Hyperon-nucleon interaction in chiral effective field theory

Johann Haidenbauer

IAS & JCHP, Forschungszentrum Jülich, Germany

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The YN interaction

- study the role of strangeness in low and medium energy nuclear physics
- test $SU(3)_{flavor}$ symmetry
- H dibaryon
  - Jaffe (1977) $\longrightarrow$ deeply bound 6-quark state with $I = 0$, $J = 0$, $S = -2$
  - many experimental searches but no convincing signal
  - Lattice QCD (2010) $\longrightarrow$ evidence for a bound H dibaryon
- prerequisite for studies of (Λ, Σ) hypernuclei and hyperons in nuclear matter
- quest for $\Lambda\Lambda$ hypernuclei and Ξ hypernuclei
  $\longrightarrow$ J-PARC, FAIR
- implications for astrophysics
  $\longrightarrow$ hyperon stars
  stability/size of neutron stars

Chiral Effective Field Theory

- Starting point: Chiral Lagrangian
  $\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{EFT} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$
- Spontaneous chiral symmetry breaking of QCD
  $\rightarrow$ pions are Goldstone bosons
- Power counting, Separation of scales

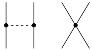
  Systematic expansion in powers of Q/Λ_χ & m_π/Λ_χ ($\Lambda_\chi \approx 1$ GeV)

  light dof (pion) $\ll \Lambda_\chi \leq$ heavy dof (ρ, ω, ... $\rightarrow$ contact terms (LECs))
- pion-pion and pion-nucleon sectors are perturbative in Q
  $\rightarrow$ chiral perturbation theory
- *NN* interaction requires non-perturbative resummation
  (bound states, large scattering lengths)
  $\rightarrow$ chirally expand V_{NN},
  use in a regularized Lippmann-Schwinger equation

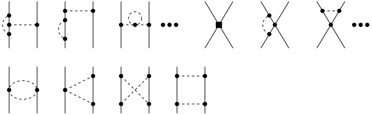
  (Weinberg (1991), van Kolck, Epelbaum/Meißner, Entem/Machleidt, ...)

NN interaction in chiral effective field theory

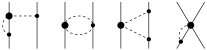
Leading order



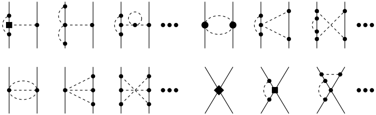
Next−to−leading order



Next−to−next−to−leading order



Next−to−next−to−next−to−leading order

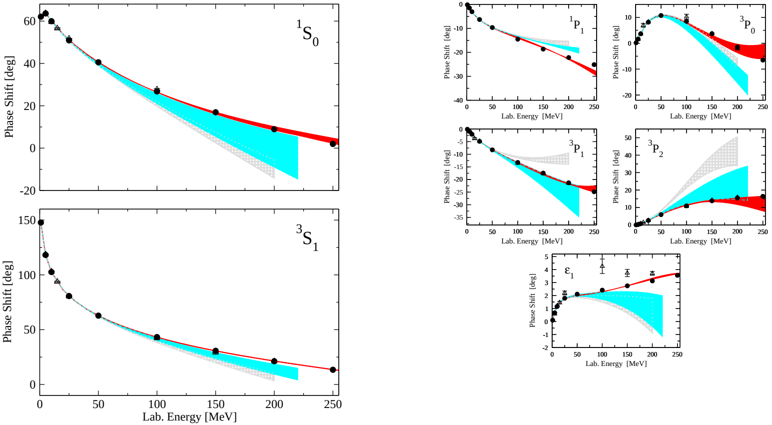


LO: 2 LECs

NLO: 7 LECs

N^3LO: 15 LECs

NN interaction in chiral effective field theory



NLO, *N^2LO*, *N^3LO*

(E. Epelbaum, W. Glöckle, Ulf-G. Meißner, NPA 747 (2005) 362)

YN interaction in chiral effective field theory

We follow the scheme of S. Weinberg (1990)
in complete analogy to the χEFT study of the NN interaction by
E. Epelbaum, W. Glöckle, U.-G. Meißner (NPA 671 (2000) 295)

