One very special twist

Evgeny Sobko

S.Leurent & E.S. [1511.08491]

GATIS Workshop, Hamburg , 2016

First Part of Introduction
"Excursion into finite temperature 4D Y-M"

Based on
D.Diakonov [0906.2456]

- Y-M theory at finite T:

$$\mathcal{Z} = \int DA_\mu \exp\{-\frac{1}{4g^2}\int d^4x F^a_{\mu\nu} F^a_{\mu\nu}\},$$

$$A_\mu(t,x) = A_\mu(t+\beta,x), \quad \beta = \frac{1}{T}$$

- Holonomy:

$$L(\mathbf{x}) = \mathcal{P}\exp\left(i\int_0^{\frac{1}{T}} dt A_4(t,\mathbf{x})\right)$$

$$L(\infty) = \mathrm{diag}\{e^{2\pi i\mu_1}, e^{2\pi i\mu_2}, ..., e^{2\pi i\mu_N}\}$$

- Effective potential as a function of $\{\mu_i\}$

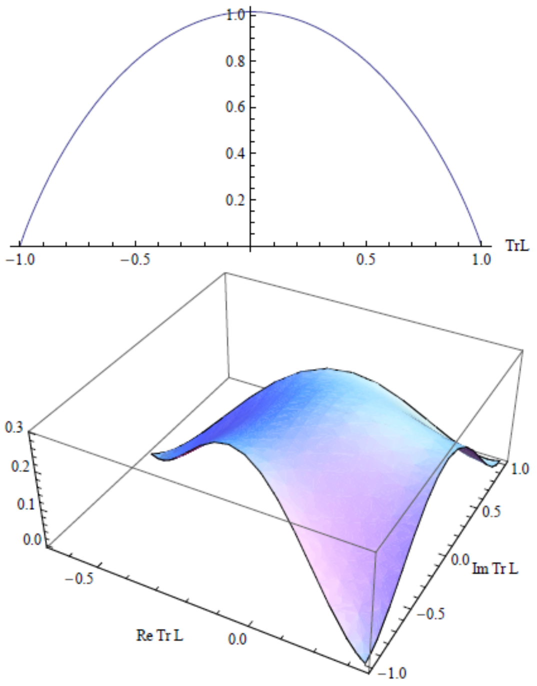


Figure 8. The perturbative potential energy as function of the Polyakov line for the $SU(2)$ (*top*) and $SU(3)$ (*bottom*) groups. It has minima where the Polyakov loop is one of the N elements of the center Z_N and is maximal at the "confining" holonomy.

[Gross, Pisarski, Yaffe], [Weiss]

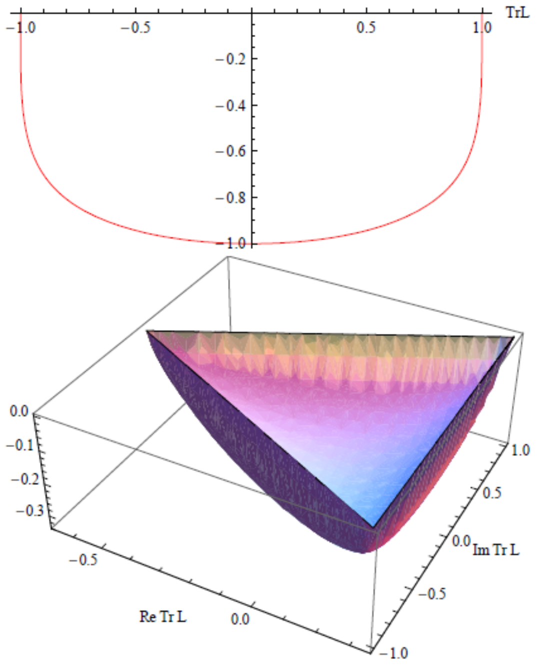


Figure 9. Dyon-induced nonperturbative potential energy as function of the Polyakov line for the $SU(2)$ (*top*) and $SU(3)$ (*bottom*) groups. Contrary to the perturbative potential energy, it has a single and non-degenerate minimum at the confining holonomy corresponding to $\mathrm{Tr}\, L = 0$.

[Diakonov, Gromov, Slizovskiy, Petrov]

• Effective potential as a function of $\{\mu_i\}$

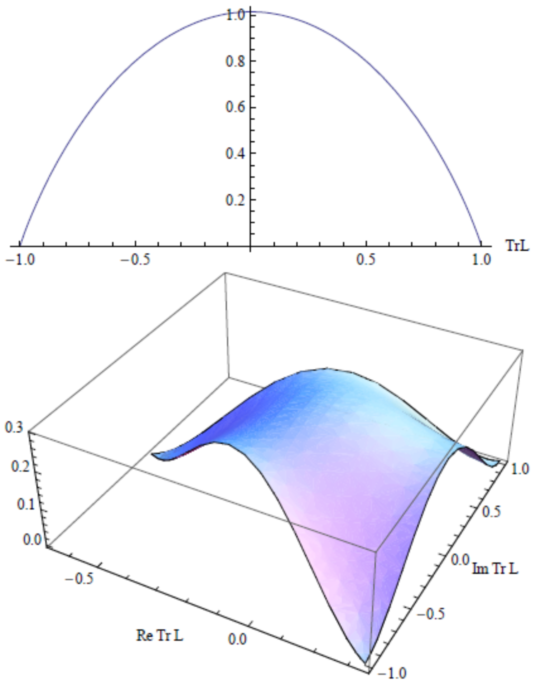


Figure 8. The perturbative potential energy as function of the Polyakov line for the $SU(2)$ (*top*) and $SU(3)$ (*bottom*) groups. It has minima where the Polyakov loop is one of the N elements of the center Z_N and is maximal at the "confining" holonomy.

