The full spectrum of planar AdS_5/CFT_4

Christian Marboe

Based on [1608.06504] + [1612.XXXXX] with Dmytro Volin

Closing workshop, December 1 2016

The spectral problem in $\mathcal{N} = 4$ SYM

- The dilatation operator

$$\hat{\mathbb{D}}\,\mathcal{O} \quad = \quad \Delta\,\mathcal{O} \qquad\qquad \mathcal{O}(x) = \mathrm{Tr}[\mathcal{D}\mathcal{Z}\mathcal{X}\Psi \ldots] + \ldots$$

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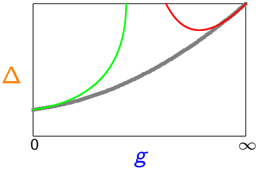
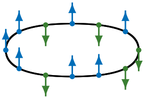
- Mathematical structure behind Δ?

$$f(\Delta, g) = 0$$



The spectral problem in $\mathcal{N} = 4$ SYM

- 1-loop: XXX spin chain [Minahan, Zarembo '02]



$$\Delta_1 \propto \sum \frac{1}{u_j^2 + \frac{1}{4}}$$

$$\left(\frac{u_j - \frac{i}{2}}{u_j + \frac{i}{2}} \right)^L = \prod \frac{u_k - u_j + i}{u_k - u_j - i}$$

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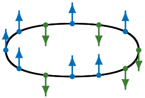

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- L-1 loops: Asymptotic Bethe Ansatz [Beisert, Staudacher '04]

$$\left(\frac{x_j^+}{x_j^-}\right)^L = \prod \frac{x_j^- - x_k^+}{x_j^+ - x_k^-} \frac{1 - \frac{g^2}{x_j^+ x_k^-}}{1 - \frac{g^2}{x_j^- x_k^+}} e^{2\, i\, \theta(u_j, u_k)} \qquad \frac{1}{x} + x = \frac{u}{g}$$

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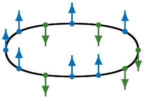

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- All loops: Thermodynamic Bethe Ansatz

[Ambjørn, Janik, Kristjansen '07]
[Gromov, Kazakov, Vieira '09]
[Arutyunov, Frolov '09]
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[Bombardelli, Fioravanti, Tateo '09]

$$\log \left(Y_{a,s}(u) \right) = \int \mathrm{d}v \; K_{a,s}^{a',s'}(u, v) \; \log(1 + Y_{a',s'}(v))$$

The spectral problem in $\mathcal{N} = 4$ SYM

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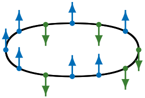

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$\downarrow$

Quantum Spectral Curve [Gromov, Kazakov, Leurent, Volin '13+'14]

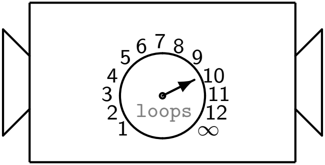
$$QQ = Q^- Q^+ - Q^+ Q^-, \qquad \tilde{Q} = \ldots$$

Our goal

My GATIS job description:

"Create a catalogue of the lowest spectrum of primary operators, their Bethe roots, and the first perturbative corrections to their anomalous dimensions..."

Our goal

$\mathrm{Tr}[\mathcal{D}^4 \mathcal{Z}^2 \mathcal{X} \Psi] \rightarrow$  $\rightarrow \Delta$

1. What to put in? Representation theory of $psu(2,2|4)$
2. Jump-starting the machine? 1-loop Q-system
3. How should the engine work?
   Quantum Spectral Curve $\rightarrow$ perturbative corrections

Our goal

1. What to put in? Representation theory of $psu(2,2|4)$
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3. How should the engine work?
   Quantum Spectral Curve $\rightarrow$ perturbative corrections

$psu(2,2|4)$

Operators come in multiplets

Multiplet $=$ irreducible representation $=$ Young diagram

Where is the grass?

$psu(2,2|4)$

$$a_1 \quad a_2 \qquad b_1 \quad b_2 \qquad f_1 \quad f_2 \quad f_3 \quad f_4$$

$$[a_i, a_j^\dagger] = [b_i, b_j^\dagger] = \{f_i, f_j^\dagger\} = \delta_{ij}$$

$$E_{ab} = \left(\begin{array}{c||c|c} -b_{\dot\alpha} b^\dagger_{\dot\beta} & -b_{\dot\alpha} a_\beta & -b_{\dot\alpha} f_j \\ \hline\hline a^\dagger_\alpha b^\dagger_{\dot\beta} & a^\dagger_\alpha a_\beta & a^\dagger_\alpha f_j \\ \hline f^\dagger_i b^\dagger_{\dot\beta} & f^\dagger_i a_\beta & f^\dagger_i f_j \end{array}\right) \qquad C = \sum_a E_{aa} = 0$$

$psu(2,2|4)$

$$a|0\rangle = b|0\rangle = f|0\rangle = 0 \qquad\qquad n_f + n_a - n_b = 2$$

$psu(2,2|4)$

$$a|0\rangle = b|0\rangle = f|0\rangle = 0 \qquad n_f + n_a - n_b = 2$$

$$f_i^\dagger f_j^\dagger |0\rangle \equiv \Phi_{ij}$$

$$a_\alpha^\dagger f_i^\dagger |0\rangle \equiv \Psi_{\alpha i} \qquad \epsilon_{ijkl} b_{\dot\alpha}^\dagger f_j^\dagger f_k^\dagger f_l^\dagger |0\rangle \equiv \bar\Psi_{\dot\alpha i}$$

$$a_\alpha^\dagger a_\beta^\dagger |0\rangle \equiv \mathcal{F}_{\alpha\beta} \qquad b_{\dot\alpha}^\dagger b_{\dot\beta}^\dagger f_1^\dagger f_2^\dagger f_3^\dagger f_4^\dagger |0\rangle \equiv \bar{\mathcal{F}}_{\dot\alpha\dot\beta}$$

$$a_\alpha^\dagger b_{\dot\alpha}^\dagger \equiv \mathcal{D}_{\alpha\dot\alpha}$$

