11D supergravity from the non-linear realisation of E11

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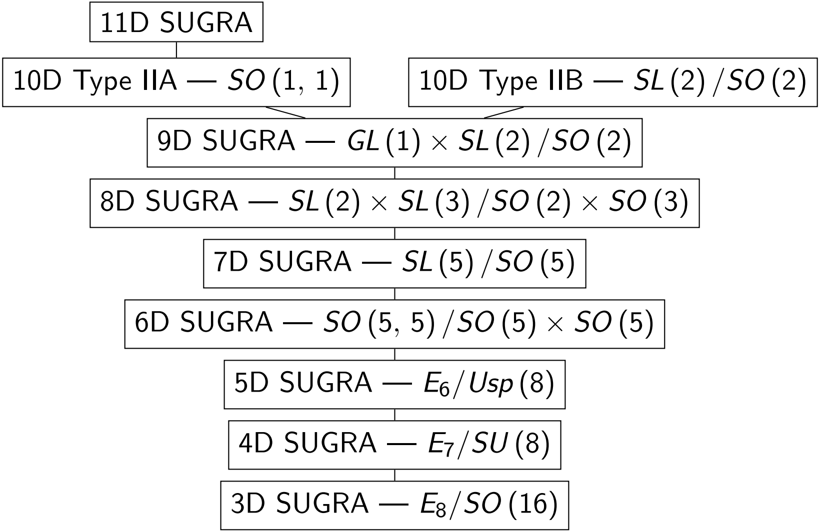
Presentation plan

- Introduction
  - E11 conjecture
  - Non-linear realisations
- E11 in 11D
- Conclusions

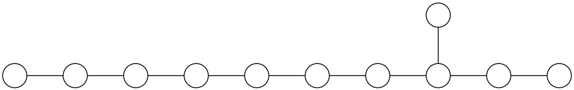
Introduction

Exceptional symmetries of maximal supergravities



Introduction

E11 Dynkin diagram



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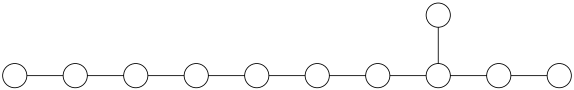
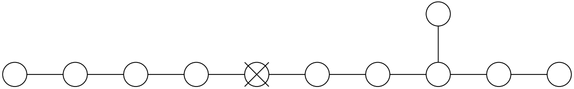


5D SUGRA — $GL(5) \times E_6$



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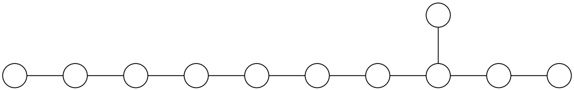

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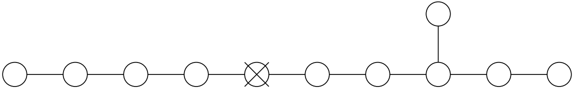
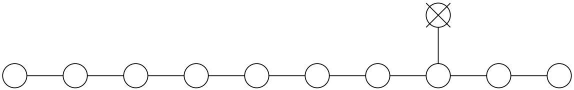


11D SUGRA — $GL(11)$



Introduction

All maximal supergravity theories in every dimension are contained in the non-linear realisation of the E11 algebra. Theories in different dimensions correspond to different decompositions of E11 into representations of its simple subalgebras.

E11 conjecture

The non-linear realisation of the E11 algebra is the low energy effective action of the theory of strings and branes.

P. West, 2001 — 0104081
P. West, 2003 — 0307098

Non-linear realisations

$G \supset H$, H - local subgroup, l_1 - vector representation of G

$$\left[R^{\alpha},\, R^{\beta}\right] = f^{\alpha\beta}{}_{\gamma}\, R^{\gamma}, \quad [R^{\alpha},\, L_A] = -\,(D^{\alpha})_A{}^B\, L_B$$

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Regular NLR: $\quad g = e^{A_\alpha\, R^\alpha}$

NLR of a space-time symmetry group: $\quad g = e^{x^A\, L_A}\, e^{A_\alpha\, R^\alpha}$

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Symmetries of NLR

- Rigid (global) $g \longrightarrow s\, g$, $s \in G$
- Local $g \longrightarrow g\, h$, $h \in H$

Non-linear realisations

The Cartan form

$$\mathcal{V} = g^{-1}\,dg = \mathcal{V}_L + \mathcal{V}_E$$

$$= dx^{\Pi}\left(E_{\Pi}{}^{A}\,L_A + G_{\Pi|\alpha}\,R^{\alpha}\right) = dx^{\Pi}\,E_{\Pi}{}^{A}\,\left(L_A + G_{A|\alpha}\,R^{\alpha}\right)$$

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Transformations of the Cartan form

- Under rigid $\mathcal{V} \longrightarrow \mathcal{V}$
- Under local $\mathcal{V} \longrightarrow h^{-1}\, \mathcal{V}\, h + h^{-1}\, dh$

