A machine to compute the planar AdS_5/CFT_4 spectrum at weak coupling

Christian Marboe

Trinity College Dublin & Humboldt Universität zu Berlin

GATIS
Gauge Theory as an Integrable System

Based on ongoing work with D. Volin

The spectral problem

- $\mathcal{N} = 4$ SYM is a CFT

The spectral problem

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- Dilatation operator

$$D\,\mathcal{O} = \Delta\mathcal{O} \qquad \mathcal{O} = \text{Tr}[\mathcal{D} Z X \Psi ...]$$
$$\Delta = \Delta_0 + \gamma(g)$$

The spectral problem

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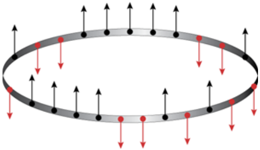
$$D\,\mathcal{O} = \Delta\mathcal{O} \qquad \mathcal{O} = \mathrm{Tr}[\mathcal{D} Z X \Psi \ldots]$$
$$\Delta = \Delta_0 + \gamma(g)$$

- $\Delta \rightarrow$ correlation functions:

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \propto \frac{1}{|x-y|^{2\Delta}}$$
$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle \propto \frac{1}{|x-y|^{\Delta_1+\Delta_2-\Delta_3}|y-z|^{\Delta_2+\Delta_3-\Delta_1}|x-z|^{\Delta_1+\Delta_3-\Delta_2}}$$

Integrability in the planar limit

- 1-loop: XXX spin chain [Minahan, Zarembo '02]



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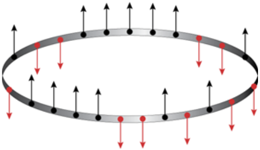

- L loops: Asymptotic Bethe Ansatz [Beisert, Staudacher '04]

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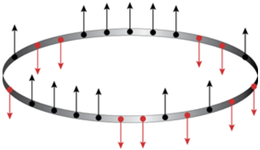


- L loops: Asymptotic Bethe Ansatz [Beisert, Staudacher '04]

- All loops:

TBA [Arutyunov, Frolov '09] [Gromov, Kazakov, Kozak, Vieira '09] [Bombardelli, Fioravanti, Tateo '09]

$$\downarrow$$

$$\vdots$$

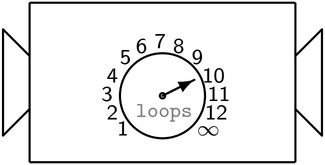
$$\downarrow$$

Quantum Spectral Curve [Gromov, Kazakov, Leurent, Volin '13]

The Goal

$S_1 \quad S_2$

$J_1 \quad J_2 \quad J_3 \quad \longrightarrow$



$\longrightarrow \quad \Delta$

Overview

1. A bit of representation theory
2. Q-systems
3. QSC
4. Examples of solutions

A bit of representation theory

$su(2)$ sector

Raising operator: R-charge

$su(2)$ sector

Raising operator: R-charge

Finite-dimensional representations

$$ZZZZ$$
$$\updownarrow$$
$$ZZZX$$
$$\updownarrow$$
$$2ZZXX + ZXZX \qquad\qquad ZZXX - ZXZX$$
$$\updownarrow$$
$$ZXXX$$
$$\updownarrow$$
$$XXXX$$

$sl(2)$ sector

Raising operator: P_μ

$sl(2)$ sector

Raising operator: P_μ

Infinite-dimensional representations

$$
\begin{array}{ccc}
ZZ & & \\
\updownarrow & & \\
\mathcal{D}ZZ & & \\
\updownarrow & & \\
\mathcal{D}^2 ZZ + \mathcal{D}Z\mathcal{D}Z & \mathcal{D}^2 ZZ - 2\mathcal{D}Z\mathcal{D}Z & \\
\updownarrow & \updownarrow & \\
\mathcal{D}^3 ZZ + \ldots & \mathcal{D}^3 ZZ + \ldots & \\
\updownarrow & \updownarrow & \\
\mathcal{D}^4 ZZ + \ldots & \mathcal{D}^4 ZZ + \ldots & \mathcal{D}^4 ZZ + \ldots \\
\updownarrow & \updownarrow & \updownarrow \\
\vdots & \vdots & \vdots
\end{array}
$$

Raising operator: supercharge, Q

$su(1|1)$ sector

Raising operator: supercharge, Q

Two-dimensional representations

Z^5

$\updownarrow$

$Z^4\Psi$

$ZZZ\Psi\Psi$

$\updownarrow$

$ZZ\Psi\Psi\Psi$

$ZZ\Psi Z\Psi$

$\updownarrow$

$Z\Psi Z\Psi\Psi$

$Z\Psi^4$

$\updownarrow$

Ψ^5

$psu(2,2|4)$

$L=2$

$$\mathcal{D}\Psi Z$$

$$\ddots \qquad \updownarrow$$

$$\mathcal{D}^2X^2 \;\leftrightarrow\; \mathcal{D}^2ZX \;\leftrightarrow\; \mathcal{D}^2Z^2$$

$L=3$

$$Z\Psi^2$$

$$\vdots \qquad \iddots$$

$$\mathcal{D}Z^2\Psi \quad \ldots$$

$L=3$

$$\ldots \quad \mathcal{D}Z^2\bar{\Psi}$$

$$\iddots \qquad \vdots$$

$$Z\bar{\Psi}^2$$

$L=4$

$$\mathcal{D}Z^3X \quad \ldots$$

$$\vdots \qquad \ddots$$

$$Z^2X^2$$

Q-systems

Q-systems

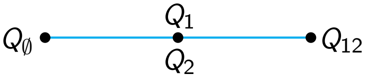
$$Q \equiv Q(u)$$

$$Q^{\pm} \equiv Q(u \pm \tfrac{i}{2})$$

$$Q^{[n]} \equiv Q(u + \tfrac{in}{2})$$

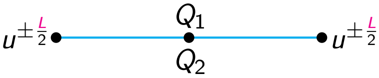
QQ-relations: $su(2)$ / $sl(2)$

$$Q_\emptyset Q_{12} \quad = \quad Q_1^+ Q_2^- - Q_1^- Q_2^+$$



QQ-relations: $su(2)$ / $sl(2)$

$$u^{\pm L} \quad = \quad Q_1^+ Q_2^- - Q_1^- Q_2^+$$



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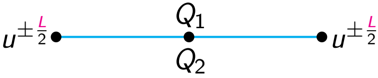

$$\left(u_\bullet - \frac{i}{2}\right)^{\pm L} = -Q_1(u_\bullet - i)Q_2(u_\bullet) \qquad \left(u_\bullet + \frac{i}{2}\right)^{\pm L} = Q_1(u_\bullet + i)Q_2(u_\bullet)$$

QQ-relations: $su(2)$ / $sl(2)$

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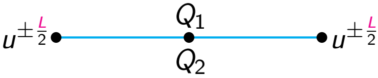
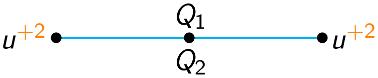


$$\frac{Q_1(u_\bullet - i)}{Q_1(u_\bullet + i)} = -\left(\frac{u_\bullet - \frac{i}{2}}{u_\bullet + \frac{i}{2}}\right)^{\pm L}$$