$psu(2,2|4)$ multiplets

$\mathcal{Z}\mathcal{Z}\mathcal{X}\mathcal{X}$

$psu(2,2|4)$ multiplets

$$\mathcal{Z}\mathcal{Z}\mathcal{X}\mathcal{X} \quad \overset{R \sim f^{\dagger} f}{\leftrightarrow} \quad \mathcal{Z}\mathcal{X}\mathcal{X}\mathcal{X} \quad \leftrightarrow \quad \mathcal{X}\mathcal{X}\mathcal{X}\mathcal{X}$$

$psu(2,2|4)$ multiplets

$$\mathcal{Z}\mathcal{Z}\mathcal{X}\Psi$$

$$\updownarrow \; Q \sim a^\dagger f$$

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$$\Updownarrow\; P \sim a^\dagger b^\dagger$$

$$\mathcal{D}\mathcal{Z}\mathcal{Z}\mathcal{X}\mathcal{X}$$

$$\Updownarrow$$

$$\mathcal{D}^2\mathcal{Z}\mathcal{Z}\mathcal{X}\mathcal{X}$$

$$\Updownarrow$$

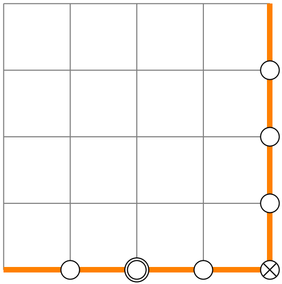
$$\vdots$$

Gradings and HWS

- Characterise multiplet by HWS:

$$E_{ab}|HWS\rangle = 0 \qquad b > a$$

$$E_{ab} = \left(\begin{array}{cc||cc|cccc}
-bb^\dagger & & & & & & & \\
 & -bb^\dagger & & & & & & \\
\hline\hline
 & & a^\dagger a & & & & & \\
 & & & a^\dagger a & & & & \\
\hline
 & & & & f^\dagger f & & & \\
 & & & & & f^\dagger f & & \\
 & & & & & & f^\dagger f & \\
 & & & & & & & f^\dagger f
\end{array}\right)$$



e.g. $\mathcal{F}_{11}^2 \mathcal{F}_{22}^2$ $\qquad \Delta_0 = 8$

Gradings and HWS

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-bb^\dagger & & & & & & & \\
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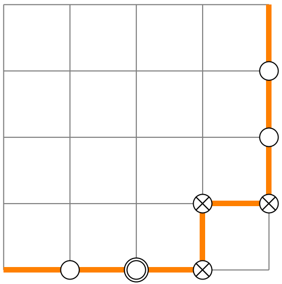
\hline\hline
 & & a^\dagger a & & & & & \\
\hline
 & & & f^\dagger f & \star & & & \\
\hline
 & & & \star & a^\dagger a & & & \\
\hline
 & & & & & f^\dagger f & & \\
 & & & & & & f^\dagger f & \\
 & & & & & & & f^\dagger f
\end{array}\right)$$



$$\Psi_{11}\mathcal{F}_{11}\mathcal{F}^2_{22} \qquad\qquad \Delta_0 = 7.5$$

Gradings and HWS

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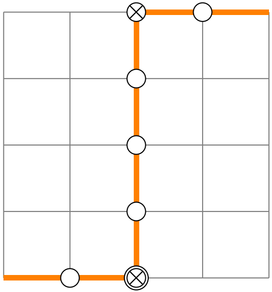
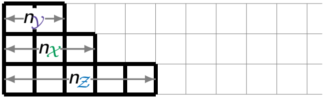


$\mathcal{Z}^2 \bar{\mathcal{Z}}^2$

$\Delta_0 = 4 \leftarrow$ minimal

Young diagrams

- Compact Young diagram, e.g. fundamental $su(3)$



$\mathcal{Z}^5 \mathcal{X}^3 \mathcal{Y}^2$

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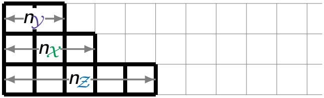

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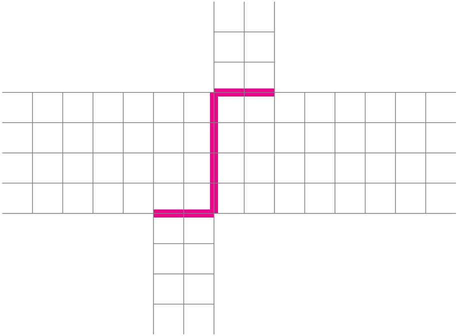

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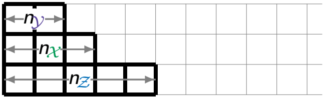

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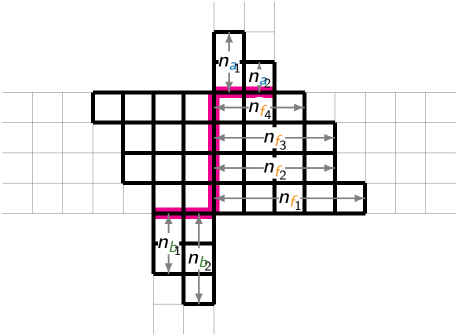

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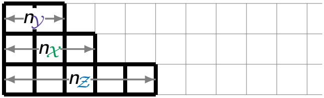

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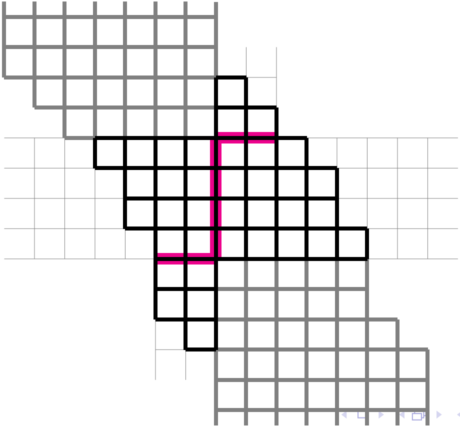