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The dynamics

- The action $S = \int d^d x\, Tr\{P^2\}$
- Direct derivation of the equations of motion

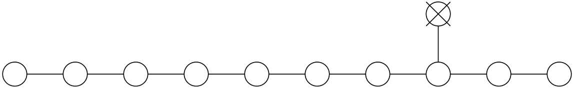
Non-linear realisations

Examples

- Pions are described by the NLR of $SU(2) \times SU(2)$ with a local subgroup $SU(2)$ (spontaneous breaking of the chiral symmetry).
- Minkowski space is described by the NLR of the Poincare group with the local subgroup $SO(1, D-1)$.
- Superspace is described by the NLR of the Super-Poincare group with the local subgroup $SO(1, D-1)$.
- Gravity can be described by the NLR of $IGL(D) = \mathbb{R}^D \rtimes GL(D)$ with the local subgroup $SO(1, D-1)$.
- In D-dimensional supergravity the scalars belong to the coset of the NLR of $E(11-D)$ ($SL(2, \mathbb{R})$ for 10D Type IIB theory).

E11 in 11D

E11 is decomposed into representations of $GL(11)$



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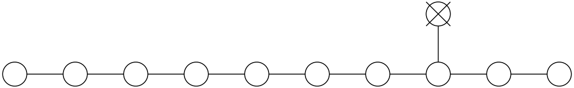

Generators of E11 and its vector representation

$$R^{\alpha} = \Big\{ \underbrace{\ldots, R_{a_1\ldots a_6}, R_{a_1 a_2 a_3}}_{\text{negative level}}, \underbrace{K^a{}_b}_{\text{level zero}}, \underbrace{R^{a_1 a_2 a_3}, R^{a_1\ldots a_6}, R^{a_1\ldots a_8, b}, \ldots}_{\text{positive level}} \Big\}$$

$$L_A = \Big\{ \underbrace{P_a}_{\text{level zero}}, \underbrace{Z^{a_1 a_2}, Z^{a_1\ldots a_5}, Z^{a_1\ldots a_7, b}, Z^{a_1\ldots a_8}, \ldots}_{\text{positive level}} \Big\}$$

E11 in 11D

The group element $g = g_L \, g_E$

$$
\begin{aligned}
g_E &= e^{A_\alpha \, R^\alpha} = \dots e^{A_{a_1 \dots a_6} \, R^{a_1 \dots a_6}} \, e^{A_{a_1 a_2 a_3} \, R^{a_1 a_2 a_3}} \, e^{h_a{}^b \, K^a{}_b} \\
g_L &= e^{x^A \, L_A} = e^{x^a \, P_a} \, e^{x_{a_1 a_2} \, Z^{a_1 a_2}} \, e^{x_{a_1 \dots a_5} \, Z^{a_1 \dots a_5}} \dots
\end{aligned}
$$

E11 in 11D

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$$g_E = e^{A_\alpha\, R^\alpha} = \ldots\, e^{A_{a_1 \ldots a_6}\, R^{a_1 \ldots a_6}}\, e^{A_{a_1 a_2 a_3}\, R^{a_1 a_2 a_3}}\, e^{h_a{}^b\, K^a{}_b}$$
$$g_L = e^{x^A\, L_A} = e^{x^a\, P_a}\, e^{x_{a_1 a_2}\, Z^{a_1 a_2}}\, e^{x_{a_1 \ldots a_5}\, Z^{a_1 \ldots a_5}} \ldots$$

Relation to the supergravity

$e_\mu{}^a = \left(e^h\right)_\mu{}^a$ - vielbein.
$A_{a_1 a_2 a_3}$ - the 3-form of 11D SUGRA, couples to the M2 brane.
$A_{a_1 \ldots a_6}$ - the 6-form of 11D SUGRA (magnetic dual of the 3-form), couples to the M5 brane.
L_A multiplet contains all the brane charges.
We choose the Cartan involution subalgebra of E11, called $I_c\,(E_{11})$, to be the local subalgebra of the NLR.
Cartan involution subalgebra is an algebra built from the generators of the following form: $E_\alpha - F_\alpha$.