[Gross, Pisarski, Yaffe], [Weiss]

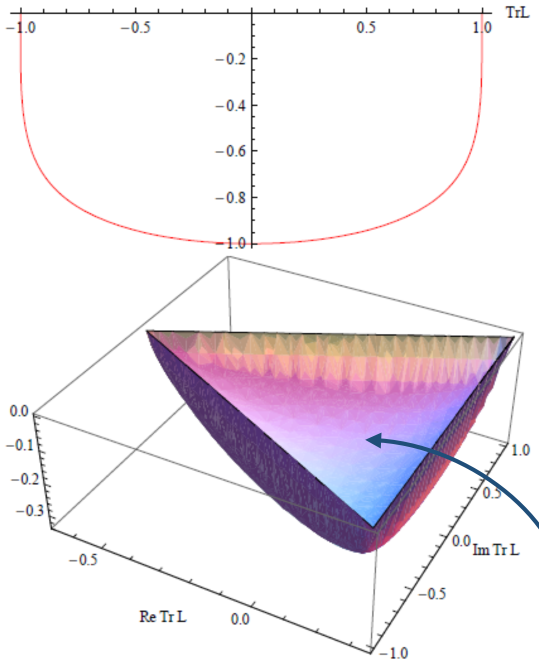


Figure 9. Dyon-induced nonperturbative potential energy as function of the Polyakov line for the $SU(2)$ (*top*) and $SU(3)$ (*bottom*) groups. Contrary to the perturbative potential energy, it has a single and non-degenerate minimum at the confining holonomy corresponding to $\mathrm{Tr}\, L = 0$.

[Diakonov, Gromov, Slizovskiy, Petrov]

$$\Omega = e^{i\frac{\pi}{N}\frac{1+(-1)^N}{2}}\mathrm{diag}\{1, e^{\frac{2\pi i}{N}}, e^{\frac{4\pi i}{N}}, \ldots, e^{\frac{2\pi(N-1)i}{N}}\}$$

$$\mathrm{Tr}\ \Omega = 0$$

- Effective potential as a function of $\{\mu_i\}$

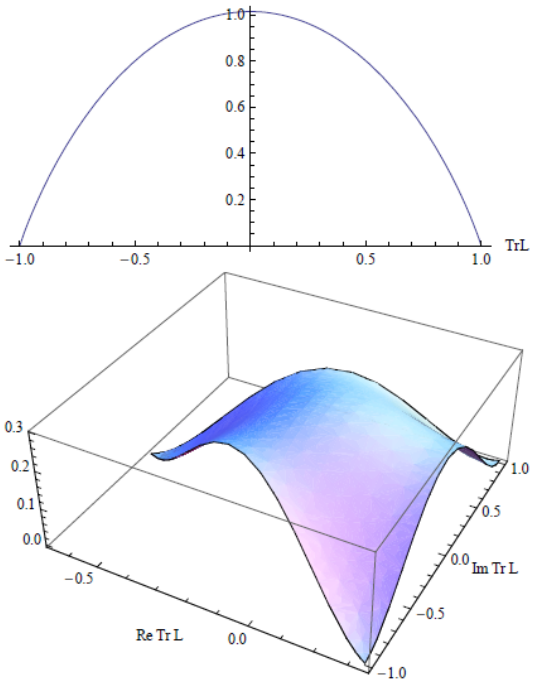


Figure 8. The perturbative potential energy as function of the Polyakov line for the $SU(2)$ (*top*) and $SU(3)$ (*bottom*) groups. It has minima where the Polyakov loop is one of the N elements of the center Z_N and is maximal at the "confining" holonomy.

[Gross, Pisarski, Yaffe], [Weiss]

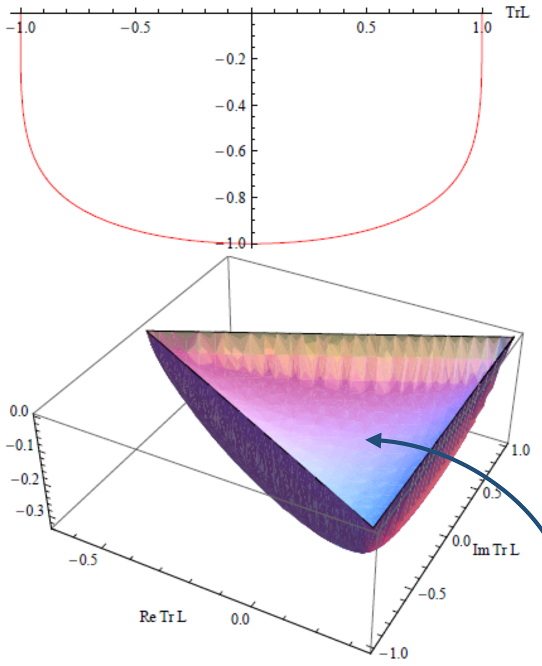


Figure 9. Dyon-induced nonperturbative potential energy as function of the Polyakov line for the $SU(2)$ (*top*) and $SU(3)$ (*bottom*) groups. Contrary to the perturbative potential energy, it has a single and non-degenerate minimum at the confining holonomy corresponding to $\mathrm{Tr}\, L = 0$.

