Chiral limit of N=4 SYM and ABJM and integrable Feynman graphs

João Caetano - LPTENS Paris

GATIS Closing Workshop - DESY

based on work to appear soon with O. Gurdogan and V. Kazakov

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All of these preserve **Integrability**!

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Idea: play with these new parameters, to kill interacting terms in the Lagrangian

$\Downarrow$

Simpler field theories,
with easier perturbation theory.
Easier to prove integrability!

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$$[A, B] \to [A, B]_\gamma = e^{i\gamma_k Q_i^A Q_j^B \epsilon^{ijk}} AB - e^{-i\gamma_k Q_i^A Q_j^B \epsilon^{ijk}} BA$$

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Cartan charges of the SU(4) R-symmetry group

R-symmetry is broken: $SU(4) \to U(1)^3$

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Deformation based on conserved charges $\rightarrow$ all-loop integrability is equally preserved

γ-deformed Lagrangian

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Lagrangian is now a function of g and γ_k

$$\mathcal{L}_{int} = N_c \mathrm{tr}\Big[\frac{g}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - g\, e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j$$
$$- e^{-\frac{i}{2}\gamma_j^-}\bar{\psi}_j\phi^j\bar{\psi}_4 + e^{+\frac{i}{2}\gamma_j^-}\bar{\psi}_4\phi^j\bar{\psi}_j + i\epsilon_{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\psi^k\phi^i\psi^j$$
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Double scaling limit [Gurdogan, Kazakov'15]

$$q_i \equiv e^{-i\frac{\gamma_i}{2}} \to \infty \qquad \text{with} \quad \xi_i \equiv g q_i \text{ finite}$$
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Chiral limit: parameters are complex hence resulting theories will be non-unitary

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$$\begin{aligned}
\mathcal{L}_{\rm int} &= N_c\,{\rm tr}\big[\xi_1^2\,\phi_2^\dagger\phi_3^\dagger\phi_2\phi_3+\xi_2^2\,\phi_3^\dagger\phi_1^\dagger\phi_3\phi_1+\xi_3^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\\
&+ i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2+\bar\psi_3\phi_1^\dagger\bar\psi_2)\\
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&+ i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1+\bar\psi_2\phi_3^\dagger\bar\psi_1)\,\big]
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Gauge fields decouple with the $g \to 0$ limit

$\xi_i = \xi$ strong β-deformation with $\mathcal{N}=1$ SUSY

$\xi_1 = \xi_2 = 0\,,\quad \xi_3 = \xi$

$$\boxed{\mathcal{L} = \frac{N_c}{2}{\rm tr}\big(\partial^\mu\phi_1^\dagger\partial_\mu\phi_1+\partial^\mu\phi_2^\dagger\partial_\mu\phi_2+2\xi^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\big)}$$

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One gets a ϕ^6 type scalar Lagrangian

$$\mathcal{L} = N_c\,\mathrm{Tr}\Big[-\partial_\mu Y_1^\dagger \partial^\mu Y^1 - \partial_\mu Y_2^\dagger \partial^\mu Y^2 - \partial_\mu Y_4^\dagger \partial^\mu Y^4 \\ + (4\pi)^2\,\frac{\xi_1\xi_2}{\xi_3} Y^1 Y_4^\dagger Y^2 Y_1^\dagger Y^4 Y_2^\dagger \Big]$$

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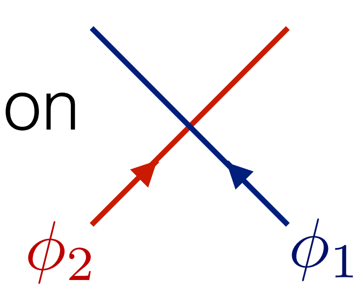
single interaction vertex



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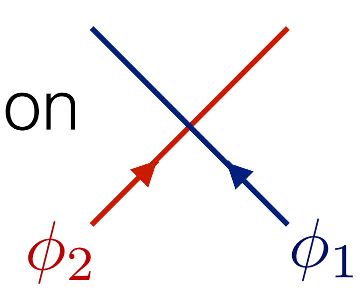

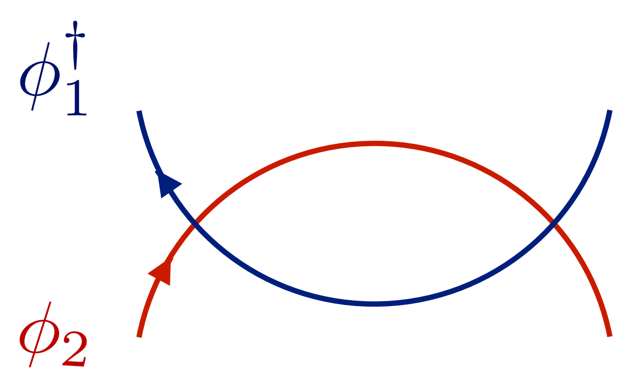

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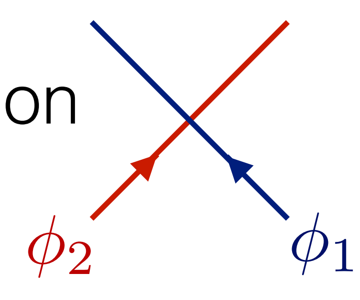

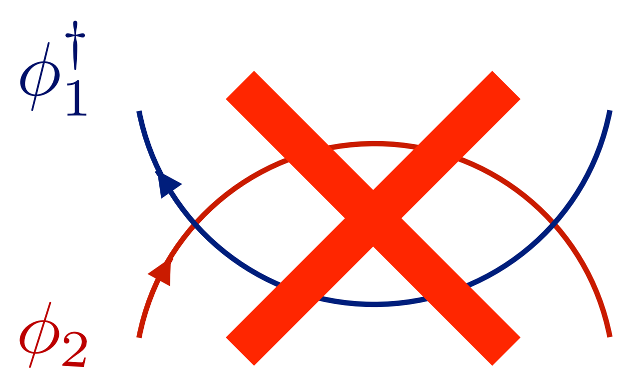

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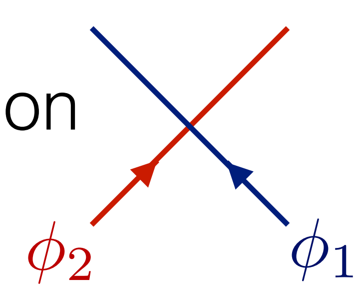

