The New Muon g-2 Experiment at Fermilab

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Goal : Measure the muon anomalous magnetic moment, a_{μ} , to 140 ppb, a fourfold improvement over the 540 ppb precision of Brookhaven E821

Anomalous part of the Magnetic Moment

• Recall non-relativistic magnetic moment interaction $H_{\text{Zeeman}} = -\mu \cdot B$

$$\boldsymbol{\mu} = -g \frac{e}{2mc} \boldsymbol{S}, \quad \boldsymbol{S} = \frac{\hbar}{2} \boldsymbol{\sigma}$$
 from quantum mechanics

 \Rightarrow Dimensionless g-factor predicted from theory; Dirac g = 2 in 1928 for spin 1/2

- 1933 Otto Stern measured proton μ_p : required g = 5.6 !
- 1947 : 0.1% discrepancies in spectroscopy. G. Breit suggests $g_e = 2(1 + \epsilon)$
- 1948 : Kusch and Foley measure g_e , it deviates from 2 $\Leftrightarrow g_e \equiv 2(1 + a_e), a_e \neq 0$!
- 1948 : Schwinger QED calculation of anomalous part of g_e factor, finds $a_e = \alpha/2\pi$



- $a_e = \alpha/2\pi \approx 0.00116$ due to *radiative corrections* from virtual particles in loops
- 1 part in 850 effect, huge success for QED !



FIG. 2 (color). Cylindrical Penning trap cavity used to confine a single electron and inhibit spontaneous emission.

- D. Hanneke, S. Fogwell, G. Gabrielse, 2008
- Penning trap for single electron
- Magnetic confinement in radial
- Electrodes for vertical confinement
- Trapped for months
- $g_e/2 = 1.001 \ 159 \ 652 \ 180 \ 73(28)$ (0.28 ppt), most precise quantity in physics
- $a_e = (g_e 2)/2$ determined to 0.24 ppb

$$\frac{g_e}{2} = 1 + C_2 \left(\frac{\alpha}{\pi}\right) + C_4 \left(\frac{\alpha}{\pi}\right)^2 + \dots + C_{10} \left(\frac{\alpha}{\pi}\right)^5 + \dots + a_{\mu,\tau} + a_{\text{hadonic}} + a_{\text{weak}}$$

• Extract α , compare with other measurements, confirms QED at ppt level

- Muons live 2.2 μ seconds why bother measuring a_{μ} ?
- Sensitivity to new physics : $\Delta a_{e,\mu}$ (New Physics) $\approx C \left(\frac{m_{e,\mu}}{\Lambda}\right)^2$
- \Rightarrow Muon mass 206 times electron mass, so new physics contribution 43,000 times larger
- \Rightarrow New physics contribution of 0.24 ppb on a_e corresponds roughly to 9000 ppb on a_{μ}
 - a_{μ} known from Brookhaven E821 to 540 ppb, hope to push at Fermilab to 140 ppb

Contributions to the Anomalous Magnetic Moment of the Muon





Low Energy Precision Frontier : The Anomalous Magnetic Moment of the Muon



Standard Model prediction, in units of 10^{-11} : (M. Davier *et al.* Eur. Phys. J. C **71**, 1515 (2011))

$a_{\mu}(QED)$	=	116 584 718.951	\pm 0.080($lpha^{5}$)
$a_{\mu}(HadVP; LO)$	=	6 923.	\pm 42(Exp)
$a_{\mu}(HadVP; LO)$	=	6 949.	\pm 43(Exp)
$a_{\mu}(HadVP; HO)$	=	-98.4	\pm 0.6(Exp) \pm 0.4(Rad)
$a_{\mu}(Had; LBL)$	=	105.	\pm 26
a_{μ} (Weak; 1 loop)	=	194.8	
a_{μ} (Weak; 2 loop)	=	-41.2	\pm 1(Had) \pm 2 \rightarrow 0(Higgs)
$\Rightarrow a_{\mu}(SM)$	=	116 591 802.	\pm 49 \times 10 ⁻¹¹ (0.42 ppm)
$\Rightarrow a_{\mu}(SM)$	=	116 591 828.	\pm 50 \times 10 ⁻¹¹ (0.43 ppm)

Brookhaven E821 g_{μ} – 2 Results (G.W. Bennett *et al.* Phys. Rev. D **73**, 072003 (2006))

