0.00006	0.17223 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21548 ± 0.00005	0.21548 ± 0.00005	0.21547 ± 0.00005
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	$1.27^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	$4.20^{+0.17}_{-0.07}$	–
m_t [GeV]	173.20 ± 0.87	yes	173.53 ± 0.82	173.53 ± 0.82	$176.11^{+2.88}_{-2.35}$
$\Delta\alpha^{(5)}_{\rm had}(M^2_Z)$ $^{(\dagger\triangle)}$	2757 ± 10	yes	2755 ± 11	2755 ± 11	2718^{+49}_{-43}
$\alpha_s(M^2_Z)$	–	yes	$0.1190^{+0.0028}_{-0.0027}$	$0.1190^{+0.0028}_{-0.0027}$	0.1190 ± 0.0027

Global Fit: Results

Pull values after the fit

- No pull value exceeds deviations of more than 3σ (consistency of SM)
- Small values for M_H, A_c, $R^0{}_c$, m_c and m_b indicate that their input accuracies exceed the fit requirements
- Largest deviations in the b-sector: $A^{0,b}{}_{FB}$ with 2.5σ (small dependence on M_H)
- $R^0{}_b$ using one-loop calculation: 0.8σ
- inclusion of M_H: largest effect on M_W prediction shifted by ~13 MeV

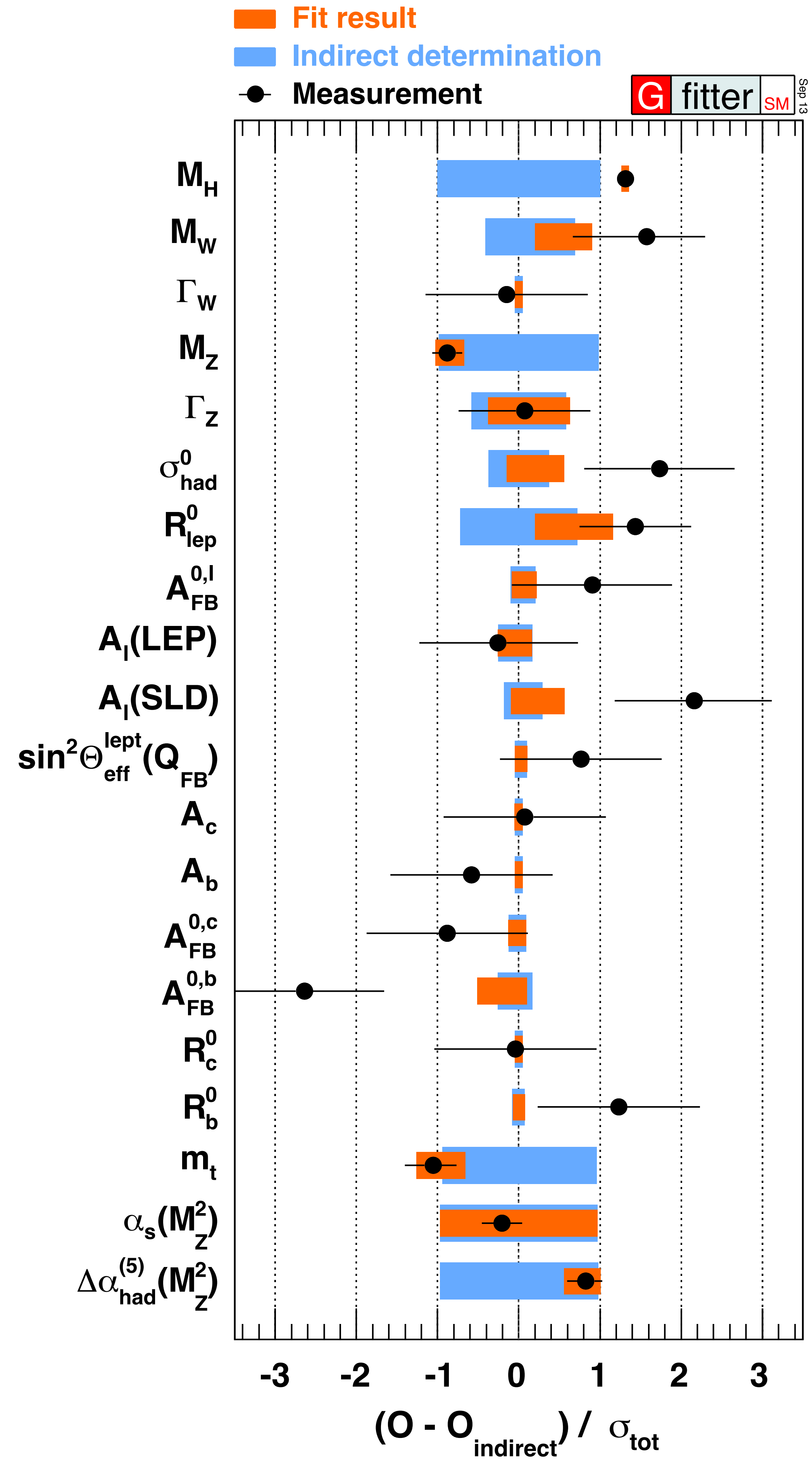


Plot inspired by Eberhardt et al. [arXiv:1209.1101]

Global Fit: Results

Indirect determination of EWPO

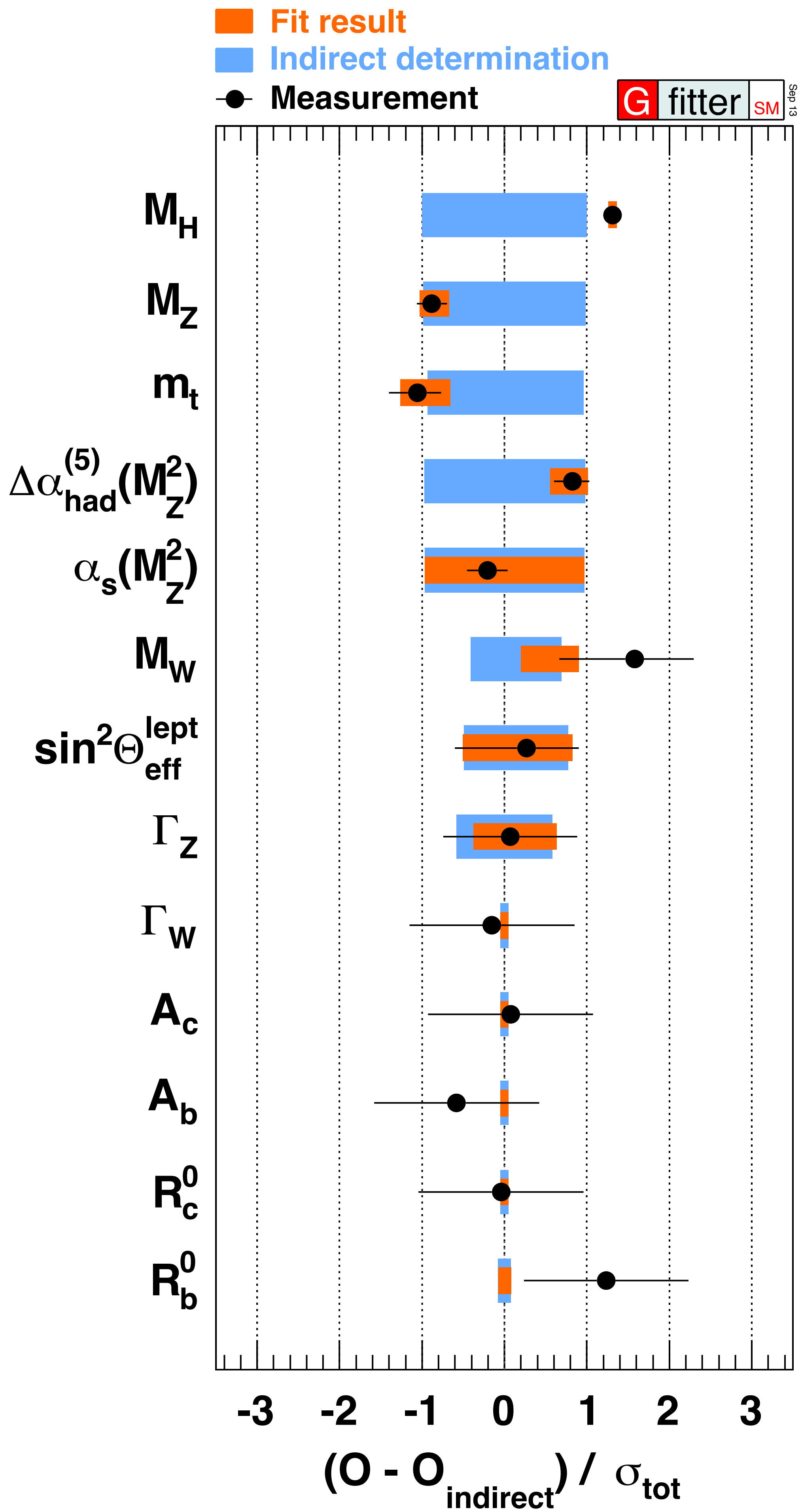
- Shown are pull values for
  - full fit
  - indirect determination
  - measurement
- deviations from indirect determination
  - divided by total error (=error from indirect and measurement)
- Fit result agrees well with the measurements
- Prediction often more precise than the measurement



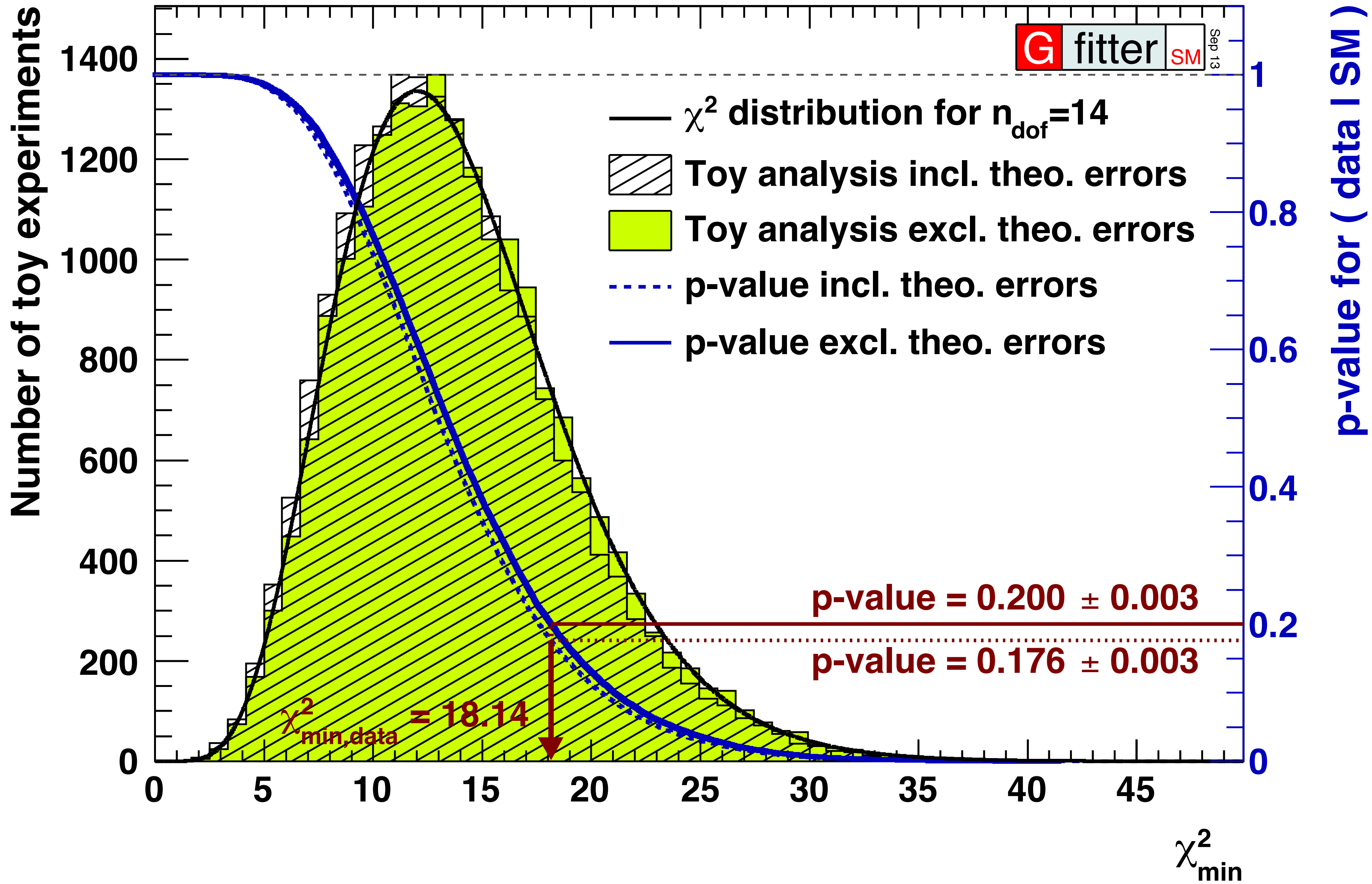
Global Fit: Results

Indirect determination of EWPO

- Shown are pull values for
  - full fit
  - indirect determination
  - measurement
- deviations from indirect determination
  - divided by total error (=error from indirect and measurement)
- Fit result agrees well with the measurements
- Prediction often more precise than the measurement
- Consistent picture when combining asymmetry observables



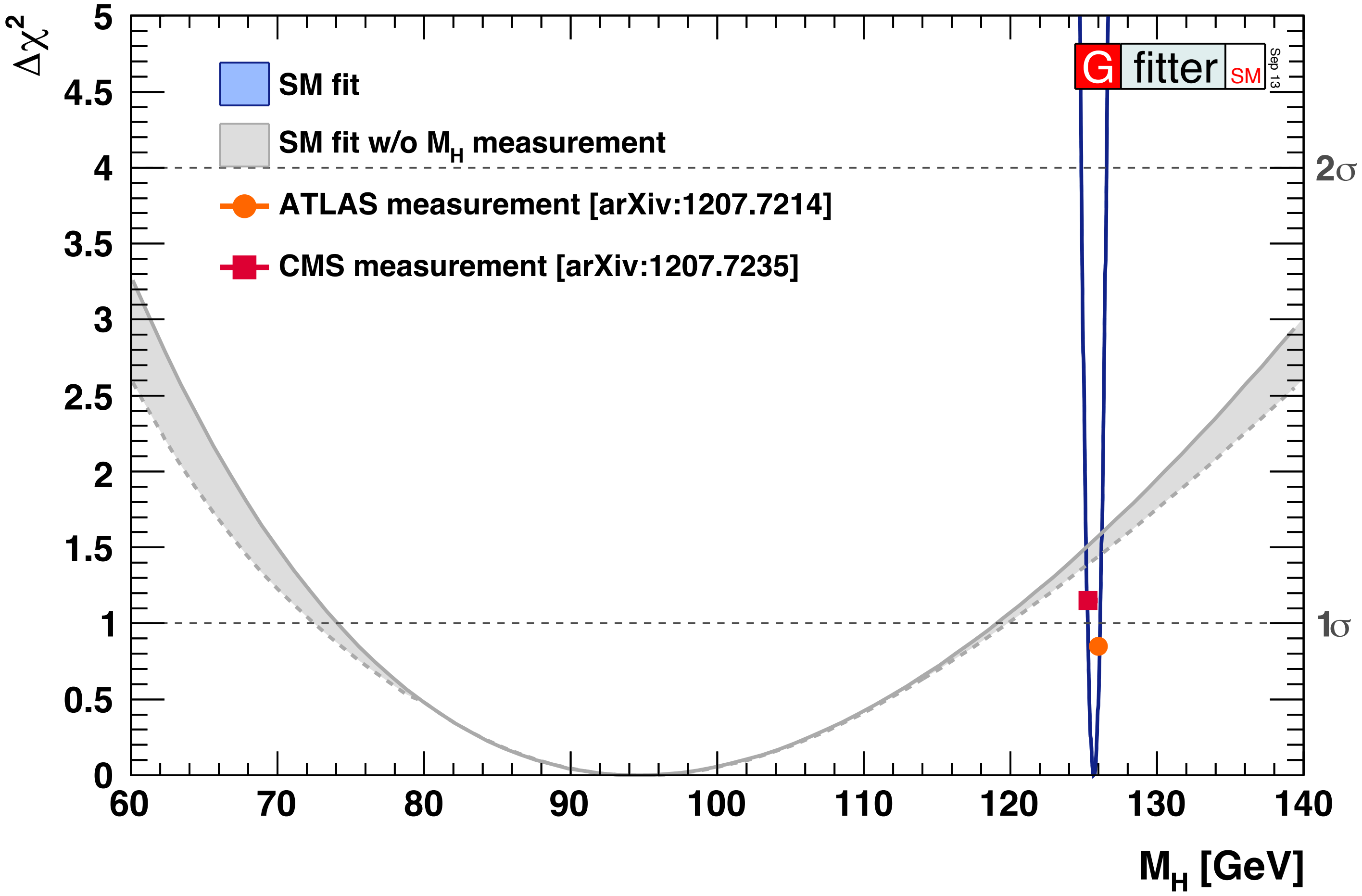
Goodness of Fit

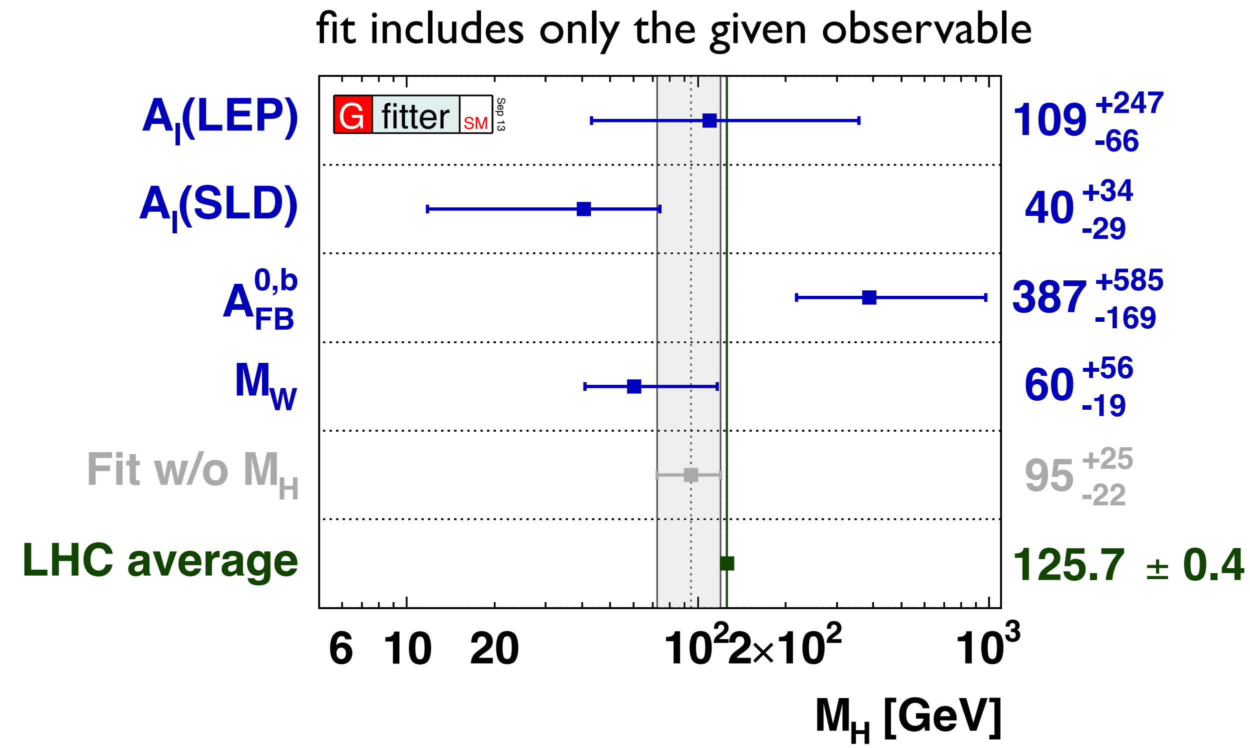


χ^2_{min}/ndf = 18.1/14 → p-value = 0.20

- value of χ^2_{min} does not change much due to inclusion of M_H measurement
- without M_H measurement: χ^2_{min} /ndf = 16.7/13 → naive p-value = 0.21
- p-value = 0.18 (exp) ± 0.02 (theo)