QQ-relations: $su(2)$ / $sl(2)$

$$u^{+4} \quad = \quad Q_1^{+} Q_2^{-} - Q_1^{-} Q_2^{+}$$



$$\frac{Q_1(u_\bullet - i)}{Q_1(u_\bullet + i)} = -\left(\frac{u_\bullet - \frac{i}{2}}{u_\bullet + \frac{i}{2}}\right)^{+4}$$

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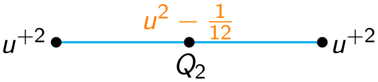

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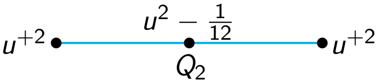

$$Q_2 = Q_1 \sum_{n=0}^{\infty} \frac{\left(u^{[2n+1]}\right)^4}{Q_1^{[2n]} Q_1^{[2n+2]}}$$

QQ-relations: $su(2)$ / $sl(2)$

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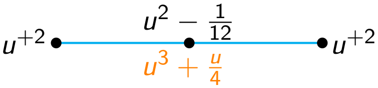

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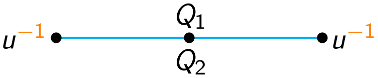
QQ-relations: $su(2)$ / $sl(2)$

$$u^{-2} = Q_1^+ Q_2^- - Q_1^- Q_2^+$$



$$\frac{Q_1(u_\bullet - i)}{Q_1(u_\bullet + i)} = -\left(\frac{u_\bullet - \frac{i}{2}}{u_\bullet + \frac{i}{2}}\right)^{-2}$$

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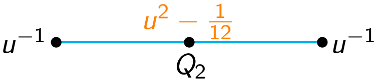

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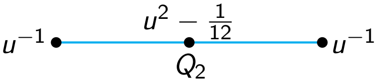

$$Q_2 = Q_1 \sum_{n=0}^{\infty} \frac{1}{(u^{[2n+1]})^2 Q_1^{[2n]} Q_1^{[2n+2]}}$$

QQ-relations: $su(2)$ / $sl(2)$

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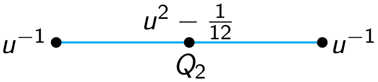

$$Q_2 = \left(u^2 - \frac{1}{12}\right) \sum_{n=0}^{\infty} \frac{1}{(u + \frac{i}{2} + in)^2} + iu$$

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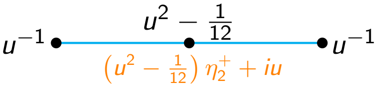

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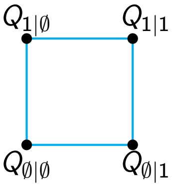
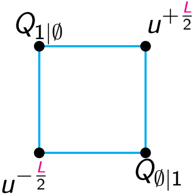
$$Q_{1|\emptyset}\, Q_{\emptyset|1} \quad = \quad Q^{+}_{1|1}\, Q^{-}_{\emptyset|\emptyset} - Q^{-}_{1|1}\, Q^{+}_{\emptyset|\emptyset}$$



QQ-relations: $su(1|1)$

$$Q_{1|\emptyset}\,Q_{\emptyset|1} = \left(\frac{u+\frac{i}{2}}{u-\frac{i}{2}}\right)^{\frac{L}{2}} - \left(\frac{u-\frac{i}{2}}{u+\frac{i}{2}}\right)^{\frac{L}{2}}$$



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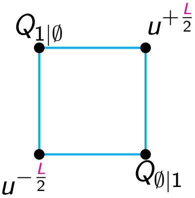

$$\left(u_\bullet + \frac{i}{2}\right)^{L} = \left(u_\bullet - \frac{i}{2}\right)^{L}$$

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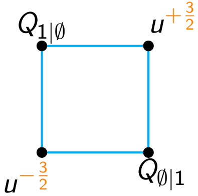

$$\left(u_\bullet + \frac{i}{2}\right)^3 = \left(u_\bullet - \frac{i}{2}\right)^3$$

QQ-relations: $su(1|1)$

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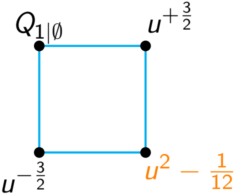

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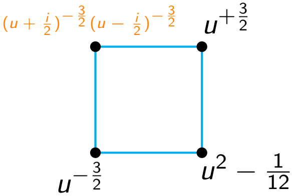
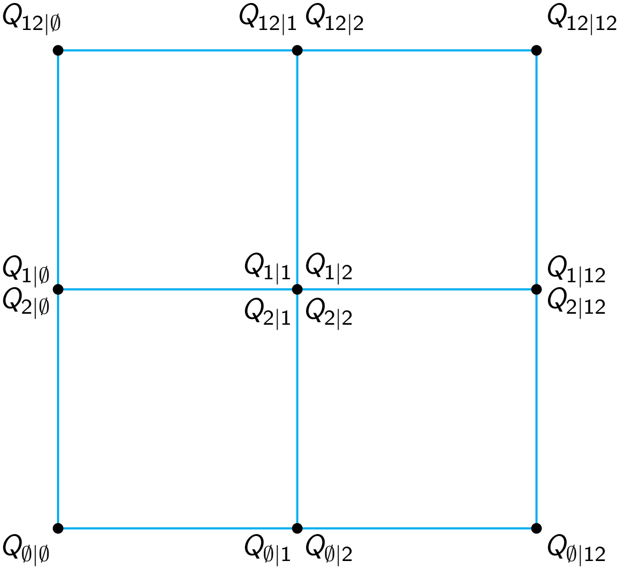


$$\left(u_\bullet+\frac{i}{2}\right)^3 = \left(u_\bullet-\frac{i}{2}\right)^3$$

$psu(1,1|2)$ Q-system



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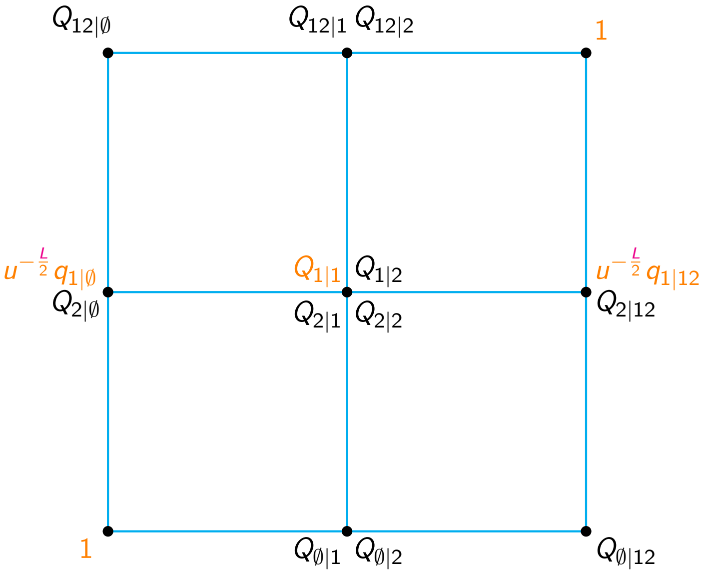