In units of 10^{-11} :



- ⇒ Theory (HVP from e^+e^- , no τ) from M. Davier *et al.*, Eur. Phys. J. C **71**, 1515 (2011), K. Hagiwara *et al.*, J. Phys. G**38**, 085003 (2011).
- ⇒ Deviation is large compared to weak contribution and uncertainty on hadronic terms
- \Rightarrow Signature of new physics?
- \Rightarrow Deviation doesn't reach 5σ threshold for discovery need to reduce uncertainties
- \Rightarrow Need to do a better experiment! Need to reduce theoretical uncertainties

• a_{μ} sensitive to variety of new physics; including many SUSY models



 MSSM one-loop contributions to anomaly from smuon and neutralino (left) and muon sneutrino and chargino (right)

$$\Delta a_{\mu}(\text{SUSY}) \simeq (\text{sgn}\mu) \times (130 \times 10^{-11}) \times \tan\beta \times \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2$$

- $\Rightarrow \mu$ and $\tan \beta$ are difficult to measure at LHC, $g_{\mu} 2$ can provide tighter constraints; complementary measurements $\tan \beta$ important test of universality, underlying structure
 - LHC : no evidence of SUSY so if SUSY is origin of deviation in a_{μ} some SUSY masses less than 700 GeV for tan $\beta < 50$ (smuons, charginos/neutralinos)
 - LHC TeV-scale limits on squarks, gluinos; looser constraints on smuons, charginos
 - Higgs at 125 GeV requires large loop corrections from large logarithm or tree-level contributions from nonminimal particles
 - Constrained MSSM seems to be excluded, some tension between all constraints
 - Split-family SUSY survives (light non-colored, heavy colored sparticles)

• a_{μ} sensitive to variety of new physics; including many SUSY models and others



- Snowmass benchmark points in SUSY parameter space show range of contributions to a_{μ}
- Some models "degenerate" parameters can't be distinguished by LHC alone, $g_{\mu}-2$ helps discriminate, can provide tighter constraints on tan β
- Regardless of final value, a_{μ} constrains the possibilities, complements other searches
- Most LHC observables chirality conserving; low energy precision observables are CPviolating (EDMs) or flavor-violating (CLFV)
- a_{μ} sensitive to flavor- and CP-conserving, chirality-flipping, loop-induced contributions

Low Energy Precision Tests : Motivation for reducing uncertainty on a_{μ}

- a_{μ} sensitive to leptonic couplings; b-, or K-physics sensitive to hadronic couplings
- CLFV $\mu \rightarrow e$ conversion depends on mass and coupling strength of new physics (several unknowns); g_{μ} help determine nature of new physics
- Dark sector models with additional light neutral gauge bosons mostly hidden from LHC, visible to $g_{\mu}-2$
- Many well motivated theories predict large Δa_{μ} new g-2 can constrain parameters
- Many well motivated theories predict tiny Δa_{μ} if large Δa_{μ} found these are excluded
- Some models predict similar signatures at LHC but distinguishable by Δa_{μ} (MSSM and UED (1D), Littlest Higgs)
- New g-2 sensitive to parameters difficult to measure at LHC [$tan(\beta)$, $sgn(\mu)$]
- Provides constraints on new physics that are independent and complementary to LHC, CLFV ($\mu \rightarrow e$), EDMs, ...
- \Rightarrow Even agreement with the Standard Model would be very interesting
- \Rightarrow Sensitivity to new particles with TeV scale mass

⇒ Many reasons to pursue a new measurement of a_{μ} at Fermilab, reduce δa_{μ} from 540 ppb → 140 ppb

E989 : New Muon g_{μ} -2 Experiment at Fermilab

- E989 will measure the Muon Anomalous Magnetic Moment to ± 140 ppb precision
- Factor of 4 improvement possible due to advantages at Fermilab



Overview of the the Experimental Method

- Produce an 8 GeV pulsed proton beam 10^{12} /pulse, direct it onto a Ni production target
- Capture pions from production target with lithium lens into long decay beam line
- Capture muons at 3.1 GeV/c, >90% polarized from "forward" pion decay $\pi^+ o \mu^+
 u_\mu$
- ullet Polarized muons enter storage ring through SC inflector that cancel storage ring $m{B}$ field
- Kick the 3.094 GeV/c muon beam onto a stored orbit radius=711.2 cm with pulsed magnets
- Measure arrival time and energy of e^+ from muon decay in ring $\mu^+ \to e^+ \bar{\nu}_{\mu} \nu_e$ for 10+ lifetimes, 700 μ s