Global Fit: Results





Scan of the $\Delta\chi^2$ profile versus M_H

- blue line: full SM fit
- grey band: fit without M_H measurement
- fit without M_H input gives $M_H = 94\,^{+25}_{-22}$ GeV
- consistent within 1.3σ with measurement

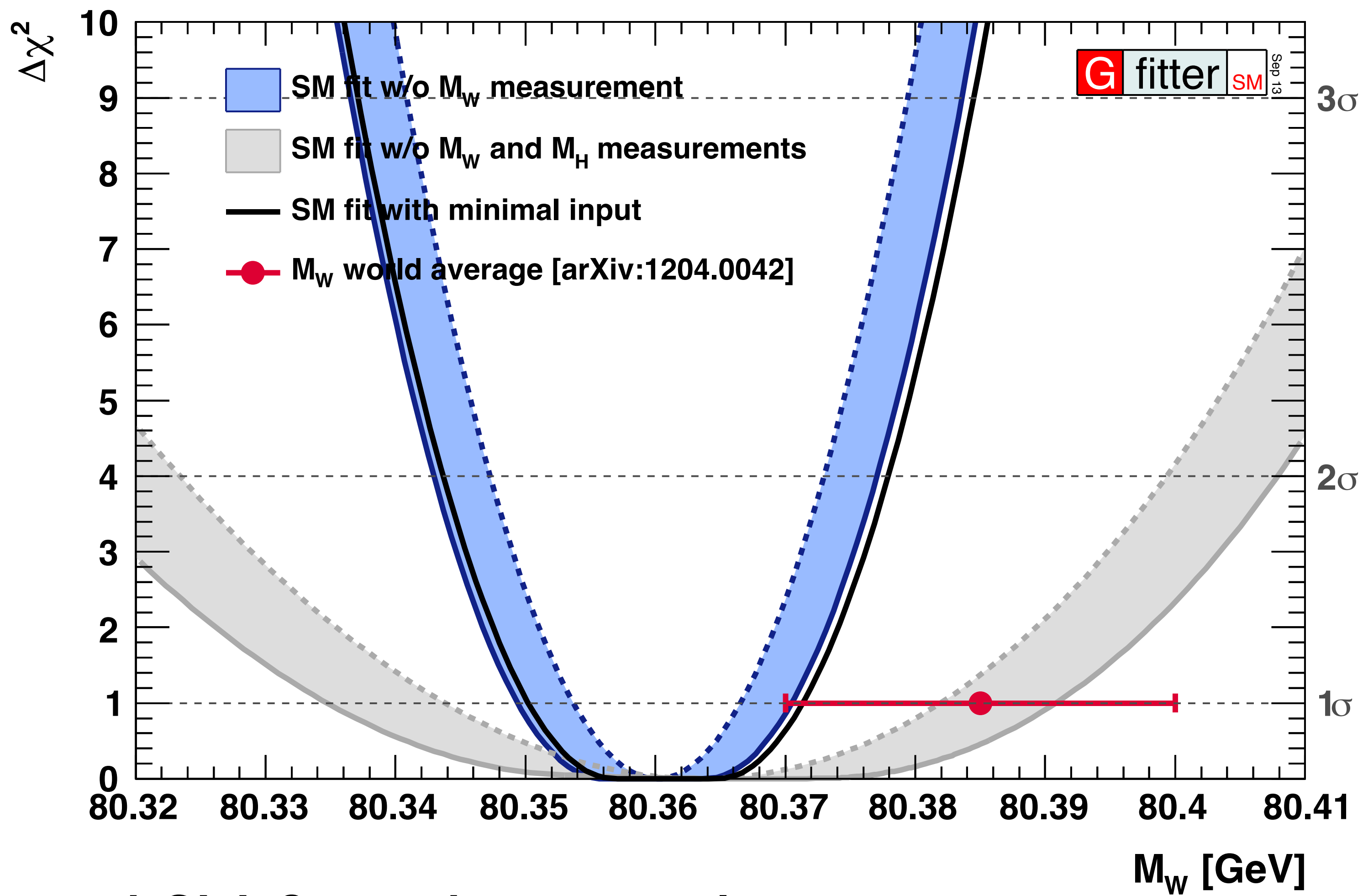
Determination of M_H removing all sensitive observables except the given one:

Tension (2.5σ) between $A^{0,b}_{FB}$, A_{lep}(SLD) and M_W visible

Indirect Determination: W Mass

Scan of the $\Delta\chi^2$ profile versus M_W

- M_H measurement allows for precise constraint of M_W
- also shown: SM fit with minimal input: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, M_H and fermion masses



- Consistency between total fit and SM fit with minimal input
- Fit result for the indirect determination of M_W:

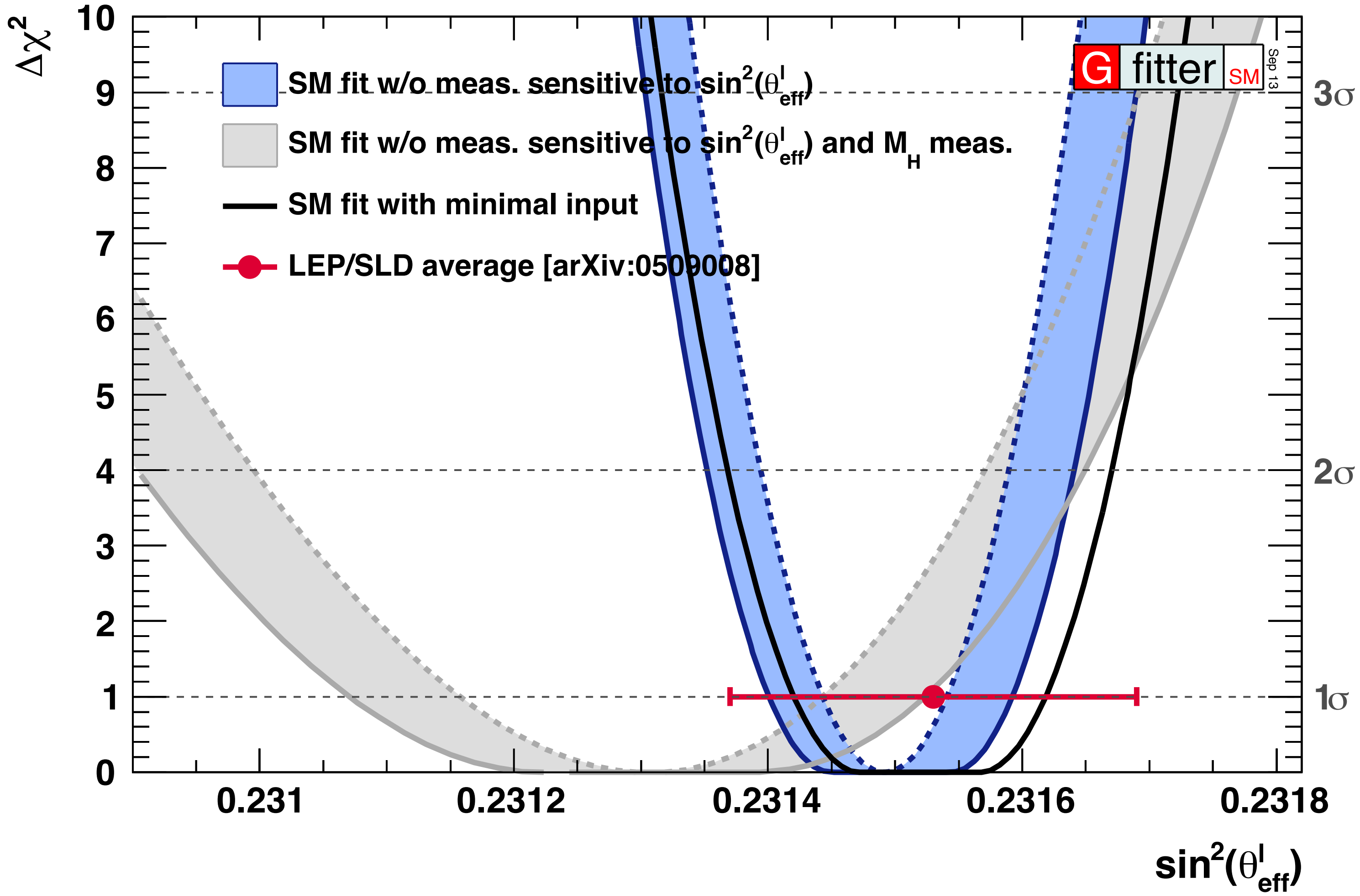
$$\begin{aligned} M_W &= 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \\ &\quad \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{\rm theo} \\ &= 80.359 \pm 0.011_{\rm tot} \end{aligned}$$

More precise than the direct measurements

The Effective Weak Mixing

Scan of the $\Delta\chi^2$ profile versus $\sin^2\theta^l_{eff}$

- all observables sensitive to $\sin^2\theta^l_{eff}$ removed from fit
- M_H measurement allows for precise constraint of $\sin^2\theta^l_{eff}$
- also shown: SM fit with minimal input

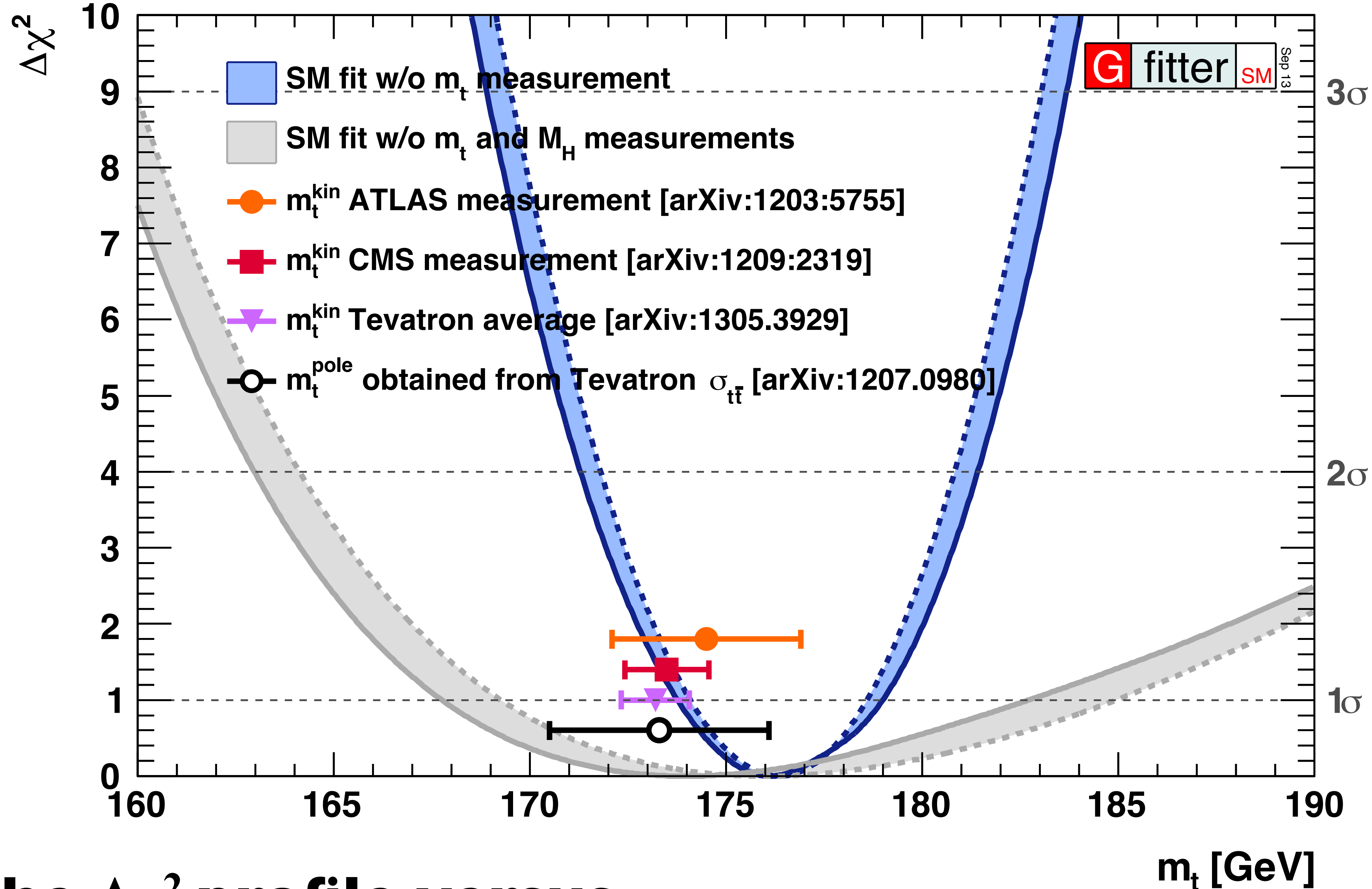


$$\sin^2\theta^\ell_{\rm eff} = 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\rm theo}\,,$$

$$= 0.23150 \pm 0.00010_{\rm tot}$$

More precise than the direct determination from LEP/SLD measurements

Indirect Determination: Top Mass



Scan of the $\Delta\chi^2$ profile versus m_t

- consistency with direct measurements
- M_H measurement allows for better constraint of m_t

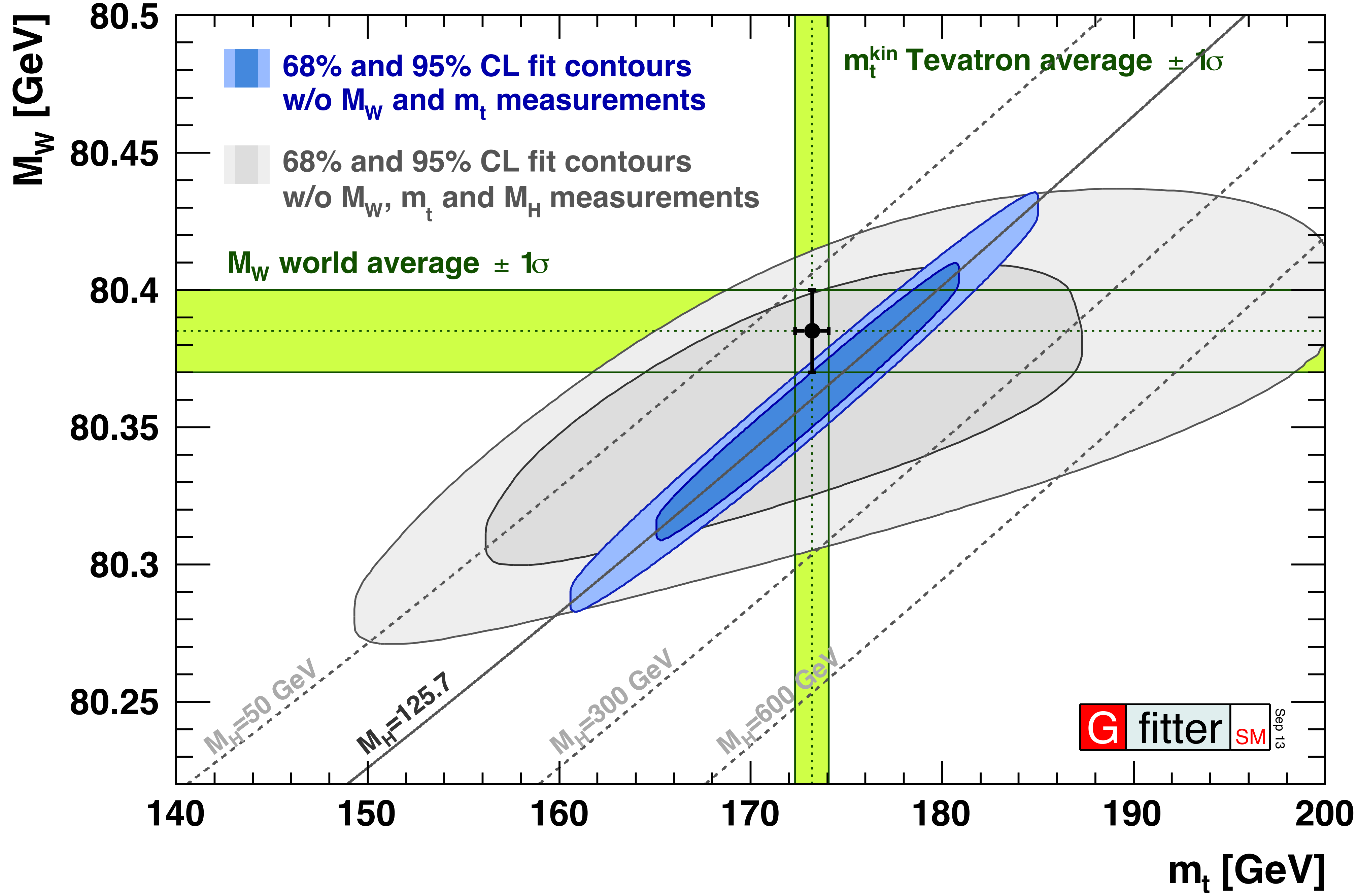
$$m_t = 175.8\,^{+2.7}_{-2.4}\ \text{GeV}$$