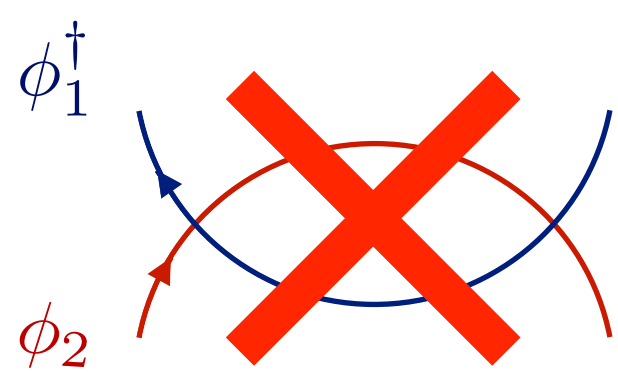

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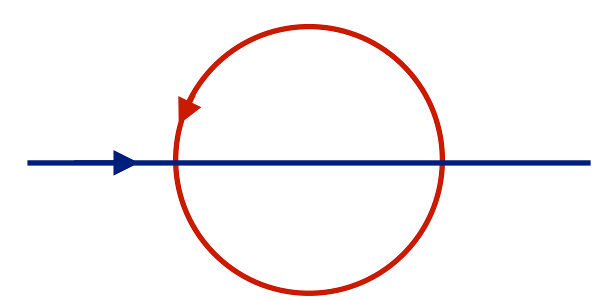

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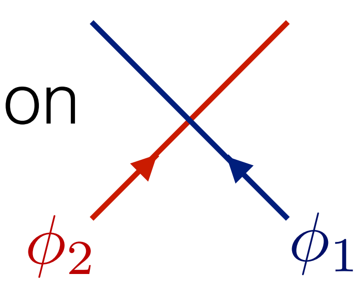

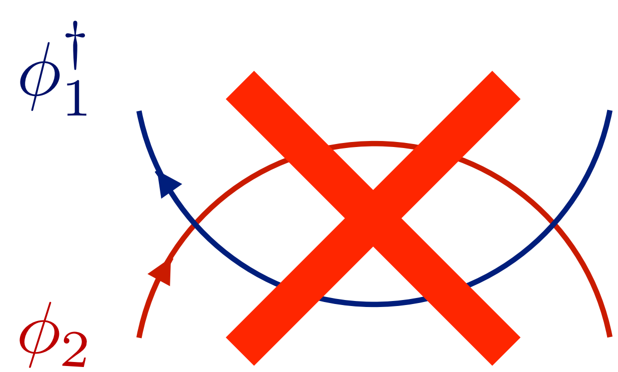

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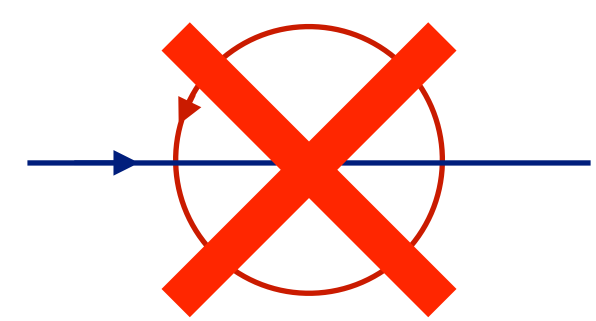

at large N,
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At 1/N…

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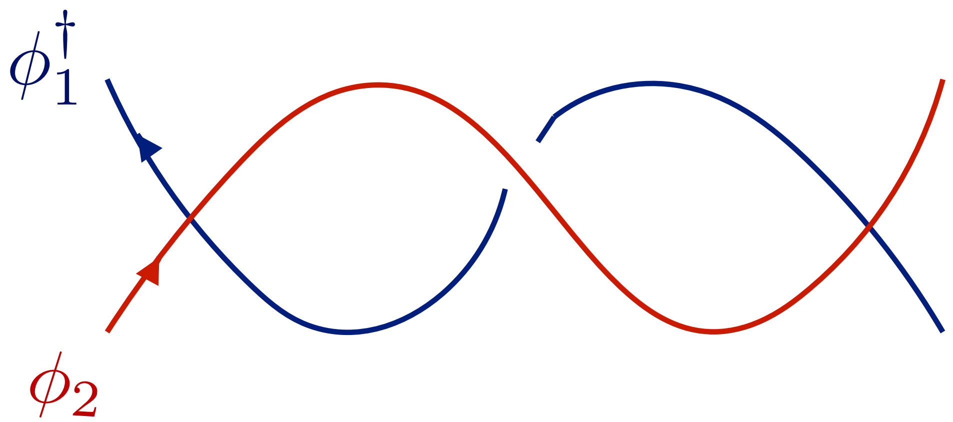

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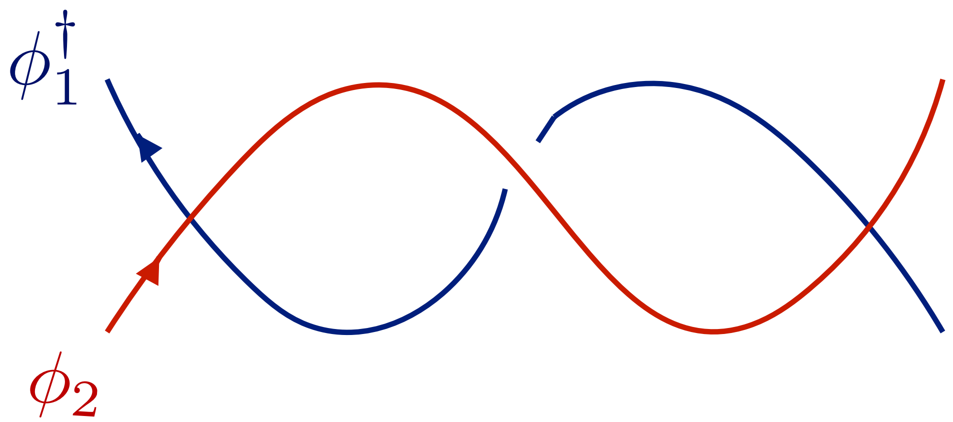

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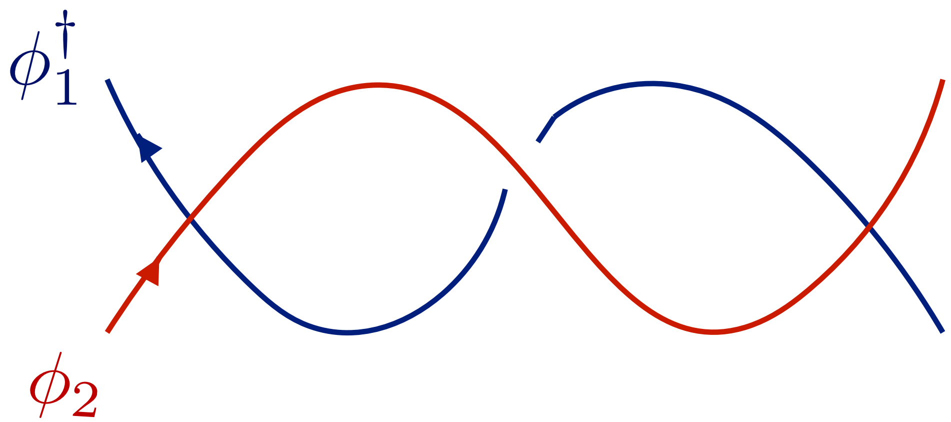