[Diakonov, Gromov, Slizovskiy, Petrov]

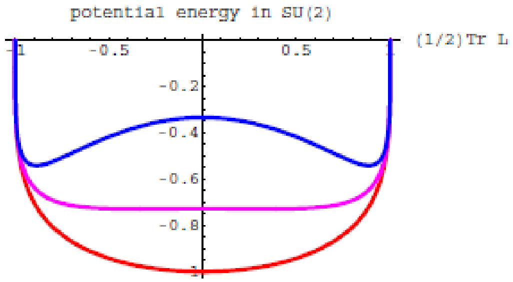


Figure 14. Potential energy of the pure $SU(2)$ YM theory as function of $\frac{1}{2}\mathrm{Tr}\, L$ at zero (*lower curve*), critical (*middle curve*) and high (*upper curve*) temperatures. At the critical temperature the curve is flat exhibiting the second order phase transition.

$$\Omega = e^{i\frac{\pi}{N}\frac{1+(-1)^N}{2}}\mathrm{diag}\{1, e^{\frac{2\pi i}{N}}, e^{\frac{4\pi i}{N}}, \ldots, e^{\frac{2\pi(N-1)i}{N}}\}$$

$$\mathrm{Tr}\ \Omega = 0$$

- Main contribution comes from the dilute gas of KvBLL calorons – neutral clusters of N dyons.

[D.Diakonov, N.Gromov, S.Slizovskiy, V.Petrov]

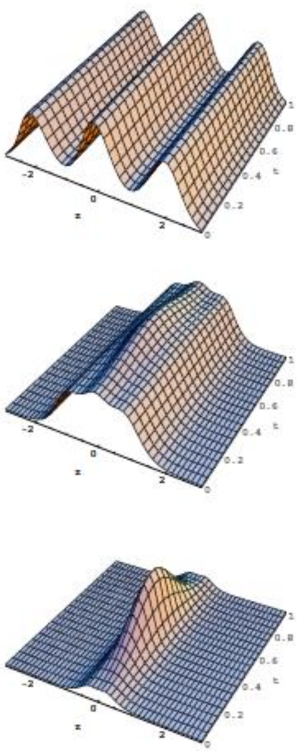


Figure 12. Action density inside the $SU(3)$ KvBLL instanton as function of time and one space coordinate, for large (*top*), intermediate (*middle*) and small (*bottom*) separations between the three constituent dyons.

Dyon-induced potential: $F = -T \log Z$

where:

$$Z = \exp(4\pi f V N (\nu_1 \nu_2 ... \nu_N)^{\frac{1}{N}})$$

$$\nu_m \equiv \mu_{m+1} - \mu_m, \quad \nu_N = 1 + \mu_1 - \mu_N$$

Minimum corresponds to: $\nu_1 = \nu_2 = ... = \nu_N = \dfrac{1}{N}$

$$\Omega = e^{i\frac{\pi}{N}\frac{1+(-1)^N}{2}} \mathrm{diag}\{1, e^{\frac{2\pi i}{N}}, e^{\frac{4\pi i}{N}}, ..., e^{\frac{2\pi(N-1)i}{N}}\}$$

<u>All criteria of confinement in this limit are satisfied</u>!

- Zero trace of holonomy
- Linear potential
- No massless modes
- etc

Second Part of Introduction
"Resurgence in 2D PCF sigma-model"

- Classical action of *SU(N)* Principle Chiral Field model:

$$S = \frac{1}{2g^2} \int_{-\infty}^{\infty} dt \int_0^L dx \; \mathrm{tr}\; \partial_\mu U \partial^\mu U^\dagger$$

- Twisted boundary conditions:

$$U(t, x+L) = e^{i\Phi} U(t,x) e^{-i\Phi}$$

- At $L \to \infty$ and periodic b.c. the quantum theory has

  - *N*-1 particles $m_k = m \frac{\sin(\frac{\pi k}{N})}{\sin(\frac{\pi}{N})}$ labeled by fundamental reps of *SU(N)*
  - $m = m_1 = \frac{\Lambda}{g} e^{-\frac{4\pi}{Ng^2}}$

- Asymptotic freedom: $g^2(L) \xrightarrow[L \to 0]{} 0$

Resurgence at the twist Ω

[Cherman, Dorigoni, Dunne, Unsal]

Reduction to QM:

$$\tilde{H} = -\frac{1}{2}\frac{d^2}{d\tilde{\theta}^2} + \frac{\xi}{2g^2}\cos\left(\sqrt{\frac{2g^2}{\xi}}\tilde{\theta}\right) . \qquad \xi \equiv \quad 2\pi/(NL)$$

Asymptotic series for energy:

$$\mathcal{E}_i(g) = \sum_{n=0}^{\infty} a_n^{[0]} g^{2n}, \qquad p_n \sim -\frac{2}{\pi}\left(\frac{1}{8\xi}\right)^n n!\left(1 - \frac{5}{2n} - \frac{13}{8n^2} + \mathcal{O}(n^{-3})\right) .$$

Not a Borel summable:

$$B\mathcal{E}_i(t) = -\frac{2}{\pi}\frac{1}{1 - \frac{t}{8\xi}} .$$

Ambiguity :

$$\mathcal{S}_{0\pm}\mathcal{E}_i \;= \mathrm{Re}\,\mathcal{S}_0\mathcal{E}_i \mp i\frac{32\pi}{g^2N}e^{-\frac{16\pi}{g^2N}}$$

Unitons - saddle points consisting of fractons:

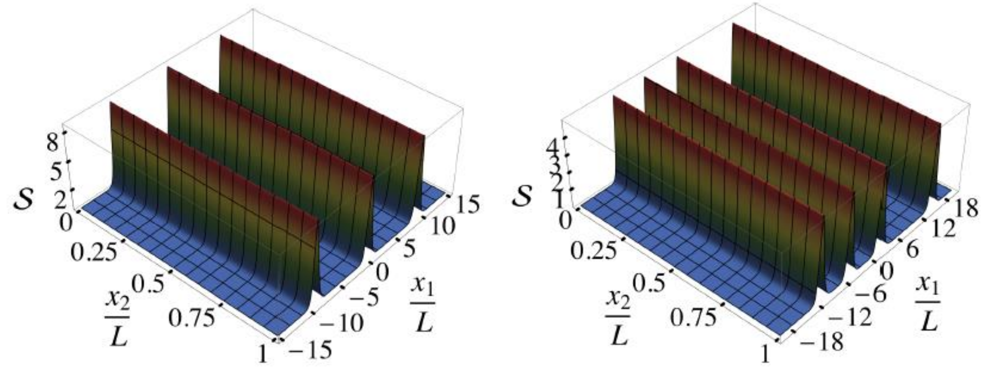


Figure 9. Action densities for the large $SU(3)$ and $SU(4)$ unitons. They split to three and four fractons, respectively.