Tevatron average: m_t = 173.20 ± 0.87 GeV

LHC average: m_t = 173.29 ± 0.95 GeV

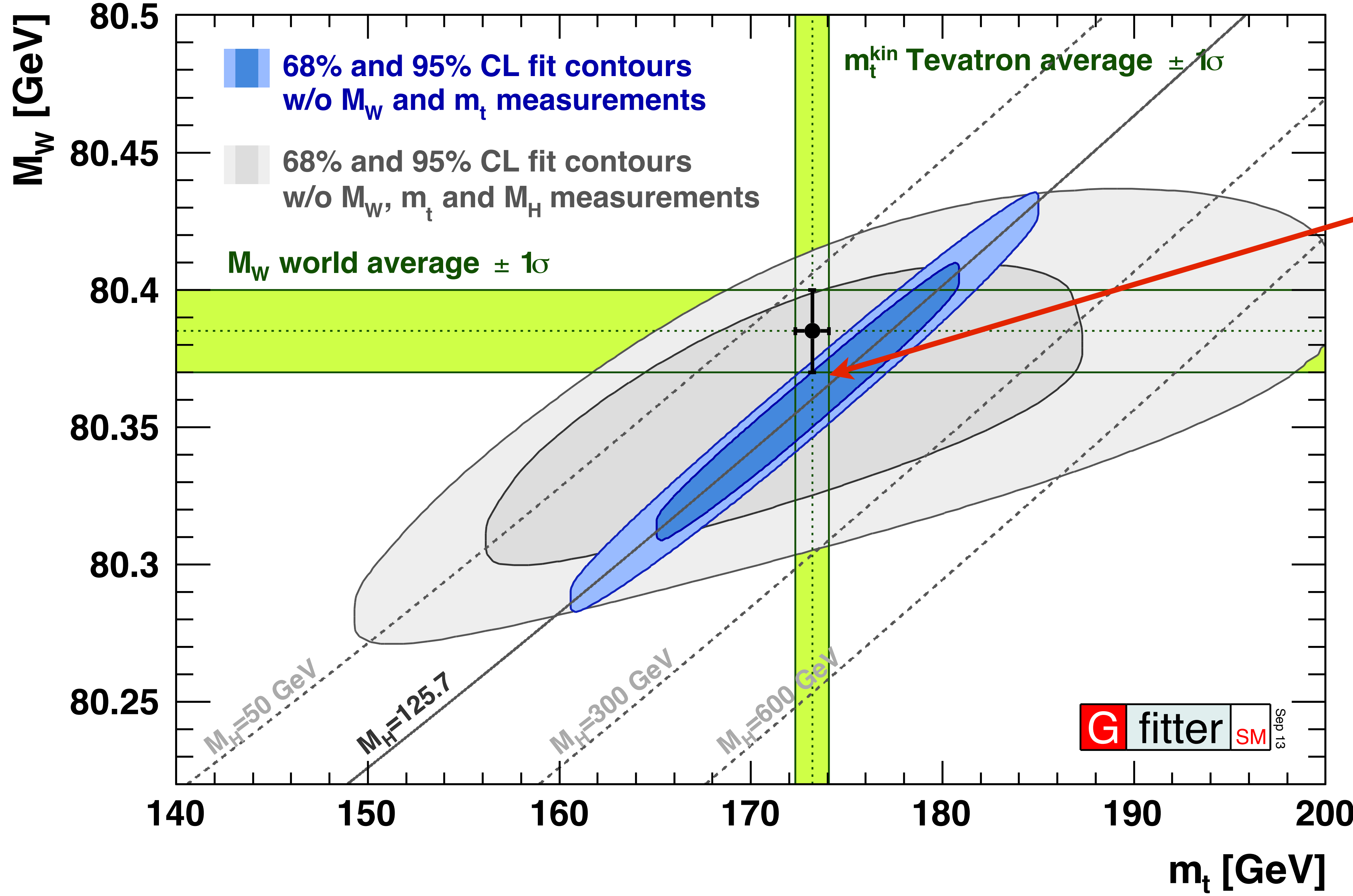
W and Top Mass

Impressive consistency of the SM



W and Top Mass

Impressive consistency of the SM



Once M_H is fixed, we cornered the SM!

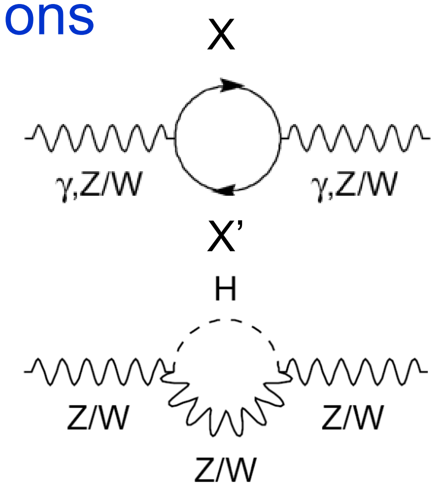
Effects of new physics through loop corrections!

⇒ improve measurements of EW precision observables

2. BSM and Higgs Coupling

Oblique Corrections

- If energy scale of NP is high, BSM physics appears dominantly through vacuum polarization corrections
  - Aka, “oblique corrections”
- Oblique corrections reabsorbed into electroweak form factors
  - $\Delta\rho$, $\Delta\kappa$, Δr parameters, appearing in: M_W^2, $\sin^2\theta_{eff}$, G_F, α, etc.
- Electroweak fit sensitive to BSM physics through oblique corrections
  - Similar to sensitivity to top and Higgs loop corrections.



- Oblique corrections from New Physics described through STU parametrization
  [Peskin and Takeuchi, Phys. Rev. D46, 1 (1991)]

$$O_{meas} = O_{SM,REF}(m_H, m_t) + c_S S + c_T T + c_U U$$

- S : New Physics contributions to neutral currents
- T : Difference between neutral and charged current processes – sensitive to weak isospin violation
- U : (+S) New Physics contributions to charged currents. U only sensitive to W mass and width, usually very small in NP models (often: U=0)

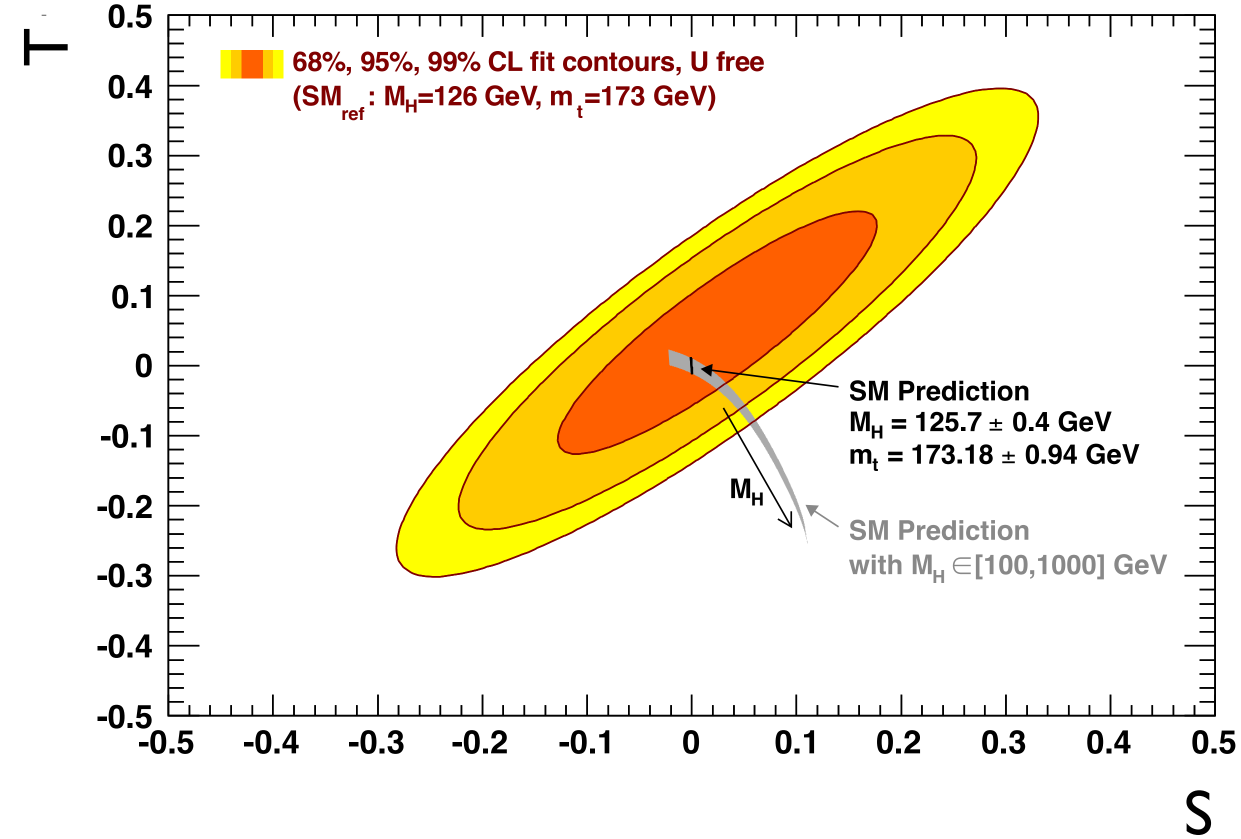
- Also implemented: extended parameters (VWX), correction to Z→bb couplings.
  [Burgess et al., Phys. Lett. B326, 276 (1994)]
  [Burgess et al., Phys. Rev. D49, 6115 (1994)]

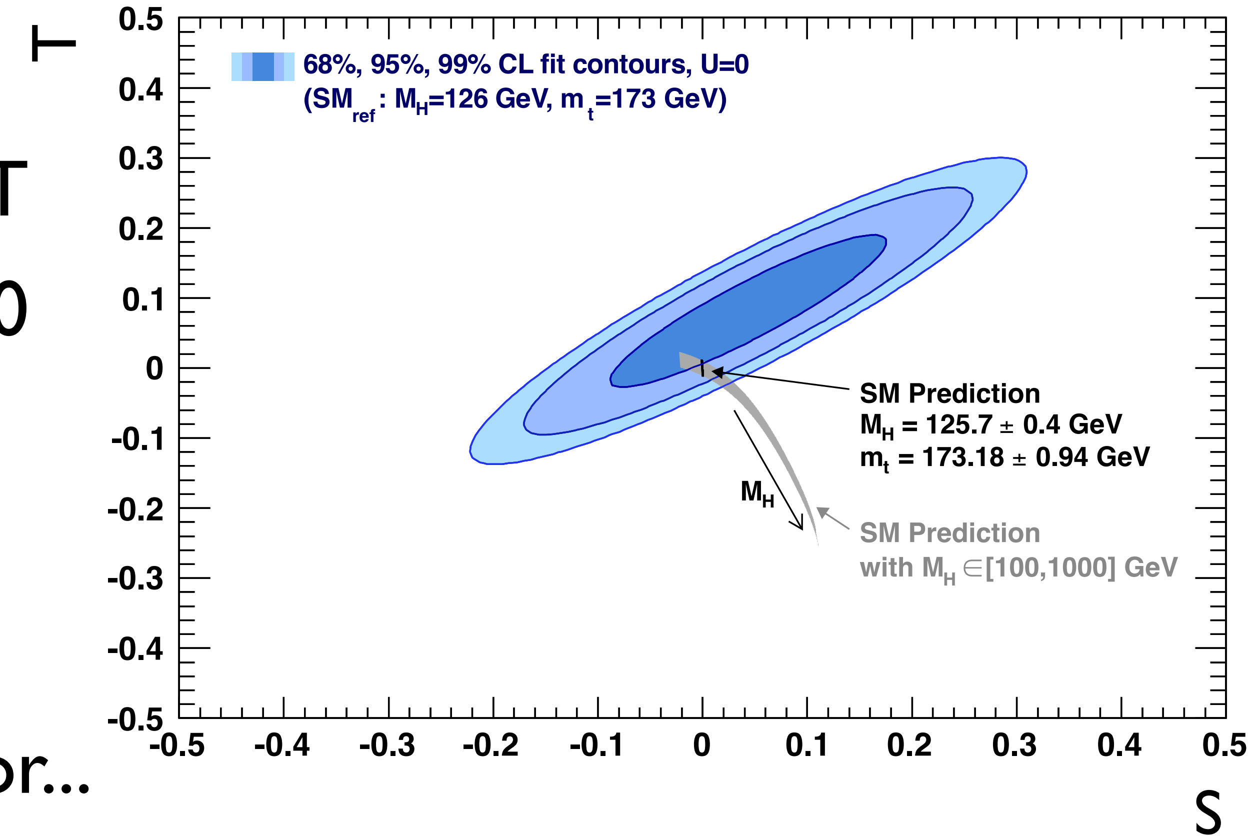
Constraints on S, T and U

- S,T,U obtained from EW fit
- SM reference chosen to be
  $M_{H,\mathrm{ref}}$ = 126 GeV
  $m_{t,\mathrm{ref}}$ = 173 GeV
  ] defines (0, 0, 0)
  - S,T depend logarithmically on M_H
- Fit result:
  S = 0.03 ± 0.10
  T = 0.05 ± 0.12
  U = 0.03 ± 0.10
  with large correlation between S and T
- Stronger constraints from fit with U=0
- Also available for $Z \to b\bar{b}$

No indication of new physics

- Constrains on 2HDM, LED, Technicolor...





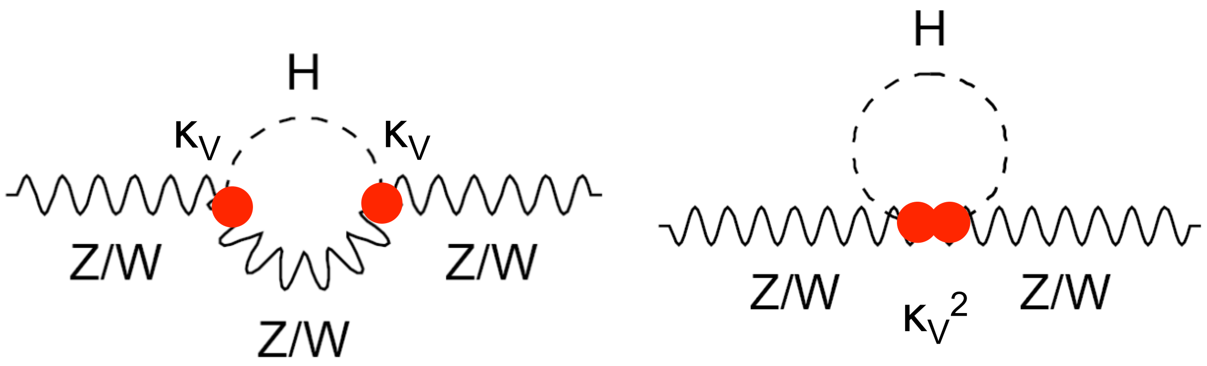
Modified Higgs Couplings

- Study of potential deviations of Higgs couplings from SM
  - BSM modelled as extension of SM through effective Lagrangian

$$L_V = \frac{h}{v}\left(2\kappa_V m_W^2 W_\mu W^\mu + \kappa_V m_Z^2 Z_\mu Z^\mu\right)$$

$$L_F = -\frac{h}{v}\left(\kappa_F m_t \bar{t}t + \kappa_F m_b \bar{b}b + \kappa_F m_\tau \bar{\tau}\tau\right)$$

- Consider leading corrections only
- Popular benchmark model:
  - Scaling of Higgs-vector boson (κ_V) and Higgs-fermion couplings (κ_F)
  - No additional loops in the production or decay of the Higgs, no invisible Higgs decays and undetectable width
- Main effect on EWPO due to modified Higgs coupling to gauge bosons (κ_V)
  - Involving the longitudinal d.o.f.



- Most BSM models: κ_V < 1
  - Additional Higgses typically give positive contribution to M_W

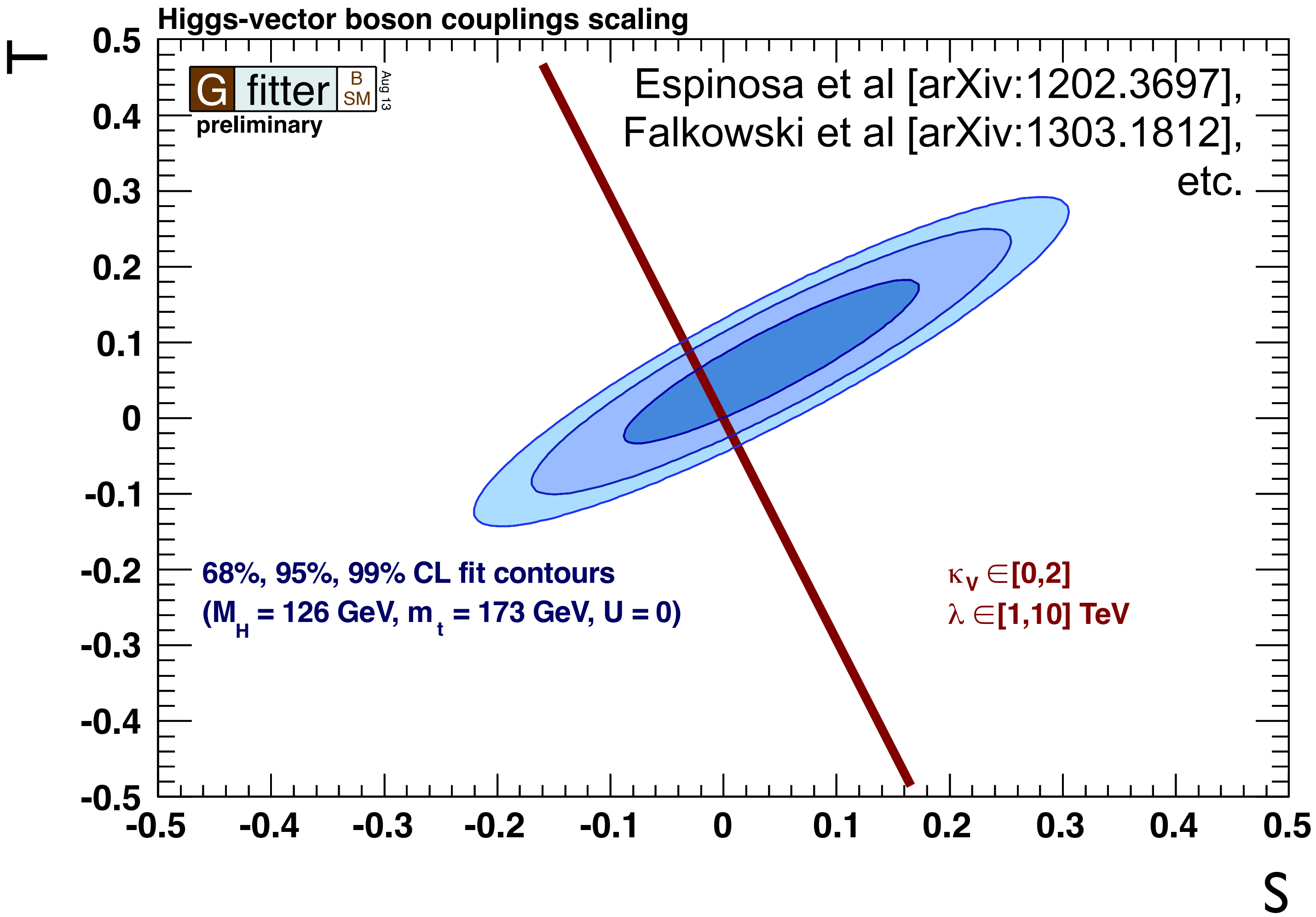
Modified Higgs Couplings

- Main effect on EWPO due to Higgs coupling to gauge bosons (κ_V)