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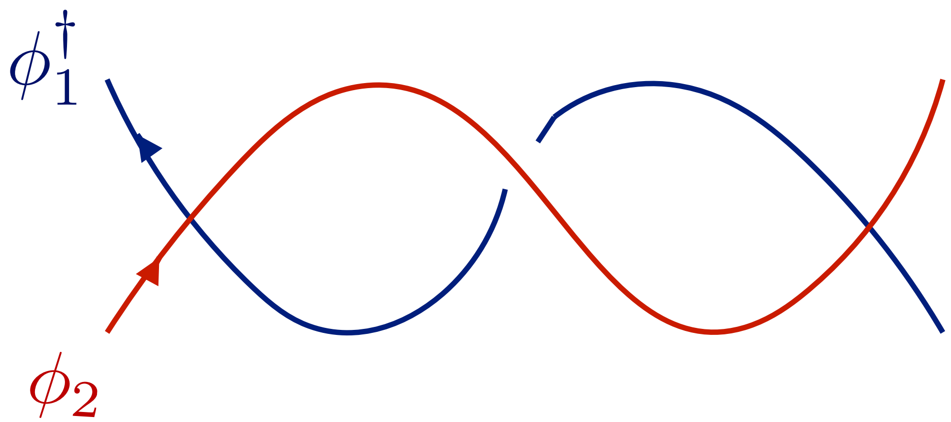

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Conformal symmetry broken even in the planar limit!

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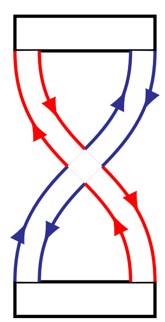
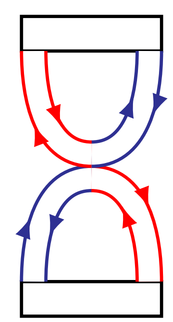
 and  same order in N

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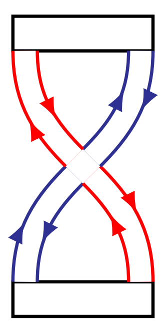

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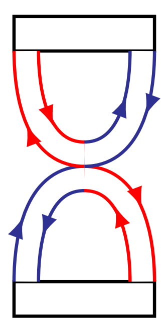

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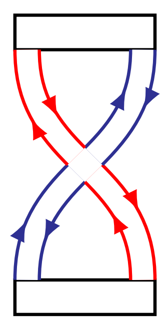

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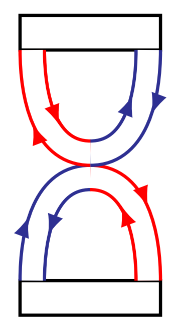

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These L=2 states play the role of **tachyons** in γ-twisted dual string theory. At strong coupling, these tachyons have been identified. [Rastelli, Pomoni'08]

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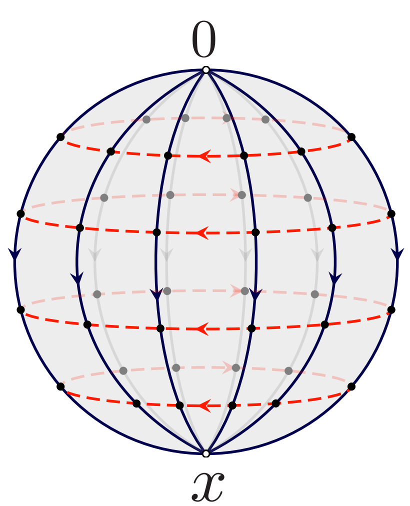
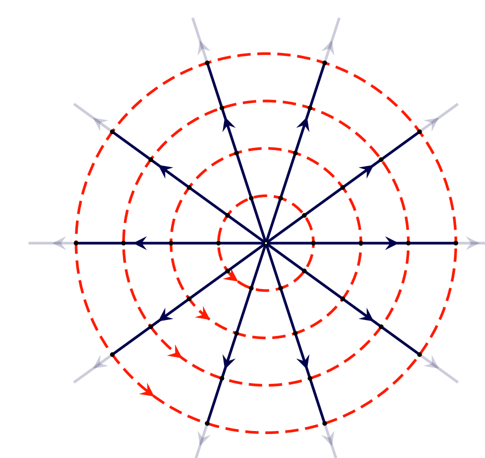


amputation of one external operator

wheel graphs



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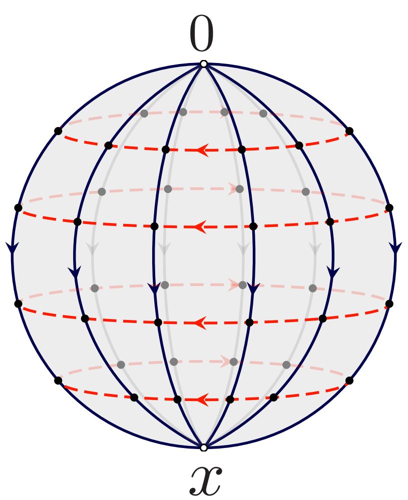
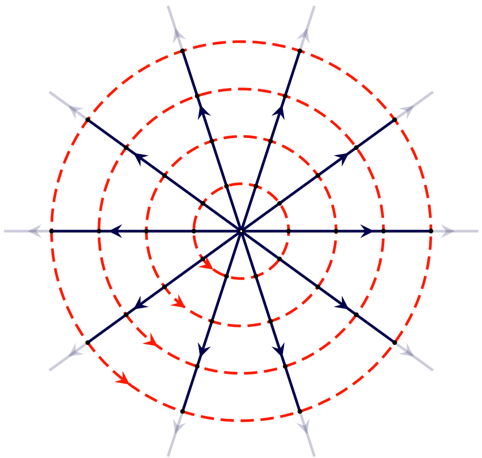


amputation of one external operator $\longrightarrow$ wheel graphs



- 1 wheel [Broadhurst, '80]

- 2 wheels from the integrability based solution of γ twisted

[Ahn et al., '11] [Gurdogan, Kazakov, '15]

A glimpse on the origin of Integrability

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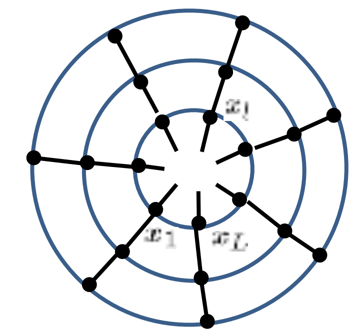

