Quantum Black holes

From macroscopics to microscopics in string theory

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Based on collaborations with

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And on the previous work of:

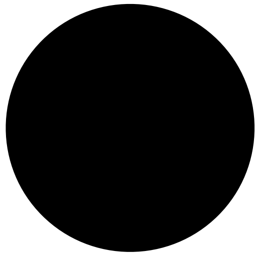
Cardoso, de Wit, Kaeppelli, Mohaupt (first computations of corrections to BH entropy formula in string theory),
Sen (Quantum entropy function program),
Ooguri, Strominger, Vafa (OSV formula)

Banerjee, Dabholkar, David, Denef, Gaiotto, Gomes, Gupta, Hama, Hosomichi, Jatkar, Lal, Mahapatra, Mandal, Moore, Nekrasov, Pestun, Pioline, Shih, Yin, …

Black holes have thermodynamic properties

Classical picture of a black hole:

Smooth, featureless object.



(M,Q,J)

[No hair theorems of GR]

(see, however, recent work [Hawking-Perry-Strominger])

Quantum picture of a black hole:

Complicated, like a piece of coal, with Temperature and Entropy.

[Benkenstein-Hawking ’70’s]

Black hole entropy is a precious clue to understand quantum gravity

$$k_B \log d_{\mathrm{micro}} = S_{\mathrm{BH}}^{\mathrm{class}} + \cdots$$

[Boltzmann]

Universal law in GR

$$S_{\mathrm{BH}}^{\mathrm{class}} = \frac{1}{4}\frac{A_{\mathrm{H}}}{\ell_{\mathrm{P}}^2} = \frac{A_{\mathrm{H}}\, c^3}{4\,\hbar\, G_N}$$

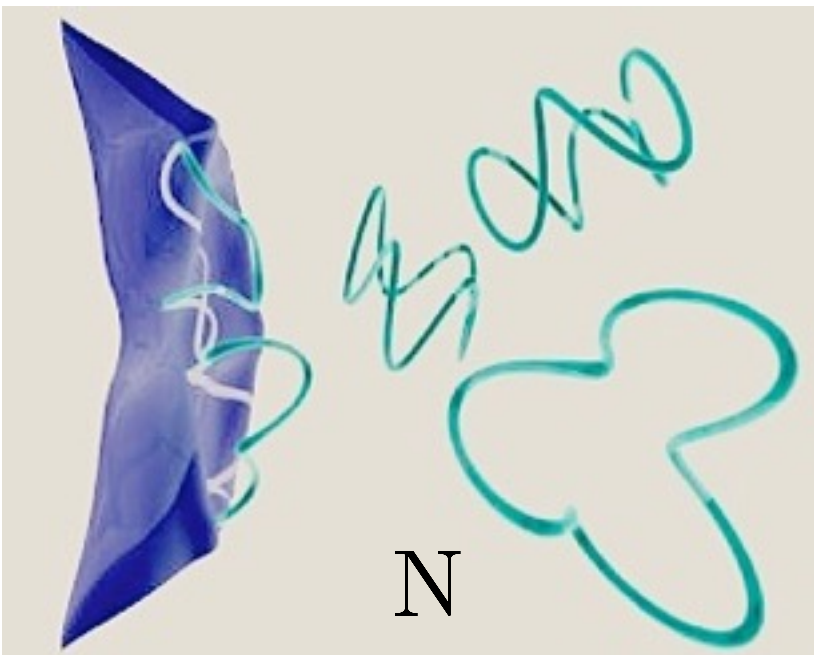
(Bekenstein-Hawking ’74)

Deviations from GR!

