1/4-BPS latitude Wilson loops in $AdS_5 \times S^5$ at strong coupling

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String sigma-model in AdS/CFT

4D $\mathcal{N}=4$ Yang-Mills theory	$\longleftrightarrow$	**Type IIB string theory in $\mathbf{AdS_5 \times S^5}$**
Wilson loop on path $\mathcal{C}$	$=$	**Partition function for string** on $\mathcal{C}$
$\langle \mathcal{W}[\mathcal{C}] \rangle$		$Z_{\rm string}[\mathcal{C}] \equiv \int [\mathcal{D}X][\mathcal{D}\Psi]\, e^{-S_{\rm string}}$

$$S_{\rm string} = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma\, G_{\mu\nu}(X)\, \partial_i X^\mu \partial^i X^\nu + \text{fermions}$$

$$\lambda \equiv g_{\rm YM}^2 N = \text{const.}$$

$$N \to \infty$$

Highly-interacting field theory, hard to quantize.

- **Non-perturbative, numerical** methods (see L. Bianchi's talk): lattice field theory to get **finite-coupling** AdS/CFT observables.

  Cusp anomaly $\quad \left\langle \mathcal{W}_{\rm null\ cusp} \right\rangle \sim e^{-\frac{1}{2} f(\lambda) V}$

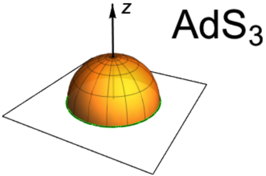
  [Bianchi, Bianchi, Forini, Leder, EV 16 + in preparation] + master student Töpfer

- **Perturbation theory** ($\lambda \gg 1$) for the string sigma-model
  $\lambda^{-1/2}$ = **small** loop-counting parameter

1/4-BPS latitude Wilson loops in $AdS_5 \times S^5$

Motivation: 1/2-BPS circular Wilson loop

$$\mathcal{W} = \frac{1}{N}\mathrm{Tr}\mathcal{P}\exp\left[\int_{\mathcal{C}}\left(iA_\mu \dot{x}^\mu + |\dot{x}|\,\Theta_I \Phi^I\right)d\tau\right]$$



Path in $\mathbb{R}^4$ $\quad x^\mu(\tau) = (\cos\tau, \sin\tau, 0, 0)$

Scalar couplings $\quad \Theta^I(\tau) = (0,0,1,0,0,0)$

[Erickson, Semenoff, Zarembo 00] [Drukker, Gross 00] [Pestun 07]

Localization $\quad \langle \mathcal{W}(\lambda, 0)\rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \overset{\lambda \gg 1}{\approx} \sqrt{\frac{2}{\pi}}\lambda^{-3/4} e^{\sqrt{\lambda}}$

σ-model pert. theory $\quad Z(\lambda, 0) \overset{\lambda\gg1}{\approx} \underbrace{\frac{1}{\sqrt{2\pi}}}_{\text{1-loop}} \times \underbrace{c\lambda^{-3/4}}_{\text{path-int. measure}} \times \underbrace{e^{\sqrt{\lambda}}}_{\text{classical area}}$

✓ Holographically ($\lambda \gg 1$) as a minimal-area surface [Drukker, Gross, Tseytlin 00].

✗ **No match** at **one-loop order** due to c (path-int. measure ambiguities). [Kruczenski, Tirziu 08] [Buchbinder, Tseytlin 14]

Goal: 1/4-BPS latitude Wilson loops

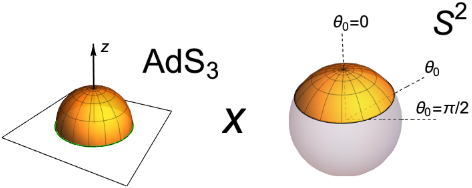
Perturbatively matching a **finite vev** is difficult (must include the constant c).

Generalize to more general WLs to circumvent this ambiguity (**parameter** $\rightarrow$ **ratio** of WLs).