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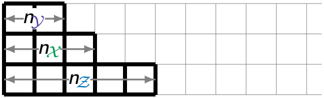

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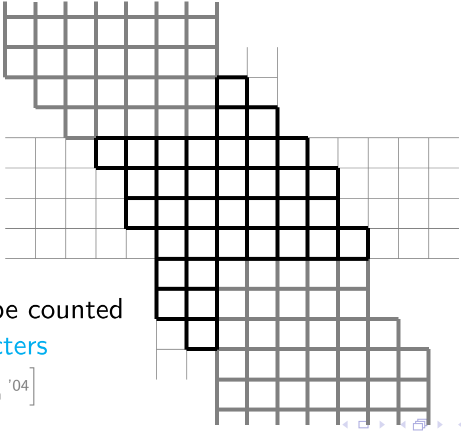

States can be counted using characters

[Beisert, Bianchi, Morales, Samtleben '04]

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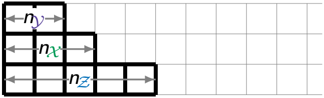

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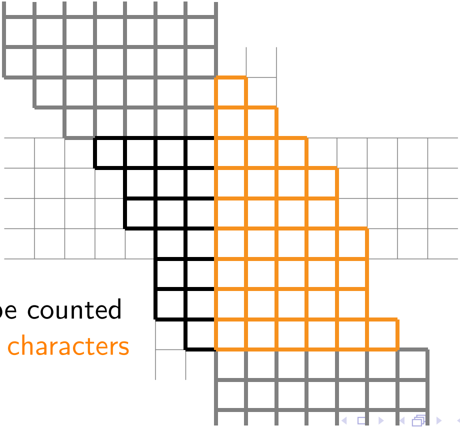
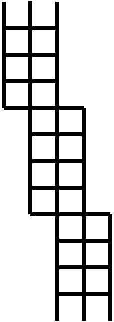


States can be counted using $su(N)$ characters

[CM, Volin '16]

The spectrum

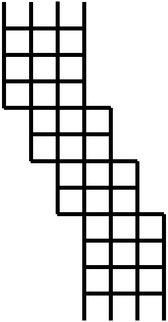
$\Delta_0 = 2$



$\mathcal{Z}\bar{\mathcal{Z}}$

1

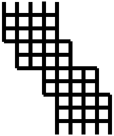
$\Delta_0 = 3$



$\mathcal{Z}^2\bar{\mathcal{Z}}$

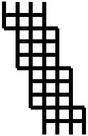
1

$\Delta_0 = 4$



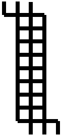
$\mathcal{Z}^3\bar{\mathcal{Z}}$

2



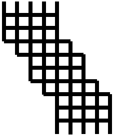
$\mathcal{D}_{12}\mathcal{Z}^2\bar{\mathcal{Z}}$

2



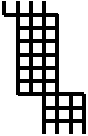
$\mathcal{D}_{12}^2\mathcal{Z}\bar{\mathcal{Z}}$

1



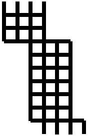
$\mathcal{Z}^2\mathcal{X}\bar{\mathcal{Z}}$

1



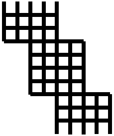
$\mathcal{Z}\bar{\mathcal{Z}}\mathcal{F}_{11}$

1



$\mathcal{Z}\bar{\mathcal{Z}}\bar{\mathcal{F}}_{22}$

1



$\mathcal{Z}^2\bar{\mathcal{Z}}^2$

2

Q-systems

Multiplet $\quad \leftrightarrow \quad$ solution to Q-system

Q-systems live on Young diagrams

Getting the machine started

Notation

$$Q^{\pm} \equiv Q(u \pm \tfrac{i}{2})$$

$$Q^{[n]} \equiv Q(u + \tfrac{i}{2} n)$$

$$Q_{abc...|ijk...} = -Q_{bac...|ijk...} = -Q_{abc...|jik...}$$

Notation

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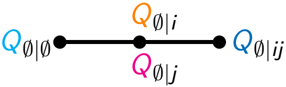
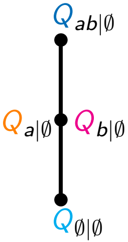
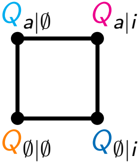
Spin chain / 1-loop SYM

$$Q(u) = \prod_{j=1}^{M} (u - u_j)$$

QQ-relations







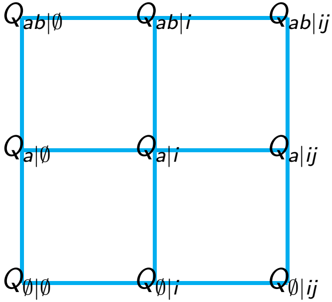
$$QQ = Q^-Q^+ - Q^+Q^-$$

Traditional way of thinking

Rank of algebra $\rightarrow$ Q-system

$su(3)$

$su(2|2)$



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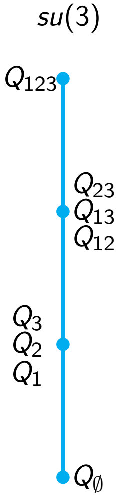
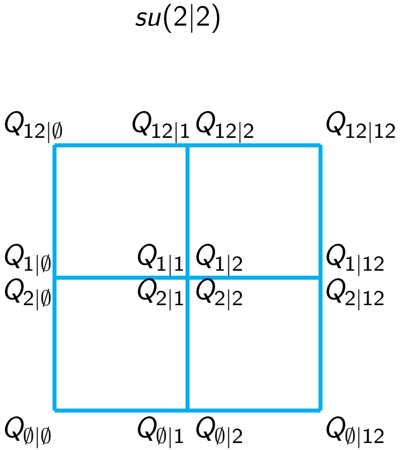