Recent progress on this front

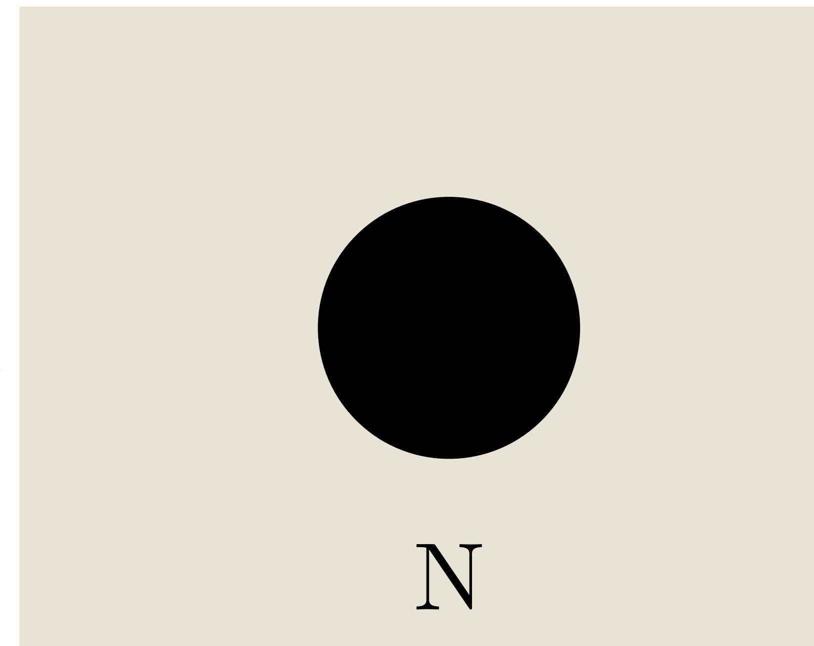
Black holes in string theory are ensembles of microscopic excitations

Microscopic



$g_s\mathrm{N} \ll 1$ $\longleftarrow$ g_s $\longrightarrow$ $g_s\mathrm{N} \gg 1$

Macroscopic



Strominger-Vafa '96

$$d_{\mathrm{micro}}(\mathrm{N}) = e^{\pi\sqrt{\mathrm{N}}} + \cdots \quad (\mathrm{N} \to \infty)$$

Bekenstein-Hawking '74

$$S_{\mathrm{BH}}^{\mathrm{class}} = \frac{A_H}{4\ell_{\mathrm{Pl}}^2} = \pi\sqrt{N}$$

$$\boxed{\log d_{\mathrm{micro}} = S_{\mathrm{BH}}^{\mathrm{class}} + \cdots} \longrightarrow S_{\mathrm{BH}}^{\mathrm{quant}}$$

(finite N)

What is new? Finite size quantum effects!

$$S_{\rm BH}^{\rm quant} = \frac{1}{4}A + a_0 \log(A) + a_1 \frac{1}{A} + a_2 \frac{1}{A^2} + \cdots$$
$$+ b_1(A) e^{-A} + \cdots$$

Can we even define the exact entropy of a black hole?

Questions

1. What is the physics of these corrections?
2. How to compute them in a concrete model?
3. Can we compare them to a similar expansion in the microscopic theory?

AdS path integrals

Supersymmetric Localization

Mock modular forms

Finite size corrections arise from quantum fluctuations in the black hole

Wald Entropy formalism

- Obeys the first law of thermodynamics
- Extends Bekenstein-Hawking area law in GR
- Applicable to any *local* effective action of gravity
- Successfully applied to BH models in supergravity

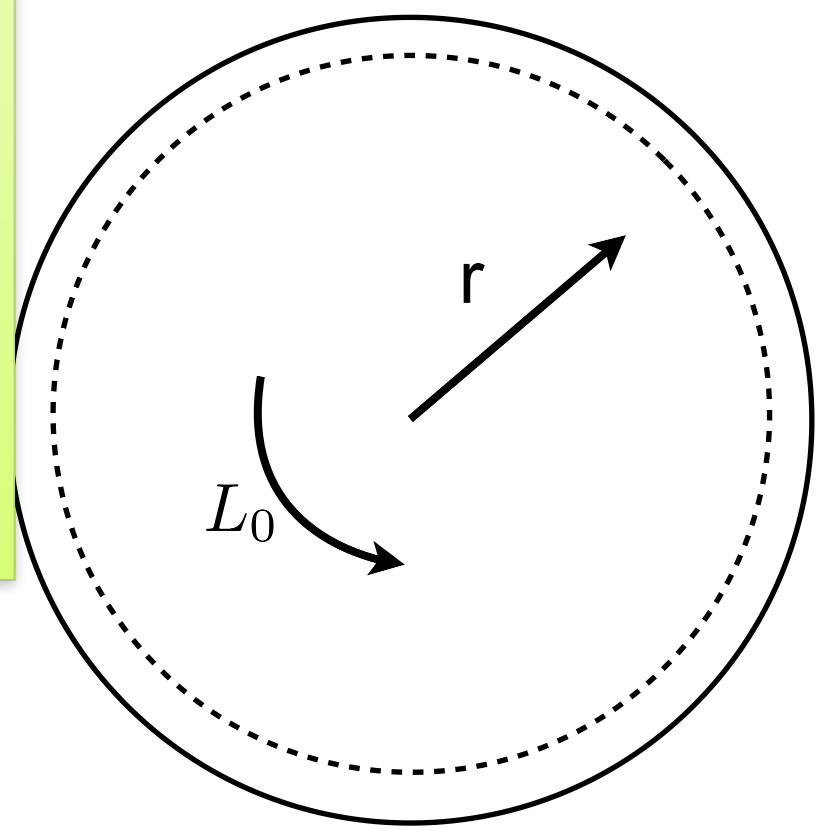
(Cardoso, de Wit, Mohaupt '99)