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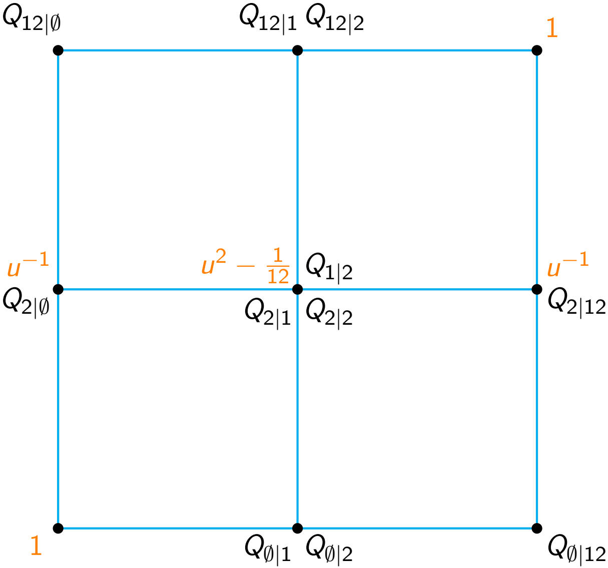

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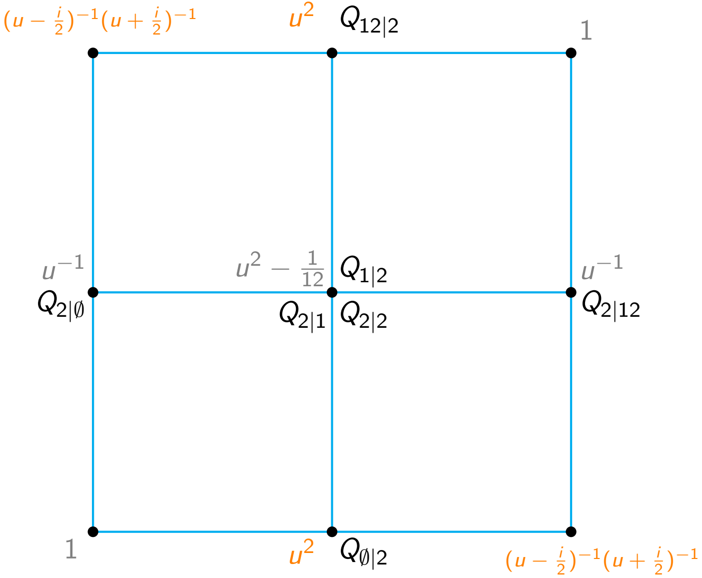

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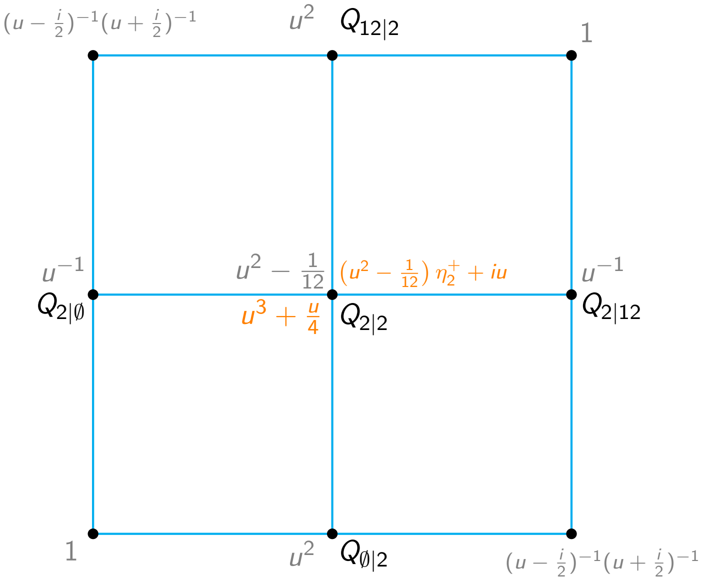

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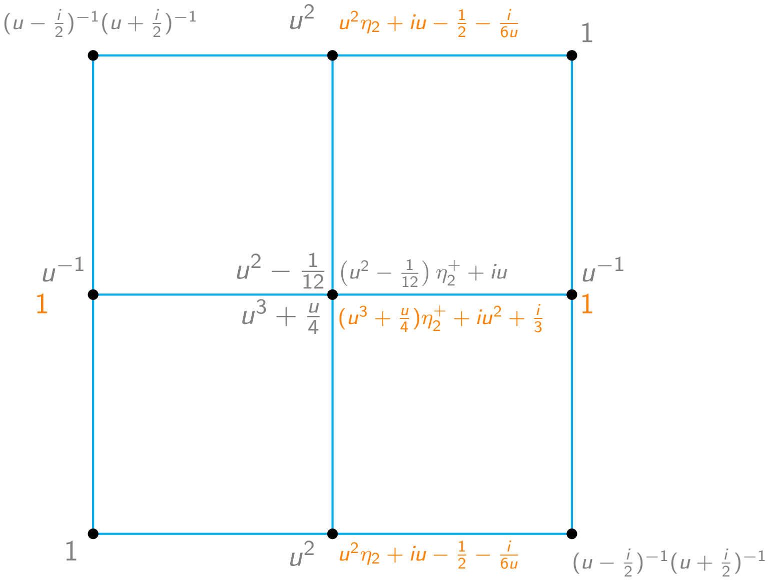