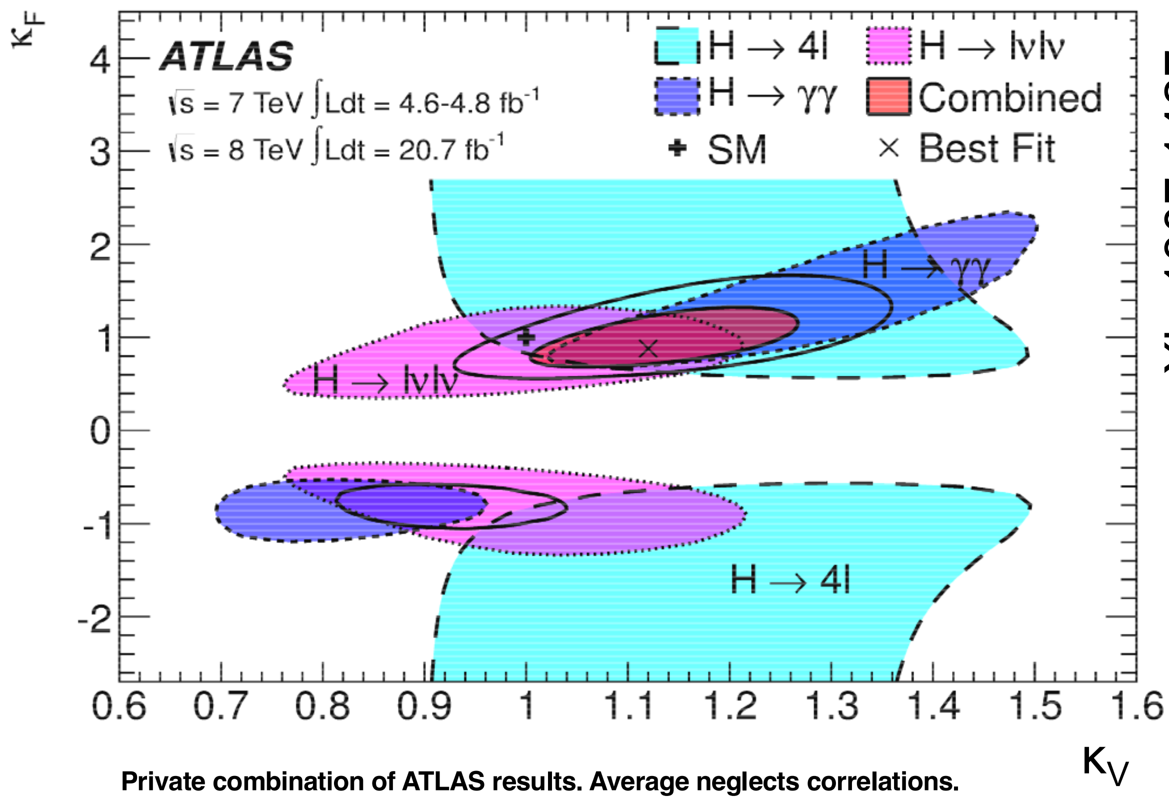
$$S = \frac{1}{12\pi}(1-\kappa_V^2)\log\left(\frac{\Lambda^2}{M_H^2}\right), \quad T = -\frac{3}{16\pi c_W^2}(1-\kappa_V^2)\log\left(\frac{\Lambda^2}{M_H^2}\right), \quad \Lambda = \frac{\lambda}{\sqrt{|1-\kappa_V^2|}}$$

Espinosa et al [arXiv:1202.3697]

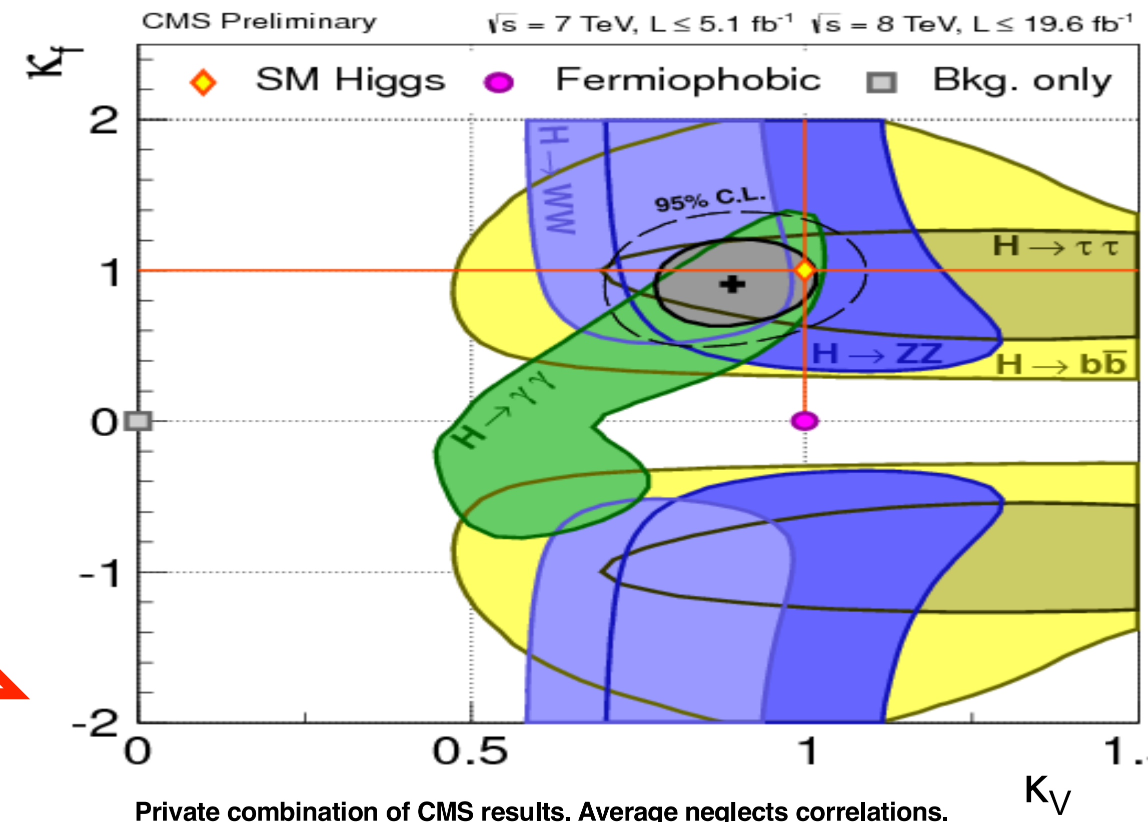
- Cut-off scale Λ represents mass scale of new states that unitarize longitudinal gauge boson couplings (as required in this model)
- λ is varied between 1-10 TeV, nominally fixed to 3 TeV ($4\pi v$)



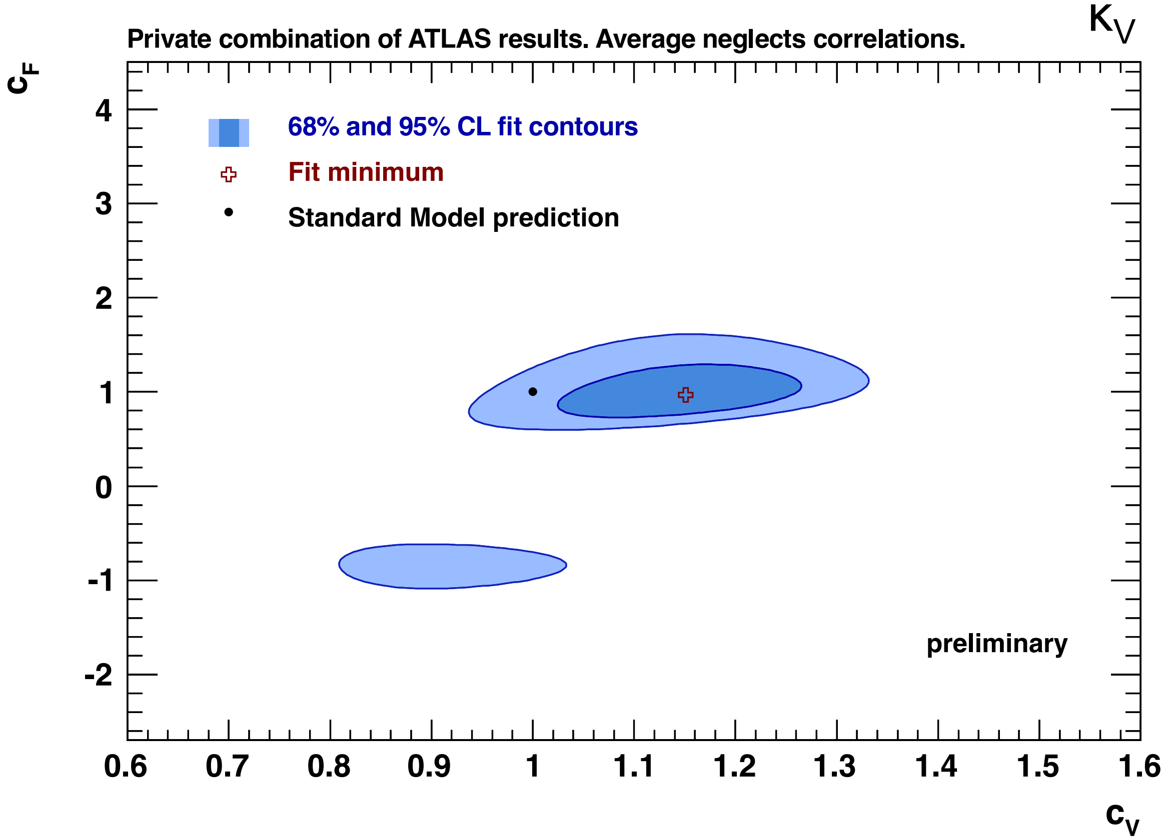
Reproduction of Experimental Results

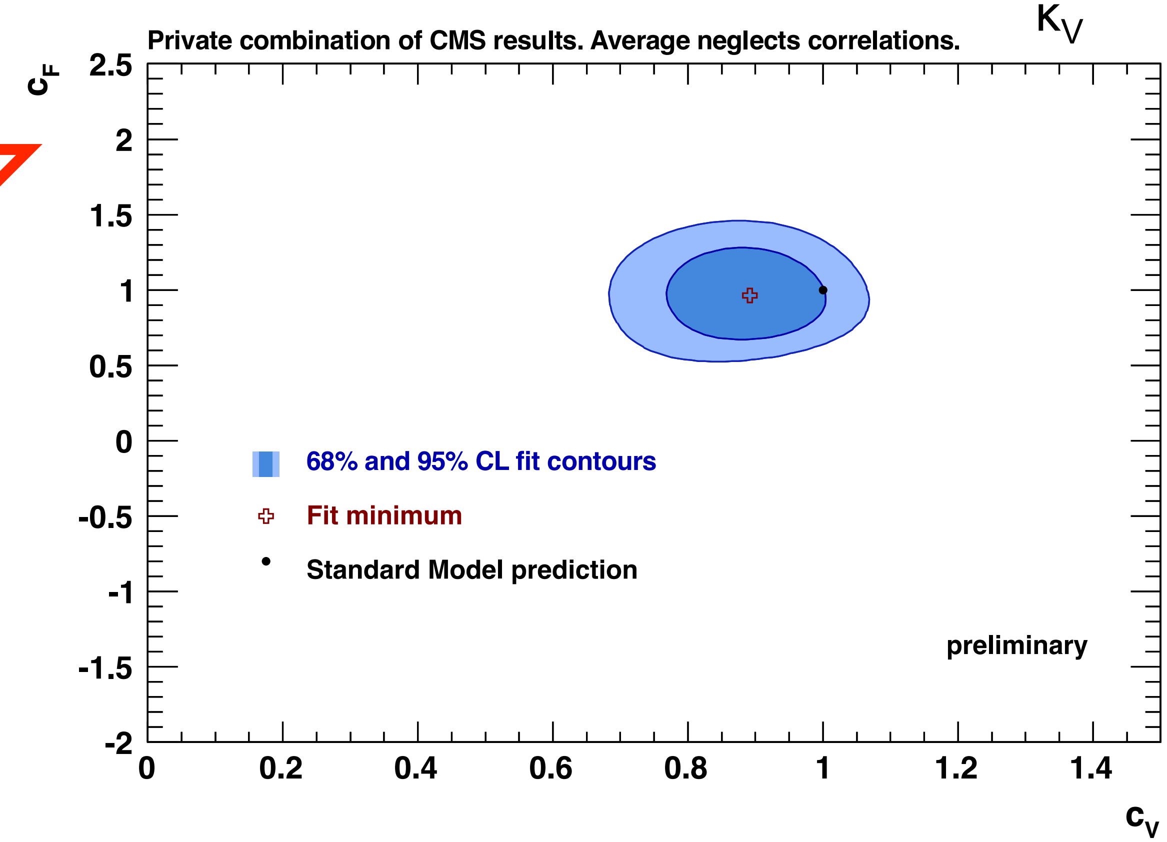


arXiv:1307.1427



CMS-PAS-HIG-13-005

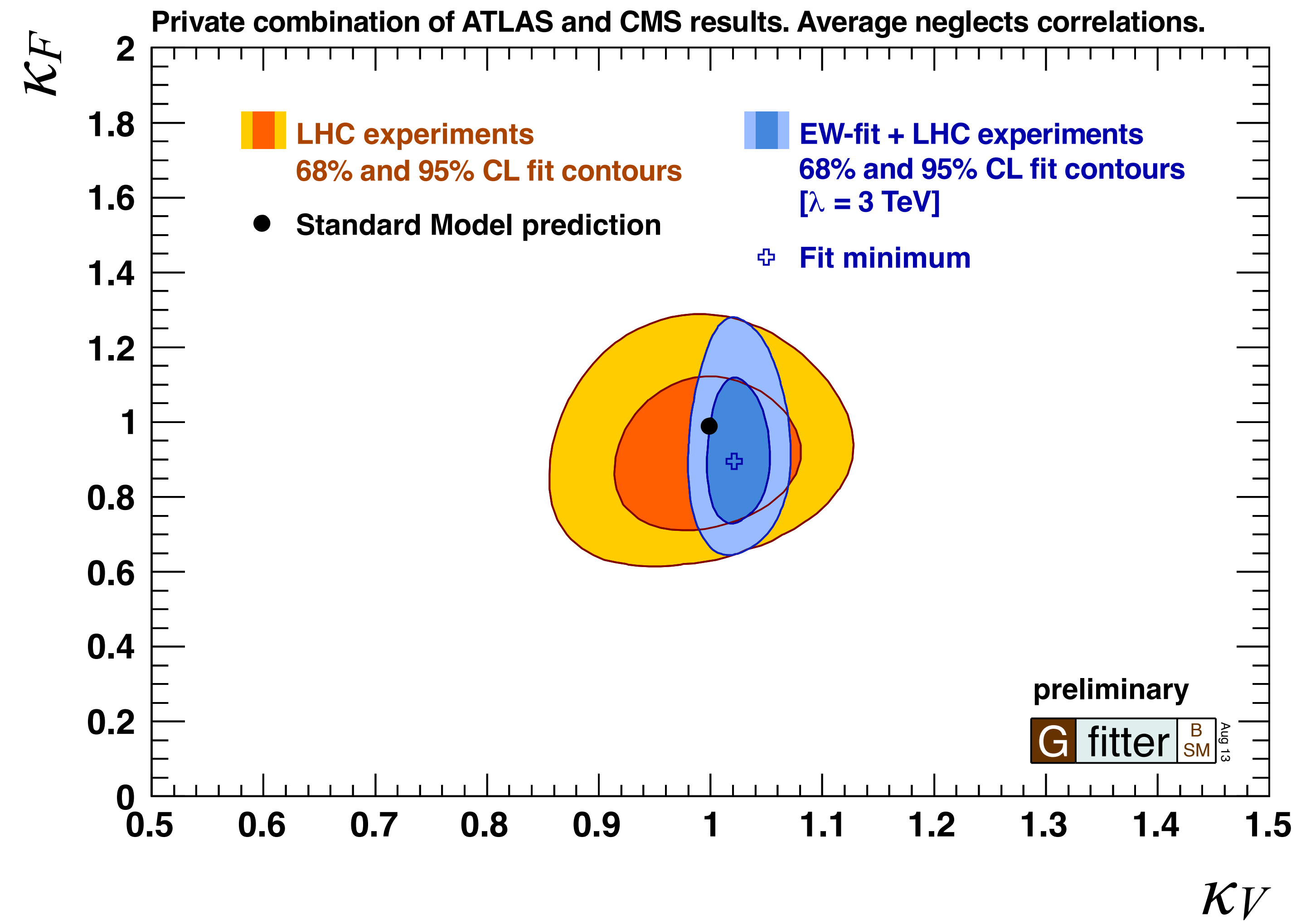




- Decent reproduction of ATLAS and CMS results within limited public-info available.

Higgs Couplings Results

- Private LHC combination:
  - $\kappa_V = 1.00 \pm 0.06$
  - $\kappa_F = 0.89 \pm 0.13$
  - perfectly consistent with SM