Path in $\mathbb{R}^4$ $\quad x^\mu(\tau) = (\cos\tau, \sin\tau, 0, 0)$ $\quad$ ($\longrightarrow$ can be mapped to a "latitude")

Scalar couplings $\quad \Theta^I(\tau) = (\sin\theta_0\cos\tau, \sin\theta_0\sin\tau, \cos\theta_0, 0, 0, 0)$

[Drukker, Fiol 05] [Drukker 06] [Drukker, Giombi, Ricci, Trancanelli 07]



1/2-BPS circular WL recovered for $\theta_0 = 0$.

Localization generalizes to any θ_0: $\langle \mathcal{W}(\lambda, \theta_0)\rangle = \frac{2}{\sqrt{\lambda}\cos\theta_0} I_1(\sqrt{\lambda}\cos\theta_0)$. [Pestun 09]

Reconcile strings with localization

Goals

Compute $(\lambda \gg 1)$ and **normalize** $Z(\lambda, \theta_0)$ to the circular case $Z(\lambda, \theta_0 = 0)$ to wash out the constant c

$$Z(\lambda, \theta_0) / Z(\lambda, 0) \qquad \text{in } \sigma\text{-model pert. theory at one loop}$$

and **snatch** the expected **one-loop θ_0-dependence**

$$\langle \mathcal{W}(\lambda, \theta_0) \rangle / \langle \mathcal{W}(\lambda, 0) \rangle \qquad \text{from localization.}$$

$$\log \frac{Z(\lambda, \theta_0)}{Z(\lambda, 0)} = \underbrace{\sqrt{\lambda}(\cos\theta_0 - 1) - \frac{3}{2}\log\cos\theta_0}_{\text{matches localization}} + \underbrace{\log\cos\frac{\theta_0}{2}}_{\text{discrepancy}} + O\left(\lambda^{-1/2}\right)$$

- Further improvements confirm this unexpected gauge/string mismatch.
[Faraggi, Pando Zayas, Silva, Trancanelli 16]

- String σ-model computations in curved backgrounds, typically **plagued by divergencies**: **symmetry-preserving** regularization scheme, observables lacking predictions.

Perturbation theory

- Type IIB **Green-Schwarz string action** in $AdS_5 \times S^5$ [Metsaev, Tseytlin 98]

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \left[\sqrt{h} + \left(\sqrt{h} h^{ij} \delta^{IJ} - i\epsilon^{ij} s^{IJ} \right) \bar{\Psi}^I \rho_i \left(D_j \Psi \right)^J + o(\Psi^I)^2 \right]$$

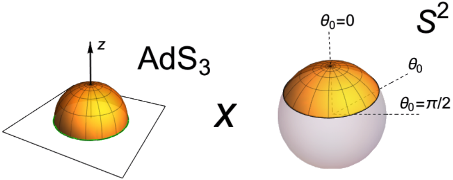
in static gauge ($\delta X \perp X_{\text{classical}}$) and standard κ-symmetry g.f. $\Psi^1 = \Psi^2$.

- **Expand** around minimal-area surface (see [Forini, Puletti, Griguolo, Seminara, EV 15])

$X = X_{\text{classical}}$

$\Psi = 0$

$S = S_{\text{classical}}$



- **Saddle-point expansion** for $\lambda \gg 1$

$$Z(\lambda, \theta_0) \equiv \int [\mathcal{D}X] [\mathcal{D}\Psi] \, e^{-S} \approx \underbrace{e^{\sqrt{\lambda}(\cos\theta_0 - 1/\epsilon)}}_{\text{exp of classical action}}$$

Perturbation theory

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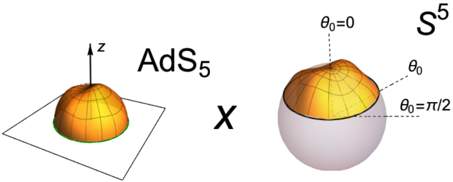
in static gauge ($\delta X \perp X_{\text{classical}}$) and standard κ-symmetry g.f. $\Psi^1 = \Psi^2$.