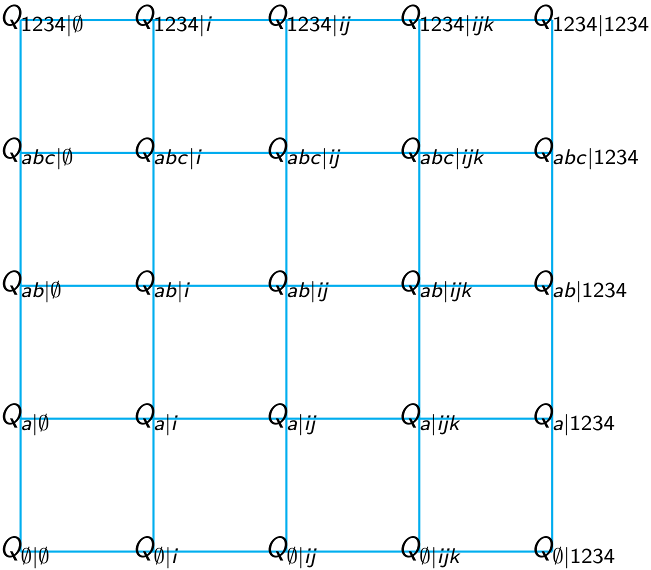
Quantum Spectral Curve

$$=$$

Q-system + some constraints

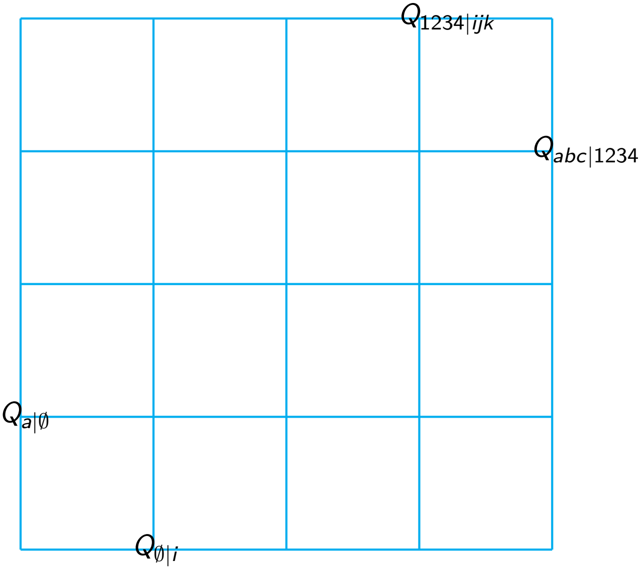
Quantum Spectral Curve

256 Q's



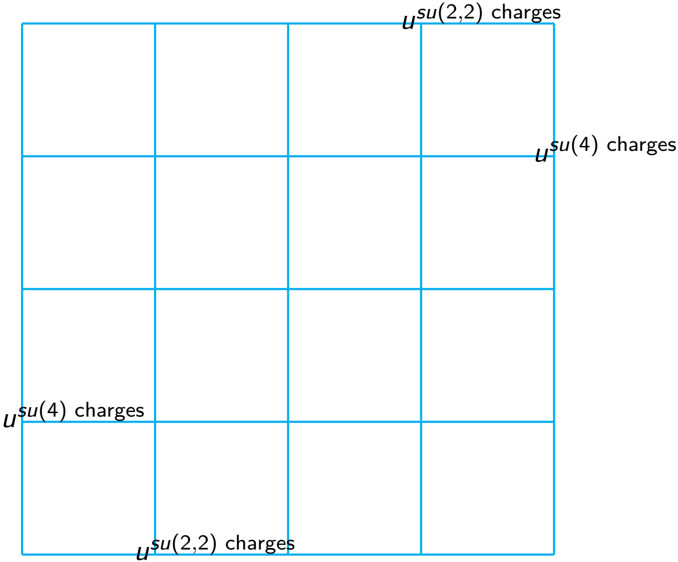
Quantum Spectral Curve

$u \to \infty$ asymptotics

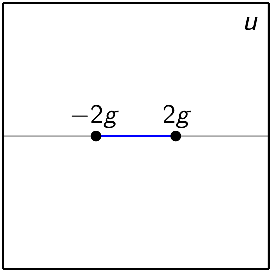


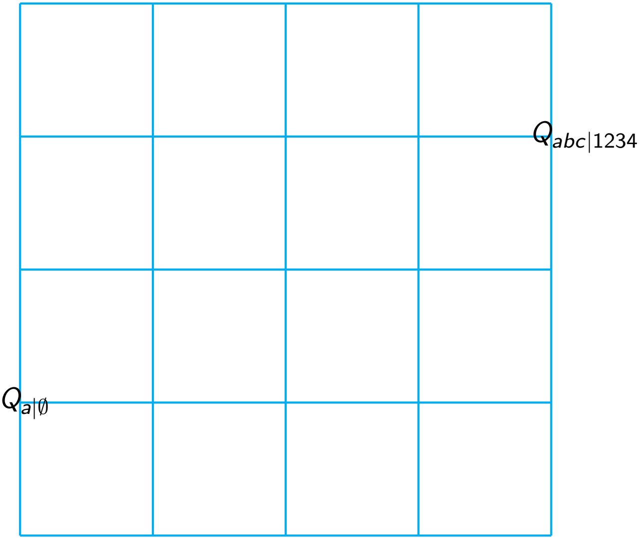
Quantum Spectral Curve

$u \to \infty$ asymptotics

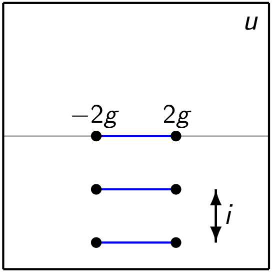


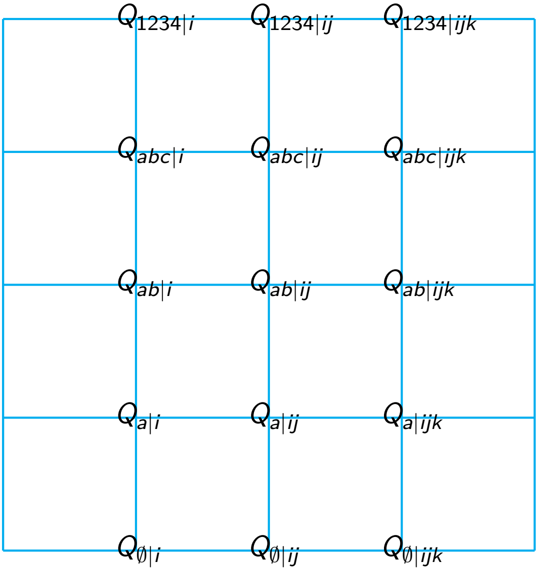
Analytic structure





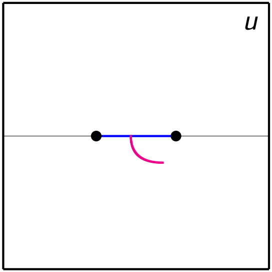
Analytic structure

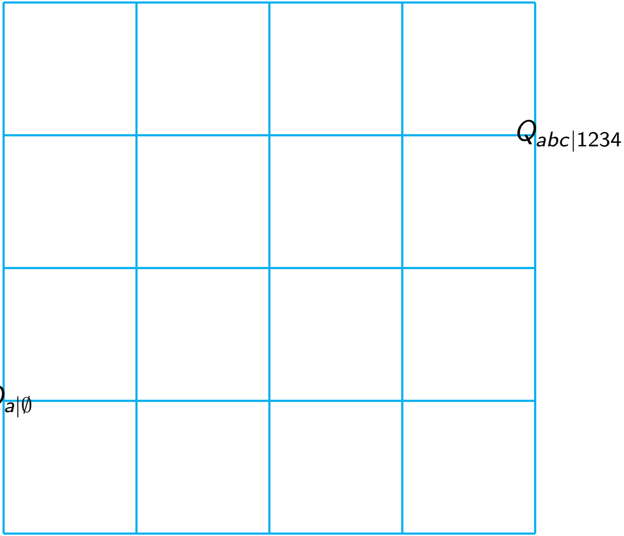




Quantum Spectral Curve

Analytic continuation





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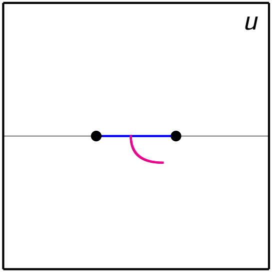
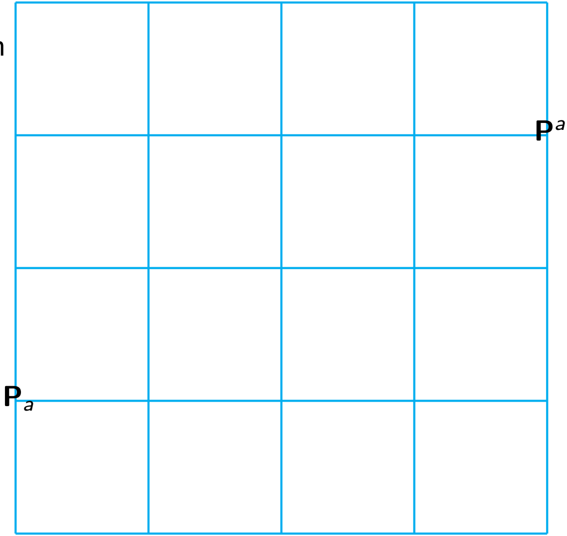
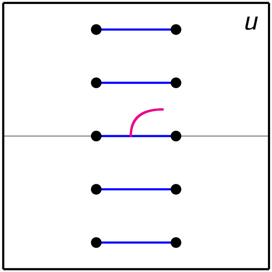
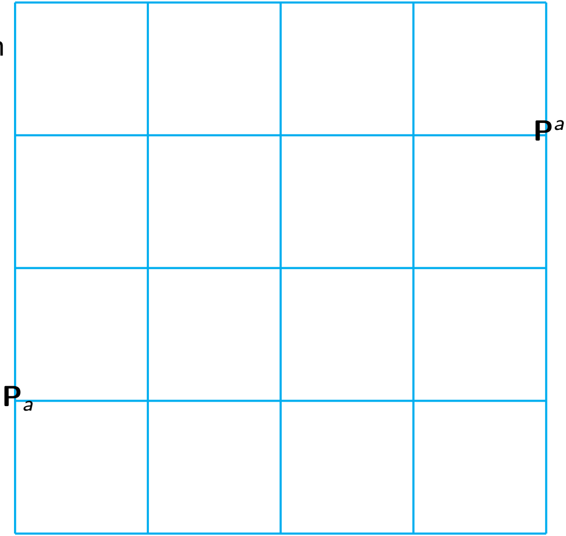




Analytic continuation

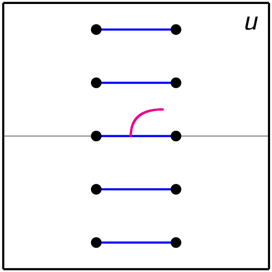




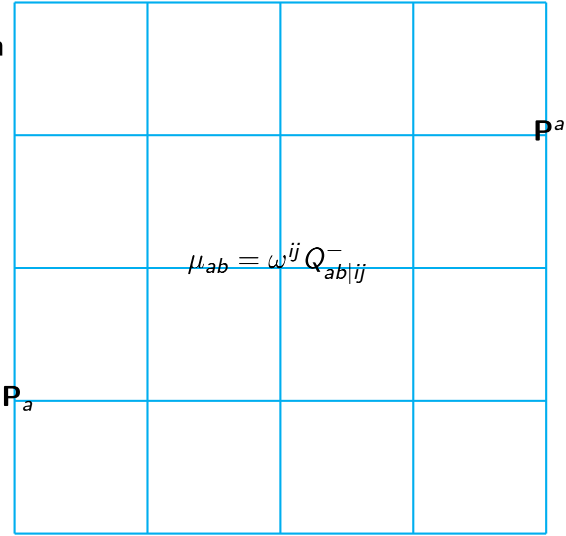
$$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^a$$

Quantum Spectral Curve