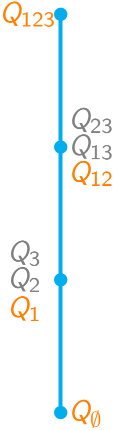
Traditional way of thinking

Rank of algebra $\to$ Q-system

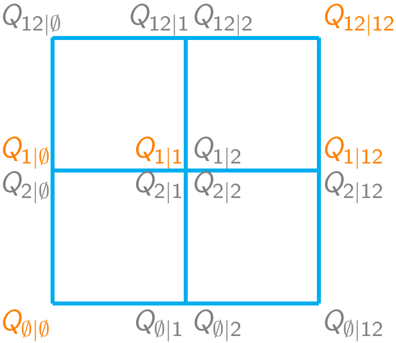
Nested Bethe equations

$$\frac{Q_\emptyset(u_{1,j}+\frac{i}{2})Q_{12}(u_{1,j}+\frac{i}{2})}{Q_\emptyset(u_{1,j}-\frac{i}{2})Q_{12}(u_{1,j}-\frac{i}{2})} = -\frac{Q_1(u_{1,j}-i)}{Q_1(u_{1,j}+i)} \quad \ldots$$

$su(3)$



$su(2|2)$



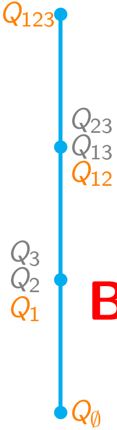
Traditional way of thinking

Rank of algebra $\to$ Q-system

Nested Bethe equations

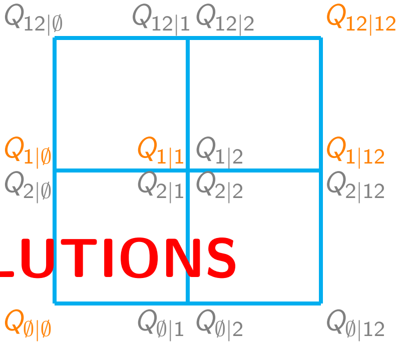
$$\frac{Q_{\emptyset}(u_{1,j}+\frac{i}{2})Q_{12}(u_{1,j}+\frac{i}{2})}{Q_{\emptyset}(u_{1,j}-\frac{i}{2})Q_{12}(u_{1,j}-\frac{i}{2})} = -\frac{Q_1(u_{1,j}-i)}{Q_1(u_{1,j}+i)} \quad \dots$$

$su(3)$



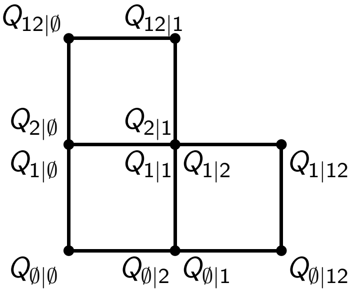
SLOW

$su(2|2)$

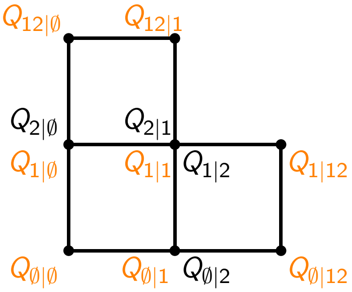


BAD SOLUTIONS

Our approach: Q-systems on Young diagrams



Our approach: Q-systems on Young diagrams



All "distinguished" Q's polynomial

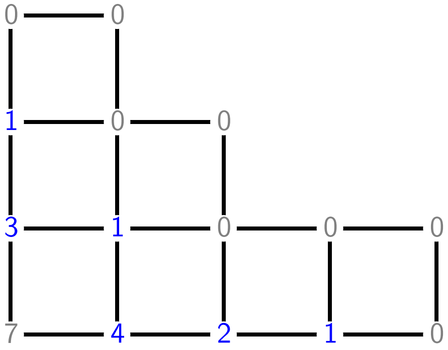
$\Downarrow$

All Q's polynomial

[CM, Volin '16]

Our approach: Q-systems on Young diagrams

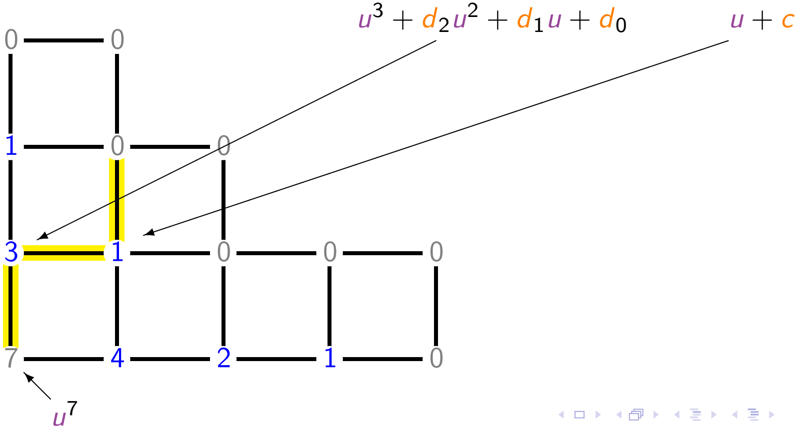
#roots = #boxes right/above



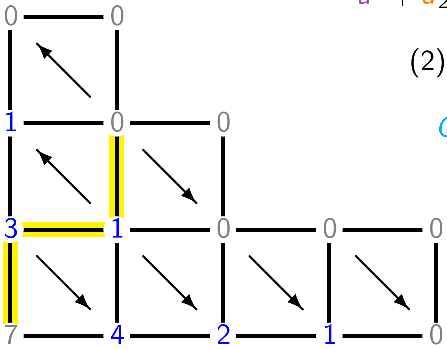
Our approach: Q-systems on Young diagrams