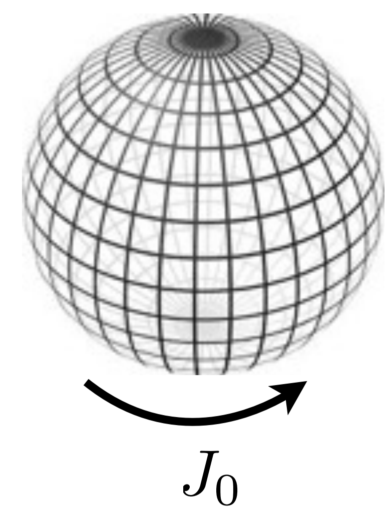
We still need a good formalism to study Quantum BH entropy including non-analytic and non-local terms.

Supersymmetric black holes and $\mathrm{AdS_2}$

4d charged extremal BH (Reissner-Nordstrom) $\Rightarrow$ near-horizon attractor $AdS_2 \times S^2$.

The near-horizon region of Susic BHs are a solution of the theory in its own right, and it can be decoupled from the rest. This means that we can actually think of assigning a certain degeneracy to it. (Unlike BHs at a non-zero temperature where we have Hawking radiation and it's not clear whether a particular particle belongs to the BH or outside. (Technically this is reflected in the fact that AdS2 is forced to have a micro canonical ensemble.)





$\mathbf{Euclidean\ AdS_2 \times S^2}$

Extremize the Lagrangian on the attractor $AdS_2 \times S^2$

$$S_{\mathrm{Wald}} = e_i q^i - \mathcal{L}(g_{\mu\nu}, A_i, \phi_\alpha)|_{\mathrm{extremum}}$$

(Sen '05)

Quantum BPS black hole entropy is an AdS_2 functional integral

(Sen '08)

$$\exp(S_{BH}^{\mathrm{qu}}(q_I)) \equiv Z_{AdS_2}(q_I) = \left\langle \exp\left[-i\, q_I \oint A^I \right] \right\rangle_{\mathrm{AdS}_2}^{\mathrm{reg}}$$

- Boundary conditions fixed by classical BH configuration
- Saddle point evaluation $\Longrightarrow$ $S_{\mathrm{BH}}^{\mathrm{qu}} = S_{\mathrm{BH}}^{\mathrm{class}} + \cdots$
- Leading logarithmic one-loop corrections systematized.

(Sen + Banerjee[2], Gupta, Mandal, '10-'14, Larsen, Keeler, Lisbão '14-'16).

$\mathrm{AdS/CFT}$ correspondence has been extremely successful...

$$\mathbf{CFT_{p+1}} \equiv \text{Quantum gravity on } \mathbf{AdS_{p+2}}$$

Fantastic progress in understanding the classical planar limit $(\mathrm{N} \to \infty)$. [cf. Work of all of you in the audience!]

...But *Quantum* gravity = 1/N effects

Dual theory for BPS BH is a collection of supersymmetric ground states

Dual $\mathbf{CFT_1}$ obtained as IR limit of brane configuration that makes up the black hole.