Analytic continuation

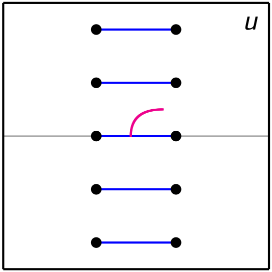


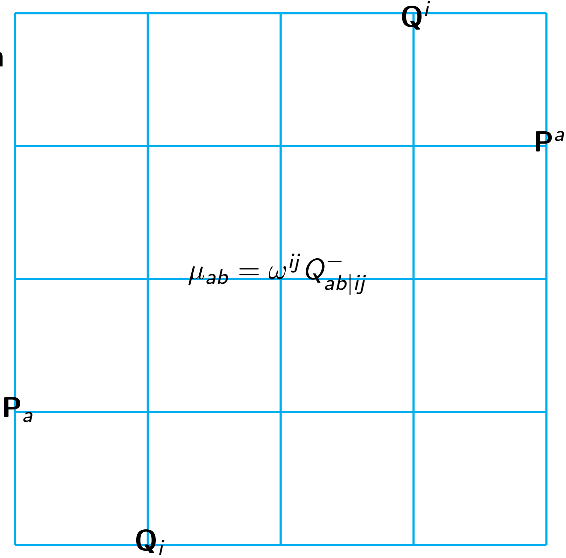
$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^a$



Quantum Spectral Curve

Analytic continuation



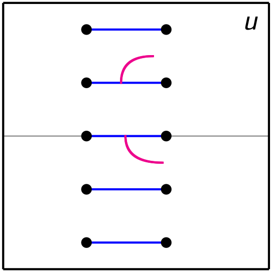


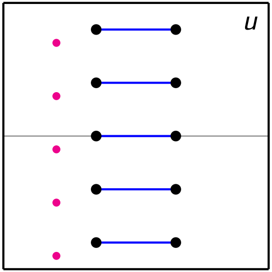
$$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^a$$

$$\tilde{\mathbf{Q}}_i = \omega_{ij}\mathbf{Q}^j$$

Quantum Spectral Curve

$$\tilde{\mu}_{ab} = \mu_{ab}^{[2]} \qquad \omega_{ij} = \omega_{ij}^{[2]}$$





Perturbative solution

$$\Delta \;=\; \Delta_0 + g^2\,\Delta_1 + g^4\,\Delta_2 + g^6\,\Delta_3 + \ldots$$

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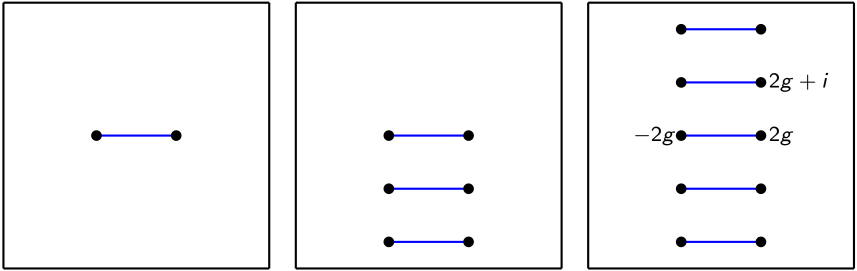
$$Q_{A|I} \;=\; Q_{A|I}^{(1)} + g^2\,Q_{A|I}^{(2)} + g^4\,Q_{A|I}^{(3)} + g^6\,Q_{A|I}^{(4)} + \ldots$$

Perturbative solution

$$\Delta \;=\; \Delta_0 + g^2\,\Delta_1 + g^4\,\Delta_2 + g^6\,\Delta_3 + \ldots$$

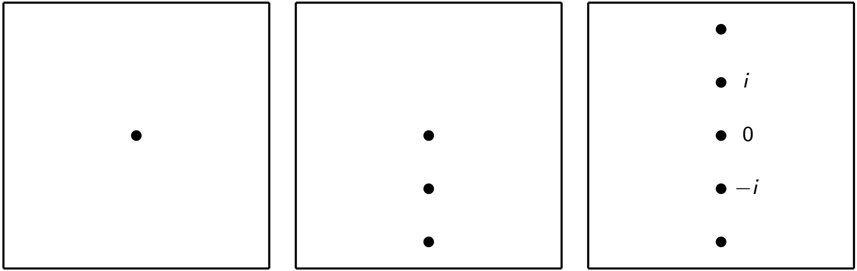
$$Q_{A|I} \;=\; Q^{(1)}_{A|I} + g^2\,Q^{(2)}_{A|I} + g^4\,Q^{(3)}_{A|I} + g^6\,Q^{(4)}_{A|I} + \ldots$$