Overview of the the Experimental Method

• Spin vector precession ω_s faster than momentum vector cyclotron precession ω_c :

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \left(\frac{mc}{p} \right)^2 \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

• Cancel term from electrostatic vertical focusing at $p_{\rm magic}=m_\mu c/\sqrt{a_\mu}pprox$ 3.094 GeV/c

- \Rightarrow Experiment measures two quantities: (1) difference in precession rates anomalous precession frequency ω_a and (2) magnetic field \vec{B} averaged over muon distribution in ring
 - Difference directly sensitive to $a_{\mu} \approx \alpha/2\pi \approx 0.00116...$, not $g_{\mu} \approx 2.00232...$



Sub-ppm corrections applied due to vertical betatron motion (pitch correction) and muons not at magic γ

- Superferric C-magnet, 680 tons of iron, 4 sets of superconducting coils with 24 windings each, 5200 A, design field 1.4513 T
- High homogeneity required need to know **B** absolutely at 70 ppb level
- Designed as shimmable kit : passive (wedges, edge shims), active (surface coils)



- BNL E821 achieved high homogeneity $\approx~\pm50$ ppm variations over azimuth
- What matters is average over azimuth \Rightarrow at 1 ppm level over muon storage volume

- To measure ω_a , need to know muon spin direction when it decayed
- Nature is kind here : muon decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ is self-analyzing due to PV
- Muon spin direction correlated with decay positron direction



- Averaged over all positron energies, forward-backward asymmetry wrt muon spin is a=1/3
- For highest energy positrons (3.1 GeV), asymmetry a=1
- $E_{\text{lab}} \approx \gamma E^* \left(1 + \cos \theta^* \right)$
- Detect decay e^+ above 1.8 GeV \Leftrightarrow cut on θ^* , reconstruct muon spin direction versus time

Figures from thesis of Alex Grossmann

Detecting the e^+ from Muon Decay : Dave Herzog UW + Collaborators



- Need fast calorimeter to detect e^+ from muon decay made from PbF₂ crystals
- Each calorimeter segmented into 9×6 individual crystals to handle pileup
- Čerenkov light detection with silicon photomultipliers (SiPMs)
- Smaller Moliere radius, greater segmentation, greater immunity to pileup then BNL E821
- Signals digitized with 800 MHz 12-bit waveform digitizers for 700+ μ s, extract e^+ signals offline



Measurement of ω_a

$$N_{\text{ideal}}(t) = N_0 \exp\left(-t/\gamma \tau_{\mu}\right) \left[1 - A\cos\left(\omega_a t + \phi\right)\right]$$



 $\leftarrow \text{Wiggle plot from BNL E821}$ • $\omega_a \approx \frac{e}{m} a_\mu B = 2\pi \times 229 \text{ kHz}$

•
$$3.6 \times 10^9 \ e^+$$
 above 1.8 GeV/c
 $\frac{\delta \omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a \gamma \tau_\mu A P \sqrt{N}}$

- $\gamma \tau \approx 64.4 \ \mu s, \ A \approx 0.4,$ $P \approx 0.95$
- Need $N \thickapprox 1.6 \times 10^{11}$ for 100 ppb
- Corrections for muon losses, pileup, coherent betatron oscillations

• Largest systematics uncertainties on ω_a from BNL E821 and FNAL E989 goals:

	<u> </u>		
Category	E821	E989 Improvement Plans	Goal
	[ppb]		[ppb]
Gain changes	120	Better laser calibration	
		low-energy threshold	20
Pileup	80	Low-energy samples recorded	
		calorimeter segmentation	40
Lost muons	90	Better collimation in ring	20
CBO	70	Higher n value (frequency)	
		Better match of beamline to ring	< 30
E and pitch	50	Improved tracker	
		Precise storage ring simulations	30
Total	180	Quadrature sum	70

• Lab-frame energies of e^+ from μ^+ decay depend on angle between μ spin and momentum vectors; highest energy when parallel