- **Expand** around minimal-area surface (see [Forini, Puletti, Griguolo, Seminara, EV 15])

$$X = X_{\text{classical}} + \frac{\delta X}{\lambda^{1/4}}$$

$$\Psi = 0 + \frac{\delta \Psi}{\lambda^{1/4}}$$

$$S = S_{\text{classical}} + S_{\text{quadratic}} + \dots$$



- **Saddle-point expansion** for $\lambda \gg 1$

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One-loop partition function

We **cannot go beyond one-loop** order with a τ, σ-**dependent** Lagrangian, e.g. also the spinning strings in [Forini, Puletti, Pawellek, EV 14].

$$Z(\lambda, \theta_0) \approx e^{\sqrt{\lambda}\cos\theta_0} \frac{\prod_{s\in\mathbb{Z}+\frac{1}{2}} \prod_{p_1,p_2,p_3=\pm 1} \mathrm{Det}_s^{1/4} \left(\mathcal{O}_{p_1,p_2,p_3}\right)^2}{\prod_{\ell\in\mathbb{Z}} \mathrm{Det}_\ell^{3/2}\mathcal{O}_1 \mathrm{Det}_\ell^{3/2}\mathcal{O}_2 \mathrm{Det}_\ell^{1/2}\mathcal{O}_{3+} \mathrm{Det}_\ell^{1/2}\mathcal{O}_{3-}}$$

τ) Surface rotational symmetry **Fourier-transforms** $-i\partial_\tau \to \ell, s$.

Impose b.c. along $\tau \in [0, 2\pi)$ $\longrightarrow$ **infinite products** $\prod_{\ell,s}$

σ) **Cutoffs** $\epsilon \ll 1$ and $R \gg 1$ to make spectral problems in $\sigma \in [0, \infty)$ well-defined.

Dirichlet b.c. at $\sigma = \epsilon, R$ $\longrightarrow$ **Gel'fand-Yaglom method** and derived technology for $\mathrm{Det}\mathcal{O}$'s [Forman 87] [Lesch Tolksdorf 98]

"SUSY-preserving" regularization

$$\log Z_{\text{1-loop}}(\theta_0) \equiv \log \frac{\prod_{s\in\mathbb{Z}+\frac{1}{2}} \prod_{p_1,p_2,p_3=\pm 1} \mathrm{Det}_s^{1/4}\left(\mathcal{O}_{p_1,p_2,p_3}\right)^2}{\prod_{\ell\in\mathbb{Z}} \mathrm{Det}_\ell^{3/2}\mathcal{O}_1 \mathrm{Det}_\ell^{3/2}\mathcal{O}_2 \mathrm{Det}_\ell^{1/2}\mathcal{O}_{3+} \mathrm{Det}_\ell^{1/2}\mathcal{O}_{3-}}$$

$$\equiv \sum_{s\in\mathbb{Z}+\frac{1}{2}} \Omega_s^F - \sum_{\ell\in\mathbb{Z}} \Omega_\ell^B$$

$$= \sum_{\ell=-\Lambda}^{\Lambda} \underbrace{\left(\frac{\Omega^F_{\ell+\frac{1}{2}} - \Omega^F_{\ell-\frac{1}{2}}}{2} - \Omega^B_\ell\right)}_{\text{pairing 1 boson with 2 fermions}} \underbrace{-\frac{\mu}{2}\Omega^F_{\frac{1}{2}} - \frac{\mu}{2}\sum_{\ell\geq 1} e^{-\mu\ell}\left(\Omega^F_{\ell+\frac{1}{2}} - \Omega^F_{\ell-\frac{1}{2}}\right)}_{\text{regularization-induced sum}}$$

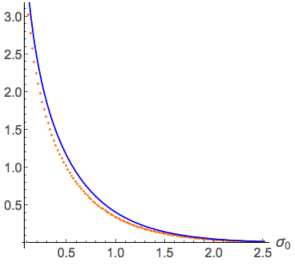
[Frolov, Parke, Tseytlin 05] [Dekel, Klose 13]

- Unphysical cutoff R drops out.
- Expected UV-divergence $\log Z_{\text{1-loop}}(\theta_0) \sim \left(\int d^2\sigma\, R\right) \log\Lambda$ from Seeley coeffs., then removed by reg. prescription.