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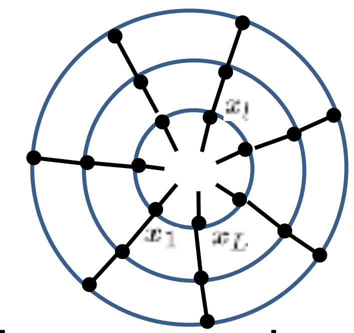

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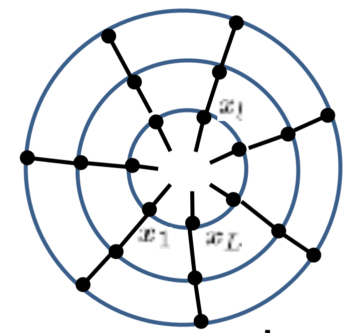

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$$[\hat{H}_L, t(u)] = 0\,, \quad \text{with} \;\; t(u) = \mathrm{Tr}[R_1(u) \ldots R_L(u)]$$

Lax operator $R(u) = uI \otimes I + T(M_{ab}) \otimes M^{ba}$

physical space in principal series irreps of SL(4)

conformal generators

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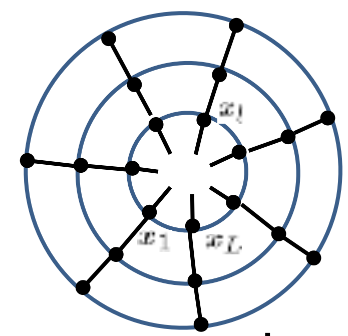

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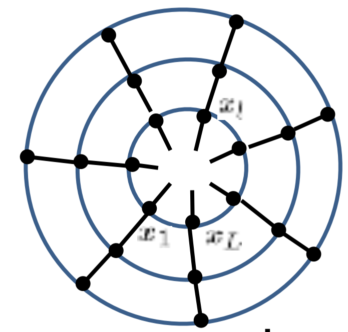

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- Other operators: modification of boundary conditions.
- Still fishnet in the bulk!

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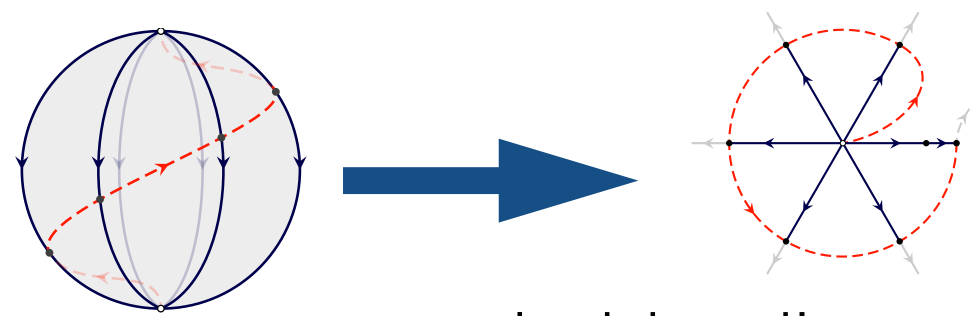

ladder diagrams

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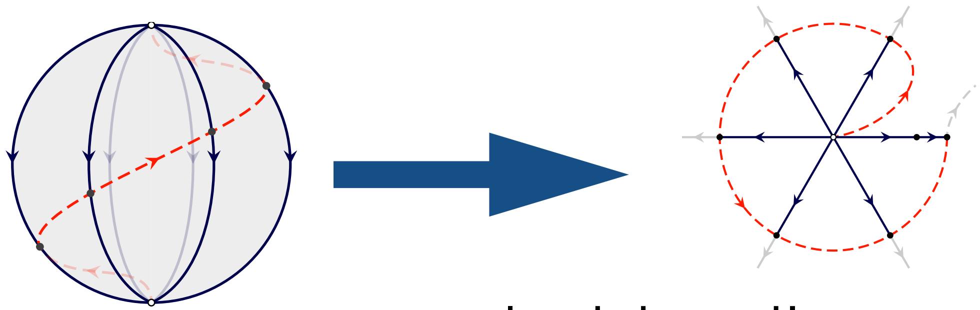

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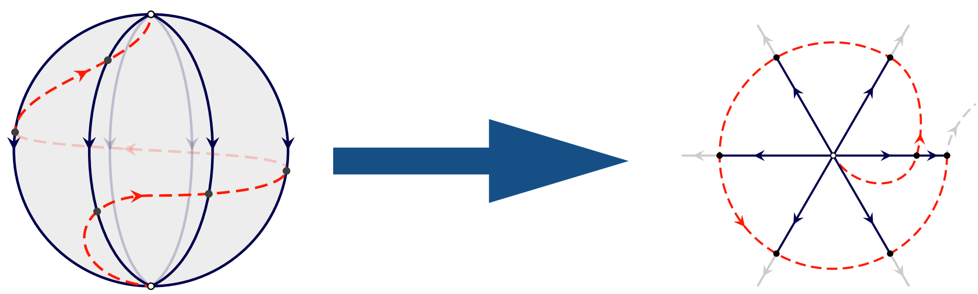

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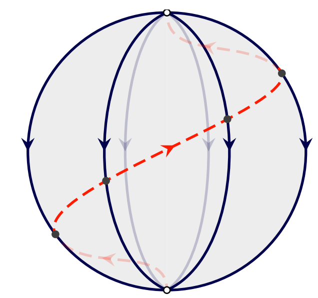

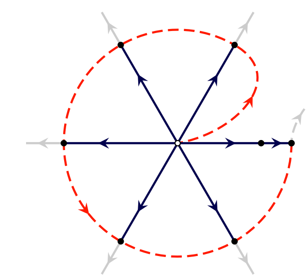

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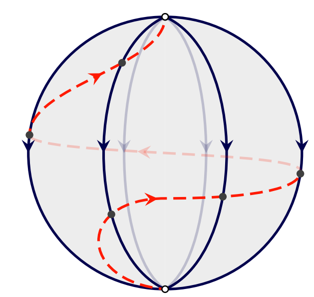

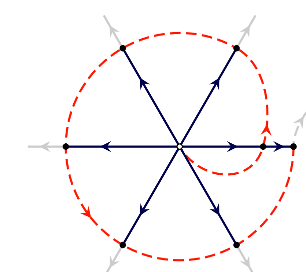

spiral diagrams

Asymptotic Bethe Ansatz: $e^{ipL} = q_3^{-2L}$

$$\gamma = -\frac{1}{2} + \frac{1}{2}\sqrt{1+4g^2\sin(p/2)^2} \to -\frac{1}{2} + \frac{1}{2}\sqrt{1-4\xi_3^2}$$