Perturbative solution

$$\Delta = \Delta_0 + g^2\,\Delta_1 + g^4\,\Delta_2 + g^6\,\Delta_3 + \dots$$

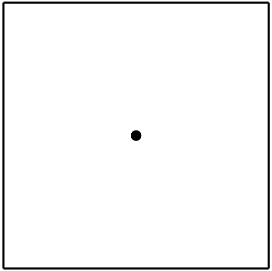
$$Q_{A|I} = Q^{(1)}_{A|I} + g^2\,Q^{(2)}_{A|I} + g^4\,Q^{(3)}_{A|I} + g^6\,Q^{(4)}_{A|I} + \dots$$



Perturbative solution

$$\Delta = \Delta_0 + g^2 \Delta_1 + g^4 \Delta_2 + g^6 \Delta_3 + \dots$$

$$Q_{A|I} = Q^{(1)}_{A|I} + g^2 Q^{(2)}_{A|I} + g^4 Q^{(3)}_{A|I} + g^6 Q^{(4)}_{A|I} + \dots$$



$\mathbf{P}_a$

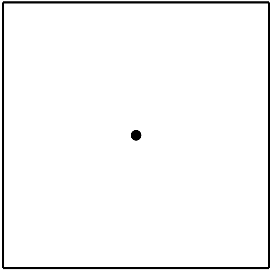
powerlike asymptotics

poles only at $u = 0$

Perturbative solution

$$\Delta = \Delta_0 + g^2 \Delta_1 + g^4 \Delta_2 + g^6 \Delta_3 + \ldots$$

$$Q_{A|I} = Q_{A|I}^{(1)} + g^2 Q_{A|I}^{(2)} + g^4 Q_{A|I}^{(3)} + g^6 Q_{A|I}^{(4)} + \ldots$$

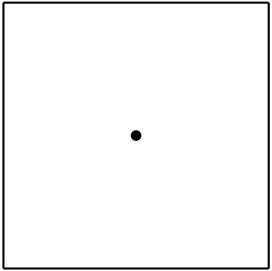


$\mathbf{P}_a$ rational

Perturbative solution

$$\Delta = \Delta_0 + g^2\,\Delta_1 + g^4\,\Delta_2 + g^6\,\Delta_3 + \dots$$

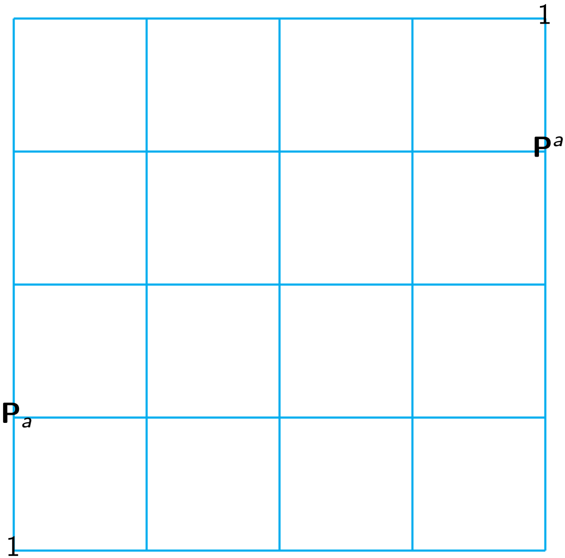
$$Q_{A|I} = Q_{A|I}^{(1)} + g^2\,Q_{A|I}^{(2)} + g^4\,Q_{A|I}^{(3)} + g^6\,Q_{A|I}^{(4)} + \dots$$



e.g. Konishi

$$\mathbf{P}_2 = -\frac{1}{u} + g^2\left(-\frac{1}{u^3} - \frac{c}{u^2} - \frac{\Delta_1}{2u}\right) + \mathcal{O}(g^4)$$

Perturbative solution

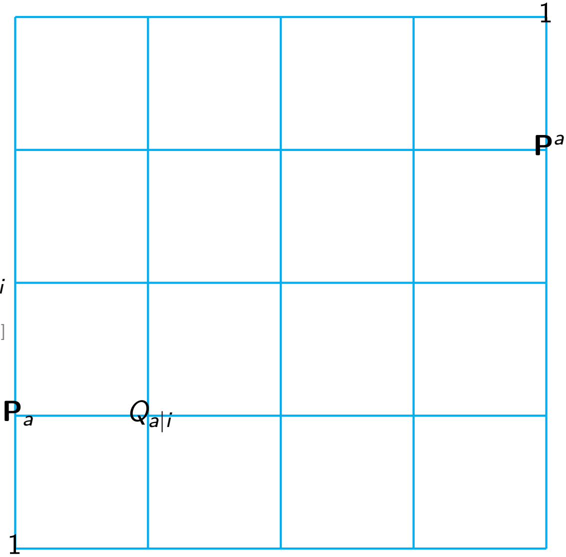


Perturbative solution

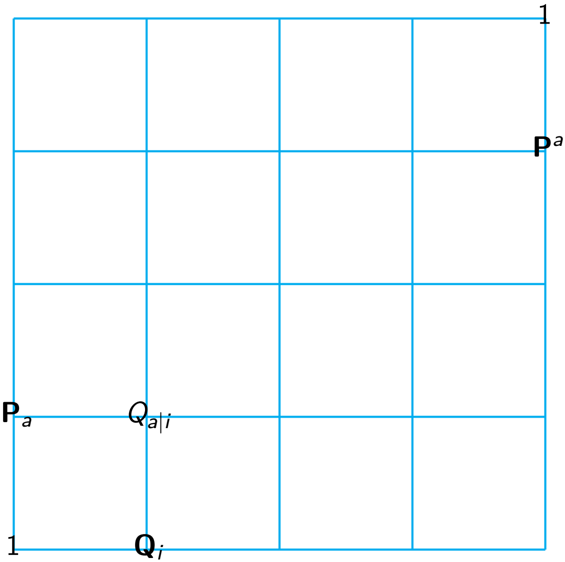
solve

$$Q^-_{a|i} - Q^+_{a|i} = \mathbf{P}_a \mathbf{P}^a Q^+_{b|i}$$

[Gromov, Levkovich-Maslyuk, Sizov '15]



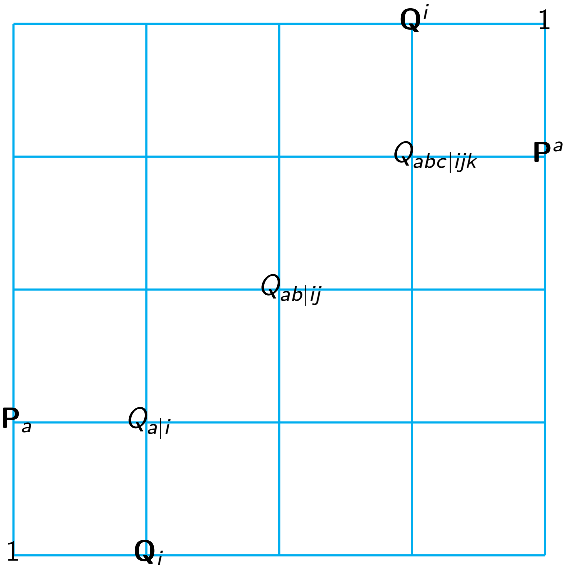
Perturbative solution



Perturbative solution

The rest are just determinants

$$Q_{ab|ij} = \begin{vmatrix} Q_{a|i} & Q_{a|j} \\ Q_{b|i} & Q_{b|j} \end{vmatrix}$$