[Förste, Ghoshal, Theisen 99] [Drukker, Gross, Tseytlin 00]

[Forini, Puletti, Griguolo, Seminara, EV 15]

Results: the latitude/circle ratio



Plot the **normalized latitude**

as a function of σ_0 ($\cos\theta_0 \equiv \tanh\sigma_0$),

inconsistent with localization.

Localization $\quad \log \frac{Z_{\text{1-loop}}(\theta_0)}{Z_{\text{1-loop}}(0)} = -\frac{3}{2}\log\cos\theta_0$

σ-model pert. theory $\quad \log \frac{Z_{\text{1-loop}}(\theta_0)}{Z_{\text{1-loop}}(0)} = \text{numerics for } \sum$

Quantify the gap with **4-decimal-digit** accuracy on **wide angular range** $0.8^\circ \leq \theta_0 \leq 89.4^\circ$.

Localization $\quad \log \frac{Z(\theta_0)}{Z(0)} \overset{\lambda\gg1}{\approx} \sqrt{\lambda}\,(\cos\theta_0 - 1) - \frac{3}{2}\log\cos\theta_0$

σ-model pert. theory $\quad \log \frac{Z(\theta_0)}{Z(0)} \overset{\lambda\gg1}{\approx} \sqrt{\lambda}\,(\cos\theta_0 - 1) - \frac{3}{2}\log\cos\theta_0 + \underbrace{\log\cos\frac{\theta_0}{2}}_{\text{remainder}}$

It defies geometric interpretation, may be θ_0-dep. path-int./regularization ambiguity?

Recent developments in [Faraggi, P. Zayas, Silva, Trancanelli 16]

- Fluctuation fields δX, $\delta\Psi$ neatly organised in multiplets of $SU(2|2) \subset SU(2,2|4)$.
- Role of symmetries in action and in summation over Fourier components.

Our paper	**Faraggi et al.**
2nd-order "Laplace" op. $\mathrm{Det}^{1/2}\,(\mathcal{O}_{p_1,p_2,p_3})^2$	1st-order "Dirac" op. $\mathrm{Det}\,(\mathcal{O}_{p_1,p_2,p_3})$
numeric sum + fitting	analytic sum
any latitude, latitude/circle	latitude/circle
✓ good control over finite+divergent part of any individual Z	✓ track modes argued to be responsible for disagreement

same "wrong" remnant for latitude/circle

Unresolved subtleties in BPS observables at strong coupling.

[Bassetto, Griguolo, Pucci, Seminara, Thambyahpillai, Young 09]

[Faraggi, Liu, Pando Zayas, Zhang 14] [Bergamin, Tseytlin 15]

Conclusions

We studied the **strong coupling behaviour** of **1/4-BPS Wilson loops** in $\mathcal{N} = 4$ SYM, perturbatively computing the **one-loop correction** to the $AdS_5 \times S^5$ classical solution.

✓ More general setup, good **pairing** of bosonic/fermionic Det's.

✓ **Delicate divergence** cancellations in the spectral problems, **vestige of SUSY**.

✗ GY method+cutoff reg. partially reproduce the **normalized latitude**, which shows traces of an unknown function of θ_0.

- 2D determinants in a **diffeo-preserving** reg. scheme treating τ, σ on equal footing.
- Desirable to develop a **higher-dimensional formalism** [Dunne, Kirsten 06] [Kirsten 10].
- Change computational setup, tailored to the (implicit) **localization regularization**.
- Quest for **ad hoc prescription** for BPS operators?
- Path-int. ambiguities depending on classical solution? $c = c\,(\theta_0)$?