- \bullet Detector gain changes affect reconstructed e^+ energy, changes phase of detected μ^+
- $\Rightarrow \phi_0 \Rightarrow \phi_0 + (d\phi/dt)\delta t \Rightarrow \cos(\omega_a t + \phi_0) \Rightarrow \cos\left[(\omega_a + d\phi/dt)t + \phi_0\right]$
 - Hadronic flash much reduced from BNL E821, detectors remain on during muon injection

Corrections to ω_a : Radial Electric Field and Pitch Correction

$$\vec{\omega}_a \approx \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) \left(\vec{\beta} \cdot \vec{B} \right) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

- Electric field correction: Not all muons at magic momentum, $p = p_m + \Delta p$,
- Storage ring momentum acceptance $\Delta p \approx \pm 0.5\% \ p_m, \ p_m \approx 3.094 \ {\rm GeV}/c$
- Measure momentum distribution from fast-rotation analysis, decay e^+ tracking chambers, muon fiber beam monitors: BNL E821 correction +470 ppb



- BNL E821 pitch correction $+270 \pm 36$ ppb
- Electric field and pitch corrections reduce observed frequency, only corrections made
- Improved E989 muon tracking brings E field and pitch uncertainties to 30 ppb level

• E989 relies on precision measurement of two quantities, ω_a and magnetic field $B \approx 1.45$ T:

$$\omega_a \approx -\frac{e}{m_\mu} a_\mu B$$

- Measure field with pulsed proton NMR: $\hbar\omega_p = 2\mu_p B$
- $\tilde{\omega}_p$: free proton precession frequency weighted by muon distribution $\approx 2\pi \times 61.79$ MHz
- \Rightarrow Goal is to determine $\tilde{\omega}_p$, reducing uncertainty from 170 ppb (BNL E821) to 70 ppb (E989)
 - E989 largely based on principles and hardware developed by Heidelberg and Yale for E821



- RF pulse at $f_{\rm ref}$ =61.74 MHz produces RF magnetic field in coil L_s around sample
- Rotates magnetization of protons in sample perpendicular to main field
- After pulse, proton spins process freely, coherently at $f_{\rm NMR} pprox 61.79$ MHz, $\omega pprox \gamma_{p'}B$
- Rotating magnetization induces V in coil L_s , signal decays exponentially, $\tau~pprox$ 1 ms

Overview of Field Measurement Tasks

⇒ Need Larmor frequency ω_p of free protons in storage volume while muons are stored
 (1) Fixed probes measure field at same time as muons stored, but outside storage volume
 (2) Field inside storage volume measured by NMR trolley, but not when muons stored

• Fixed probes are cross-calibrated when trolley goes by; can infer field inside storage volume when muons stored from fixed probes



Fixed probes on vacuum chambers

Trolley with matrix of 17 NMR probes



(3) Trolley probes calibrated in terms of free proton frequency by an absolute calibration probe



← FID from E821 and Fourier transform

- Signals typically last 1 ms
- Signal : noise ≥ 100 : 1
- Frequency resolution \approx linewidth/[S/N] ≈ 130 Hz / 100 = 1.3 Hz
- Fractional resolution: $\delta f_{\rm NMR}/f_{\rm NMR} = 1.3$ Hz/61.79 MHz \Leftrightarrow 20 ppb resolution on field
- Corrections necessary to get from measurements in NMR probes to ω_p of *free* proton
- \Rightarrow Need absolute calibration of probes in terms of free proton precession frequency; demonstrated at level of 34 ppb (see X. Fei *et al.*, Nucl. Inst. Meth. A **394**, 349 (1997))
- ⇒ Main challenge of field measurement : effectively transfer high accuracy absolute calibration to many probes providing high resolution monitoring field over long periods in which muons are stored

E989 : Fermilab offers advantages, factor 4 improvement possible

Recycler

• Rebunches 8 GeV protons from booster

Target Station

• Target + focusing lens

Decay Line

- Target to M2 to M3 to delivery ring
- ⇒ 1900 m long decay channel for $\pi \Rightarrow \mu$ reduced π and p in ring, factor 20 reduction in hadronic flash
- \Rightarrow 4× higher fill frequency than E821
- \Rightarrow Muons per fill about the same
- \Rightarrow 21 times more detected $e^+,~2\times 10^{11}$
- ⇒ Better temperature control in experimental hall
- ⇒ Reduction in systematics by factor of 3 without major modifications



From BNL E821 to E989 at Fermilab

- 650 ton magnet iron yoke and pole pieces are disassembled, transported by truck to FNAL
- 8 ton, 15 m diameter superconducting coils must be transported in one piece
- Keep accel <1 g, protect delicate superconductor, cooling lines, heat shield, G10 straps,



From BNL E821 to E989 at Fermilab

 Coils moved by barge up Mississippi. Constant monitoring of acceleration, always <12 hrs to safe harbor. Accel <1g, tilt <30°.