E11 in 11D

AT, Peter West, 2015 — 1601.03974

Level 1 $I_c\,(E_{11})$ transformation

$$g \longrightarrow g\,h, \quad h = 1 - \Lambda_{a_1 a_2 a_3}\left(R^{a_1 a_2 a_3} - \eta^{a_1 b_1}\,\eta^{a_2 b_2}\,\eta^{a_3 b_3}\,R_{b_1 b_2 b_3}\right)$$

E11 in 11D

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First order vector equation — duality relation between the 3-form and the 6-form

$$E_{a_1 \ldots a_4} = G_{[a_1,\,a_2 a_3 a_4]} - \frac{1}{48}\,\varepsilon_{a_1 \ldots a_4}{}^{b_1 \ldots b_7}\,G_{b_1,\,b_2 \ldots b_7} = 0$$

$$\delta\,E_{a_1 \ldots a_4} = \frac{1}{24}\,\varepsilon_{a_1 \ldots a_4}{}^{b_1 \ldots b_7}\,\Lambda_{b_1 b_2 b_3}\,E_{b_4 \ldots b_7} + \text{gravity part}$$

E11 in 11D

Second order vector equations — 3-form equation of 11D SUGRA

$$E^{a_1 a_2 a_3} = \frac{1}{2}\, G_{b,\,c}{}^{c}\, G^{[b,\,a_1 a_2 a_3]} - 3\, G_{b,\,c}{}^{[a_1|}\, G^{[b,\,c|a_2 a_3]]} - G_{c,\,b}{}^{c}\, G^{[b,a_1 a_2 a_3]}$$

$$+ (\det e)^{\frac{1}{2}}\;\, e_b{}^{\mu} \partial_\mu\, G^{[b,\,a_1 a_2 a_3]} + \frac{1}{48}\, \varepsilon^{a_1 a_2 a_3 b_1 \dots b_8}\, G_{[b_1,\,b_2 b_3 b_4]}\, G_{[b_5,\,b_6 b_7 b_8]} = 0$$

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Second order vector equations — 3-form equation of 11D SUGRA

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$$\delta\, E^{a_1a_2a_3} = \frac{1}{24}\, \varepsilon^{\mu_1\ldots\mu_4\lambda_1\ldots\lambda_7}\, e^{a_1}_{\mu_1} e^{a_2}_{\mu_2} e^{a_3}_{\mu_3}\, \partial_{\mu_4}\, \left((\det e)^{-\frac{1}{2}}\, E_{\lambda_1\ldots\lambda_4}\, \Lambda_{\lambda_5\lambda_6\lambda_7} \right)$$

$$+ 210\, E_{b_1\ldots b_4}\, G^{[b_1,\,b_2b_3b_4}\, \Lambda^{a_1a_2a_3]} + \frac{3}{2}\, E_b{}^{[a_1}\, \Lambda^{a_2a_3]b}$$

$E_a{}^b$ is the gravity (Einstein) equation

E11 in 11D

Gravity equation

$$E_a{}^b = \underbrace{(\det e)\; R_a{}^b}_{\text{gravity}} - \underbrace{48\, G_{[a,\, c_1 c_2 c_3]}\, G^{[b,\, c_1 c_2 c_3]} + 4\, \delta_a^b\, G_{[c_1,\, c_2 c_3 c_4]}\, G^{[c_1,\, c_2 c_3 c_4]}}_{\text{stress-energy tensor}}$$

$$R_a{}^b = e_a{}^\mu \partial_\mu\, \omega_{\nu,}{}^{bd}\, e_d{}^\nu - e_a{}^\mu \partial_\nu\, \omega_{\mu,}{}^{bd}\, e_d{}^\nu + \omega_{a,}{}^b{}_c\, \omega_{d,}{}^{cd} - \omega_{d,}{}^b{}_c\, \omega_{a,}{}^{cd}$$

Ultimate check — variation of the gravity equation

$$\begin{aligned} \delta\, E_{ab} = &- 72\, \Lambda^{c_1 c_2}{}_{(a}\, E_{b) c_1 c_2} + 8\, \eta_{ab}\, \Lambda^{c_1 c_2 c_3}\, E_{c_1 c_2 c_3} \\ &- 4\, \varepsilon_{(a}{}^{c_1 \ldots c_{10}}\, G_{[b),\, c_1 c_2 c_3]}\, E_{c_4 \ldots c_7}\, \Lambda_{c_8 c_9 c_{10}} \\ &+ \frac{1}{3}\, \eta_{ab}\, \varepsilon^{c_1 \ldots c_{11}}\, G_{[c_1,\, c_2 c_3 c_4]}\, E_{c_5 \ldots c_8}\, \Lambda_{c_9 c_{10} c_{11}} \end{aligned}$$

Conclusions

- On the low levels of the NLR of E11 we find the entire field content of the bosonic sector of 11D SUGRA. The equations of motion are uniquely determined by the NLR.
- The same kind of analysis can be repeated for any $D \leq 11$. For the 5D case see *AT, Peter West, 2015 — 1512.01644*
- Apart from the usual supergravity fields the E11 theory contains an infinite number of higher level fields and coordinates. Some classes of the fields represent well understood extensions of the supergravity theories (magnetic duals of supergravity fields, gauged supergravities), while other might represent some new phenomena (example: dual graviton).
- The NLR of E11 is a vast theory that contains all maximal supergravities at the same time. This means that E11 approach could be the correct way of looking at the low-energy limit of the theory of strings and branes.

Thank you