In d=0+1, no space for long-wavelength fluctuations.

$$Z_{\mathrm{CFT}_1}(N) = \mathrm{Tr}_{\mathcal{H}(N)}\, 1 = d_{\mathrm{micro}}(N)$$

AdS/CFT correspondence

$$Z_{\mathrm{AdS}_2}(N) = d_{\mathrm{micro}}(N)$$

Prototype: N=8 string theory in 4d (macro)

Macroscopic description: d=4 supergravity coupled to 28 U(1) gauge fields + superpartner scalars + fermions.

1/8 BPS dyonic BH solutions. (Cvetic, Youm ’96)

BH Charges $(q_I, p^I),\ I = 1, \ldots, 28\,,$

U-duality symmetry $E_{7,7}(\mathbb{Z}) \Longrightarrow \mathcal{Q}_a\,, a = 1, \cdots 56\,.$

Quartic invariant $N(\mathcal{Q}) \,=\, C^{abcd}\mathcal{Q}_a\mathcal{Q}_b\mathcal{Q}_c\mathcal{Q}_d$

Classical BH Entropy $S_{BH} = \pi\sqrt{N} + \cdots$

Prototype: N=8 string theory in 4d (micro)

Microscopic degeneracies $d_{\mathrm{micro}}(\mathrm{N})$ computed using representation as D1-D5-P-K system in Type II string theory. (Maldacena, Moore, Strominger '99)

They depend only on U-duality invariant N.

With $q = e^{2\pi i\tau}$,

Modular form!

$$\sum_{\mathrm{N}} d_{\mathrm{micro}}(\mathrm{N})\, e^{2\pi i \mathrm{N}\tau} = \theta(\tau)/\eta(\tau)^6$$

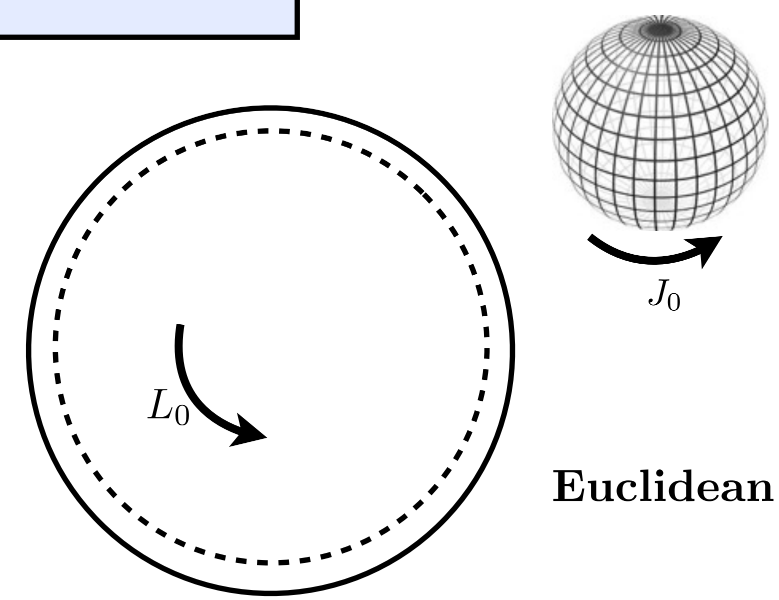
$$= q^{-1} + 2 + 8q^3 + 12q^4 + 39q^7 + 56q^8 + \cdots$$

$$d_{\mathrm{micro}}(N) \approx e^{\pi\sqrt{N}} \quad \text{as } N \to \infty$$

Beyond large N: exact functional integral

$$\exp(S^{\text{qu}}_{BH}(q_I)) \equiv Z_{AdS_2}(q_I) = \Big\langle \exp\big[-i\, q_I \oint A^I\big] \Big\rangle^{\text{reg}}_{\text{AdS}_2}$$

$$\Rightarrow \quad Z_{\text{AdS}_2}(q_I) \equiv \int_{\mathcal{M}} d\mu\, \mathcal{O}\, e^{-\mathcal{S}}$$



$\mathbf{Euclidean\ AdS_2 \times S^2}$

$\mathcal{M}$: Field space of supergravity.

$d\mu$: Measure on this field space.

$\mathcal{O}$: Wilson line.

$\mathcal{S}$: Action of graviton and other massless fields.