RECIPE

(1) Make poly. ansatz on path



Our approach: Q-systems on Young diagrams



RECIPE

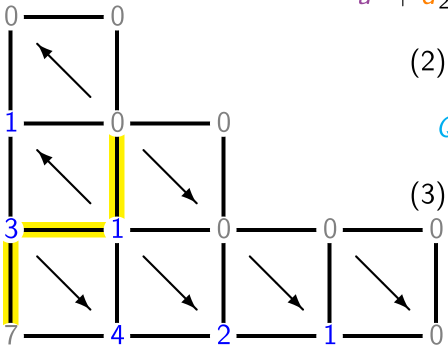
(1) Make poly. ansatz on path

$$u^3 + d_2 u^2 + d_1 u + d_0 \qquad\qquad u + c$$

(2) Generate rest by polynomial division

$$Q \propto \texttt{Quotient}\left[\frac{Q^+ Q^- - Q^- Q^+}{Q}\right]$$

Our approach: Q-systems on Young diagrams



RECIPE

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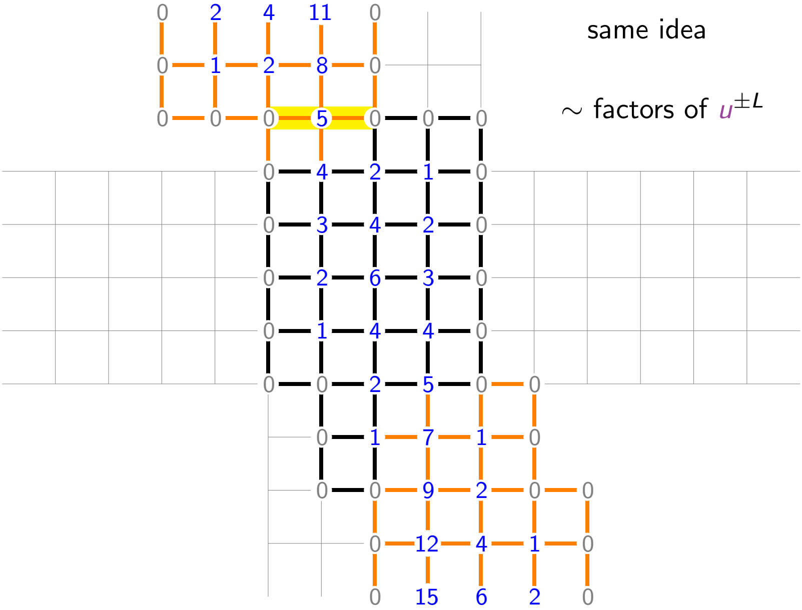
(2) Generate rest by polynomial division