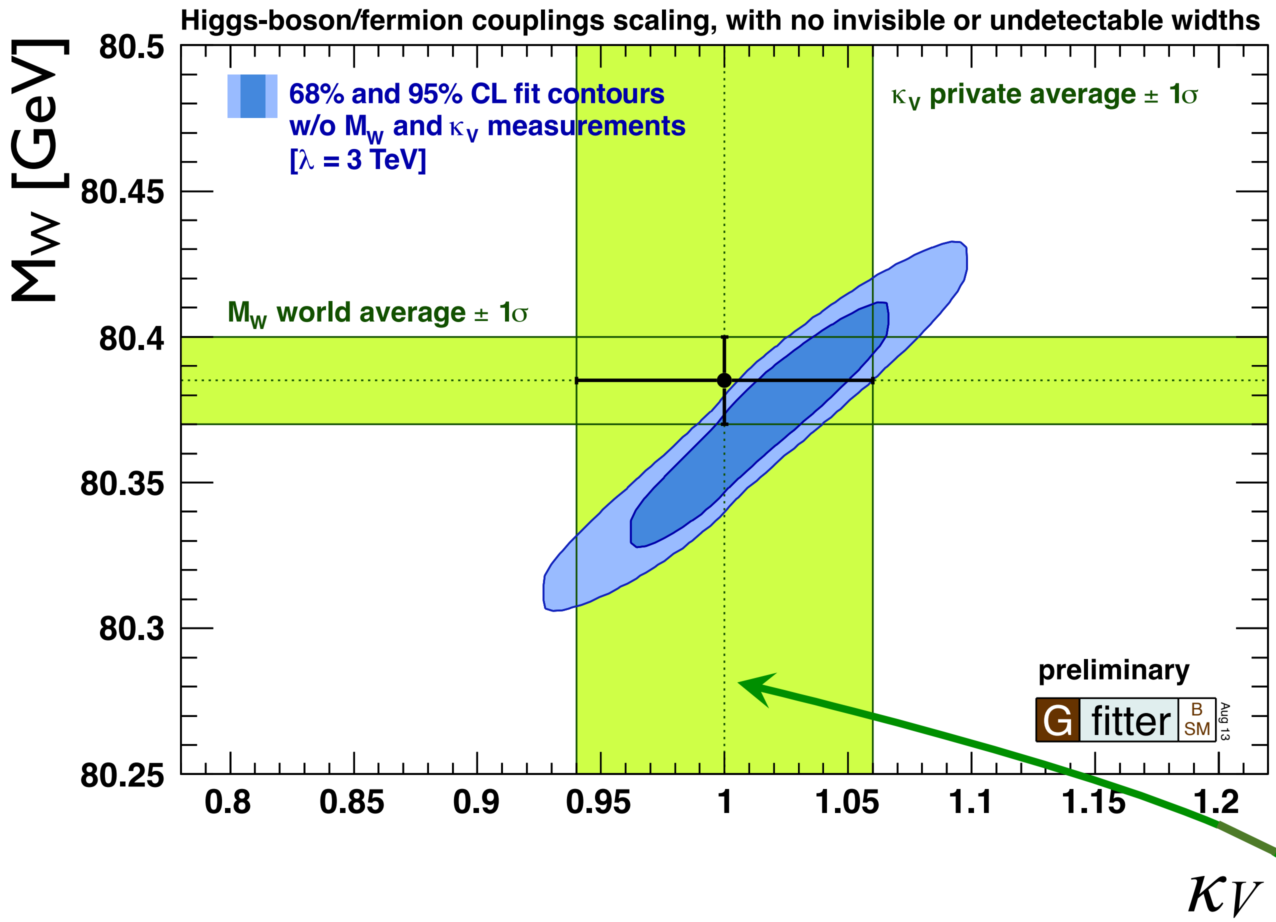


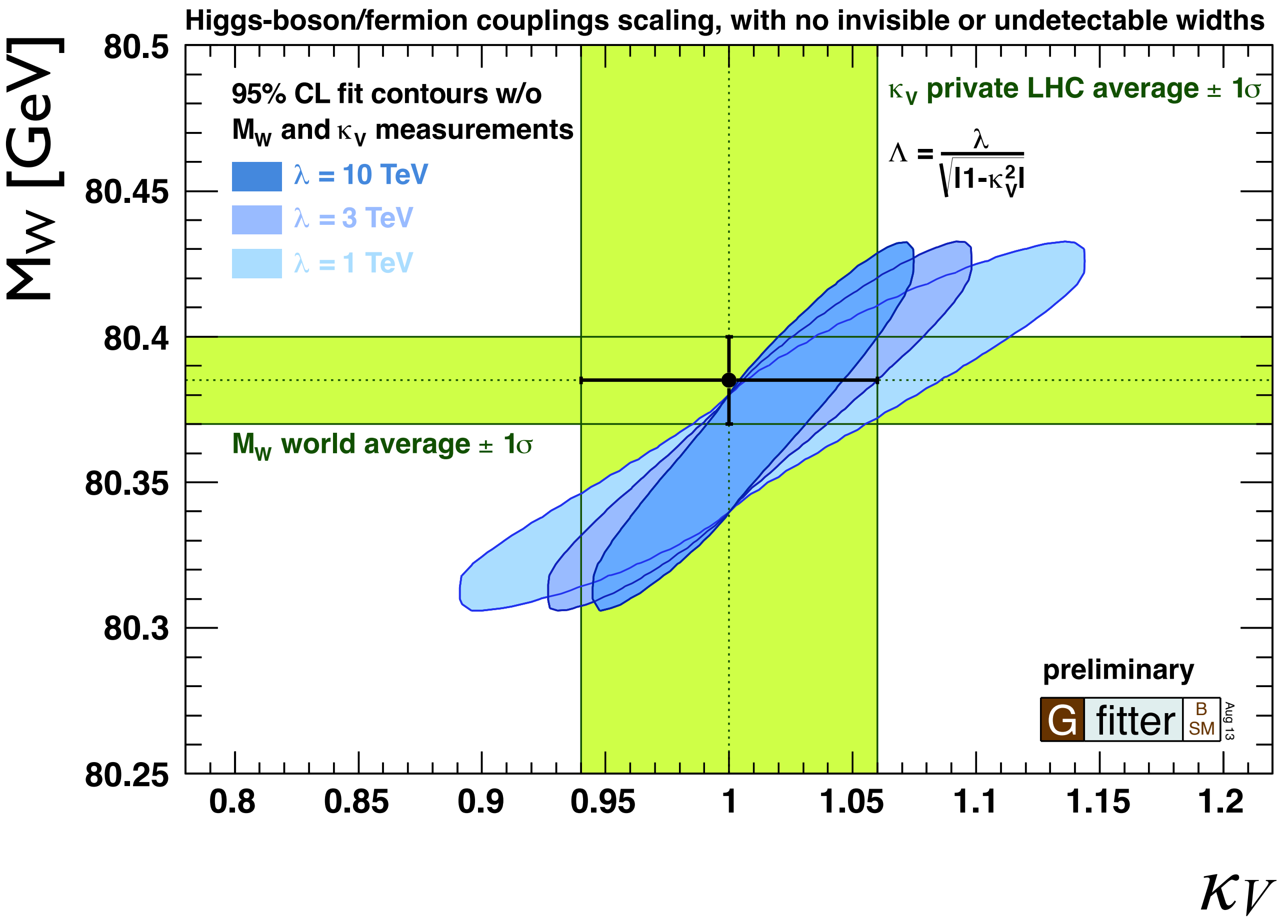
- Results from stand-alone EW fit
  - $\kappa_V = 1.03^{+0.04}_{-0.03}$ (λ = 1 TeV)
  - $\kappa_V = 1.02^{+0.02}_{-0.02}$ (λ = 3 TeV)
  - $\kappa_V = 1.02^{+0.02}_{-0.01}$ (λ = 10 TeV)

- EW fit sofar more precise result for κ_V than current LHC experiments
- EW fit results in positive deviation of κ_V from 1.0 (Many BSM models: $\kappa_V < 1$)

Higgs Couplings Results

- EW fit: positive deviation of κ_V from one driven by small tension in W mass prediction versus measurement

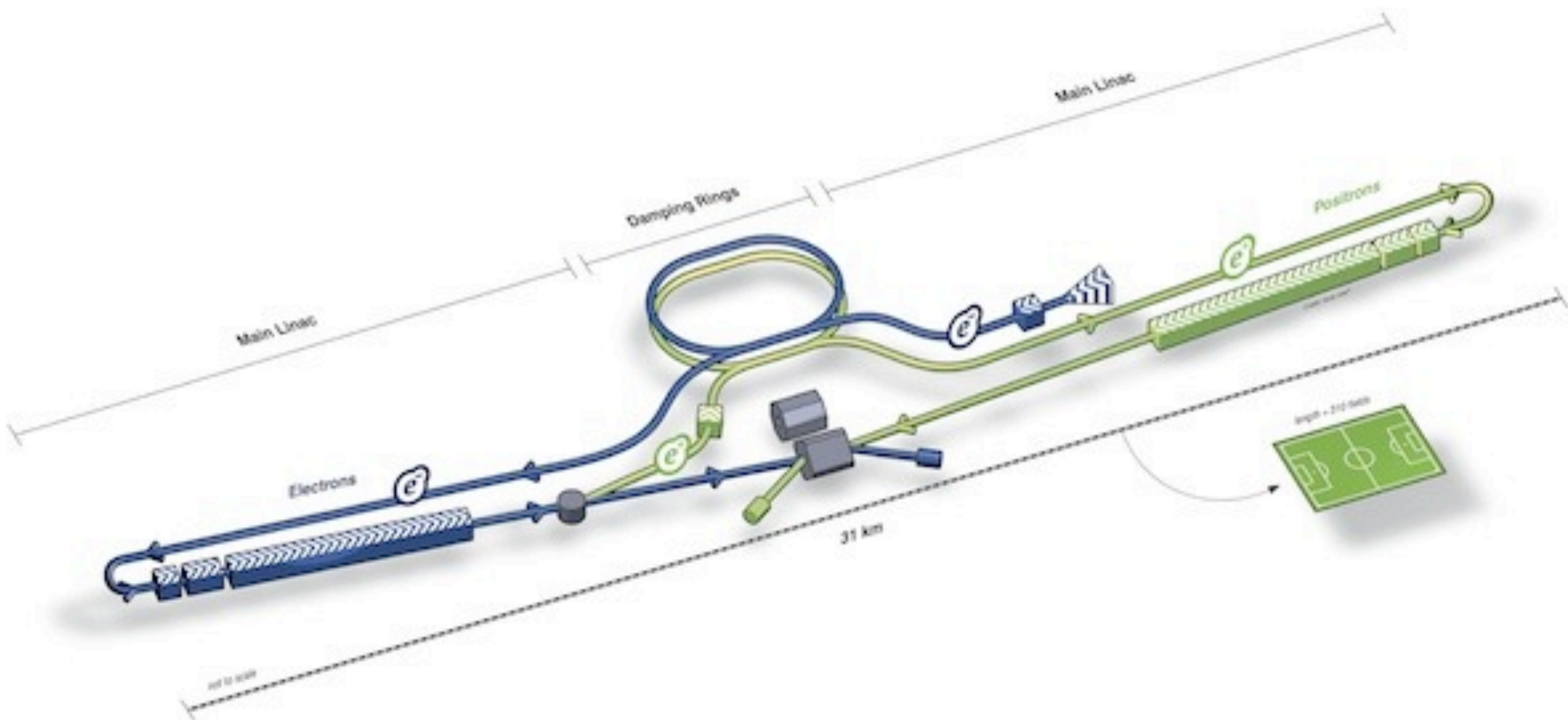




- Private LHC combination:
  - $\kappa_V = 1.00 \pm 0.06$
  - $\kappa_F = 0.89 \pm 0.13$

- Above: dependency on λ
- (Will be interesting to see how these measurements develop.)

3. Prospects of the EW Fit



ILC Scheme | © www.form-one.de

Future Prospects of the EW Fit

Two future scenarios are studied

- LHC Run-2+3
  - Final W and top mass measurements, combination with LEP and Tevatron
    δM_W : 15 → 8 MeV, δm_t : 0.9 → 0.6 GeV
  - H→ZZ and H→WW couplings: measured at 4.5% precision
  - (possibly optimistic scenario, but not impossible)

- ILC with GigaZ option
  - Operation of ILC at lower energies like Z-pole or WW threshold. Allows to perform precision measurements of EW sector
  - At Z-pole, several billion Z's can be studied within 1-2 months
  - H→ZZ and H→WW couplings: measured at 1% precision

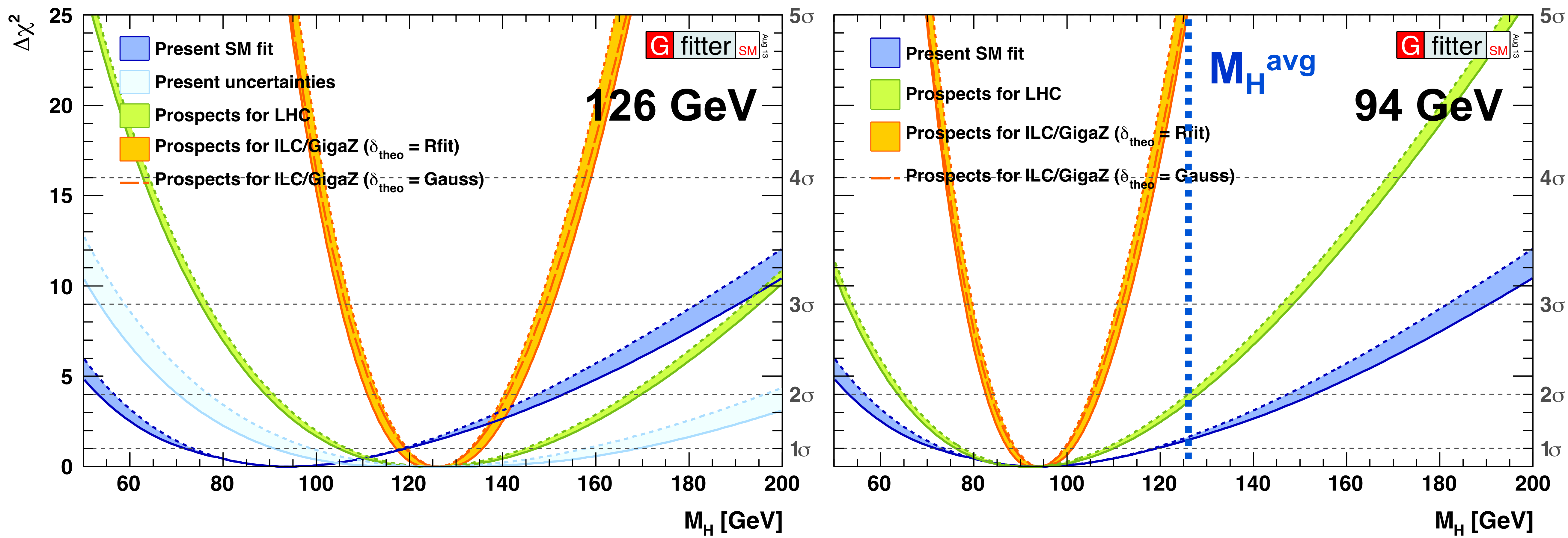
- Common improvement: theory
  - Assuming ~25% of today's theoretical uncertainties on M_W and $\sin^2\theta^l_{eff}$
    Implies three-loop EW calculations!
  - δM_W (4→1 MeV), $\delta\sin^2\theta^l_{eff}$ (4.7x10-5 → 1x10-5)

Future Prospects of the EW Fit

In following: central values of input measurements adjusted to M_H = 126 GeV

Parameter	Experimental input [$\pm 1\sigma$]: Present	LHC	ILC/GigaZ	
M_H [GeV]	0.4 →	< 0.1	< 0.1	
M_W [MeV]	15 →	8 →	5	WW threshold
M_Z [MeV]	2.1	2.1	2.1	
m_t [GeV]	0.9 →	0.6 →	0.1	$t\bar{t}$ threshold scan
$\sin^2\theta^\ell_{\rm eff}$ [$\cdot 10^{-5}$]	16	16 →	1.3	$\delta A^{0,f}_{LR}$: $10^{-3} \to 10^{-4}$
$\Delta\alpha^5_{\rm had} M_Z^2$ [$\cdot 10^{-5}$]	10 →	4.7	4.7	low energy data
R^0_l [$\cdot 10^{-3}$]	25	25 →	4	high statistics on Z-pole
$\delta_{\rm th} M_W$ [MeV]	4 →	1	1	three-loop calculations
$\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}$ [$\cdot 10^{-5}$]	4.7 →	1	1	three-loop calculations

Prospects for the EW Fit

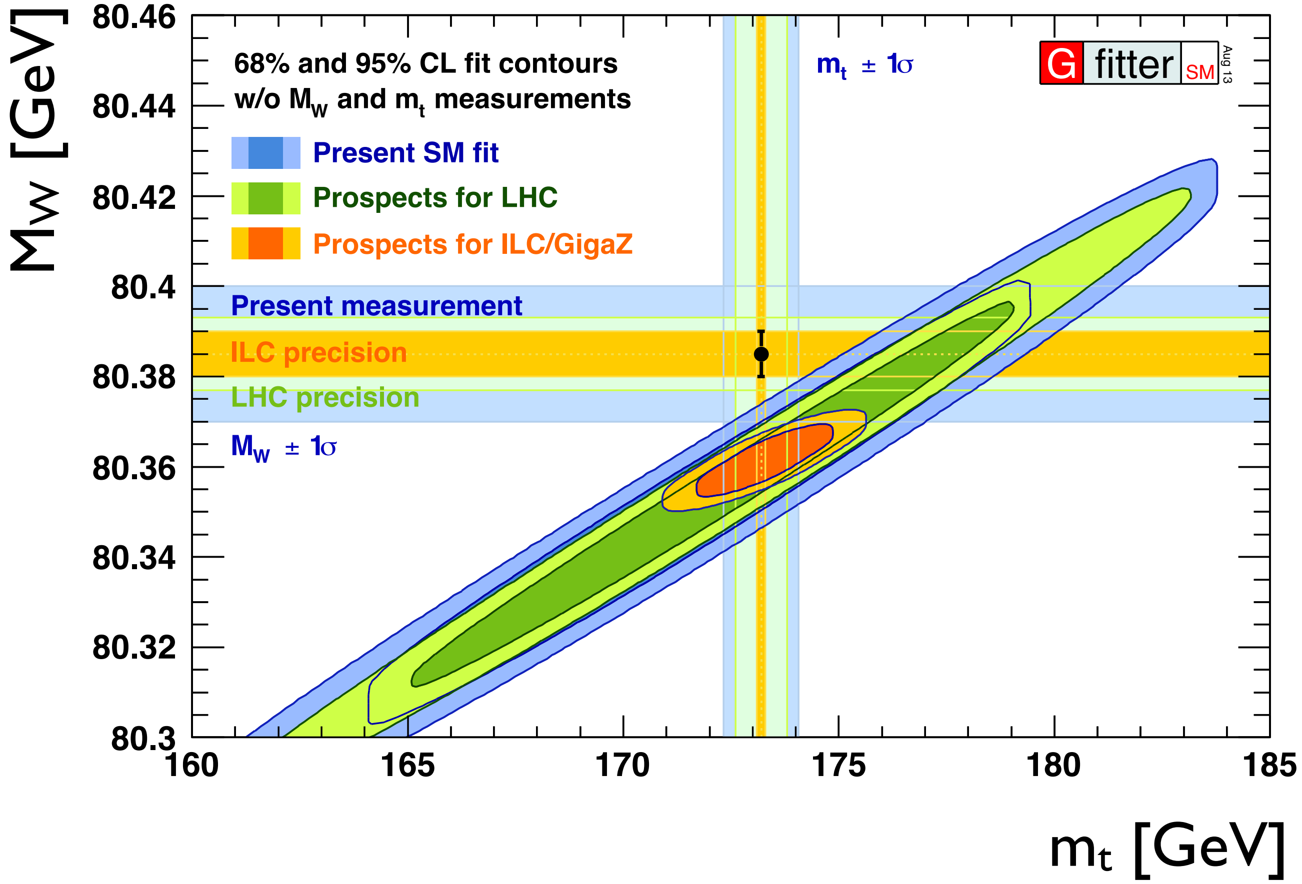


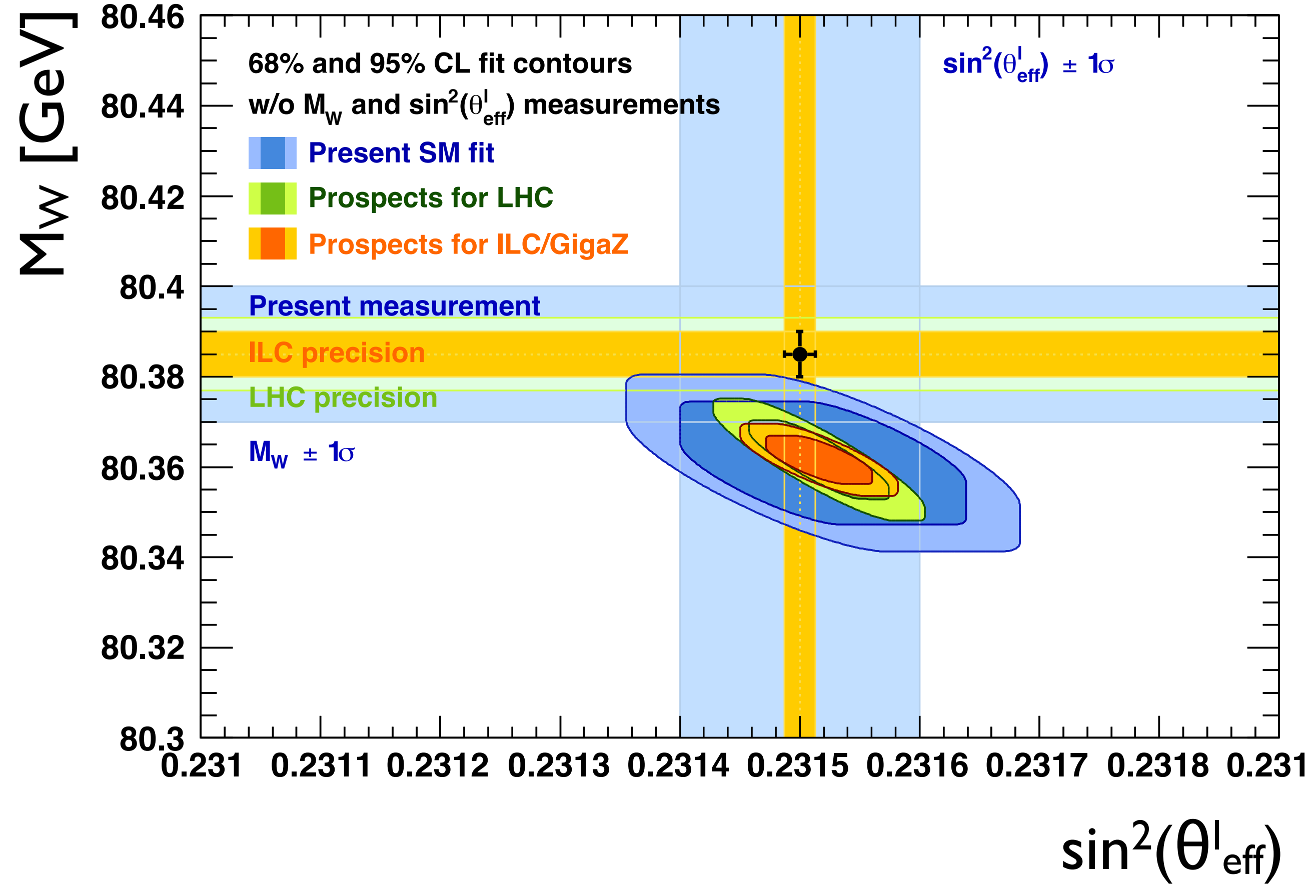
Logarithmic dependency on M_H → cannot compete with direct M_H meas.