Ambiguity from fraction-antifracton:

$$\begin{aligned}[\mathcal{F}_i\overline{\mathcal{F}}_i]_{\theta=0\pm} &= \mathrm{Re}\,[\mathcal{F}_i\overline{\mathcal{F}}_i] + i\,\mathrm{Im}\,[\mathcal{F}_i\overline{\mathcal{F}}_i]_{\theta=0\pm} \qquad (7)\\ &= \left[\log\left(\frac{\lambda}{16\pi}\right) - \gamma\right]\frac{16}{\lambda}e^{-\frac{16\pi}{\lambda}} \pm i\frac{32\pi}{\lambda}e^{-\frac{16\pi}{\lambda}}\end{aligned}$$

Cancelation of ambiguities and confined mass-gap:

$$\Delta = (E_- - E_+) \sim \frac{2\pi}{LN}\sqrt{\frac{16}{g^2N}}e^{-\frac{8\pi}{g^2N}} \sim \frac{1}{LN}e^{-S_{\mathrm{uniton}}/N} \sim \Lambda(\Lambda LN) ,$$

Last Section of Introduction
“What do I want?”

- I want to analyze this twisted PCF using Integrability and check is there something special about that particular twist at any coupling constant.

Twisted TBA

- Vacuum energy:

$$E_0^d = -\lim_{R\to\infty} \frac{\log \mathrm{tr}(e^{-H(L)R})}{R}$$

- Using usual logic of TBA we go to the mirror model:

$$\mathrm{Tr}(e^{-H(L)R}) = \mathrm{Tr}(e^{-\tilde{H}(R)L} \qquad)$$

Twisted TBA

- Vacuum energy:

$$E_0^d = -\lim_{R\to\infty} \frac{\log \mathrm{tr}(e^{-H(L)R})}{R}$$

- Using usual logic of TBA we go to the mirror model:

Twist as a defect-line operator

$$\mathrm{Tr}(e^{-H(L)R}) = \mathrm{Tr}(e^{-\tilde{H}(R)L} e^{i\Phi})$$

- In the mirror model the twist $e^{i\boldsymbol{\Phi}}$ acts on one –particle states as $e^{i\Phi\otimes 1 - 1\otimes i\Phi}$, $e^{i\Phi} = \mathrm{diag}[e^{i\phi_1}, e^{i\phi_2}, ..., e^{i\phi_N}]$

Twisted TBA

- Vacuum energy:

$$E_0^d = -\lim_{R\to\infty} \frac{\log \mathrm{tr}(e^{-H(L)R})}{R}$$

- Using usual logic of TBA we go to the mirror model:

Twist as a defect-line operator

$$\mathrm{Tr}(e^{-H(L)R}) = \mathrm{Tr}(e^{-\tilde{H}(R)L} e^{i\Phi})$$

- In the mirror model the twist $e^{i\Phi}$ acts on one –particle states as $e^{i\Phi\otimes 1 - 1\otimes i\Phi}$, $e^{i\Phi} = \mathrm{diag}[e^{i\phi_1}, e^{i\phi_2}, ..., e^{i\phi_N}]$
- TBA equations ($a \in [1, ..., N-1]$):

$$\mu^{a,0} = mL\frac{\sin\frac{\pi a}{N}}{\sin\frac{\pi}{N}}\cosh\frac{2\pi}{N}u - \log\frac{1}{Y^{a,0}} + \sum_{a'} \mathcal{K}^{(a',0),(a,0)} * \log(1+Y^{a',0}) + \sum_{s'\neq 0, a'} \mathcal{K}^{(a',s'),(a,0)} * \log(1+\frac{1}{Y^{a',s'}}),$$

$$\mu^{a,s\neq 0} = -\log Y^{a,s} + \sum_{a'} \mathcal{K}^{(a',0),(a,s)} * \log(1+Y^{a',0}) + \sum_{s'\neq 0, a'} \mathcal{K}^{(a',s'),(a,s)} * \log(1+\frac{1}{Y^{a',s'}})$$

where chemical potentials defined as:

$$\begin{cases} \mu_{a,s} = -is(\phi_{a+1}-\phi_a), & s>0 \\ \mu_{a,s} = i|s|(\phi_{a+1}-\phi_a), & s<0 \\ \mu_{a,s} = 0, & s=0 \end{cases}$$

As usual:

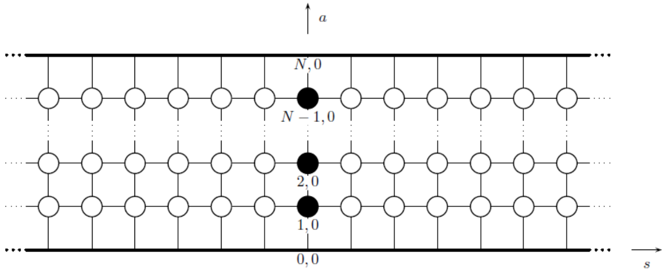
$$Y^{a,0} = \frac{\rho^{a,0}}{\bar{\rho}^{a,0}}, \quad Y^{a,s\neq 0} = \frac{\bar{\rho}^{a,s}}{\rho^{a,s}},$$

$$[f*g](u) = \int_{-\infty}^{\infty} dv\, f(u-v)g(v),$$

$$\mathcal{K} \sim \partial_u \log S(u-v).$$

- TBA can be rewritten as a Y-system:

$$\frac{Y_{a,s}^{+}Y_{a,s}^{-}}{Y_{a,s+1}Y_{a,s-1}} = \frac{1+1/Y_{a,s+1}}{1+1/Y_{a+1,s}}\frac{1+1/Y_{a,s-1}}{1+1/Y_{a-1,s}}$$