$$Q \propto \texttt{Quotient}\left[\frac{Q^+Q^- - Q^-Q^+}{Q}\right]$$

(3) Impose vanishing remainders

$$\texttt{Remainder}\left[\frac{Q^+Q^- - Q^-Q^+}{Q}\right] = 0$$

Non-compact case



same idea

$\sim$ factors of $u^{\pm L}$

Quantum Spectral Curve

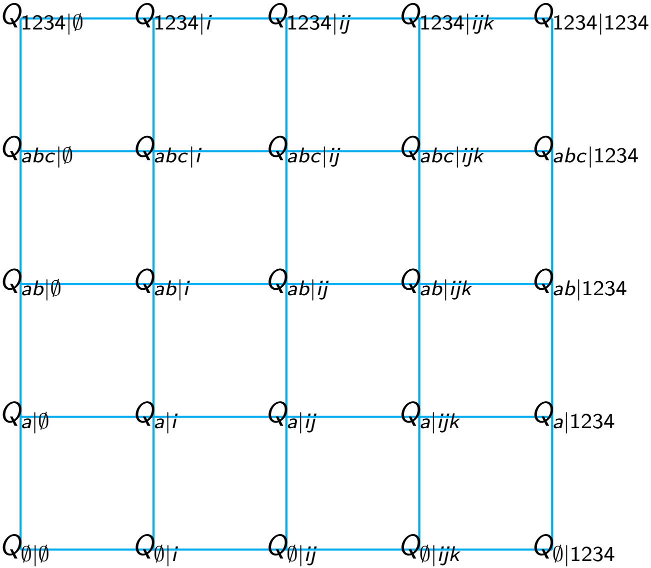
$=$

Q-system $+$ analytic structure

The engine

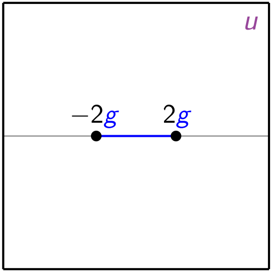
Quantum Spectral Curve

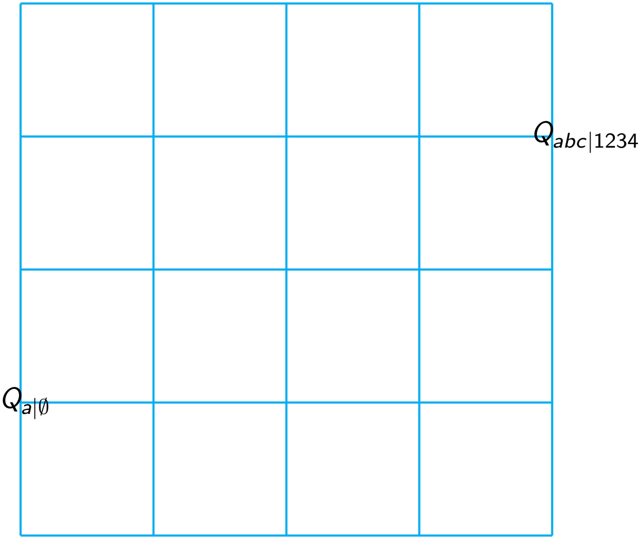
256 Q's



Quantum Spectral Curve

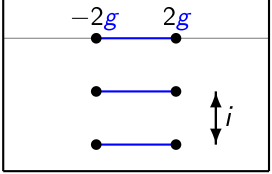
Cut structure

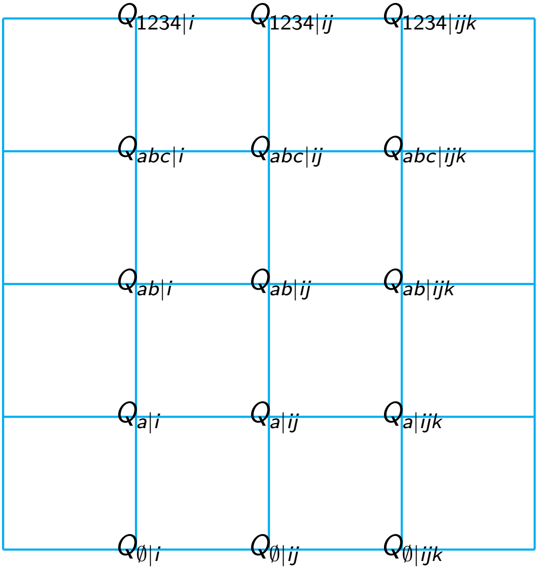




Quantum Spectral Curve

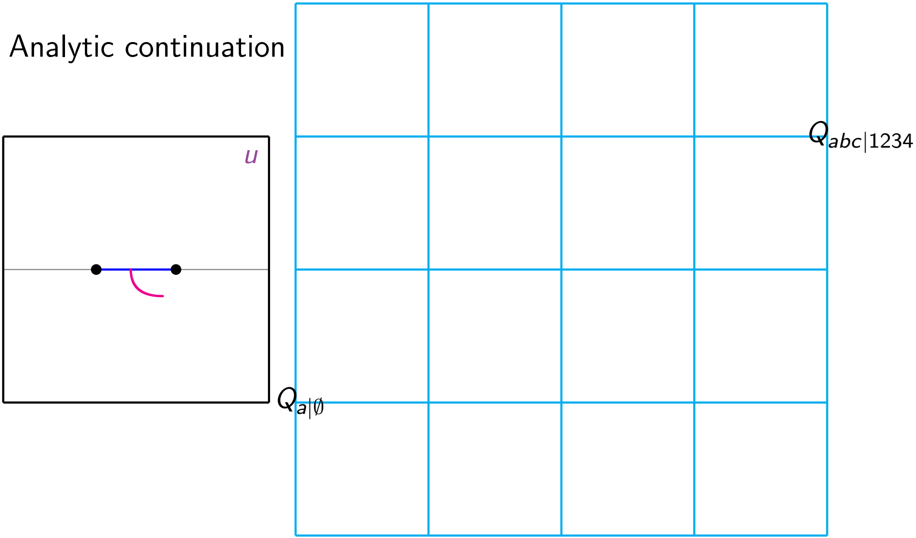
Cut structure





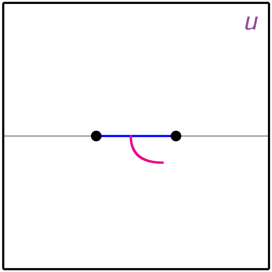
Quantum Spectral Curve

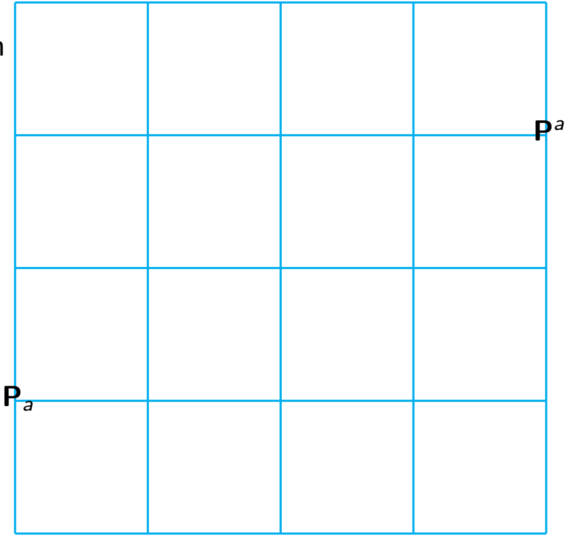
Analytic continuation



Quantum Spectral Curve

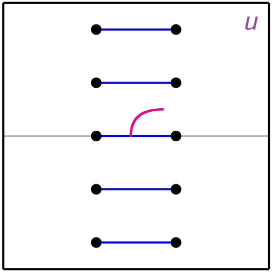
Analytic continuation

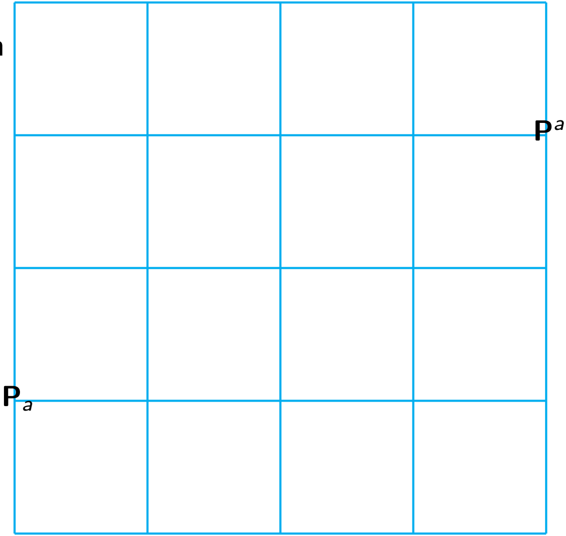




Quantum Spectral Curve

Analytic continuation

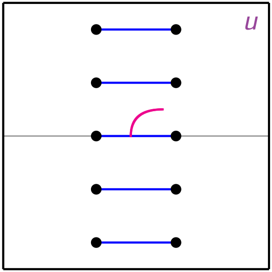


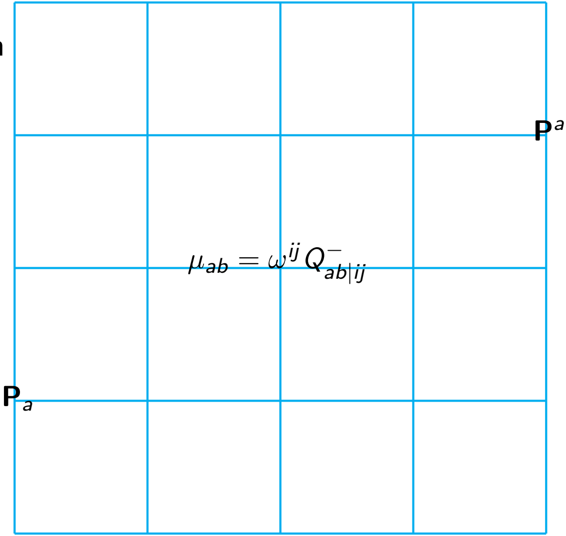