- no theory uncertainty: $M_H = 126 \pm 7$ GeV
- present day theory uncertainty: $M_H = 126\,^{+20}_{-17}$ GeV
- future theory uncertainty (Rfit): $M_H = 126\,^{+10}_{-9}$ GeV

If EWPO central values unchanged, i.e. keep favouring low value of M_H (94 GeV), ~5σ discrepancy with measured Higgs mass

Prospects for the SM EW Fit





- Huge reduction of uncertainty on indirect determinations of m_t, M_W, and $\sin^2\theta^l_{eff}$ by a factor of 3 or more
- Assuming central values of m_t and M_W do not change (at ILC), a deviation between the SM prediction and the direct measurements would be prominently visible

Prospects for the BSM EW Fit

- Breakdown of individual contributions to errors of M_W and $\sin^2\theta^l_{eff}$
- Parametric uncertainties (not the full fit)

				error due to uncertainty $(\pm 1\sigma)$						
Parameter	Scenario	$\delta_{\rm meas}$	$\delta_{\rm pred}$	$\delta_{\rm exp}$	δM_H	δM_Z	δm_t	$\delta\Delta\alpha_{\rm had}$	$\delta\alpha_S$	$\delta_{\rm theo}$
M_W [MeV]	Present	15	10.3	6.3	0.2	2.6	5.2	1.8	1.7	4.0
	LHC	8	5.8	4.8	–	2.6	3.6	0.9	1.7	1.0
	ILC	5	3.8	2.8	–	2.6	0.6	0.9	0.4	1.0
$\sin^2\theta^\ell_{\rm eff}$ $^{(\circ)}$	Present	16	9.5	4.8	0.2	1.5	2.8	3.5	1.0	4.7
	LHC	16	4.1	3.1	–	1.5	1.9	1.6	1.0	1.0
	ILC	1.3	3.2	2.2	–	1.5	0.3	1.6	0.2	1.0

$^{(\circ)}$In units of 10^{-5}.

Prospects for the BSM EW Fit

- Breakdown of individual contributions to errors of M_W and $\sin^2\theta^l_{\mathrm{eff}}$
- Parametric uncertainties (not the full fit)

Parameter	Scenario	δ_{meas}	δ_{pred}	error due to uncertainty ($\pm 1\sigma$) δ_{exp}	δM_H	δM_Z	δm_t	$\delta\Delta\alpha_{\mathrm{had}}$	$\delta\alpha_S$	δ_{theo}
M_W [MeV]	Present	15	10.3	6.3	0.2	2.6	5.2	1.8	1.7	4.0
	LHC	8	5.8	4.8	–	2.6	3.6	0.9	1.7	1.0
	ILC	5	3.8	2.8	–	2.6	0.6	0.9	0.4	1.0
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $^{(\circ)}$	Present	16	9.5	4.8	0.2	1.5	2.8	3.5	1.0	4.7
	LHC	16	4.1	3.1	–	1.5	1.9	1.6	1.0	1.0
	ILC	1.3	3.2	2.2	–	1.5	0.3	1.6	0.2	1.0

$^{(\circ)}$In units of 10^{-5}.

Prospects for the BSM EW Fit

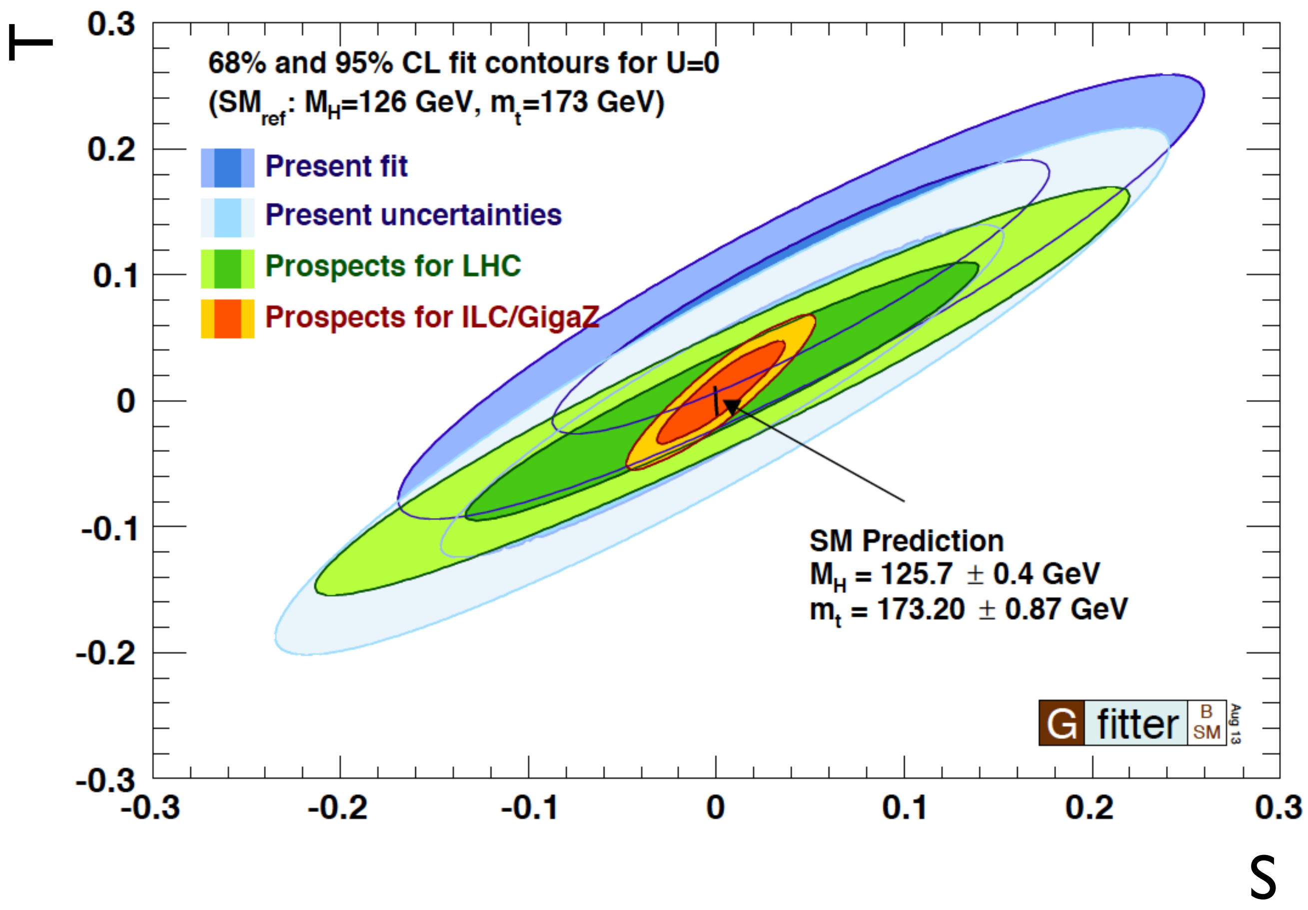
- Breakdown of individual contributions to errors of M_W and $\sin^2\theta^l_{eff}$
- Parametric uncertainties (not the full fit)

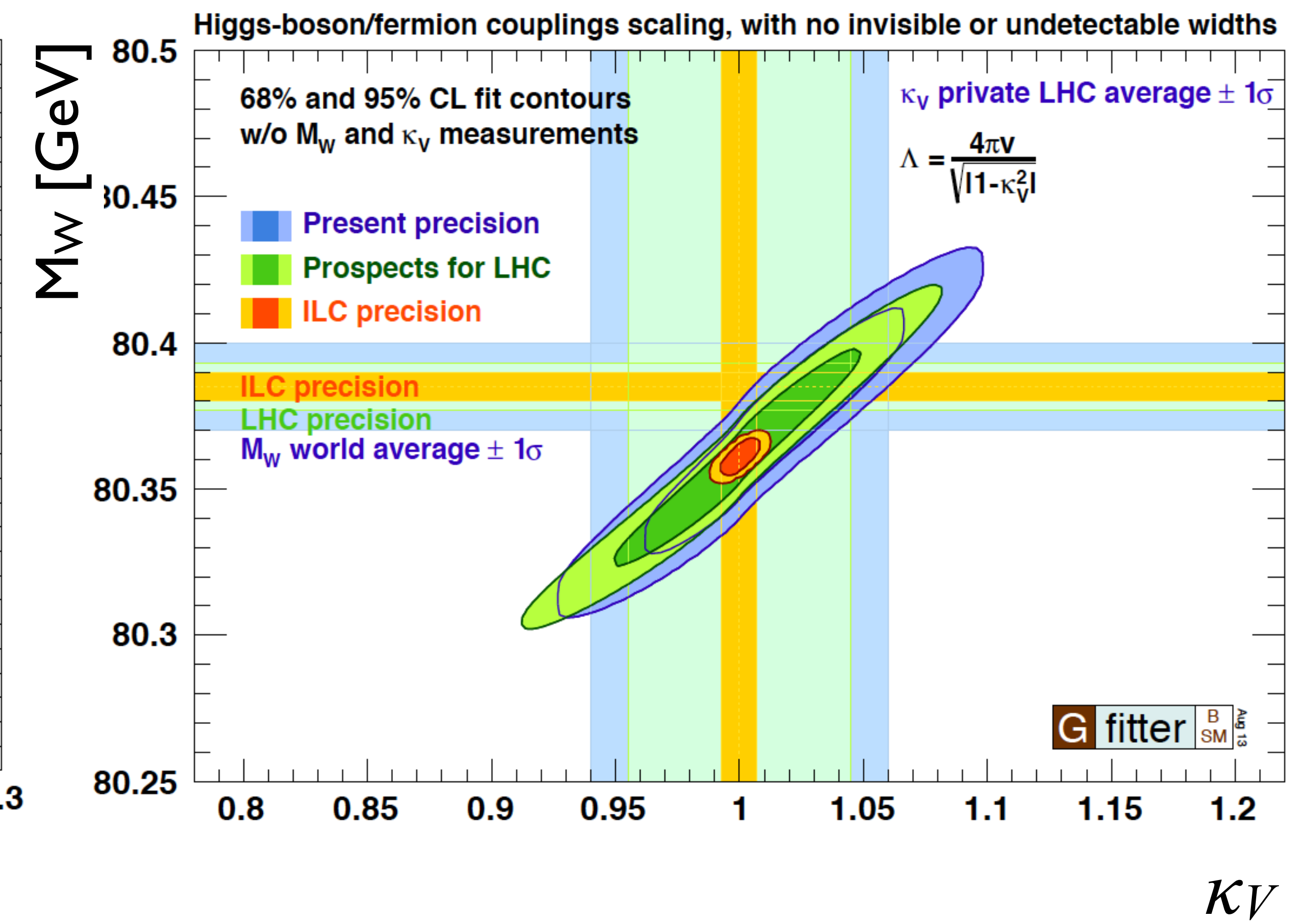
Parameter	Scenario	δ_{meas}	δ_{pred}	error due to uncertainty ($\pm 1\sigma$) δ_{exp}	δM_H	δM_Z	δm_t	$\delta\Delta\alpha_{had}$	$\delta\alpha_S$	δ_{theo}
M_W [MeV]	Present	15	10.3	6.3	0.2	2.6	5.2	1.8	1.7	4.0
	LHC	8	5.8	4.8	–	2.6	3.6	0.9	1.7	1.0
	ILC	5	3.8	2.8	–	2.6	0.6	0.9	0.4	1.0
$\sin^2\theta^\ell_{eff}$ $^{(\circ)}$	Present	16	9.5	4.8	0.2	1.5	2.8	3.5	1.0	4.7
	LHC	16	4.1	3.1	–	1.5	1.9	1.6	1.0	1.0
	ILC	1.3	3.2	2.2	–	1.5	0.3	1.6	0.2	1.0

$^{(\circ)}$In units of 10^{-5}.

- M_W and $\sin^2\theta^l_{eff}$ will be sensitive probes of new physics
- At the ILC/GigaZ: precision of M_Z will become important again! (current uncertainty: δM_Z = 2.1 MeV)

Prospects for the BSM EW Fit



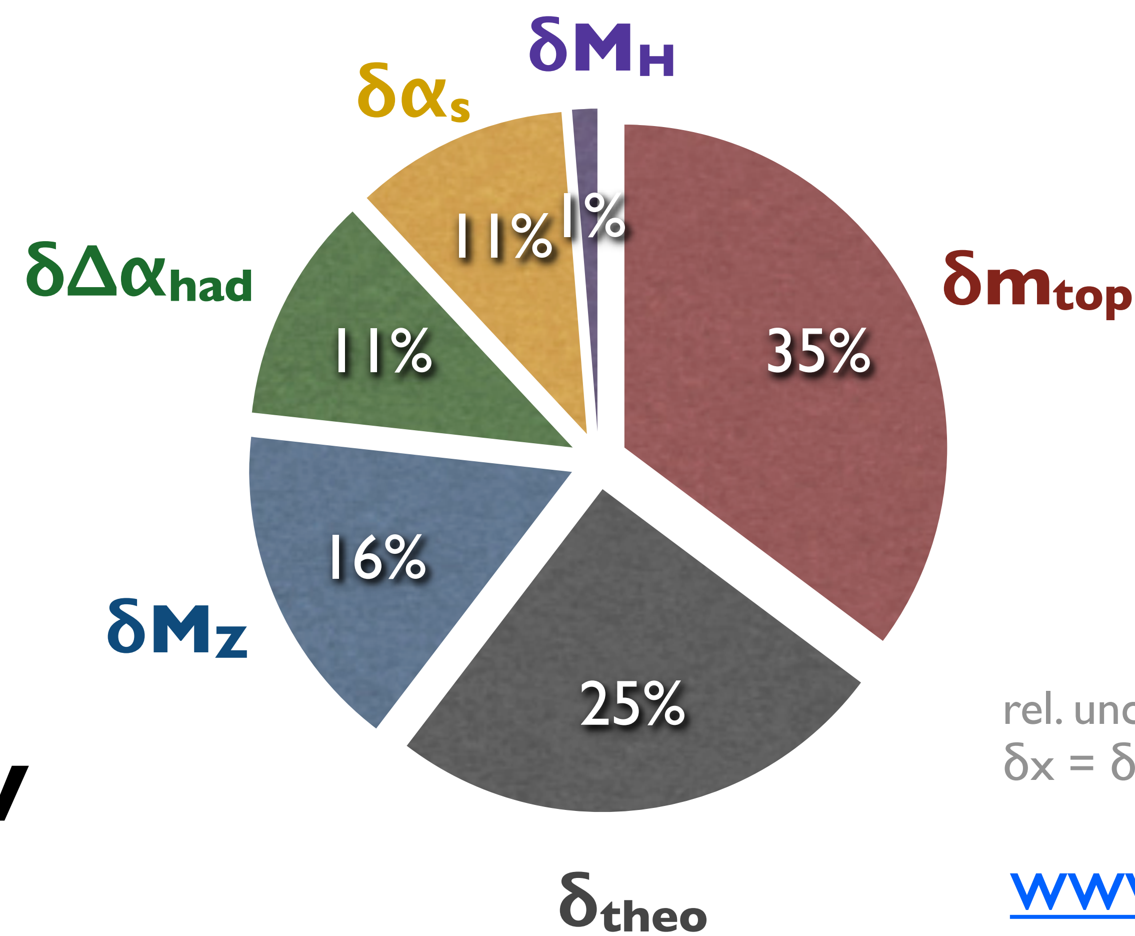


- For STU parameters, improvement of factor of >5 is possible at ILC
- Again, at ILC a deviation between the SM predictions and direct measurements would be prominently visible.
- Competitive results between EW fit and Higgs coupling measurements! (level of 1%.)