Transporting the coils to FNAL : E-ZPass



- Trailer with coils passes toll arches with 6" clearance on each side
- "Nature is hard and unyielding" Martin Perl, *Reflections on Experimental Science*
- We were lucky this time

• MC1 Building ready for some parts of installation. Expect cryoplant ready Jan-Mar 2015



The Fermilab E989 Collaboration as of Oct 2013



Summary

- Experiment under construction to measure a_{μ} to 140 ppb, fourfold improvement over BNL E821
- Reduction in statistical uncertainty by factor 4; reduce ω_a , ω_p systematics by factor 3
- Magnet cold and energized in early 2015, shim and install detectors by 2016, first data 2017
- Hope to motivate improvements in theory and more exp. work :
 - Currently $\delta a_{\mu}(\text{HadVP,LO}) = 0.36$ ppm, and $\delta a_{\mu}(\text{Had,LBL}) = 0.23$ ppm
- Before E821 (1983), expt. known to 7 ppm, theory to 9 ppm : now 0.54 and 0.42 ppm
- Regardless of where final result for a_{μ} lands :
 - Precision test Standard Model
 - Determine parameters $(tan(\beta))$ or viability of many new physics models predicting $\Delta a_{\mu} \neq 0$ (SUSY models)
 - UED (1D) predict tiny effects incompatible with $\Delta a_{\mu} << 300 \times 10^{-11}$
 - Constraint on all future models
 - Provide complementary information to direct searches at LHC, CLFV, EDMs

E989 : New Muon g-2 Experiment at Fermilab



- Need to know muon beam distribution :
 - Finite momentum spread : not all muons at magic momentum, need ppm-level corrections ω_a from *E*-field
 - Betatron motion : ppm-level correction because muon momentum not always \perp to $\vec{\mathbf{B}}$ (pitch correction)
 - Betatron motion of beam leads to time-dependent acceptance changes in calorimeters, must be corrected
 - Muon distribution convoluted with magnetic field map to determine effective magnetic field seen by muons
- Muon beam distribution determined with straw tube trackers

Determining the Stored Muon Distribution : Straw Tube Trackers

- Two tracker stations planned, reside in vacuum chambers 180° and 270° from injection
- 1216 aluminized mylar straws/station, 12 cm long, 1 atm, 80:20 Argon:CO₂, 1400 V, $\pm 7.5^{\circ}$ angle from vertical
- Vertical angular resolution < 10 mrad, momentum resolution < 3.5% at 1 GeV
- Brendan Casey FNAL and collaborators



What to do with a_{μ} (HadVP,LO) based on τ data?

- a_{μ} (HadVP,LO) can be evaluated using dispersion integral from threshold to τ mass from hadronic τ decays
- Can relate decay rate $\tau^- \to \pi^- \pi^0 \nu_\tau$ to $e^+ e^- \to \pi^+ \pi^-$:



- \bullet Attractive option since lots of precision τ data from LEP, CLEO
- Caveats : need to invoke CVC, and apply isospin corrections
- au data has only isovector component, insert by hand isoscalar contribution present in e^+e^-
- Needs more attention : lattice calculation would be great, suggestion that $\rho-\gamma$ mixing reduces discrepancies
- \Rightarrow Lattice QCD experts suggest percent level determination of a_{μ} (HadVP,LO) possible.

What about $a_{\mu}(\text{Had}; \text{LBL}) = 105 \pm 26 \ (\times \ 10^{-11})$?