$$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^b$$

Quantum Spectral Curve

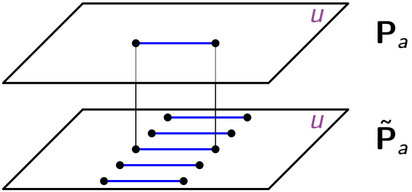
Analytic continuation





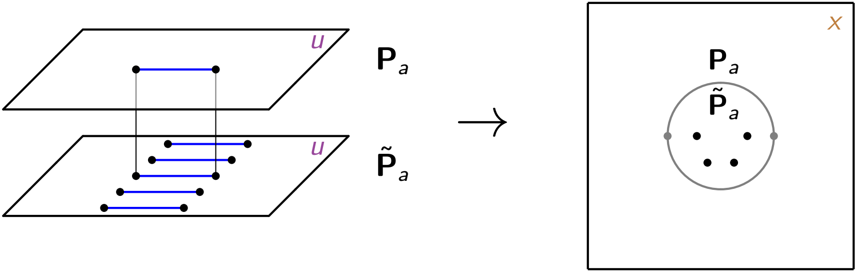
$$\tilde{\mathbf{P}}_a = \mu_{ab} \mathbf{P}^b$$

Key property: structure of **P**



Key property: structure of **P**

$$x + \frac{1}{x} = \frac{u}{g}$$



$$\mathbf{P}(x) = \sum_{k=\#}^{\infty} \frac{c_k}{x^k}$$

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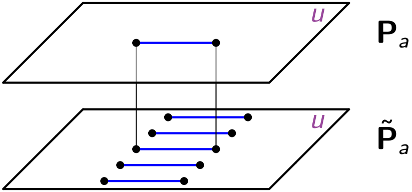
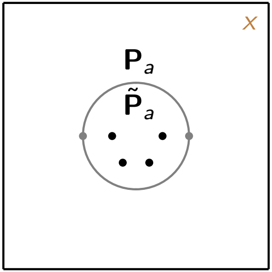


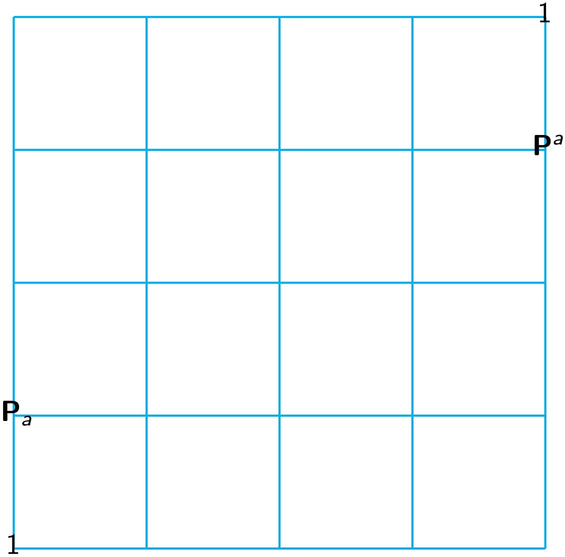
$\longrightarrow$



$$\mathbf{P}(u) = -\frac{1}{u} + g^2\left(-\frac{1}{u^3} - \frac{c}{u^2} - \frac{\Delta_1}{2u}\right) + \ldots \qquad \longleftarrow \qquad \mathbf{P}(x) = \sum_{k=\#}^{\infty} \frac{c_k}{x^k}$$

finite # of constants at each loop

Perturbative solution



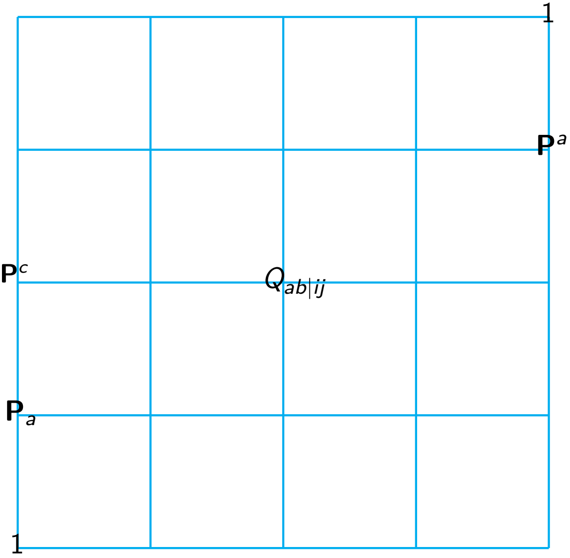
Perturbative solution

solve

$$Q^-_{ab|ij} - Q^+_{ab|ij} = \mathbf{P}_{[a} Q^-_{b]c|ij} \mathbf{P}^c$$

[CM, Volin '14]

[Gromov, Lev.-Masl., Sizov '15]