- The asymptotic solution at large *L* :

$$Y_{a,s} = \frac{\chi_{a,s+1}(e^{-i\Phi})\chi_{a,s-1}(e^{-i\Phi})}{\chi_{a+1,s}(e^{-i\Phi})\chi_{a-1,s}(e^{-i\Phi})}, \quad s>0$$

$$Y_{a,0} = \chi_{a,1}(e^{i\Phi})\chi_{a,1}(e^{-i\Phi})e^{-Lp_a(u)}, \quad s=0$$

$$Y_{a,s} = \frac{\chi_{a,|s|+1}(e^{i\Phi})\chi_{a,|s|-1}(e^{i\Phi})}{\chi_{a+1,|s|}(e^{i\Phi})\chi_{a-1,|s|}(e^{i\Phi})}, \quad s<0$$

$$p_a(\theta) = m\frac{\sin\frac{a\pi}{N}}{\sin\frac{\pi}{N}}\cosh\frac{2\pi}{N}\theta$$

- In order to reproduce the right chemical potentials in TBA we need a special epsilon prescription:

$$e^{\pm i\Phi} \to \mathrm{diag}\{e^{\pm i\phi_1}e^{\epsilon_1}, e^{\pm i\phi_2}e^{\epsilon_2}, ..., e^{\pm i\phi_N}e^{\epsilon_N}\}$$

where $0 > \epsilon_1 > \epsilon_2 > ... > \epsilon_N$, $\frac{\epsilon_i}{\epsilon_{i+1}}$ - fixed and $\epsilon_N \to 0$.

- Let's see how this prescription works at large L :

$$\mu_{a,s} = -\log(Y_{a,s}) - \sum_{s'=1}^{\infty} min(s,s')\log(1+\frac{1}{Y_{a-1,s'}})+$$
$$+\sum_{s'=1}^{\infty}(2min(s,s')-\delta_{s,s'})\log(1+\frac{1}{Y_{a,s'}}) - \sum_{s'=1}^{\infty} min(s,s')\log(1+\frac{1}{Y_{a+1,s'}}) + O(e^{-mL})$$

- Exponentiating r.h.s we get:

$$e^{\text{r.h.s.}} = \lim_{p\to\infty}\left(\frac{\chi_{a-1,p+1}(e^{i\Phi})}{\chi_{a-1,p}(e^{i\Phi})}\frac{\chi_{a,p}(e^{i\Phi})^2}{\chi_{a,p+1}(e^{i\Phi})^2}\frac{\chi_{a+1,p+1}(e^{i\Phi})}{\chi_{a+1,p}(e^{i\Phi})}\right)^s$$

- Epsilon prescription leads to

$$\lim_{p\to\infty}\frac{\chi_{a,p+1}}{\chi_{a,p}} = e^{-i\phi_1}e^{-i\phi_2}...e^{-i\phi_a}$$

- And finally:

$$e^{\text{r.h.s.}} = \left(\frac{e^{-i\phi_{a+1}}}{e^{-i\phi_a}}\right)^s = e^{\mu_{a,s}}$$

Exact solution for Vacuum at twist Ω

- The characters of Ω in all fundamental representations are zero: $\chi_{a,0}(\Omega) = 0$. What gives $Y_{a,0} = 0$.
- Zero-characters decouple Y-system in two independent wings, as e^{-mL} does in the large *L* limit. It means that the large *L* ansatz turns out to be exact solution at any *L* :

$$E_{\Omega}^{exact}(L) = \lim_{\epsilon_i \to 0} -\frac{1}{N} \sum_{a=1}^{N-1} \int_{-\infty}^{\infty} d\theta p_a(\theta) \log(1 + o(\epsilon_i)) = 0$$

Exact solution for Vacuum at twist Ω

- The characters of Ω in all fundamental representations are zero: $\chi_{a,0}(\Omega)=0$. What gives $Y_{a,0}=0.$
- Zero-characters decouple Y-system in two independent wings, as e^{-mL} does in the large *L* limit. It means that the large *L* ansatz turns out to be exact solution at any *L* :

$$E_{\Omega}^{exact}(L)=\lim_{\epsilon_i\to 0}-\frac{1}{N}\sum_{a=1}^{N-1}\int_{-\infty}^{\infty}d\theta p_a(\theta)\log(1+o(\epsilon_i))=0$$

Exact solution for Vacuum at twist Ω

- The characters of Ω in all fundamental representations are zero: $\chi_{a,0}(\Omega)=0$. What gives $Y_{a,0}=0$.
- Zero-characters decouple Y-system in two independent wings, as e^{-mL} does in the large *L* limit.
  It means that the large *L* ansatz turns out to be exact solution at any *L* :

$$E_{\Omega}^{exact}(L)=\lim_{\epsilon_i\to 0}-\frac{1}{N}\sum_{a=1}^{N-1}\int_{-\infty}^{\infty}d\theta p_a(\theta)\log(1+o(\epsilon_i))=0$$

<u>Comparing with week coupling (small L)</u>

- One loop Casimir energy at arbitrary twist: $E_{e^{i\Phi}}^{1-loop}=-\frac{1}{\pi L}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\left|tr(e^{in\Phi})\right|^2-1\right)$ [Cherman, Dorigoni, Unsal]
- In the periodic case it gives well known result: $E_{periodic}^{1-loop}=-\frac{1}{\pi L}\sum_{n=1}^{\infty}\frac{1}{n^2}(N^2-1)=-\frac{\pi(N^2-1)}{6L}$
- At twist Ω it gives zero: $E_{\Omega}^{1-loop}=-\frac{1}{\pi L}\left(\sum_{k=1}^{\infty}\frac{1}{(Nk)^2}(N)^2-\sum_{n=1}^{\infty}\frac{1}{n^2}\right)=0$

An interesting open question:
What is the symmetry behind this cancelation?