Supercharge Q with

$$Q^2 = L_0 - J_0\,.$$

The functional integral localizes onto the solutions of the off-shell BPS equations

Duistermaat-Heckmann, Atiyah-Singer-Bott, Berligne-Vergne, Witten,…Nekrasov, Pestun

Off-shell symmetry algebra $Q^2 = H \Longleftarrow$ Compact $U(1)$

$$I := \int_{\mathcal{M}} d\mu \, \mathcal{O} \, e^{-\mathcal{S}}$$

The functional integral localizes onto the solutions of the off-shell BPS equations

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Off-shell symmetry algebra $Q^2 = H \longleftarrow$ Compact $U(1)$

$$I(\lambda) = \int_{\mathcal{M}} d\mu \, \mathcal{O} \, e^{-\mathcal{S}+\lambda Q\mathcal{V}} \,, \qquad \mathcal{V} = \sum_{\psi} \int d^4x \, \overline{\psi} \, Q \, \psi$$

$$\frac{d}{d\lambda} I(\lambda) = \int_{\mathcal{M}} d\mu \, \mathcal{O} \, Q\mathcal{V} \, e^{-\mathcal{S}+\lambda Q\mathcal{V}} = 0$$

$$\Longrightarrow \; I(0) = I(\infty) = \int_{\substack{\mathcal{M}_Q \\ = \{Q\Psi = 0\}}} d\mu_Q \, \mathcal{O} \, e^{-\mathcal{S}} \, Z_{\text{1-loop}}(Q\mathcal{V})$$

How to compute the BH functional integral (Short version)

(A.Dabholkar, J.Gomes, S.M. '10, '11, '14)

1. Describe the supergravity theory in a formalism which admits off-shell susy algebra $Q^2 = L_0 - J_0$.

2. Find all solutions of localization equations $Q\,\Psi\ =\ 0$, subject to $AdS_2 \times S^2$ boundary conditions.

3. Evaluate full supergravity action on these solutions (including all higher derivative terms).

4. Compute one-loop determinant and quantum measure on the solution space.

Off-shell conformal N=2 supergravity

(de Wit, van Holten, Van Proeyen '80)

The susy transformations are specified once and for all.
They do not need to be modified as one modifies the action
e.g. with higher derivative terms .

Gravity multiplet + $n_{\rm v}$ vector multiplets.

$$\boxed{\begin{aligned} \delta\psi_\mu^i &= 2\mathcal{D}_\mu\epsilon^i + \mathcal{V}_{\mu}^{i}{}_{j}\,\epsilon^j - \frac{1}{4}\sigma^{\rho\nu}\,T_{\rho\nu}^{ij}\,\gamma_\mu\epsilon_j - \gamma_\mu\eta^i\,, \\ \delta\Omega_i^I &= 2\gamma^\mu D_\mu X^I\epsilon_i + Y_{ij}^I\,\epsilon^j + \sigma^{\mu\nu}\mathcal{F}_{\mu\nu}^{I-}\varepsilon_{ij}\,\epsilon^j + 2X^I\eta_i\,. \end{aligned}}$$

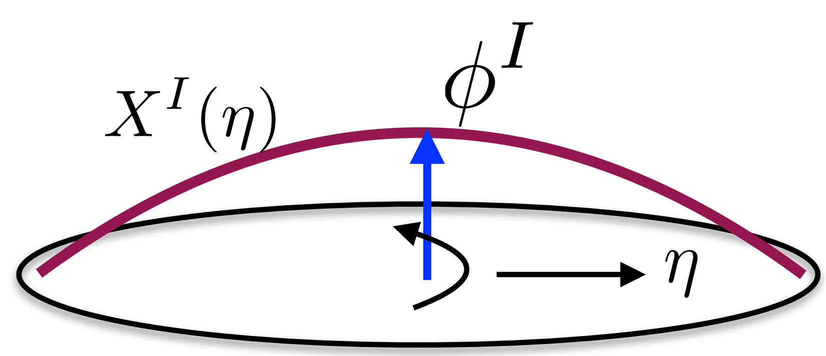
BPS equations + Euclidean AdS_2 boundary conditions
extremely constraining.

The space of off-shell BPS solutions

- In the gravity multiplet sector, only solution ★ is the (off-shell) fluctuation of the conformal factor of $AdS_2 \times S^2$.