Perturbative solution

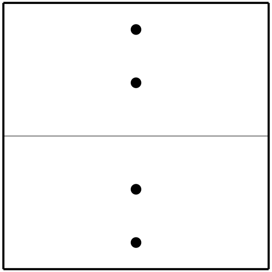
Fix

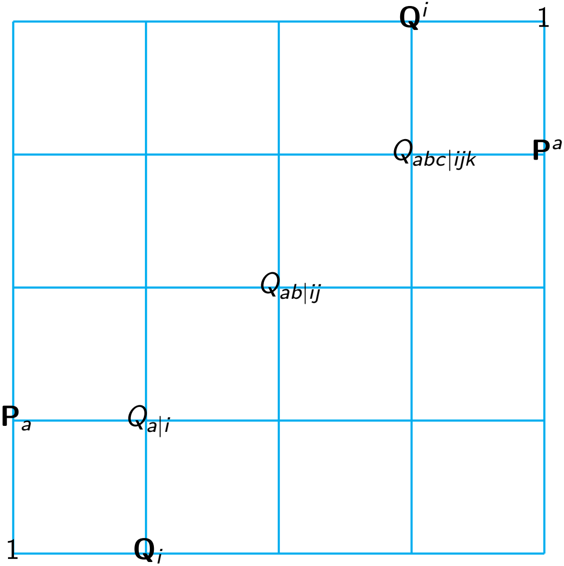
$\mu_{ab} = \omega^{ij} Q^{-}_{ab|ij}$

from regularity of

$\mu + \tilde{\mu}$

at $u = 0$





Perturbative solution

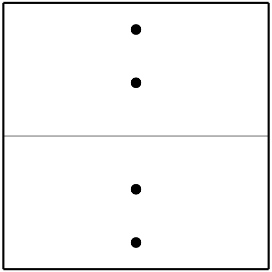
Fix

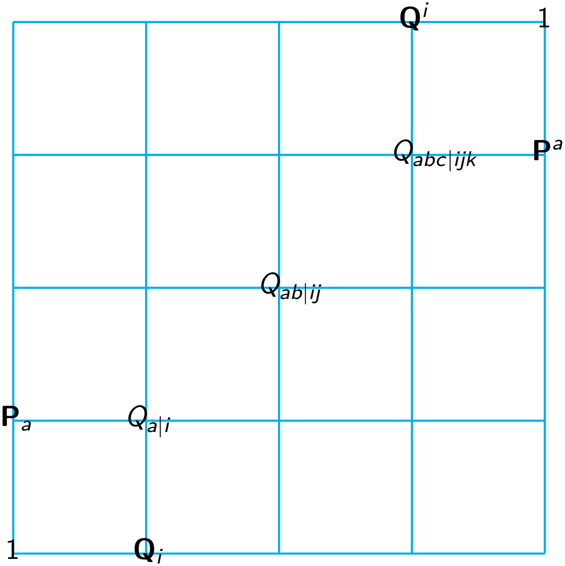
$\mu_{ab} = \omega^{ij} Q^{-}_{ab|ij}$

from regularity of

$\mu + \mu^{[2]}$

at $u = 0$





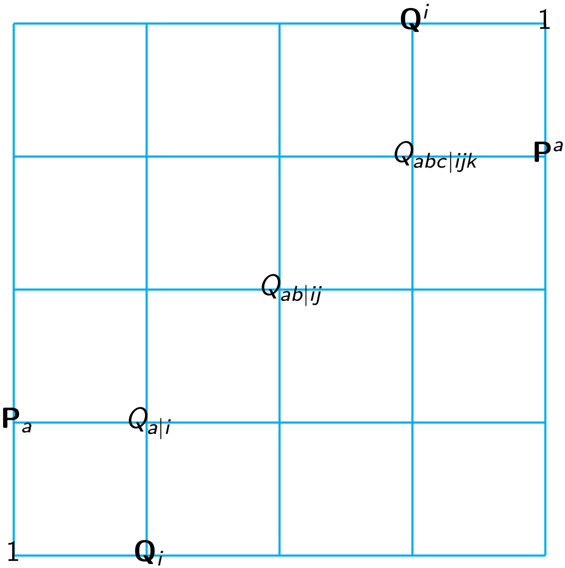
Perturbative solution

"Glue"

$\mathbf{P}$ and $\tilde{\mathbf{P}} = \mu \mathbf{P}$

at $u = 0$

to fix Δ



Perturbative solution

$$\mathbf{P}_2 = -\frac{1}{u}$$

$$Q_{12|12} = u^2 - \frac{1}{12}$$

$$\Delta = 4 + 12g^2$$

Perturbative solution

$$\mathbf{P}_2 = -\frac{1}{u} + g^2\left(-\frac{1}{u^3} - \frac{6}{u}\right)$$

$$Q_{12|12} = u^2 - \frac{1}{12} + g^2\left(-\frac{7}{12} - 9u^2 + i(1 - 12u^2)\eta_1^+\right)$$

$$\Delta = 4 + 12g^2 - 48g^4$$

Perturbative solution

$$\mathbf{P}_2 = -\frac{1}{u} + g^2\left(-\frac{1}{u^3} - \frac{6}{u}\right) + g^4\left(-\frac{2}{u^5} - \frac{9}{u^3} + \frac{24}{u}\right)$$

$$\begin{aligned} Q_{12|12} = \; & u^2 - \frac{1}{12} + g^2\left(-\frac{7}{12} - 9u^2 + i(1 - 12u^2)\eta_1^+\right) \\ & + g^4\left(-\frac{5}{2} + 99u^2 + i(3 + 156u^2)\eta_1^+ + (6 - 72u^2)\eta_2^+ \right. \\ & \left. + i(\frac{9}{8} - \frac{27}{2}u^2)\eta_3^+ + i(12 - 144u^2)\eta_{1,1}^+\right) \end{aligned}$$

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6$$

Perturbative solution

$$\mathbf{P}_2 = -\frac{1}{u} + g^2\left(-\frac{1}{u^3} - \frac{6}{u}\right) + g^4\left(-\frac{2}{u^5} - \frac{9}{u^3} + \frac{24}{u}\right)$$
$$+g^6\left(-\frac{5}{u^7} - \frac{21}{u^5} - \frac{27-12\zeta_3}{u^3} - \frac{168}{u}\right)$$

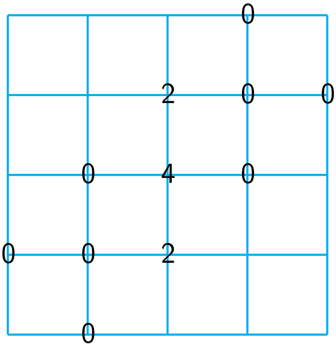
$$Q_{12|12} = u^2 - \frac{1}{12} + g^2\left(-\frac{7}{12} - 9u^2 + i(1-12u^2)\eta_1^+\right)$$
$$+g^4\left(-\frac{5}{2} + 99u^2 + i(3+156u^2)\eta_1^+ + (6-72u^2)\eta_2^+\right.$$
$$\left.+i(\frac{9}{8} - \frac{27}{2}u^2)\eta_3^+ + i(12-144u^2)\eta_{1,1}^+\right) + g^6(\dots)$$