From the Integrability point of view this zero is very similar to the vanishing of anomalous dimension of BPS operators in the undeformed limit of N=4 SYM. In this case it has very simple physical explanation – undeformed case has unbroken supersymmetry. But in our case there is no SUSY which could make such miracle cancelation…

ABA. Any twist, any excited state.

- Y-system can be rewritten as Hirota ($Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}$):

$$T^+_{a,s}T^-_{a,s} = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}$$

where T-functions are defined up to gauge-transformation: $T_{a,s} \to \chi_1^{[a+s]}\chi_2^{[a-s]}\chi_3^{[-a+s]}\chi_4^{[-a-s]}T_{a,s}$, $\quad f^{[n]}(\theta) \equiv f(\theta + n\frac{i}{2})$.

- We can choose either $T_{a,1} \sim e^{-Lp_a(\theta)}$ or $T_{a,-1} \sim e^{-Lp_a(\theta)}$. What leads to two natural gauges - left(L) and right(R):

$$\underbrace{T^{(R)}_{a,-1} \ll 1 \qquad , T^{(L)}_{a,1} \ll 1}_{1 \le a \le N-1} \qquad , T^{(R)}_{a,s\ge 0} \sim 1 \qquad , T^{(L)}_{a,s\le 0} \sim 1$$

- The large L limit corresponds to the spin-chain limit and $T^R_{1,s}$ can be found from generating function:

$$\hat{W}^R = \frac{1}{(1 - e^{-i\phi_N}X^R_{(N)}(\theta)e^{i\partial_\theta})} \cdot \ldots \cdot \frac{1}{(1 - e^{-i\phi_1}X^R_{(1)}(\theta)e^{i\partial_\theta})} = \sum_{s=0}^{\infty} \frac{T^R_{1,s}(\theta + \frac{i}{2}(s-1))}{\varphi(\theta - i\frac{N}{4})} e^{is\partial_\theta}$$

and generating function for $T^L_{1,-s}$ differs by substitution $\phi_i \to -\phi_i$

$$X^{(W)}_{(k)} = \frac{Q^{(W)\,[N/2-k-1]}_{k-1}}{Q^{(W)\,[N/2-k+1]}_{k-1}} \frac{Q^{(W)\,[N/2-k+2]}_{k}}{Q^{(W)\,[N/2-k]}_{k}}, \quad k = 1, 2, \ldots, N$$

$$Q^{(R)}_k(\theta) = \prod_{j=1}^{J^{(R)}_k} \left(\theta - u^{(k)}_j\right), \qquad Q^{(L)}_k(\theta) = \prod_{j=1}^{J^{(L)}_k} \left(\theta - v^{(k)}_j\right), \qquad (k = 1, \cdots, N-1)$$

$$Q^{(R,L)}_N(\theta) \equiv \varphi(\theta) = \prod_{j=1}^{\mathcal{N}} (\theta - \theta_j), \qquad Q^{(R,L)}_0(\theta) \equiv 1\,.$$

- Cancelling poles $T^R_{1,1}$ at $w - \frac{i}{2}(\frac{N}{2} - k + 1)$ we get twisted auxiliary Bethe equations:

$$-e^{i(\phi_{k-1}-\phi_k)} = \frac{Q^R_{k-2}(w-\frac{i}{2})Q^R_{k-1}(w+i)Q^R_k(w-\frac{i}{2})}{Q^R_{k-2}(w+\frac{i}{2})Q^R_{k-1}(w-i)Q^R_k(w+\frac{i}{2})}$$

and similar for the left wing.

- Asymptotic form of the middle-node Y-functions reads as:

$$Y_{a,0}(\theta) \sim e^{-mLp_a} \frac{T_{a,1}T^L_{a,-1}}{T_{a+1,0}T_{a-1,0}} \frac{\varphi^{[-\frac{N}{2}-a+1]}\varphi^{[-\frac{N}{2}-a+1]}}{\varphi^{[-\frac{N}{2}+a-1]}\varphi^{[-\frac{N}{2}+a+1]}} \frac{1}{\Pi_a\left((S^{[-\frac{N}{2}]})^2\chi^{[-\frac{N}{2}]}_{CDD}\right)}$$

and it leads to the massive Bethe equation $Y_{1,0}(\theta_j + i\frac{N}{4}) = 0$:

$$-1 = \frac{e^{-imL\sinh\frac{2\pi}{N}\theta_j}}{\chi_{CDD}S^2(\theta_j)} \frac{Q^R_{N-1}(\theta_j-\frac{i}{2})Q^L_{N-1}(\theta_j-\frac{i}{2})}{Q^R_{N-1}(\theta_j+\frac{i}{2})Q^L_{N-1}(\theta_j+\frac{i}{2})}$$

Finite L

- General solution of Hirota system can be represented through the Wronskian determinants:

[Krichever, Lipan, Wiegmann, Zabrodin]
[Zabrodin]
[Zabrodin]

$$T^{(R)}_{a,s}(\theta) = i^{\frac{N(N-1)}{2}} \mathrm{Det}(c_{j,k})_{1\le j,k\le N}$$

$$\text{where } c_{j,k} = \begin{cases} e^{i\phi_j(s/2-k)}\overline{q_j}^{[s+a+1+\frac{N}{2}-2k]} & \text{if } k \le a \\ e^{i\phi_j(-s/2-k)} q_j^{[-s+a+1+\frac{N}{2}-2k]} & \text{if } k > a\,, \end{cases}$$

and similar for the left wing.