(R.Gupta, S.M. '12; C. Klare, A. Zaffaroni '13)

- In vector multiplet sector, scalar field fluctuates (off-shell) around attractor solution. (A.Dabholkar, J.Gomes, S.M. '10)



Localization manifold: $\{\phi^I \equiv X^I(0)\,, I = 0, 1, \cdots, n_{\rm v}\}$

Evaluation of Wilson line

The Wilson line expectation value in supergravity localizes to the finite integral:

$$\boxed{Z_{AdS_2}(q,p) = \int \prod_{I=0}^{n_{\rm v}} [d\phi^I] \exp(\mathcal{S}_{\rm ren}(\phi, p, q))}$$

- $\mathcal{S}_{\rm ren}$ is the renormalized action of N=2 supergravity evaluated on the localizing manifold.
- The measure $[d\phi^I]$ is that of the supergravity scalar field space combined with the one-loop determinant.

N=2 supergravity action (effective theory)

Chiral superspace integrals

$$\int d^4x \int d^4\theta \, \mathbf{X}$$

Holomorphic prepotential function $F(\mathbf{X^I}, \mathbf{W^2})$

Computed by topo-logical string on CY

$$F(X^I, W^2) = \sum_{n=0}^{\infty} F_g(X^I)\,(W^2)^g$$

Full superspace integrals

$$\int d^4x \int d^4\theta \, d^4\bar{\theta} \, \mathbf{e}\,\mathbf{X}\,\bar{\mathbf{X}}$$

Action governed by F

(de Wit, van Holten, Van Proeyen '80)

$$
\begin{aligned}
S \;=\;& (-i(X^I\bar{F}_I - F_I\bar{X}^I))\cdot(-\frac{1}{2}R) + \Big[i\nabla_\mu F_I\nabla^\mu\bar{X}^I \\
+\;& \frac{1}{4}iF_{IJ}(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^{ij}_{ab}\,\varepsilon_{ij})(F^{-abJ} - \frac{1}{4}\bar{X}^J T^{ij}_{ab}\,\varepsilon_{ij}) \\
-\;& \frac{1}{8}iF_I(F^{+I}_{ab} - \frac{1}{4}X^I T_{abij}\,\varepsilon^{ij})T^{ij}_{ab}\,\varepsilon_{ij} - \frac{1}{8}iF_{IJ}Y^I_{ij}Y^{Jij} - \frac{i}{32}F\,(T_{abij}\,\varepsilon^{ij})^2 \\
+\;& \frac{1}{2}iF_{\widehat{A}}\widehat{C} - \frac{1}{8}iF_{\widehat{A}\widehat{A}}(\varepsilon^{ik}\varepsilon^{jl}\widehat{B}_{ij}\widehat{B}_{kl} - 2\widehat{F}^{-}_{ab}\widehat{F}^{-}_{ab}) \\
+\;& \frac{1}{2}i\widehat{F}^{-ab}F_{\widehat{A}I}(F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^{ij}_{ab}\,\varepsilon_{ij}) - \frac{1}{4}i\widehat{B}_{ij}F_{\widehat{A}I}Y^{Iij} + \text{h.c.}\Big] \\
-\;& i(X^I\bar{F}_I - F_I\bar{X}^I)\cdot(\nabla^a V_a - \frac{1}{2}V^a V_a - \frac{1}{4}|M_{ij}|^2 + D^a\Phi^i{}_\alpha D_a\Phi^\alpha{}_i)\,.
\end{aligned}
$$

(For the moment, go on with this action…)

Simple formula for exact entropy of $\frac{1}{2}$-BPS BH in theories with 8 supercharges

(A.Dabholkar, J.Gomes, S.M. '10, '11) (c.f. Ooguri-Strominger-Vafa '04)

4d N=2 supergravity coupled to $n_{\rm v}$ vector multiplets,

BH carrying charges (p^I, q_I) $I = 0, 1, \cdots, n_{\rm v}$

$$Z_{AdS_2}(q,p) = \int \prod_{I=0}^{n_{\rm v}} [d\phi^I] \exp(\mathcal{S}_{\rm ren}(\phi, p, q))$$

$$\mathcal{S}_{\rm ren}(\phi, p, q) = -\pi q_I \phi^I + {\rm Im} F(\phi^I + i p^I)$$

$F(X^I)$: holomorphic prepotential of d=4 N=2 supergravity.