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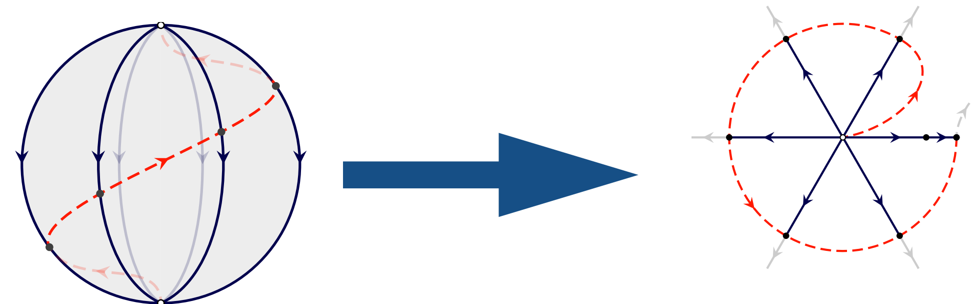

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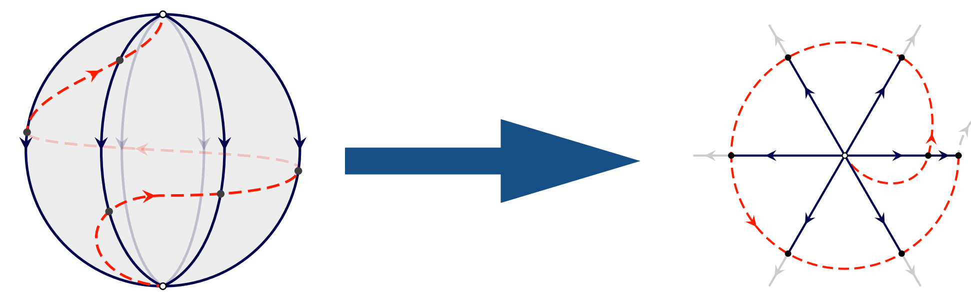

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Resummation of ladder diagrams [Broadhurst, '93, Gross, Mikhailov, Roiban' 02]

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- The anomalous dimension is: $\gamma = \sum_{k=1}^{N} i\left(u_k + \frac{i}{2}\right)$

What can we say about integrals?

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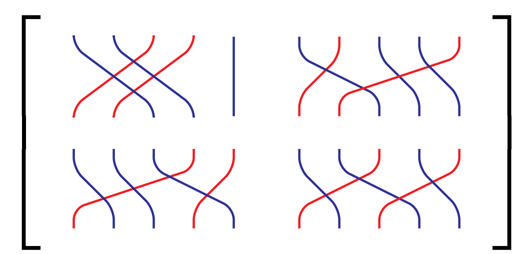
Only 1 diagram for each pair (α, β) at a given loop order

Say at four loops for L=5:

$$\mathcal{G}_{\alpha\beta}\Big|_{\xi^8} =$$



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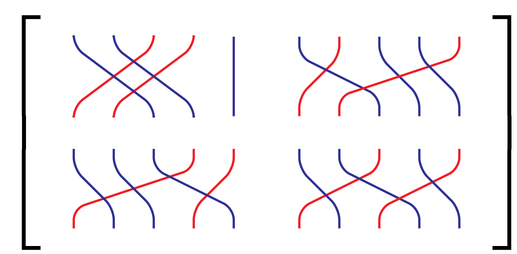

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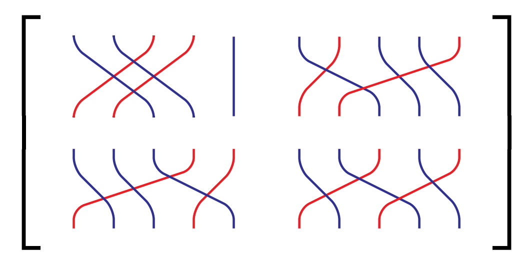
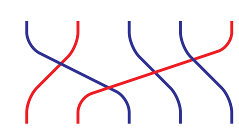
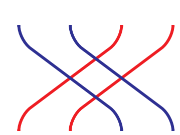
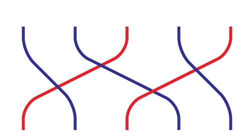


Spectrum provides relations between integrals

Spectrum + input of lower loop integrals + 

fixes completely  and  up to $1/\epsilon$ term

Predictions at five-loops

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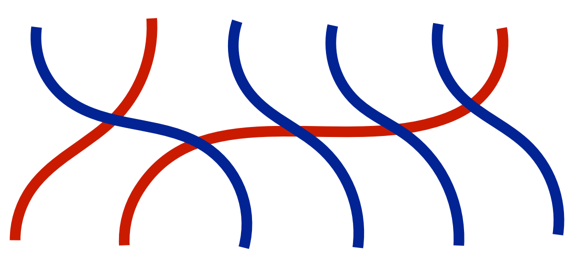
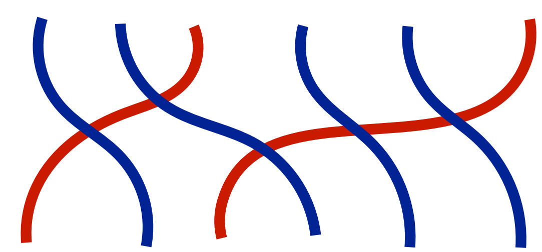
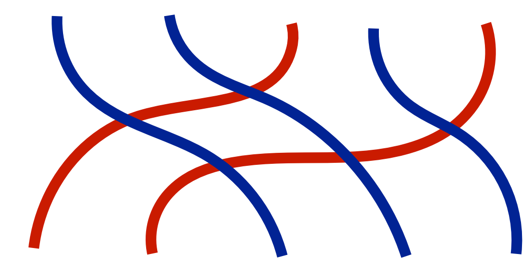
Spectrum provides relations between these 5-loop integrals and lower loop ones



I_a



I_b



I_c

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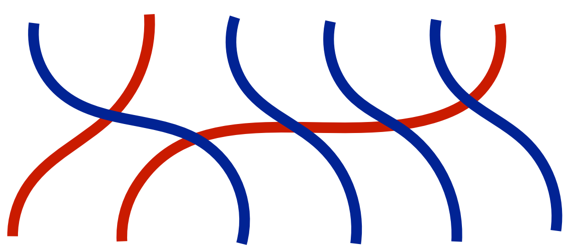

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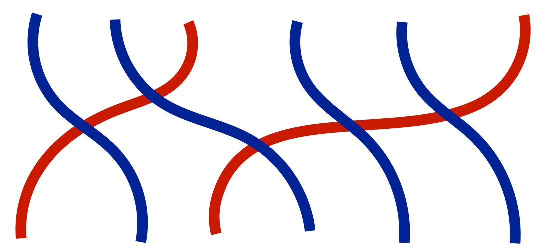
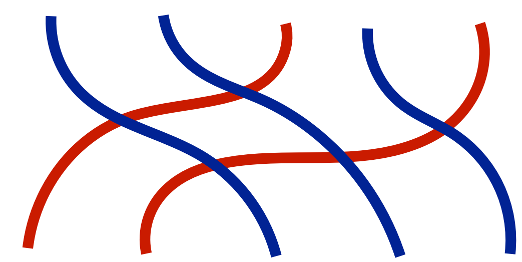