Summary

- **Paradigm shift for EW fit:**
  From Higgs mass prediction to consistency tests of the Standard Model
- **LHC has only added one parameter to the EW fit**
  Knowledge of M_H dramatically improves SM prediction of key observables
- **Higgs coupling measurements and ILC/GigaZ**
  Expect further exploration of Higgs couplings in the EW fit

δM_W (indirect) = **= 11 MeV**



rel. uncertainty:
$\delta x = \delta M_{W,x} / \delta M_{W,tot}$

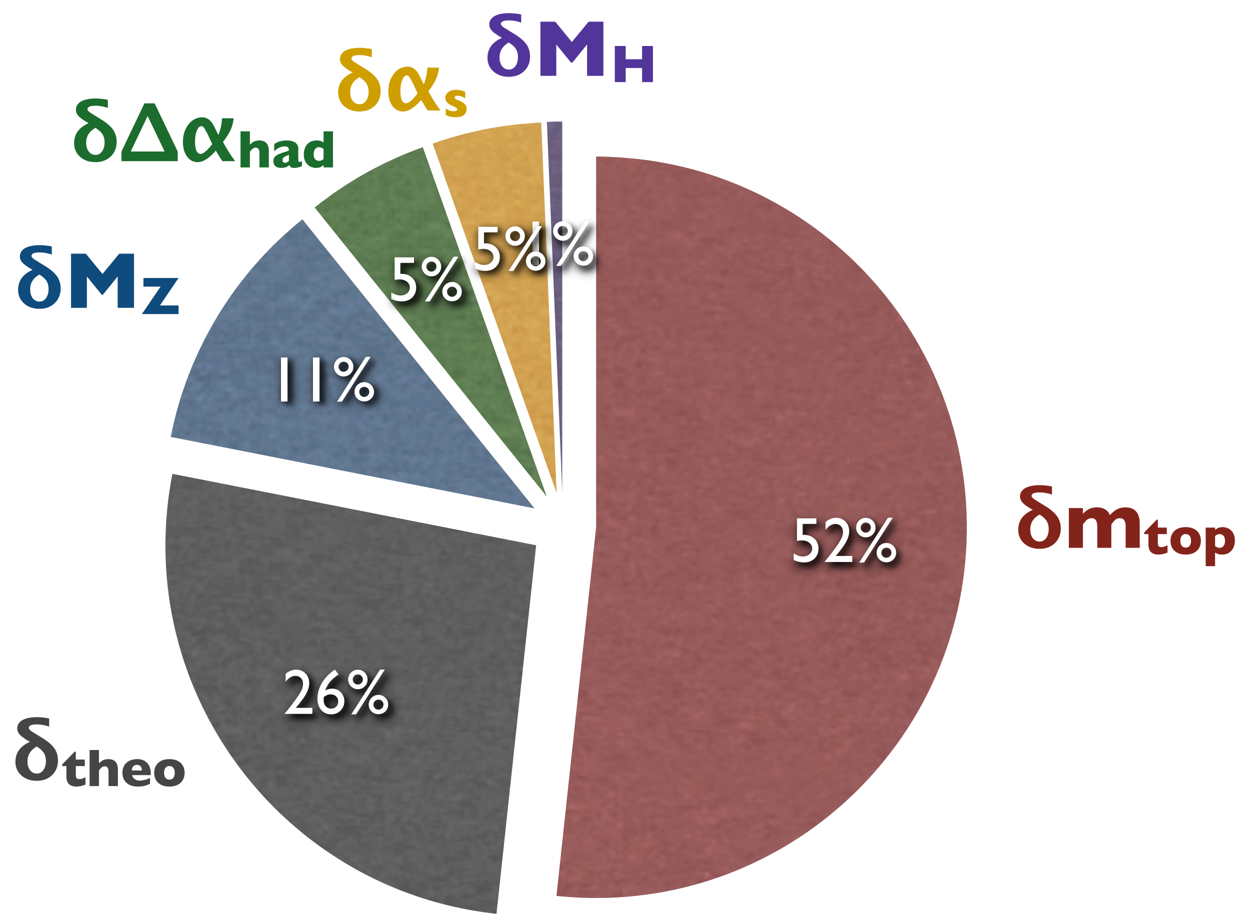
δM_W (exp) = 15 MeV

www.cern.ch/gfitter

Additional Material

Error on M_W

δM_W (indirect) =



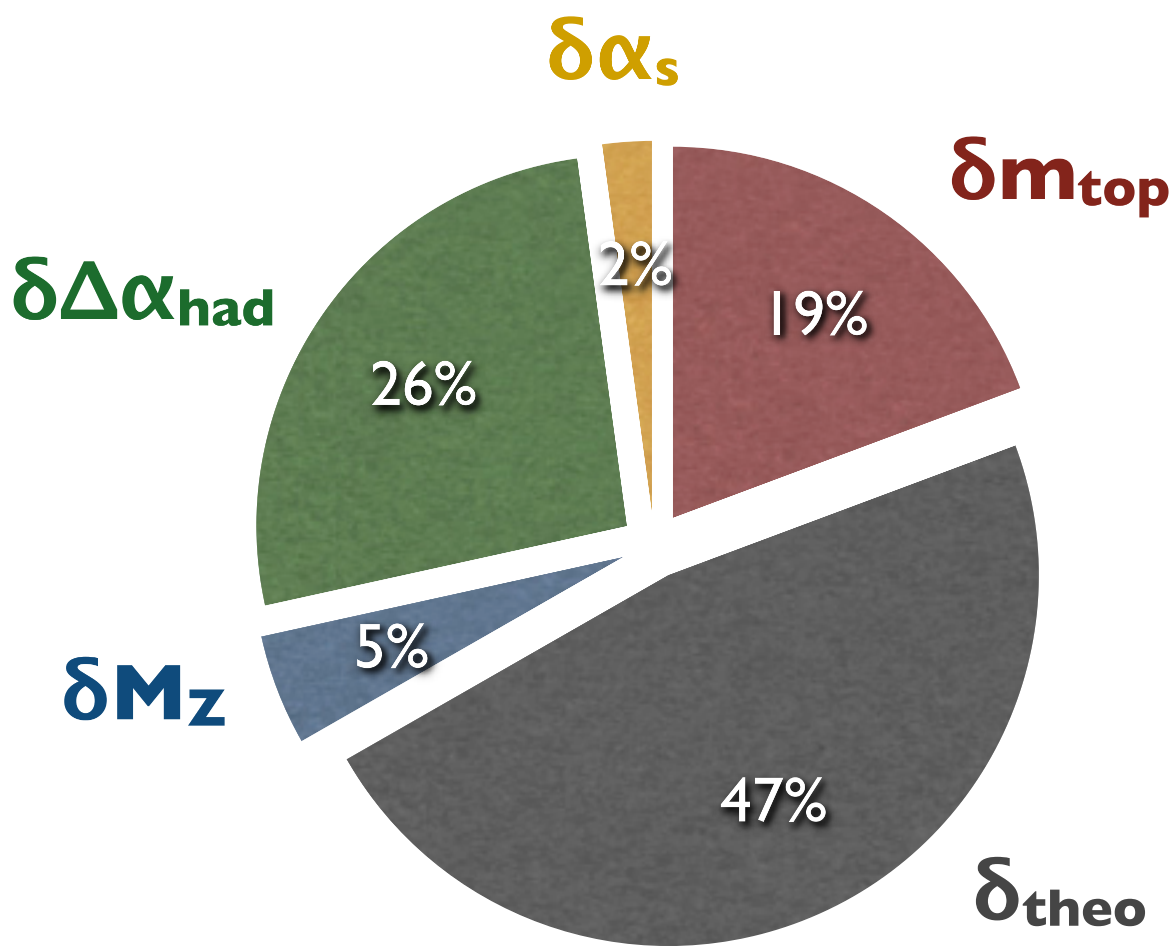
rel. uncertainty:
$\delta x = (\delta M_{W,x})^2 \,/\, (\Sigma_i\, \delta M_{W,i}^2)$

δM_W (indirect) = 11 MeV

δM_W (exp) = 15 MeV

Error on $\sin^2(\theta^l_{eff})$

$\delta\sin^2(\theta^l_{eff})$ (indirect) =



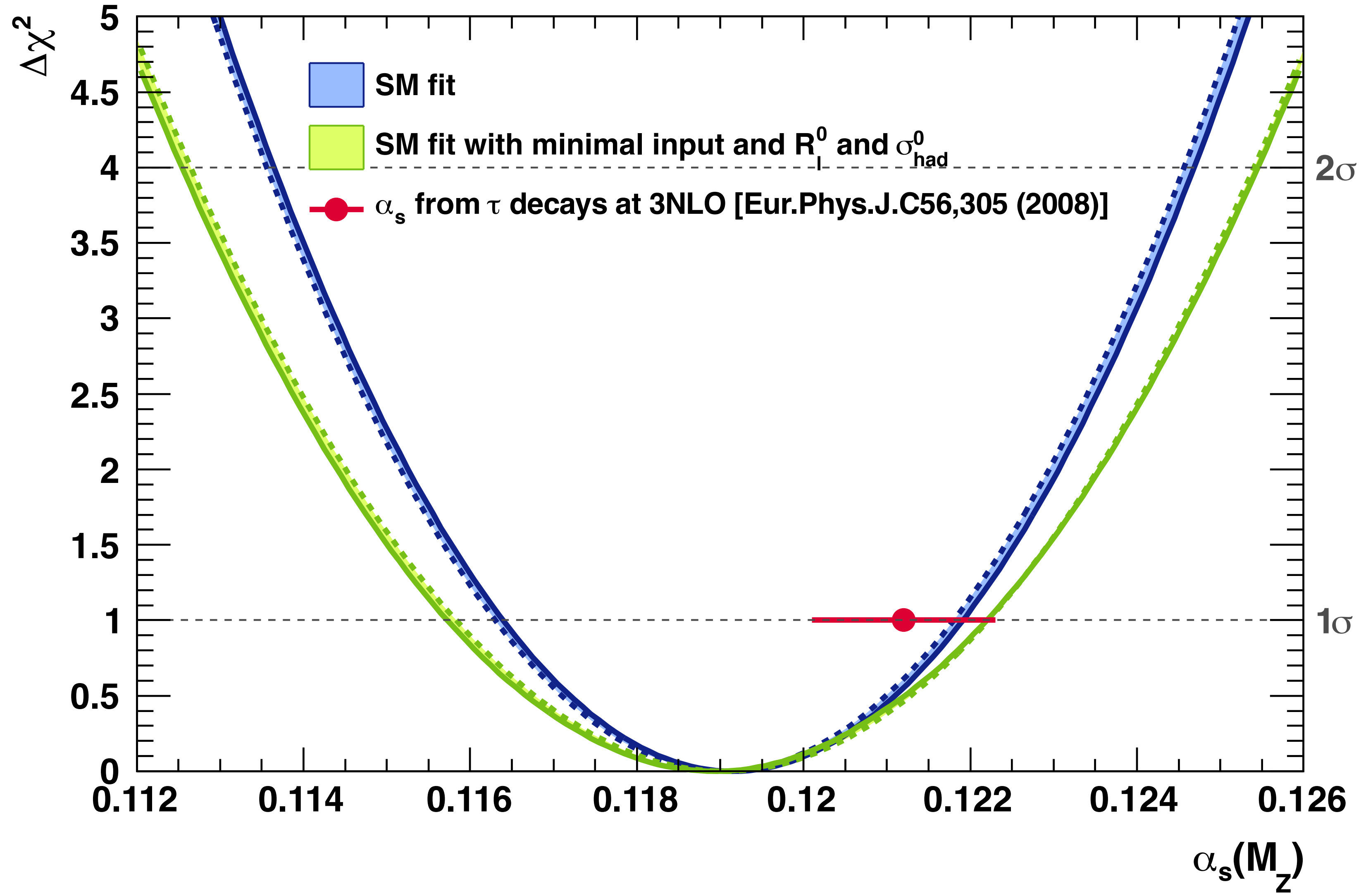
rel. uncertainty:
$\delta x = (\delta M_{W,x})^2 / (\Sigma_i\ \delta M_{W,i}{}^2)$

$\delta\sin^2(\theta^l_{eff})$ (indirect) = $1 \cdot 10^{-4}$

$\delta\sin^2(\theta^l_{eff})$ (exp) = $1.6 \cdot 10^{-4}$

$\alpha_s(M_Z)$ from Z→hadrons

- Determination of α_s at NNNLO
- most sensitivity through total hadronic cross section σ^0_{had} and the partial leptonic width R^0_l
- Theory uncertainty obtained by scale variation, per-mille level



$$\alpha_s(M_Z) = 0.1191 \pm 0.0028\,(\text{exp.}) \pm 0.0001\,(\text{theo.})$$

- Good agreement with value from τ decays, also at N^3LO

Improvement in precision only with ILC/GigaZ expected

ILC with GigaZ

A future linear collider would tremendously improve the precision of electroweak observables

- $t\bar{t}$ threshold
  - obtain m_t indirectly from production cross section: δm_t = 1 → 0.1 GeV
- Z peak measurements
  - polarised beams, uncertainty $\delta A^{0,f}_{LR}$: $10^{-3} \rightarrow 10^{-4}$ translates to $\delta\sin^2\theta^{l}_{\mathrm{eff}}$: $10^{-4} \rightarrow 1.3 \cdot 10^{-5}$
  - high statistics: 10^9 Z decays: $\delta R^0_{\mathrm{lep}}$: $2.5 \cdot 10^{-2} \rightarrow 4 \cdot 10^{-3}$
- WW threshold
  - from threshold scan: δM_W : 15 → 6 MeV
- Low energy data
  - $\Delta\alpha_{\mathrm{had}}$: more precise cross section data for low energy ($\sqrt{s}$ < 1.8 GeV) and around $c\bar{c}$ resonance (BES-III), improved α_s, improvements in theory: $10^{-4} \rightarrow 4.7 \cdot 10^{-5}$

Measurements at the Z-Pole

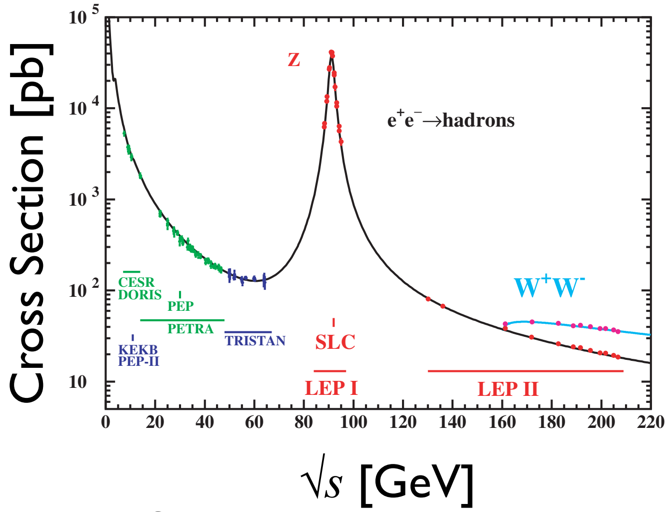
Total cross section

- Express in terms of partial decay width of initial and final state

$$\sigma^Z_{f\bar{f}} = \sigma^0_{f\bar{f}} \frac{s\Gamma_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \frac{1}{R_{\rm QED}} \quad \text{with} \quad \sigma^0_{f\bar{f}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

Corrected for QED radiation

- Full width: $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\rm had} + \Gamma_{\rm inv}$
- Highly correlated set of parameters



Less correlated set of parameters

- Z mass and width: M_Z, Γ_Z
- Hadronic pole cross section $\sigma^0_{\rm had} = 12\pi/M_Z^2 \cdot \Gamma_{ee}\Gamma_{\rm had}/\Gamma_Z^2$
- Three leptonic ratios (lepton univ.) $R^0_\ell = R^0_e = \Gamma_{\rm had}/\Gamma_{ee}\ (= R^0_\mu = R^0_\tau)$
- Hadronic width ratios R^0_b, R^0_c

Measurements at the Z-Pole

Definition of Asymmetry

- Distinguish axial and axial-vector couplings of the Z

$$A_f = \frac{g_{L,f}^2 - g_{R,f}^2}{g_{L,f}^2 + g_{R,f}^2} = \frac{2 g_{V,f}\, g_{A,f}}{g_{V,f}^2 + g_{A,f}^2}$$

- Directly related to $\sin^2\theta_{\mathrm{eff}}^{f\bar{f}} = \frac{1}{4Q_f}\left(1 + \mathcal{R}e\left(\frac{g_{V,f}}{g_{A,f}}\right)\right)$

Observables

- In case of no beam polarisation (LEP) use final state angular distribution to define forward/backward asymmetry

$$A_{FB}^f = \frac{N_F^f - N_B^f}{N_F^f + N_B^f} \qquad A_{FB}^{0,f} = \frac{3}{4} A_e A_f$$

- Polarised beams (SLC): define left/right asymmetry

$$A_{LR}^f = \frac{N_L^f - N_R^f}{N_L^f + N_R^f} \frac{1}{\langle |P|_e \rangle} \qquad A_{LR}^0 = A_e$$

- Measurements: $A_{FB}^{0,\ell}$, $A_{FB}^{0,c}$, $A_{FB}^{0,b}$, A_ℓ, A_c, A_b

The Electromagnetic Coupling

Running of the EM coupling

- The EW fit requires precise knowledge of $\alpha(M_Z)$ (better than 1%)
- Conventionally parametrised as ($\alpha(0)$ = fine structure constant)

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalisation scale

$$\Delta\alpha(s) = \Delta\alpha_{\mathrm{lep}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s)$$

- Leptonic term known up to three loops for $q^2 \gg m_l$ [M. Steinhauser, Phys. Lett. B429, 158 (1998)]
- Top quark contribution known up to two loops, small: $-0.7 \cdot 10^{-4}$
- Hadronic contribution difficult, cannot be obtained from pQCD alone
  - analysis of low energy e^+e^- data
  - usage of pQCD if lack of data

$$\Delta\alpha_{\mathrm{had}}(M_Z^2) = (274.2 \pm 1.0) \cdot 10^{-4}$$

[M. Davier et al., Eur. Phys. J. C71, 1515 (2011)]

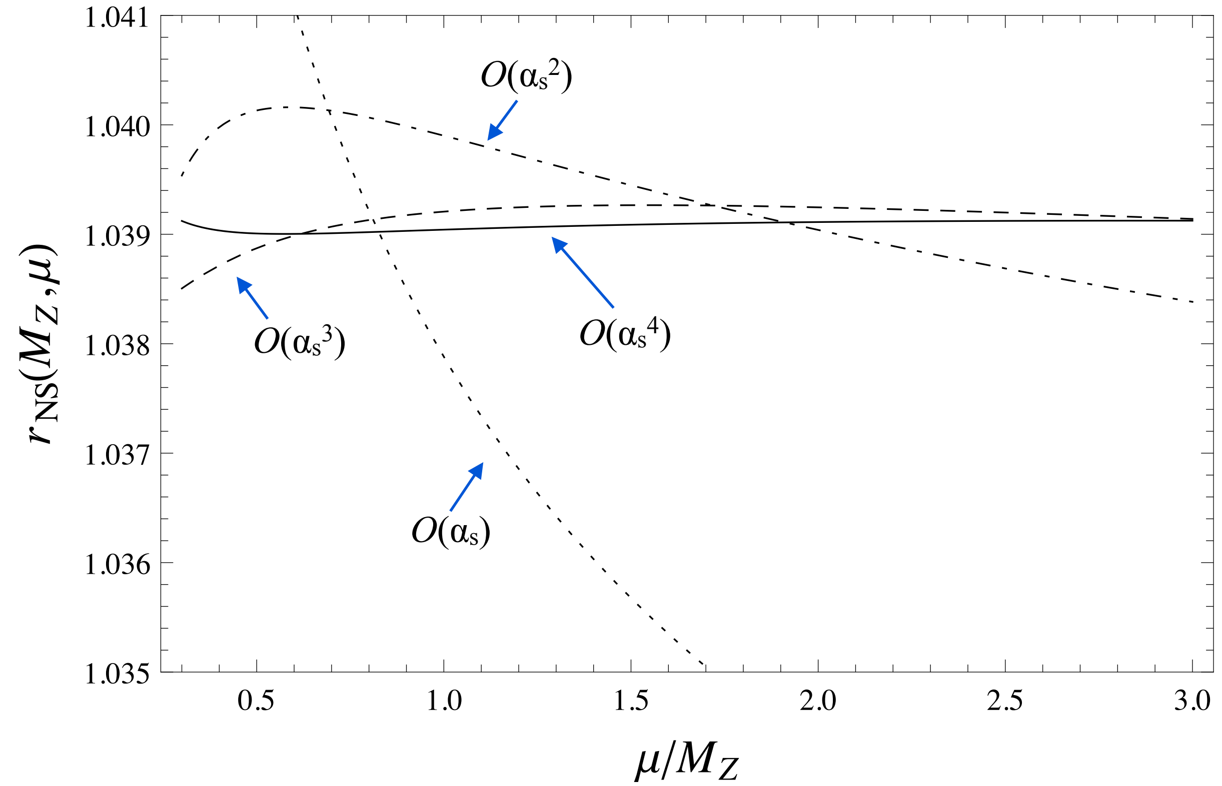
Radiator Functions

- Partial widths are defined inclusively: they contain QCD and QED contributions
- Corrections can be expressed as radiator functions $R_{A,f}$ and $R_{V,f}$