Extra slides

The worldsheet geometry

Endowing the $AdS_5 \times S^5$ space with a **Lorentzian metric**

$$ds^2_{\rm 10D} \equiv G_{MN} dx^M dx^N = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho \, \frac{dy_m^2}{(1+\frac{y^2}{4})^2} + \frac{dz_n^2}{(1+\frac{z^2}{4})^2},$$

$$y^2 \equiv \sum_{m=1}^{3} y_m^2, \qquad z^2 \equiv \sum_{n=1}^{5} z_n^2$$

the classical configuration is the **spacelike surface**

$$t=0, \quad \rho=\rho(\sigma), \quad y_1 = 2\sin\tau, \quad y_2 = 2\cos\tau, \quad y_3 = 0\,,$$
$$z_1 = z_2 = 0, \quad z_3 = 2\cos\theta(\sigma), \quad z_4 = 2\sin\theta(\sigma)\sin\tau, \quad z_5 = 2\sin\theta(\sigma)\cos\tau\,.$$

The worldsheet geometry

It implements the correct boundary geometry and minimizes the area functional for

$$\sinh\rho(\sigma) = \frac{1}{\sinh\sigma}, \qquad \cosh\rho(\sigma) = \frac{1}{\tanh\sigma},$$
$$\sin\theta(\sigma) = \frac{1}{\cosh(\sigma+\sigma_0)}, \qquad \cos\theta(\sigma) = \tanh(\sigma+\sigma_0),$$
$$\cos\theta_0 \equiv \tanh\sigma_0, \qquad \tau\in[0,2\pi), \qquad \sigma\in[0,\infty).$$

The **induced worldsheet metric**

$$ds^2_{\mathrm{2D}} \equiv h_{\tau\tau}d\tau^2 + h_{\sigma\sigma}d\sigma^2 = \Omega^2(\sigma)\left(d\tau^2+d\sigma^2\right)$$

shows a conformal factor $\Omega^2(\sigma) \equiv \sinh^2\rho(\sigma) + \sin^2\theta(\sigma)$ depending on the latitude polar angle θ_0 .

One-loop partition function

Functional determinants of scalar-/matrix-valued operators.

$$\mathcal{O}_1 \equiv \frac{1}{\Omega^2}\left(-\partial_\tau^2 - \partial_\sigma^2 + \frac{2}{\sinh^2\sigma}\right)$$

$$\mathcal{O}_2 \equiv \frac{1}{\Omega^2}\left(-\partial_\tau^2 - \partial_\sigma^2 + \frac{2}{\cosh^2(\sigma+\sigma_0)}\right)$$

$$\mathcal{O}_{3\pm} \equiv \frac{1}{\Omega^2}\big[-\partial_\tau^2 \mp 2i\left[1-\tanh(2\sigma+\sigma_0)\right]\partial_\tau - \partial_\sigma^2 \\ -1 - 2\tanh(2\sigma+\sigma_0) + 3\tanh^2(\sigma+\sigma_0)\big]$$

$$\mathcal{O}_{p_1,p_2,p_3} \equiv \frac{i}{\Omega}\left(\Gamma_4\partial_\tau + \Gamma_3\partial_\sigma - a_{34}\Gamma_3 - ip_2 a_{56}\Gamma_4\right) \\ + \frac{1}{\Omega^2}\left(-\frac{p_1p_2}{\cosh^2(\sigma+\sigma_0)}\Gamma_{034} - \frac{ip_1}{\sinh^2\sigma}\Gamma_0\right) \qquad p_1,\ p_2,\ p_3 = \pm 1$$

$$\Omega^2 \equiv \frac{1}{\sinh^2\sigma} + \frac{1}{\cosh^2(\sigma+\sigma_0)}$$
$$\cos\theta_0 \equiv \tanh\sigma_0$$

The normal bundle gauge connection is not flat [Forini, Puletti, Griguolo, Seminara, EV 15].