- $a_{\mu}(Had; LBL)$ non-perturbative, high order correction
- Important might dominate theoretical uncertainty soon
- KLOE will measure $\gamma^*\gamma^* \ \rightarrow \pi^0$ at low Q^2, dominant contribution
- Might reduce leading uncertainty on $a_{\mu}(Had; LBL)$
- \Rightarrow PrimEx effort to reduce uncertainty on π^0 decay width, pion polarizability measurement important
- Lattice QCD effort underway as well (T. Blum and collaborators)



Preliminary Schedule (as of Oct 2013)



- Extensive review process. Total project cost is \approx \$40 million : nearly 1/2 is for upgrade of accelerator and new multipurpose facilities at FNAL
- First data in 2017? Run for 2 or more years
- Precision measurements take a lot of patience but they're worth it

Winding the Coils



Contributions to Had-VP Dispersion Integral



• Dispersion integral weights cross-section ratio R(s) as $1/s^2$, low energy important

• K. Hagiwara *et al.*, J. Phys. G38, 085003 (2011) : contribution to $a_{\mu}^{\rm had,LO}$ and uncertainty



- Have to get the muon beam into the storage ring from zero field area outside to 1.45 T inside - beam strongly deflected unless we cancel this field
- Use a superconducting flux-exclusion tube? Perturbations in storage region too large



Superconducting Inflector

• Base plan: use double- $\cos \theta$ design from BNL E821, $\int \vec{B} \cdot d\vec{L} = 2.55 \text{ T} \cdot \text{m}$



- Procedure :
 - Warm inflector+Type II SC shield, turn on main magnet, flux penetrates inflector and SC shield
 - Cool inflector and shield, since $H > H_{C1}$ field fully penetrates shield
 - Energize coils cancels field in beam channel, eddy currents in passive shield prevents flux leaking out
 - Cancels B field in beam channel, no perturbation to field outside SC shield

• Versions of superconducting inflector with closed and open ends



- BNL E821 inflector closed ends, significant multiple scattering, aperture 18×56 mm², injection efficiency $\approx 2\%$
- New inflector : open ends, $40 \times 56 \text{ mm}^2$ (storage aperture $\pm 45 \text{mm}$) $\rightarrow 4 \times$ more stored muons, could do μ^- and μ^+

- Muons exit the inflector, enter storage region at radius 77 mm outside ideal closed orbit
- ullet Muons cross ideal orbit $pprox~90^\circ$ later in azimuth, angle off by 10.8 mrad
- Including momentum spread, multiple scattering in inflector, need 14 mrad kick
- Temporarily reduce **B** by 280 Gauss, $\int \vec{B} \cdot d\vec{L} = 1.4$ kG·m for 14 mrad kick
- \bullet Pulse width 80 ns $<\tau<\!\!149$ ns, 100 Hz, 10% homogeneity



The Fast Muon Kicker



- New geometry yields 33%-50% higher field/current than BNL E821
- 3×1.7 m stripline kickers, Blumlein PFN
- Tracking studies determine optimal shape
- Dave Rubin and collaborators at Cornell



Storing the Muon Beam : Vertical Focusing Electric Quadrupoles

- Storage ring is a weak-focusing betatron using electric quadrupoles for linear restoring force in vertical, $\kappa = dE_y/dy$, field index $n = \kappa R_0/\beta B_0 \approx 0.137$
- Uniform quadrupole field leads to simple harmonic motion radial x and vertical y betatron oscillations of beam



Stored Beam Dynamics and Related Systematic Uncertainties

$$x = x_e + A_x \cos\left(\nu_x \frac{s}{R_0} + \delta_x\right), \ y = A_y \cos\left(\nu_y \frac{s}{R_0} + \delta_y\right)$$

 $\nu_x = \sqrt{1-n}, \ \nu_y = \sqrt{n}, \ n \approx 0.137, \ f_x = f_C \sqrt{(1-n)} \approx 0.929 f_C, \ f_y = f_C \sqrt{n} \approx 0.37 f_C$

Quantity	Expression	Frequency [MHz]	Period $[\mu s]$
f_a	$\frac{e}{2\pi mc}a_{\mu}B$	0.228	4.37
f_C	$\frac{v}{\pi R_0}$	6.7	0.149
f_x	$\sqrt{1-n}f_c$	6.23	0.160
$ f_y$	$\sqrt{n}f_c$	2.48	0.402
$f_{\rm CBO}$	$f_c - f_x$	0.477	2.10
$\int f_{\rm VW}$	$\int f_c - 2f_y$	1.74	0.574

- \bullet Perturbations of stored muon beam from ideal circular orbit affect ω_a
- \Rightarrow Resonances in ring cause muon beam losses, distort ^{0.40} time spectrum
 - Resonances occur if $L\nu_x + M\nu_y = N$ where L, M, N integers. Operating points have $\nu_x^2 + \nu_y^2 = 1$