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + g^8(-2496 + 576\zeta_3 - 1440\zeta_5)$$

Examples

Examples

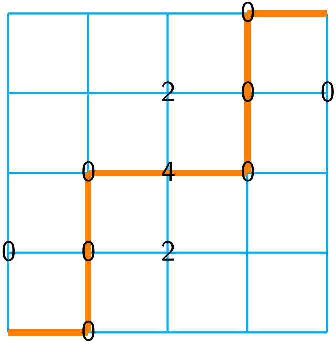
$\mathcal{D}^4 Z^2$ $\quad$ $sl(2)$



Examples

$\mathcal{D}^4 Z^2$

$sl(2)$



$$Q_{12|12} = u^4 - \frac{13}{14}u^2 + \frac{27}{560}$$

Examples

$$\mathcal{D}^4 Z^2$$

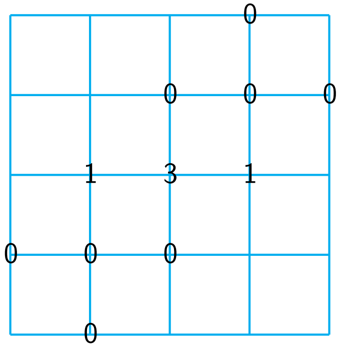
$$\begin{aligned}\Delta &= 6 + \frac{50}{3}g^2 - \frac{1850}{27}g^4 + \frac{241325}{486}g^6 \\ &+g^8\left(-\frac{8045275}{2187} + \frac{114500\zeta_3}{81} - \frac{25000\zeta_5}{9}\right) \\ &+g^{10}\left(\frac{3007398125}{157464} + \frac{24048500\zeta_3}{729} - \frac{125000\zeta_3^2}{9} - \frac{3357500\zeta_5}{81} + \frac{175000\zeta_7}{3}\right) \\ &+\dots\end{aligned}$$

10 loops known [CM, Volin '14]

Examples

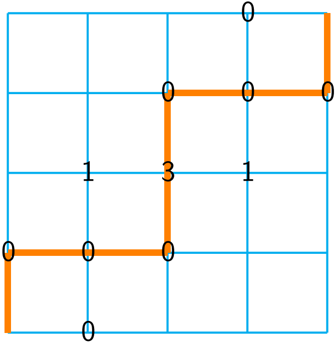
$Z^3 X^3$

$su(2)$



Examples

$Z^3 X^3$ $su(2)$



$Q_{12|12} = u^3 + \frac{u}{4}$

Examples

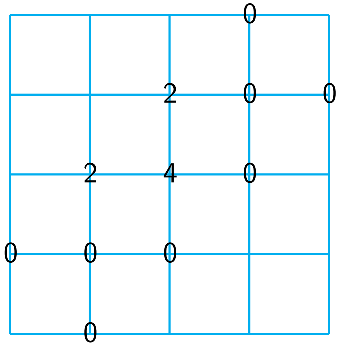
$$Z^3 X^3$$

$$\begin{aligned}\Delta \quad = \quad & 6 + 12g^2 - 36g^4 + 252g^6 \\ & -2484g^8 \\ & +g^{10}\left(28188 - 288\zeta_3\right) \\ & +g^{12}\left(-339012 + 7776\zeta_3 + 12096\zeta_5 - 18144\zeta_9\right) \text{ [Arutyunov, Frolov, Sfondrini '12]} \\ & + \ldots\end{aligned}$$

Examples

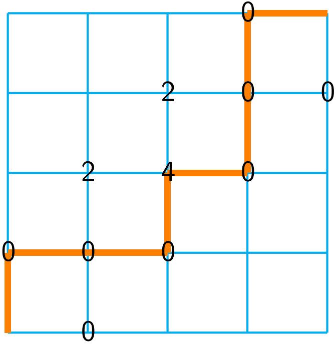
$Z\Psi^4$

$su(1|1)$



Examples

$Z\Psi^4$ $\qquad$ $su(1|1)$



$$Q_{12|12} = u^4 - \frac{u^2}{2} + \frac{1}{80}$$

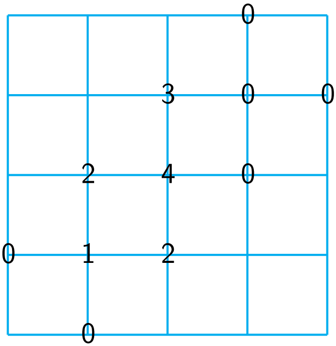
Examples

$Z\Psi^4$

$$\begin{aligned}\Delta \quad = \quad & 7 + 20g^2 - 80g^4 + 580g^6 \\ & + g^8 \left(-5180 - 320\zeta_3\right) \\ & + g^{10} \left(52220 + 4800\zeta_3 + 3200\zeta_5\right) \\ & + g^{12} \left(-\frac{1580960}{3} + \frac{121280\zeta_3}{3} - \frac{248320\zeta_5}{3} - \frac{347200\zeta_7}{3} - 50400\zeta_9\right) \\ & + \dots\end{aligned}$$

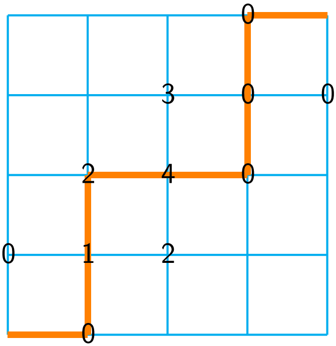
Examples

$$\mathcal{D}^2 Z \Psi_{11} \Psi_{12} + \dots$$



Examples

$$\mathcal{D}^2 Z \Psi_{11} \Psi_{12} + \ldots$$



$$Q_{12|12} = u^4 - \frac{7u^2}{10} + \frac{7}{240}$$

$$Q_{12|1} = u^2 + \frac{4}{15}$$

$$Q_{1|1} = u$$

Examples

$$\mathcal{D}^2 Z \Psi_{11} \Psi_{12} + \dots$$

$$\begin{aligned}
\Delta \quad = \quad & 6 + 18g^2 - 81g^4 + 630g^6 \\
& + g^8 \left(-\frac{18333}{4} + 3132\zeta_3 - 5400\zeta_5 \right) \\
& + g^{10} \left(10773 + 61452\zeta_3 - 87480\zeta_5 - 29160\zeta_3^2 + 120960\zeta_7 \right) \\
& + g^{12} \left(\frac{5067495}{8} - 1940706\zeta_3 + 1019898\zeta_5 - 361584\zeta_3^2 \right. \\
& \qquad \left. + 1144395\zeta_7 - 2054808\zeta_3\zeta_5 - \frac{17477289}{8}\zeta_9 \right) \\
& + \dots
\end{aligned}$$

Conclusion

- Automatised solution of the spectral problem at weak coupling
- Limitations
  - Bethe/Baxter equations hard to solve
  - Bethe roots in general complicated algebraic numbers