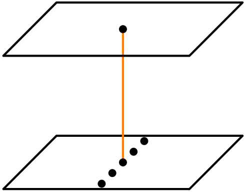


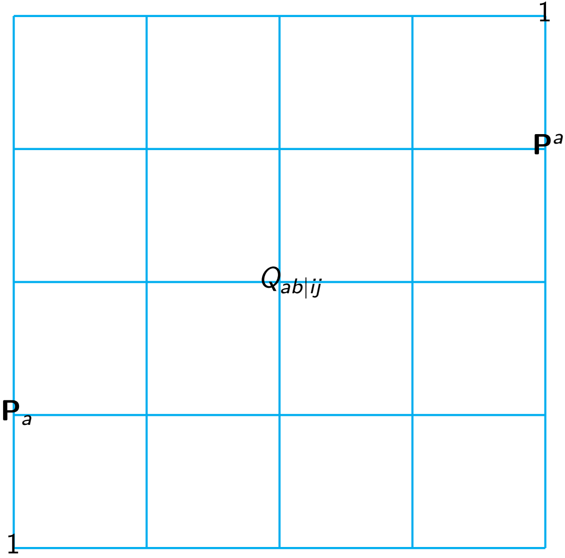
Perturbative solution

Glue

$\mathbf{P}$ and $\tilde{\mathbf{P}}$

at $u = 0$





Perturbative solution

$\Delta = 4$

Perturbative solution

$\Delta = 4 + 12g^2$

Perturbative solution

$$\Delta = 4 + 12g^2 - 48g^4$$

Perturbative solution

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6$$

Perturbative solution

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + g^8\left(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\right)$$

Perturbative solution

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + g^8\big(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\big) + g^{10}\big(15168 + 6912\,\zeta_3 - 5184\,\zeta_3^2 - 8640\,\zeta_5 + 30240\,\zeta_7\big)$$

Perturbative solution

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Perturbative solution

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$$-489888\,\zeta_9\big) + g^{14}\big(-2135040 + 5230080\,\zeta_3 - 421632\,\zeta_3^2 + 124416\,\zeta_3^3 - 229248\,\zeta_5 + 411264\,\zeta_3\,\zeta_5$$
$$-993600\,\zeta_5^2 - 1254960\,\zeta_7 - 1935360\,\zeta_3\,\zeta_7 - 835488\,\zeta_9 + 7318080\,\zeta_{11}\big)$$

Perturbative solution

$$
\begin{aligned}
\Delta &= 4 + 12g^2 - 48g^4 + 336g^6 + g^8\big(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\big) + g^{10}\big(15168 + 6912\,\zeta_3 - 5184\,\zeta_3^2 \\
&-8640\,\zeta_5 + 30240\,\zeta_7\big) + g^{12}\big(-7680 - 262656\,\zeta_3 - 20736\,\zeta_3^2 + 112320\,\zeta_5 + 155520\,\zeta_3\,\zeta_5 + 75600\,\zeta_7 \\
&-489888\,\zeta_9\big) + g^{14}\big(-2135040 + 5230080\,\zeta_3 - 421632\,\zeta_3^2 + 124416\,\zeta_3^3 - 229248\,\zeta_5 + 411264\,\zeta_3\,\zeta_5 \\
&-993600\,\zeta_5^2 - 1254960\,\zeta_7 - 1935360\,\zeta_3\,\zeta_7 - 835488\,\zeta_9 + 7318080\,\zeta_{11}\big) + g^{16}\Big(54408192 - 83496960\,\zeta_3 \\
&+7934976\,\zeta_3^2 + 1990656\,\zeta_3^3 - 19678464\,\zeta_5 - 4354560\,\zeta_3\,\zeta_5 - 3255552\,\zeta_3^2\,\zeta_5 + 2384640\,\zeta_5^2 + 21868704\,\zeta_7 \\
&-6229440\,\zeta_3\,\zeta_7 + 22256640\,\zeta_5\,\zeta_7 + 9327744\,\zeta_9 + 23224320\,\zeta_3\,\zeta_9 + \frac{65929248}{5}\,\zeta_{11} - 106007616\,\zeta_{13} - \frac{684288}{5}\,Z_{11}^{(2)}\Big)
\end{aligned}
$$

Perturbative solution

$$\begin{aligned}\Delta &= 4 + 12g^2 - 48g^4 + 336g^6 + g^8\left(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\right) + g^{10}\left(15168 + 6912\,\zeta_3 - 5184\,\zeta_3^2\right.\\
&\left.-8640\,\zeta_5 + 30240\,\zeta_7\right) + g^{12}\left(-7680 - 262656\,\zeta_3 - 20736\,\zeta_3^2 + 112320\,\zeta_5 + 155520\,\zeta_3\,\zeta_5 + 75600\,\zeta_7\right.\\
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&+g^{18}\Big(-1014549504 + 1140922368\,\zeta_3 - 51259392\,\zeta_3^2 - 20155392\,\zeta_3^3 + 575354880\,\zeta_5 - 14294016\,\zeta_3\,\zeta_5\\
&-26044416\,\zeta_3^2\,\zeta_5 + 55296000\,\zeta_5^2 + 15759360\,\zeta_3\,\zeta_5^2 - 223122816\,\zeta_7 + 34020864\,\zeta_3\,\zeta_7 + 22063104\,\zeta_3^2\,\zeta_7\\
&-92539584\,\zeta_5\,\zeta_7 - 113690304\,\zeta_7^2 - 247093632\,\zeta_9 + 119470464\,\zeta_3\,\zeta_9 - 245099520\,\zeta_5\,\zeta_9 - \frac{186204096}{5}\,\zeta_{11}\\
&-278505216\,\zeta_3\,\zeta_{11} - 253865664\,\zeta_{13} + 1517836320\,\zeta_{15} + \frac{15676416}{5}\,Z_{11}^{(2)} - 1306368\,Z_{13}^{(2)} + 1306368\,Z_{13}^{(3)}\Big)\\
&+\dots\end{aligned}$$

Conclusion

- Automatic solution of the spectral problem
  - Weak coupling analytically to arbitrary order
  - Numerically at any g [Gromov, Sizov, Levkovich-Maslyuk '15]

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  - BFKL limit, quark/antiquark potential...
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- Missing piece: strong coupling ($\frac{1}{g}$-expansion)