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left(|g_{A,f}|^2 R_{A,f} + |g_{V,f}|^2 R_{V,f} \right)^2$$

[D. Bardin, G. Passarino, "The Standard Model in the Making", Clarendon Press (1999)]

- High sensitivity to the strong coupling α_s
- Recently full four-loop calculation of QCD Adler function became available (N^{3}LO)
- Much reduced scale dependence
- Theoretical uncertainty of 0.1 MeV, compare to experimental uncertainty of 2.0 MeV



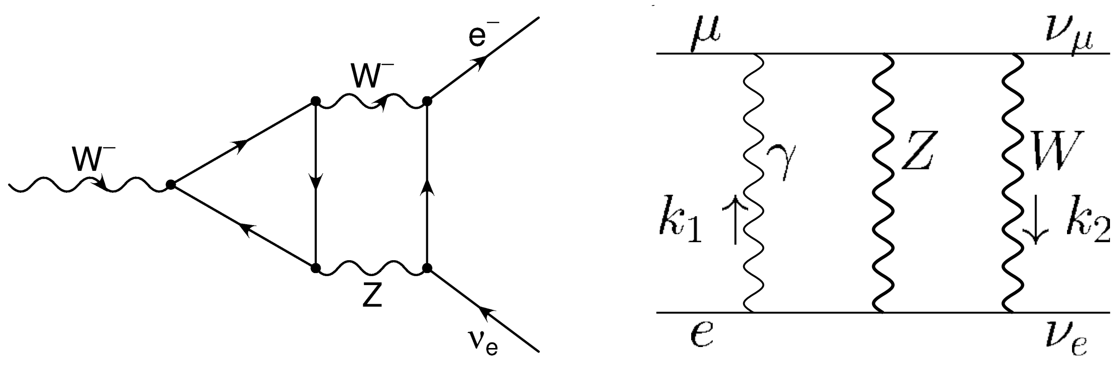
[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]
[P. Baikov et al Phys. Rev. Lett. 104, 132004 (2010)]

Calculation of M_W

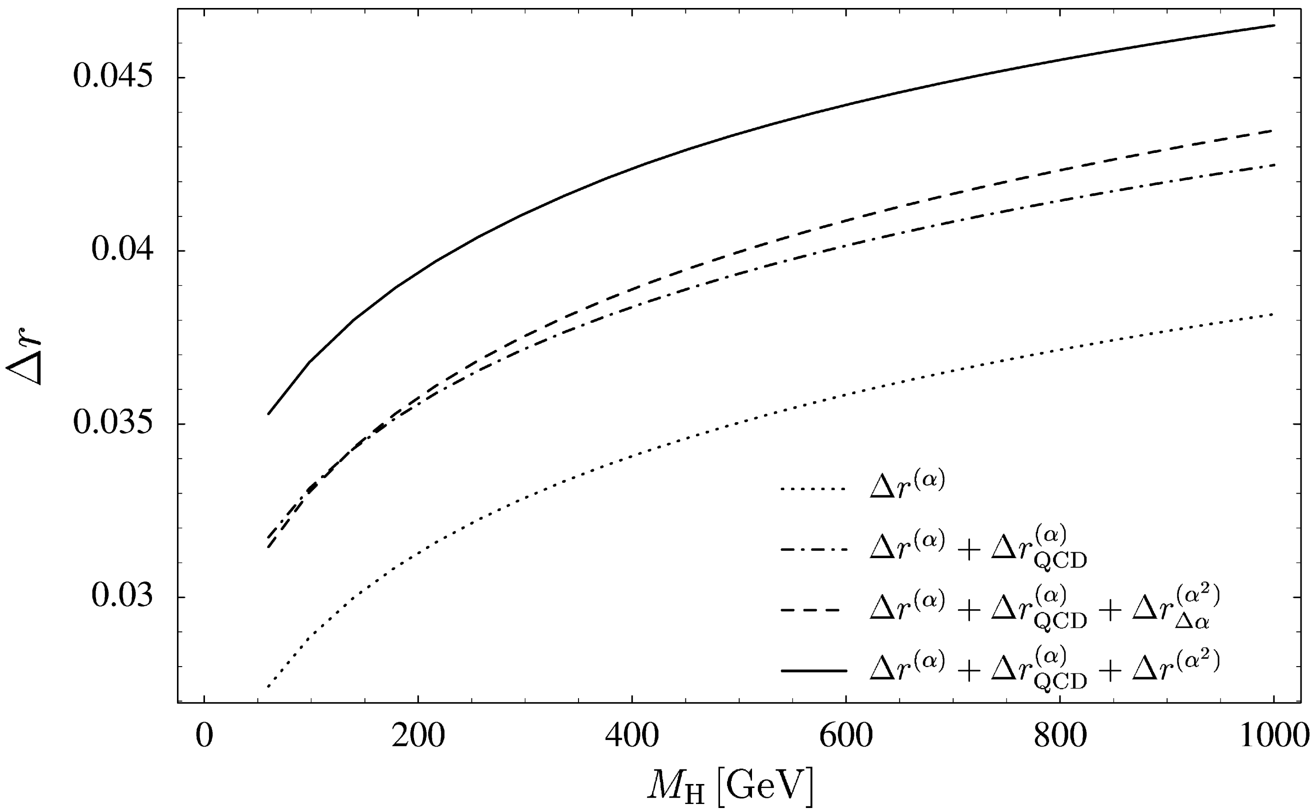
- Full EW one- and two-loop calculation of fermionic and bosonic contributions
- One- and two-loop QCD corrections and leading terms of higher order corrections
- Results for Δr include terms of order $O(\alpha)$, $O(\alpha\alpha_s)$, $O(\alpha\alpha_s^2)$, $O(\alpha^2_{\rm ferm})$, $O(\alpha^2_{\rm bos})$, $O(\alpha^2\alpha_s m_t^4)$, $O(\alpha^3 m_t^6)$
- Uncertainty estimate:
  - missing terms of order $O(\alpha^2\alpha_s)$: about 3 MeV (from $O(\alpha^2\alpha_s m_t^4)$)
  - electroweak three-loop correction $O(\alpha^3)$: < 2 MeV
  - three-loop QCD corrections $O(\alpha\alpha_s^3)$: < 2 MeV
  - Total: $\delta M_W \approx$ 4 MeV

[M Awramik et al., Phys. Rev. D69, 053006 (2004)]

[M Awramik et al., Phys. Rev. Lett. 89, 241801 (2002)]



A Freitas et al., Phys. Lett. B495, 338 (2000)]



Calculation of $\sin^2(\theta^l_{eff})$

- Effective mixing angle:

$$\sin^2\theta_{\rm eff}^{\rm lept} = (1 - M_W^2/M_Z^2)(1+\Delta\kappa)$$

- Two-loop EW and QCD correction to $\Delta\kappa$ known, leading terms of higher order QCD corrections
- fermionic two-loop correction about 10^{-3}, whereas bosonic one 10^{-5}
- Uncertainty estimate obtained with different methods, geometric progression:

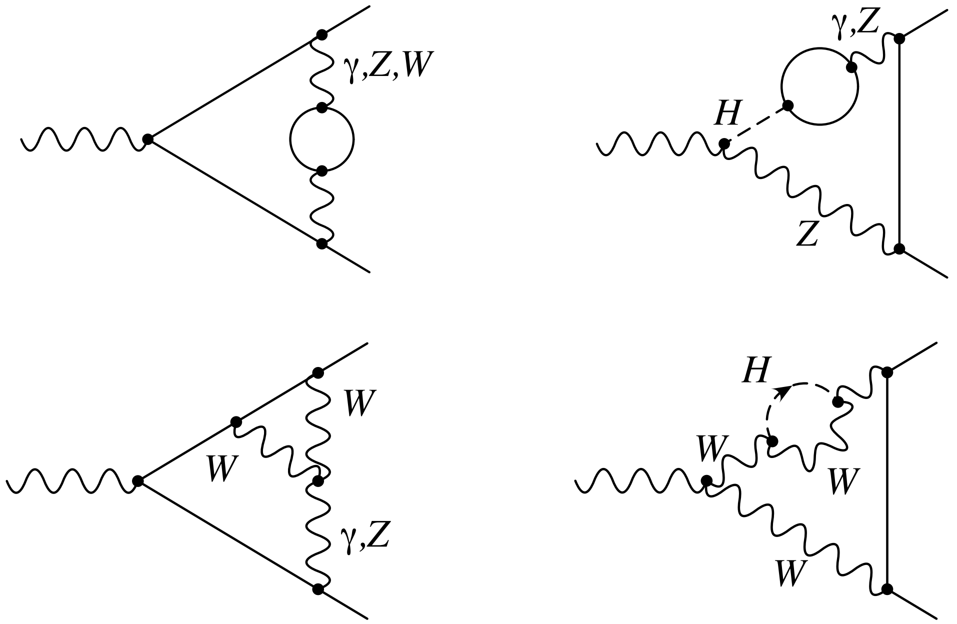
$$\mathcal{O}(\alpha^2\alpha_s) = \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}\,\mathcal{O}(\alpha\alpha_s).$$

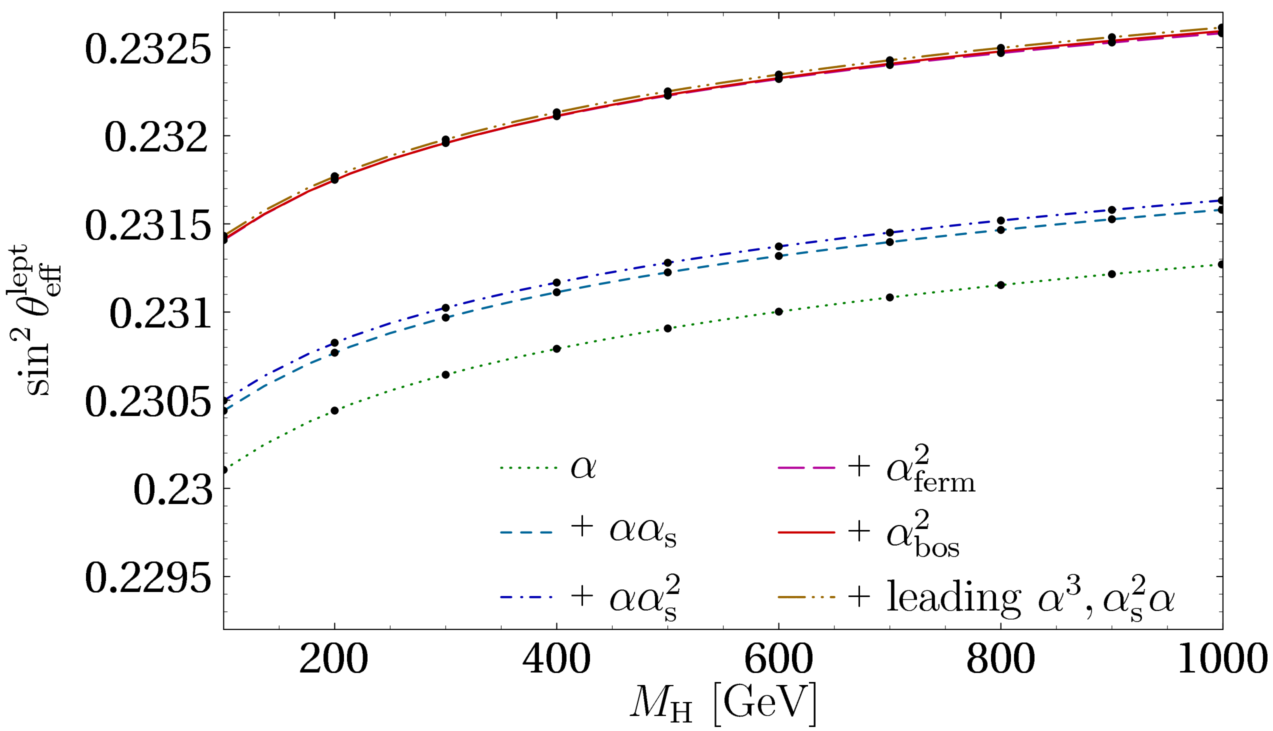
$\mathcal{O}(\alpha^2\alpha_s)$ beyond leading m_t^4	$3.3\ldots2.8\times10^{-5}$
$\mathcal{O}(\alpha\alpha_s^3)$	$1.5\ldots1.4$
$\mathcal{O}(\alpha^3)$ beyond leading m_t^6	$2.5\ldots3.5$

Total: $\delta\sin^2\theta^l_{eff} \approx 4.7\ 10^{-5}$

[M Awramik et al, Phys. Rev. Lett. 93, 201805 (2004)]

[M Awramik et al., JHEP 11, 048 (2006)]



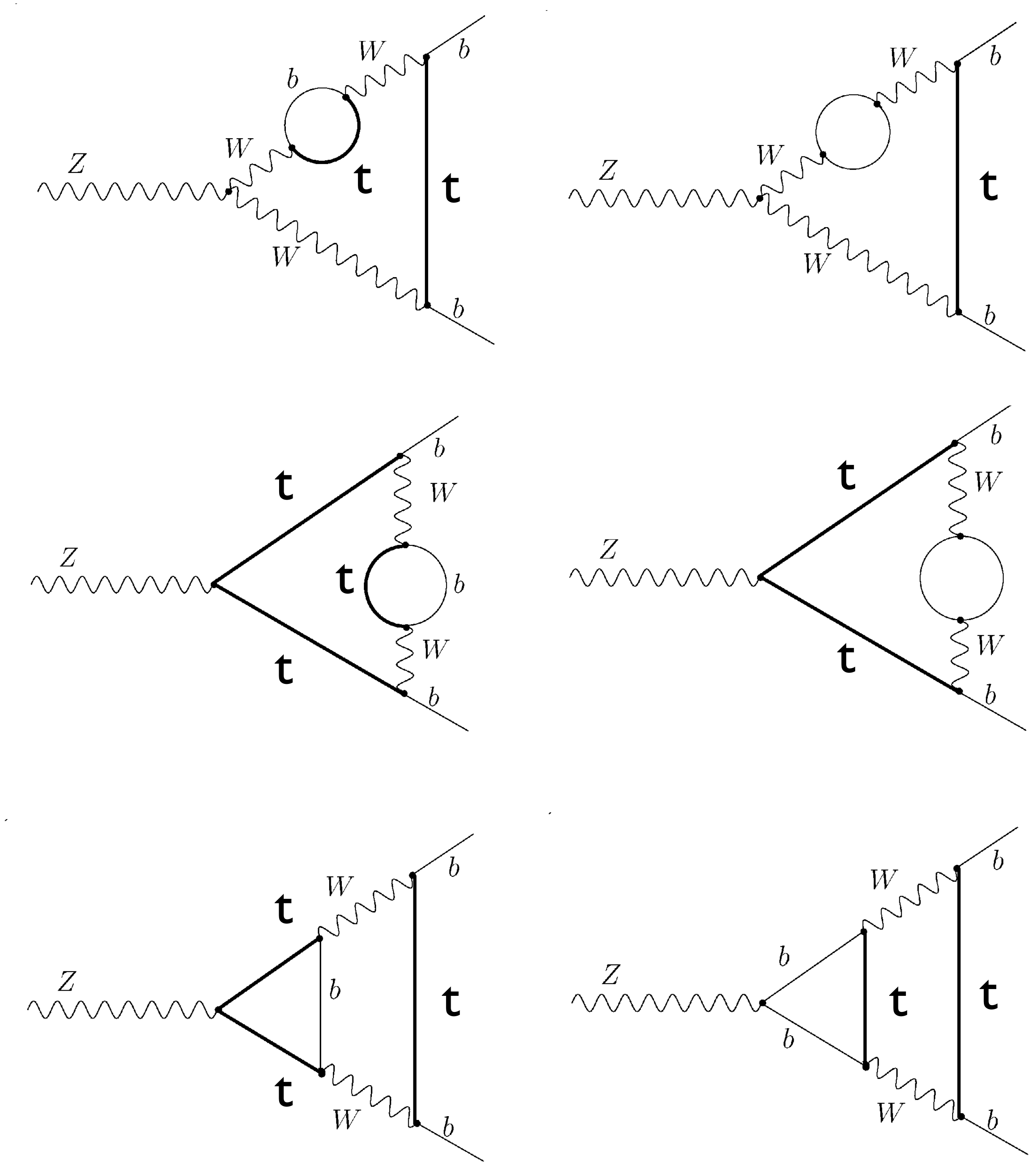


New Calculation of $\sin^2(\theta^{bb}_{eff})$

[M Awramik et al, Nucl. Phys. B813, 174 (2009)]

- Calculation of $\sin^2\theta_{eff}$ for b-quarks more involved, because of top quark propagators in the $Z\to b\bar{b}$ vertex
- Investigation of known discrepancy between $\sin^2\theta_{eff}$ from leptonic and hadronic asymmetry measurements
- Two-loop EW correction only recently completed, effect of $O(10^{-4})$
- Now $\sin^2\theta^{bb}_{eff}$ known at the same order as $\sin^2\theta_{eff}$ for leptons and light quarks
- Uncertainty assumed to be of same size as for $\sin^2\theta_{eff}$:

$$\delta\sin^2\theta^{bb}_{eff} \approx 4.7\ 10^{-5}$$



New Calculation of R^0_b

Full two-loop calculation of $Z \to b\bar{b}$

[A. Freitas et al., JHEP 1208, 050 (2012) Erratum ibid. 1305 (2013) 074]

- The branching ratio R^0_b: partial decay width of $Z \to b\bar{b}$ and $Z \to q\bar{q}$

$$R_b \equiv \frac{\Gamma_b}{\Gamma_{\rm had}} = \frac{\Gamma_b}{\Gamma_d + \Gamma_u + \Gamma_s + \Gamma_c + \Gamma_b} = \frac{1}{1 + 2(\Gamma_d + \Gamma_u)/\Gamma_b}$$

- Contribution of same terms as in the calculation of $\sin^2\theta^{bb}_{\rm eff}$
  → cross-check the two results, found good agreement
- Two-loop corrections are small compared to experimental uncertainty ($6.6 \cdot 10^{-4}$) and one-loop corrections

	1-loop EW and QCD correction to FSR	2-loop EW correction	2-loop EW and 2+3-loop QCD correction to FSR	1+2-loop QCD correction to gauge boson selfenergies
$M_{\rm H}$ [GeV]	$\mathcal{O}(\alpha) + {\rm FSR}_{\alpha,\alpha_{\rm s},\alpha_{\rm s}^2}$ $[10^{-4}]$	$\mathcal{O}(\alpha^2_{\rm ferm})$ $[10^{-4}]$	$\mathcal{O}(\alpha^2_{\rm ferm}) + {\rm FSR}_{\alpha_{\rm s}^3,\alpha\alpha_{\rm s},m_b^2\alpha_{\rm s},m_b^4}$ $[10^{-4}]$	$\mathcal{O}(\alpha\alpha_{\rm s}, \alpha\alpha_{\rm s}^2)$ $[10^{-4}]$
100	-35.66	-0.856	-2.496	-0.407
200	-35.85	-0.851	-2.488	-0.407
400	-36.09	-0.846	-2.479	-0.406