I_b



I_c

Spectrum + input lower loop integrals fixes everything up to a constant. The nontrivial part are $1/\epsilon$ terms:

$$I_b\big|_{1/\epsilon} = -I_a\big|_{1/\epsilon} - \frac{160\zeta(3)}{9} + \frac{53\pi^4}{72} - \frac{187}{5} - \frac{25\pi^2}{12}$$

$$I_c\big|_{1/\epsilon} = I_a\big|_{1/\epsilon} + \frac{418\zeta(3)}{45} + \frac{121\pi^4}{360} + \frac{2\pi^2}{9} - \frac{112}{5}$$

Other observables

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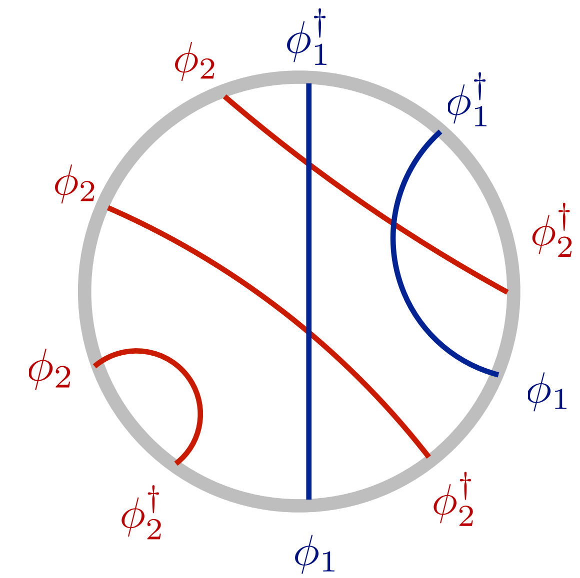
Scattering amplitudes: for a given ordering of external fields in the disk -> single Feynman diagram



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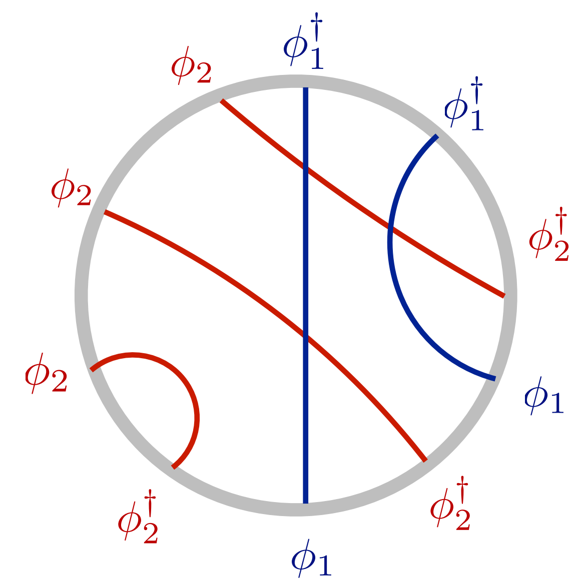

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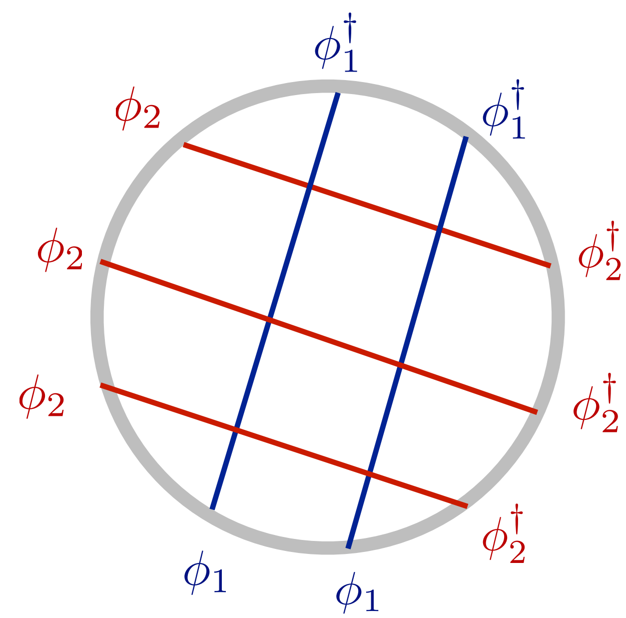

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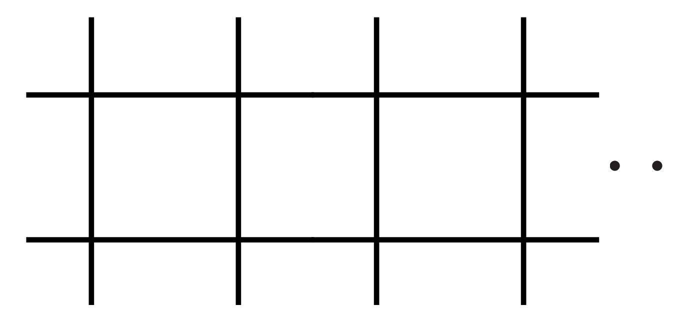

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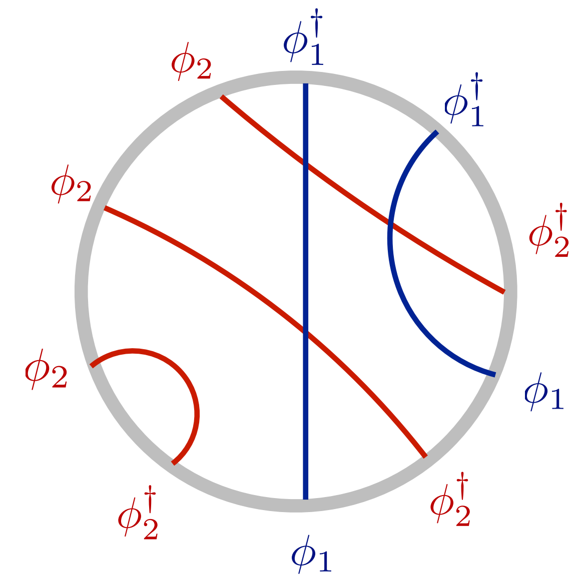

