Supersymmetric type IIB Supergravity solutions with an AdS_5 factor and vanishing self dual flux

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Outline

1. Motivation and background
   - AdS_5 Supergravity Solutions
   - Sasaki Einstein solutions and the Pilch Warner solution
2. Generalizing to vanishing self dual field strength
   - G-Structure analysis
   - Application of G Structure analysis
   - The Metric
   - Differential equations
3. NATD-T Dual $AdS_5 \times T^{(1,1)}$ Solution

Introduction

- Via the AdS/CFT correspondence it is theorirised that supergravity solutions with AdS_{d+1} factors in the metric are dual to d dimensional SCFT's. For this reason solutions with AdS_{d+1} factors are sought after.
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with M_5 compact and no self dual flux.

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Notation

- Bosonic Fields of Type IIB Supergravity in $SU(1,1)$ formalism are;

$$g_{\mu\nu},\ P,\ G,\ F = *F \tag{1}$$

- We specialize to the case where $F = 0$. Physically this means that there is no D3 brane charge.
- Supersymmetry requires the existence of a Killing spinor satisfying:

$$\delta\psi_M \approx D_M\epsilon - \frac{1}{96}\left(\Gamma_M{}^{P_1..P_3} G_{P_1..P_3} - 9\Gamma^{P_1 P_2} G_{MP_1 P_2}\right)\epsilon^c = 0$$

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- The Solution has a non-trivial G, constant dilaton and axion and a non vanishing F.
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- A G-structure may be characterized by the existence of globally defined G invariant tensors. A tensor is G-invariant if it is invariant under rotations of the orthonormal frame.
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G-structure analysis part 2

- A Killing spinor defines a G-strucutre by way of its isotropy group. One constructs the invariant tensors by forming spinor bilinears.
- Using Fierz identities we obtain algebraic conditions for the invariant tensors.
- Using the SUSY equations we obtain differential conditions for the bilinears.
- Using the above differential equations we find which EOM's are satisified.
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Spinor decomposition

- We want bosonic supersymmetric solutions with a warped metric of the form;

$$\mathrm{d}s^2 = e^{2\Delta(y)}(\,\mathrm{d}s^2_{AdS_5} + \mathrm{d}s^2_{M_5}(y))$$

preserving the $SO(4,2)$ isometry of AdS_5.

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- We decompose the Killing spinor schematically as;

$$Spin(9,1) \to Spin(4,1) \times Spin(5) \quad \epsilon \to \psi \otimes \xi$$

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Method Part 2

- Construct all spinor bilinears using the Spin(5) spinor, e.g.

$$K_5 = \frac{1}{2}(\bar{\xi}_1 \gamma_{(1)} \xi_1 + \bar{\xi}_2 \gamma_{(1)} \xi_2)$$

- Fierz identities imply algebraic equations for the bilinears. Determines G in terms of the bilinears.
- SUSY equations give differential equations for the bilinears.
- One finds that K_5 is a Killing vector. Corresponds to the $U(1)$ R symmetry of the SCFT.

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Obtaining the metric

- Introducing a frame one can construct the metric in terms of the spinor bilinears.
- The metric $\mathrm{d}s^2_M$ after redefinitions is given by;

$$9m^2\,\mathrm{d}s^2_M = (\tau\mu)^2\,\mathrm{d}\psi^2 + \frac{1}{1-(\tau\mu)^2}(\mu^3\sigma\otimes\sigma^* + \mu^2\beta^2 + \mu^2\,\mathrm{d}\tau^2).$$

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Differential equations

- From the SUSY equations we are left with only 3 first order differenital equations for one forms to solve and two algebraic equations.
- Moreover one finds that the SUSY equations imply all the equations of motion and Bianchi identities.
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Does the classification work?

- We show a solution in [Macpherson, Núñez, Zayas, Rodgers, Whiting 2014] satisifes this classification.
- These solutions were obtained by performing a Non-Abelian T duality followed by a T duality to Sasaki Einstein solutions.
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Summary

- SUSY encodes a lot of information about the solution. We have shown that it implies the equations of motion and Bianchi identities and gives 'simpler' equations to solve.
- All SUSY type IIB AdS_5 solutions are now classified by this work or the 2005 work.
- Outlook
  - Can we find non-singular solutions?
  - What are the dual SCFT's to these geometries?
  - T-Dual to II A classification? [F. Apruzzi, M. Fazzi, A. Passias, A. Tomasiello]

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Any Questions?