- Detector acceptance depends on muon radius at decay coherent radial motion modulates electron time spectrum
- Radial betatron wavelength (blue line) is longer than circumference (cyclotron wavelength), $f_x < f_{
 m C}$
- ullet At fixed detector location, each pass of bunched beam appears at different radius moving at $f_{
 m CBO}$
- CBO frequency $f_{\rm CBO} = f_{\rm C} f_x$ must be kept far from f_a



- Cyclotron wavelength marked by black lines, single detector by black block, betatron oscillations in blue
- ullet Red line : apparent radial breathing in and out of beam at $f_{
 m CBO}$
- Effect nearly cancels when all detectors added together

Coherent Betatron Oscillations (CBO)

- In BNL E821 2000 data taken when CBO frequency close to f_a can be seen in residual to 5 parameter fit
- In 2001, field index n changed to move $f_{\rm CBO}$ away from f_a



$$\vec{\omega}_a \approx \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) \left(\vec{\beta} \cdot \vec{B} \right) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

- Not all muons at magic momentum, $p=p_m+\Delta p$,
- Storage ring momentum acceptance $\Delta p \approx \pm 0.5\% \ p_m, \ p_m \approx 3.094 \ {\rm GeV}/c$

$$\frac{p - p_m}{p_m} = (1 - n) \left[\frac{R - R_0}{R_0} \right] = (1 - n) \frac{x_e}{R_0}$$
$$\frac{\omega'_a - \omega_a}{\omega_a} = \frac{\Delta \omega_a}{\omega_a} = -2 \frac{\beta E_r}{c B_y} \left(\frac{\Delta p}{p_m} \right) = -2n(1 - n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

- Momentum distribution from fast-rotation (de-bunching) analysis, decay e^+ tracking chambers, muon beam fiber monitors
- ⇒ Correction determined from detailed tracking analysis using actual discontinuous quad geometry

• Momentum distribution from fast-rotation (de-bunching) analysis, decay e^+ tracking chambers, muon beam fiber monitors



Corrections to ω_a : Pitch Correction

$$\vec{\omega}_{a} \approx \vec{\omega}_{S} - \vec{\omega}_{C} = -\frac{e}{m} \left[a_{\mu}\vec{B} - a_{\mu} \left(\frac{\gamma}{\gamma+1} \right) \left(\vec{\beta} \cdot \vec{B} \right) \vec{\beta} - \left(a_{\mu} - \frac{1}{\gamma^{2}-1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$
• Vertical betatron motion
$$\rightarrow \vec{\beta} \text{ not perpendicular to } \vec{B}$$

$$\omega_{a}' \approx \omega_{a} \left(1 - \frac{\psi^{2}}{2} \right),$$

$$C_{p} = -\frac{\langle \psi^{2} \rangle}{2} = -\frac{n}{2} \frac{\langle \psi^{2} \rangle}{R_{0}^{2}}$$

- Electric field and pitch corrections reduce observed frequency
- BNL E821 pitch correction $+0.27 \pm 0.036$ ppm
- Electric field and pitch are the only corrections made to the ω_a data
- Improved E989 muon tracking reduces uncertainties ± 0.05 ppm $\Rightarrow 0.03$ ppm level

- Measure field using pulsed NMR to induce and detect free induction decay (FID) of protons in a water sample¹
- Typical NMR probe shown below (field direction vertical, perpendicular to L_s coil axis) :



- RF pulse at f_{ref} =61.74 MHz produces RF magnetic field in coil L_s around sample
- Rotates magnetization of protons in sample perpendicular to main field
- After pulse, proton spins process freely, coherently at $f_{\rm NMR} \approx 61.79$ MHz, $\omega \approx \gamma_{p'} B$
- ullet Rotating magnetization induces V in coil L_s , signal decays exponentially, au~pprox 1 ms

¹May use petroleum jelly (CAS 8009-03-08) : long $T_2 \approx 40$ ms, doesn't evaporate, low temp. coefficient