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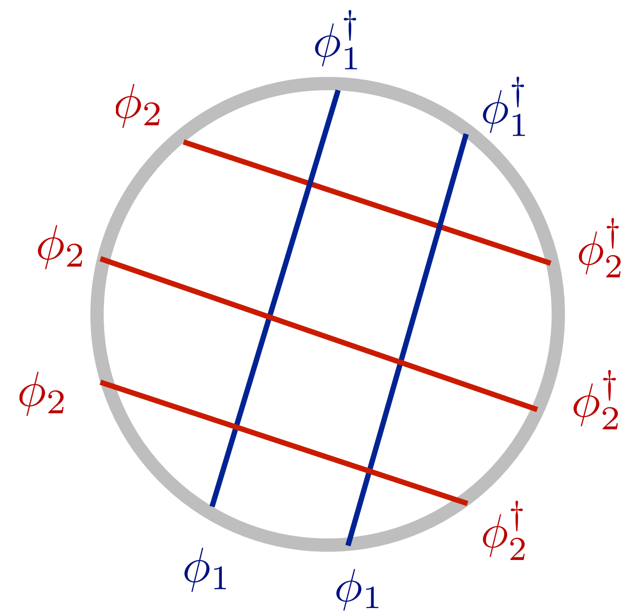

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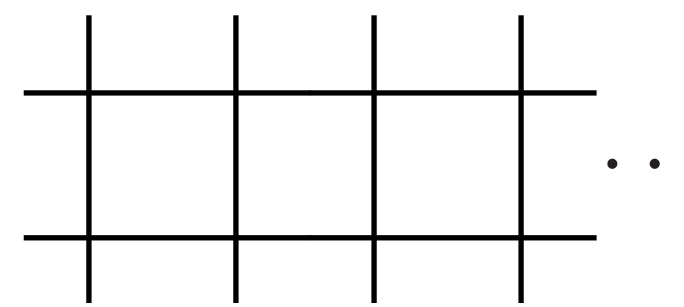

Amplitudes are **finite** and manifestly **dual conformal invariant!**

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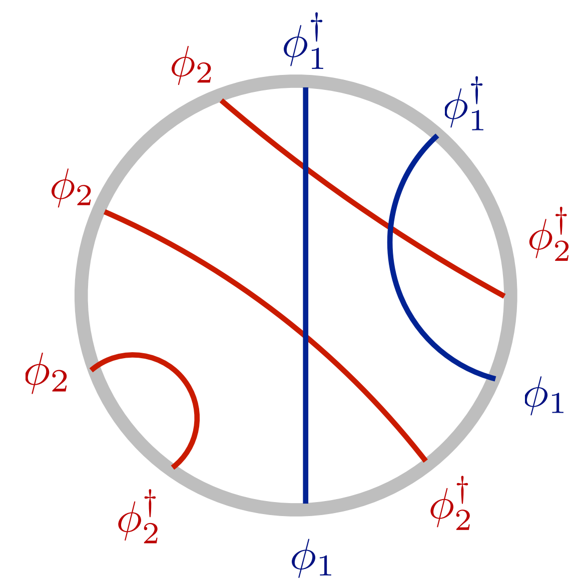

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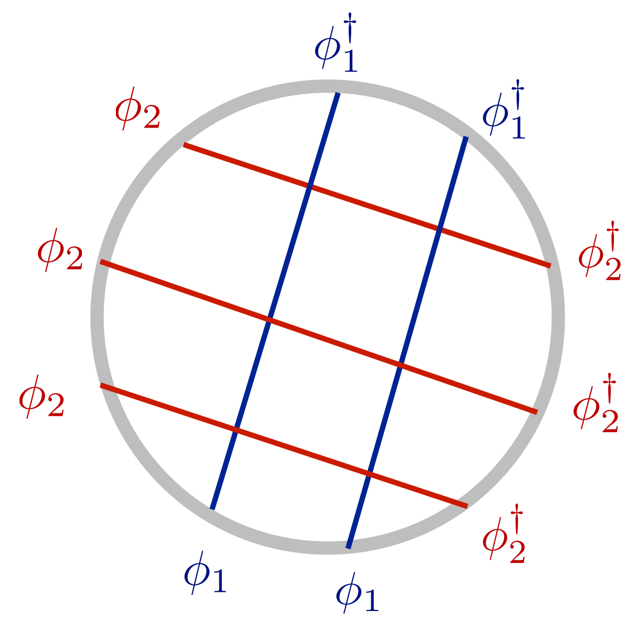

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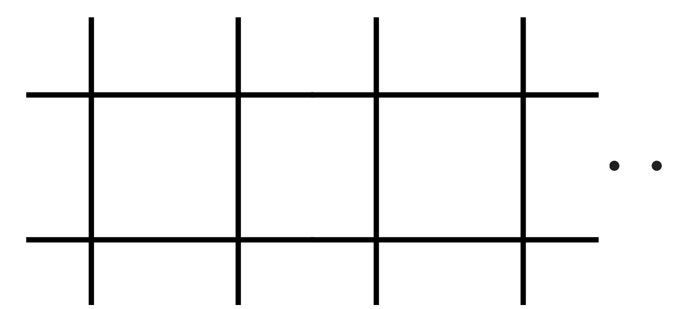

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Three point functions: simple perturbation theory. Hexagon program for bi-scalar model? [Basso, Komatsu, Vieira,' 15]

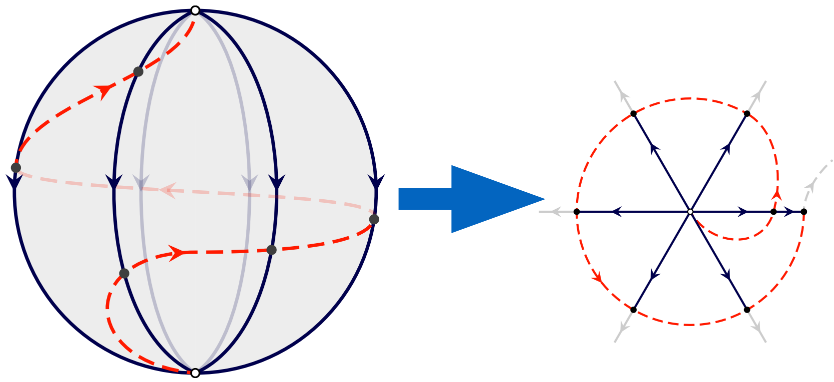
Conclusions

- New 4D and 3D integrable field theories.
- Shed light on the origins of Integrability. First explicit demonstration of all-loop integrability in a corner of N=4 SYM - related to fishnet graphs.
- How to take the DS limit at the level of the Quantum Spectral Curve?
- Can we derive from first principles Thermodynamic Bethe Ansatz and/or Quantum Spectral Curve?
- Structure constants, scattering amplitudes, four-point functions?
- Can integrability further help in computing more integrals?
- Integrability to compute beta function?
- Dual string theory?

Thank you!