- q_j (resp $\bar{q}_j$) are analytic on the lower (resp upper) half plane. In addition $q_j - \bar{q}_j$ decreases at large θ as $e^{-L\cosh(2\pi/N\theta)}$. It allows us to introduce the following parametrization:

$$P_j + i\,\mathcal{C} * f_j = \begin{cases} \bar{q}_j & \text{if } \mathrm{Im}(\theta) > 0 \\ q_j & \text{if } \mathrm{Im}(\theta) < 0 \end{cases}$$

where $\mathcal{C} \equiv \frac{1}{2i\pi\theta}$ is Cauchy kernel, $f_j \equiv i(\bar{q}_j - q_j)$ is a real jump density and P_j's are polynomials.

SU(2) case. Vacuum and particle in the rest.

- Some states like vacuum or one particle in the rest Θ_0 have extra symmetry: $T^L_{a,-s}(\theta) = (-1)^{\mathcal{N}} T^R_{N-a,s}(-\theta)$

  where $\mathcal{N}$ denotes number of Bethe roots.
- General twist in SU(2) case:

$$e^{i\Phi} = \mathrm{diag}[e^{i\phi}, e^{-i\phi}]$$

- <u>Vacuum</u> :

$$P_1 = 1,\ P_2 = 1$$

- <u>One particle in the rest</u> :

$$P_1 = 1 \quad P_2(\theta) = \theta + c \quad c = -\frac{1}{2\tan\phi}$$

- Gauge freedom allows to set $q_1 = 1$ and then one can notice that $T_{1,-1} = -i(\bar{q}_2 - q_2) = -f_2$ and write:

$$T^{(R)}_{1,s} = ie^{-i(s+1)\phi}\left((\theta^{[s+1]} + c)^{\mathcal{N}} + i\mathcal{C}^{[s+1]} * T^{(R)}_{1,-1}\right) - ie^{i(s+1)\phi}\left((\theta^{[-s-1]} + c)^{\mathcal{N}} + i\mathcal{C}^{[-s-1]} * T^{(R)}_{1,-1}\right),$$

which holds for $|\mathrm{Im}(\theta)| < s + 1$

Final equation:

$$T_{1,-1}^{(R)} = e^{-mL\cosh(\pi\theta)} \frac{(-1)^{\mathcal{N}} |\hat{T}_0^{[+1+0]}|^2 \{T_{1,1}^{R}\}}{\left(|\hat{T}_0^{[+2]}\{\hat{T}_0^{[+2]}\}|^2\right)^{*s}} \qquad \bigstar$$

where $\{f\}(u) \equiv f(-u)$, $\quad f^{*s} = \exp(\log f * (1/(2\cosh(\pi\theta))))$ and

$$\hat{T}_0(\theta) = \begin{cases} T_{2,0}^{(R)}(\theta - \frac{i}{2}) & \text{if } \mathrm{Im}(\theta) > \frac{1}{2} \\ T_{1,0}^{(R)}(\theta) & \text{if } \mathrm{Im}(\theta) \in]-\frac{1}{2}, \frac{1}{2}[\\ T_{0,0}^{(R)}(\theta + \frac{i}{2}) & \text{if } \mathrm{Im}(\theta) < -\frac{1}{2}\,. \end{cases}$$

We can express all T's through $T_{1,-1}^{R}$ and equation ($\bigstar$) has a form of $T_{1,-1}^{R} = F(T_{1,-1}^{R})$

F is contraction mapping and the equation can be solved iteratively.

Numerics

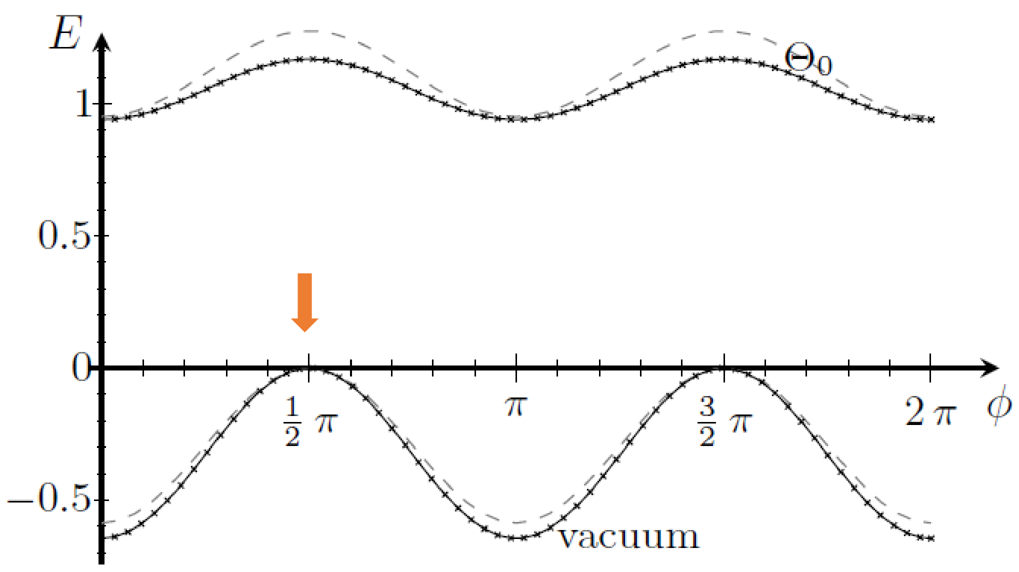


FIG. 1. Energies of vacuum and Θ_0 as functions of twist at $L = 1$. Dashed gray lines are the large L expression of energy as a function of twist. In all numerical calculations we put $m = 1$.