- NMR signal at $f_{\rm NMR}$ goes to low noise amplifier, mixed with $f_{\rm ref}=61.74$ MHz from synthesizer
- Difference frequency $f_{\rm NMR} f_{\rm ref} \equiv f_{\rm FID}$ ranges from 45-55 kHz, dependent on local field
- Difference of 62 Hz in $f_{
 m FID}$ corresponds to 1 ppm difference in field
- Count zero crossings of this free induction decay (FID) and ticks of clock running at 20 MHz till signal decays to roughly 1/e of peak, ≈ 1 ms



- \Rightarrow Local field characterized by Larmor frequency, $f_{
 m NMR} = f_{
 m ref} + f_{
 m FID}$
 - Single shot resolution on $f_{
 m NMR}$ pprox 0.020 ppm
 - Depends on signal duration, S/N
 - See R. Prigl *et al.*, Nucl. Inst. Meth. A 374, 118 (1996).

• Block diagram of the proposed NMR electronics shown.



• Multiplexer connects to 20 NMR probes, and contains a duplexer and preamplifier

• DL611 frequency counter, NIM modules, multiplexers, NMR probes from E821 will be refurbished for E989; parts shaded red are new

- Construct absolute calibration probe with spherical water sample at known temperature
- \Rightarrow Larmor frequency of proton in spherical water sample related to that of free proton by :

$$\omega_p(\mathrm{sph} - \mathrm{H}_2\mathrm{O}, T) = [1 - \sigma(\mathrm{H}_2\mathrm{O}, T)] \,\omega_p(\mathrm{free}),$$

- $\sigma(H_2O,T) \approx$ 26 ppm, is the temperature-dependent diamagnetic shielding of the proton in a water molecule
- E821 absolute calibration probe properties known well enough to determine fields in terms of free protons to accuracy of 0.034 ppm



- E821 used this probe with accuracy of 0.050 ppm (limited in part by temp. uncertainties)
- E989 will repeat and improve study of probe properties, *improve temperature stability* and *monitoring* to reduce temperature related uncertainties, calibration goal is 0.035 ppm

- Systematic errors on E821 field measurements from 1999, 2000, 2001 listed below
- The final column lists the uncertainties anticipated for E989

Source of uncertainty	R99	R00	R01	E989
	[ppm]	[ppm]	[ppm]	[ppm]
Absolute calibration of standard probe	0.05	0.05	0.05	0.035
Calibration of trolley probes	0.20	0.15	0.09	0.03
Trolley measurements of B_0	0.10	0.10	0.05	0.03
Interpolation with fixed probes	0.15	0.10	0.07	0.03
Uncertainty from muon distribution	0.12	0.03	0.03	0.01
Inflector fringe field uncertainty	0.20	-	—	_
Time dependent external B fields	—	_	—	0.005
Others †	0.15	0.10	0.10	0.03
Total systematic error on ω_p	0.4	0.24	0.17	0.070
Muon-averaged field [Hz]: $\tilde{\omega}_p/2\pi$	61 791 256	61791595	61 791 400	_

- [†]Higher multipoles, trolley temperature ($\leq 0.05 \text{ ppm}/^{\circ} \text{C}$) and power supply voltage response (0.4 ppm/V, ΔV =50 mV), and eddy currents from the kicker.
- Note the steady reduction in uncertainties achieved in E821

Rate Estimates

Item	Factor	Value per fill
Protons on target		10 ¹² p
Positive pions captured in FODO, $\delta p/p = \pm 0.5\%$	$1.2 imes 10^{-4}$	1.2×10^8
Muons captured and transmitted to SR, $\delta p/p = \pm 2\%$	0.67%	$8.1 imes 10^5$
Transmission efficiency after commissioning	90%	$7.3 imes 10^5$
Transmission and capture in SR	$(2.5 \pm 0.5)\%$	$1.8 imes 10^4$
Stored muons after scraping	87%	$1.6 imes 10^4$
Stored muons after 30 μs	63%	1.0×10^4
Accepted positrons above $E = 1.86 \text{ GeV}$	10.7%	1.1×10^3
Fills to acquire 1.6×10^{11} events (100 ppb)		1.5×10^8
Days of good data accumulation	17 h/d	202 d
Beam-on commissioning days		150 d
Dedicated systematic studies days		$50 \mathrm{d}$
Approximate running time		$402\pm80~{\rm d}$
Approximate total proton on target request		$(3.0 \pm 0.6) \times 10^{20}$

Table 5.1: Event rate calculation using a bottom-up approach.