Extra Slides

Spiral graphs



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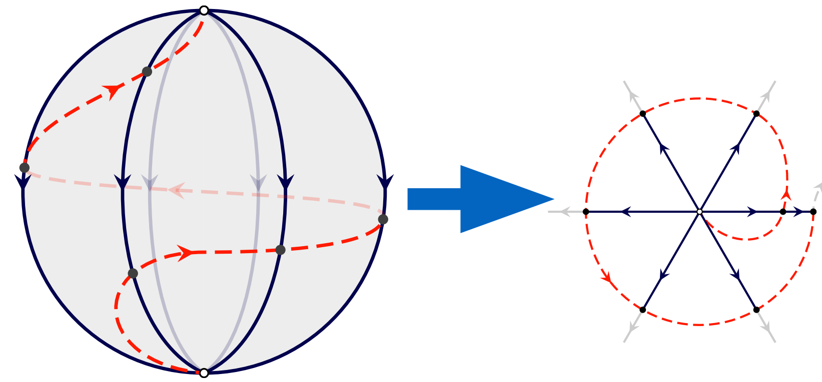

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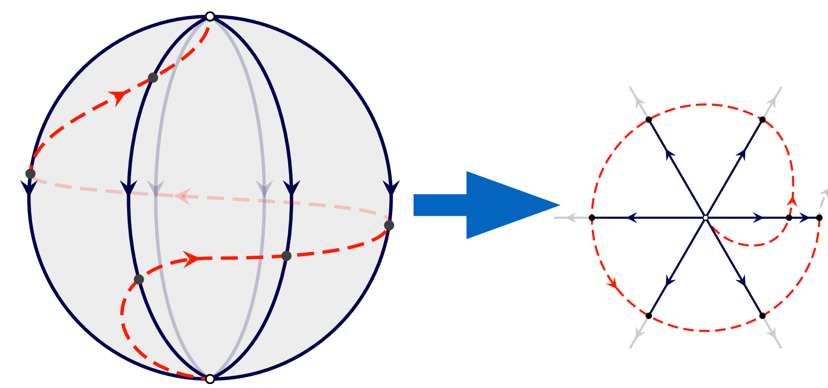

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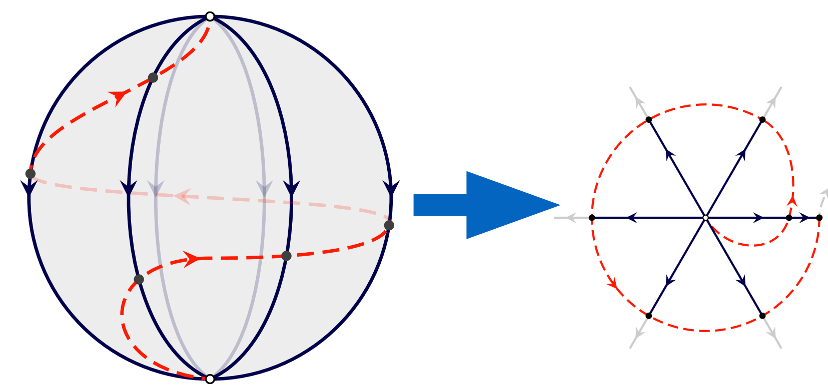
Anomalous dimension is exactly known
(including wrapping) in β-deformed theory by TBA:

[Gromov, Levkovich-Maslyuk,' 10]

$$(\delta\gamma)_{\rm wrap} = 4\xi^{2L}\sum_{k=3}^{L-1}\binom{2(-k+L[k/2]-1)}{L-k}\zeta(2(-k+L+[k/2]-1)+1)$$

Spiral graphs



For these operators, $\mathrm{Tr}[\phi_1\phi_1\ldots\phi_1{\color{red}\phi_2}\phi_1\ldots\phi_1]$

strongly β-deformed $=$ strongly γ-deformed

(They are given by the same graphs in both theories)

Anomalous dimension is exactly known
(including wrapping) in β-deformed theory by TBA:

[Gromov, Levkovich-Maslyuk,' 10]

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We can easily predict this integral in dim-reg

wrapping integral = ladder integral up to $1/\epsilon$ (by exponentiation)

$+1/\epsilon$ term fixed by anomalous dimension from TBA

N-magnon states

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- In N=4 SYM, magnons are characterised by set of rapidities $\{u_1, u_2, \ldots, u_N\}$ satisfying Asymptotic Bethe Ansatz equations [Beisert, Eden, Staudacher,' 05]
- The anomalous dimension is then given by the sum of N dispersion relations evaluated on these solutions.

$$\gamma = ig\sum_j \left(\frac{1}{x^+(u_j)} + \frac{1}{x^-(u_j)}\right)$$

About ABJM

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Same set of ideas apply for DS limit in ABJM

In particular for single excited states $\mathrm{Tr}[(Y^1 Y_4^\dagger)^{L-1}(Y^2 Y_4^\dagger)]$

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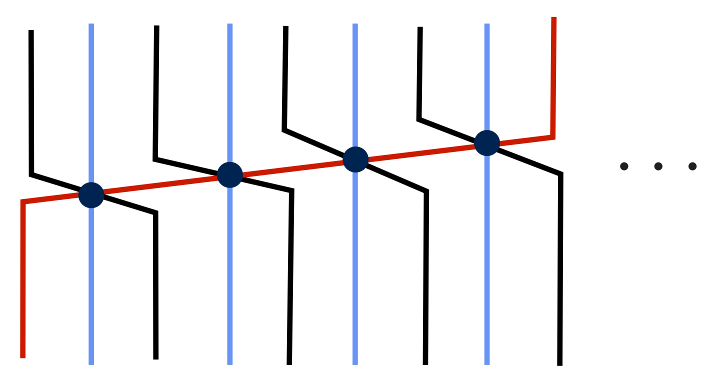
Resums 3D ladders



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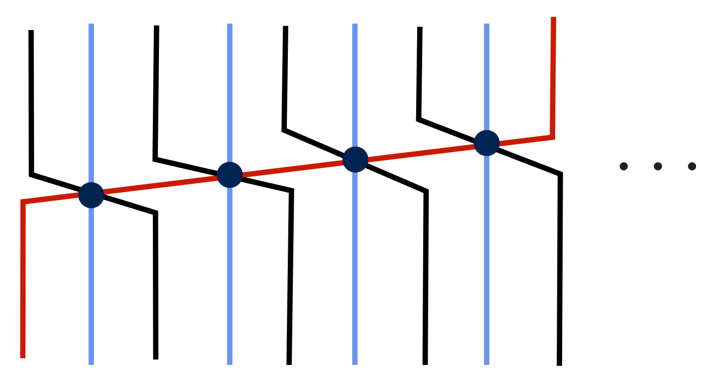

Also Bethe Ansatz for the states $\mathrm{Tr}[(Y^1 Y_4^\dagger)^{L-N}(Y^2 Y_4^\dagger)^N]$

$$\xi^{-L}\,(u_j^2 + 1/4)^L \quad = \quad \prod_{j\neq k}^{N}\left[\frac{u_k - u_j + i}{u_k - u_j - i}\,\sigma_0^{m,m}(u_k, u_j)\right]$$

Can be used for 3D Feynman integrals