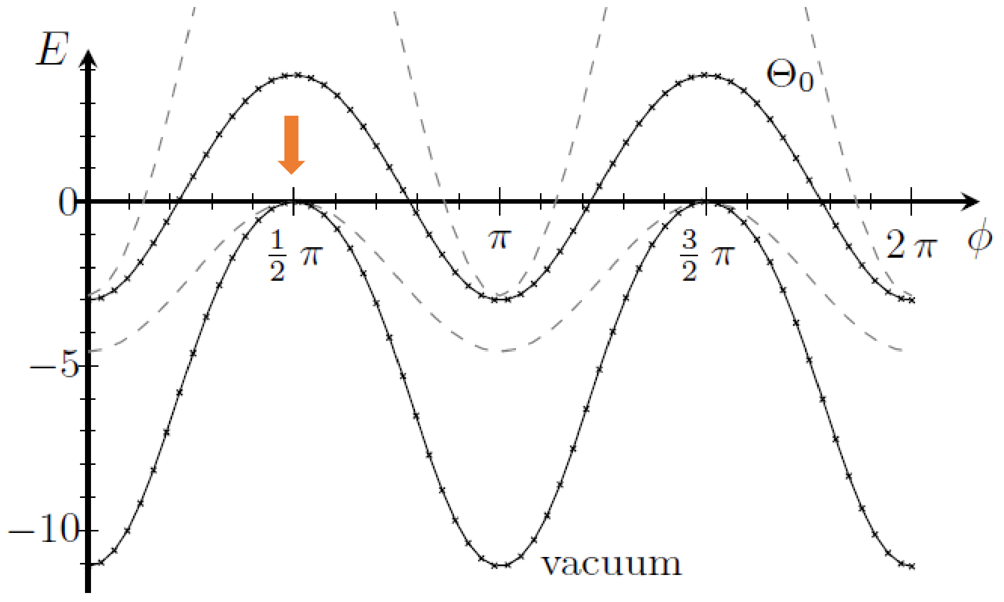


FIG. 2. Energies of vacuum and Θ_0 as functions of twist at $L = 1/10$. Deviation from the dashed gray lines shows the importance of finite-size effects.

Does one-particle in the rest give us a mass-gap?

- Making reduction to QM at small $g^2(L) \simeq \dfrac{2\pi}{|\log L|}$ and using WKB approach or resurgence one

would expect the confined form of the mass-gap:

$$\Delta \sim \frac{1}{L} e^{-\frac{4\pi}{g^2}} \sim L$$

- However for the one particle in the rest

we see ideal perturbative mass-gap : $\Delta \sim \dfrac{g^2}{L} \sim \dfrac{1}{L \log L}$

<u>Open question</u>: Is there contradiction and who is mass-gap?

<u>Potential answer</u>: Probably in the twisted case the mass gap realizes by another state.

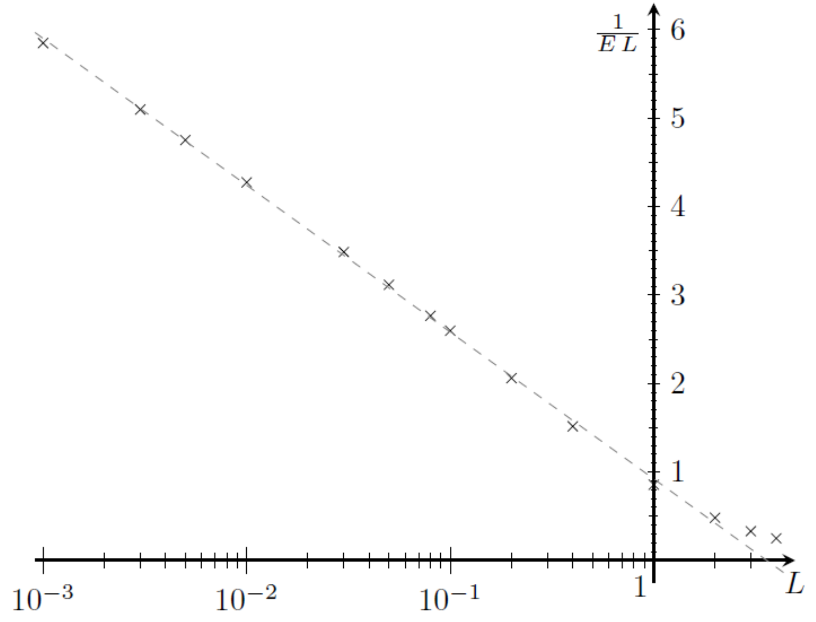


FIG. 3. Energy of the state Θ_0 at twist $\phi = \pi/2$. Crosses are numeric results and the dashed gray line is a linear fit. In spite of the limited numeric precision, it is manifest that at small L, $\frac{1}{EL}$ scales linearly with $\log L$.

Outlook

- We constructed finite system of equations for (generally)twisted PCF.
- At the special twist Ω large L solution turns out to be exact. It gives exactly zero vacuum energy.
- In case of SU(2) energies of vacuum and one particle in the rest was calculated numerically for arbitrary twist and different L.

Wish list

- Full analysis of all states in the twisted PCF. Clarifying the nature of mass-gap.
- What is the mechanism behind vanishing vacuum energy at the twist Ω ?
- Generalization to other sigma-models. What happens at twist Ω in supersymmetric theories?
- Build connection between integrability and resurgence.
- …

Thank you!