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor

- only about 40 data points
- no polarization data $\Rightarrow$ no phase shift analysis
- need to fix the LECs by a fit directly to YN data
- constraints from hypernuclei ($^3_\Lambda$H binding energy)

$\rightarrow$ impose $SU(3)_f$ constraints

(J.H, N. Kaiser, U. Meißner, A. Nogga, S. Petschauer, W. Weise, NPA 915 (2013) 24)

Contact terms for BB

e.g., LO contact terms for BB:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right) \quad \Rightarrow \quad \begin{aligned} \mathcal{L}^1 &= \tilde{C}_i^1 \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_b \left(\Gamma_i B\right)_a \right\rangle , \\ \mathcal{L}^2 &= \tilde{C}_i^2 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle , \\ \mathcal{L}^3 &= \tilde{C}_i^3 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \right\rangle \left\langle \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \end{aligned}$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

a, b ... Dirac indices of the particles

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

C_i, $\tilde{C}_i$ ... low-energy constants (LECs)

Contact terms for BB up to NLO

spin-momentum structure of the contact term potential:

BB contact terms without derivatives (LO):

$$V^{(0)}_{BB\to BB} = C_{S,BB\to BB} + C_{T,BB\to BB}\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$

BB contact terms with two derivatives (NLO):

$$\begin{aligned}V^{(2)}_{BB\to BB} &= C_1\vec{q}^{\,2} + C_2\vec{k}^{\,2} + (C_3\vec{q}^{\,2} + C_4\vec{k}^{\,2})(\vec{\sigma}_1\cdot\vec{\sigma}_2)\\ &+ \frac{i}{2}C_5(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k}) + C_6(\vec{q}\cdot\vec{\sigma}_1)(\vec{q}\cdot\vec{\sigma}_2)\\ &+ C_7(\vec{k}\cdot\vec{\sigma}_1)(\vec{k}\cdot\vec{\sigma}_2) + \frac{i}{2}C_8(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k})\end{aligned}$$

note: $C_i \to C_{i,BB\to BB}\cdots$ but constrained by $SU(3)$ symmetry

$\vec{q} = \vec{p}\,' - \vec{p}$; $\vec{k} = (\vec{p}\,' + \vec{p})/2$

$SU(3)$ structure of contact terms for BB

$SU(3)$ structure for scattering of two octet baryons →
$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

can re-express $C_{S,i}$, $C_{T,i}$, $C_{1,i}$, etc., in terms of LECs corresponding to the $SU(3)_f$ irreducible representations: C^1, C^{8_a}, C^{8_s}, C^{10^*}, C^{10}, C^{27}

	Channel	I	V_{1S0}, $V_{3P0,3P1,3P2}$	V_{3S1}, $V_{3S1-3D1}$, V_{1P1}	$V_{1P1-3P1}$
$S = \ \ 0$	$NN \to NN$	0	–	C^{10^*}	–
	$NN \to NN$	1	C^{27}	–	–
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{-1}{\sqrt{20}}C^{8_s8_a}$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}}C^{8_s8_a}$ $\frac{-1}{\sqrt{20}}C^{8_s8_a}$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}}C^{8_s8_a}$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C^{27}	C^{10}	–

No. of contact terms: LO: 2 (NN) + 3 (YN) + 1 (YY)
NLO: 7 (NN) + 11 (YN) + 4 (YY)

Pseudoscalar-meson exchange

$\mathrm{SU(3)}_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} = -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\left\{\partial_\mu P, B\right\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\left[\partial_\mu P, B\right]\right\rangle$$

$$f = g_A/(2F_\pi); \quad g_A \simeq 1.26, \quad F_\pi \approx 93 \text{ MeV}$$

$$\alpha = F/(F+D) \text{ with } g_A = F + D$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\begin{array}{llllll}
f_{NN\pi} = & f & f_{NN\eta_8} = & \frac{1}{\sqrt{3}}(4\alpha-1)f & f_{\Lambda NK} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f \\
f_{\Xi\Xi\pi} = & -(1-2\alpha)f & f_{\Xi\Xi\eta_8} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f & f_{\Xi\Lambda K} = & \frac{1}{\sqrt{3}}(4\alpha-1)f \\
f_{\Lambda\Sigma\pi} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma\Sigma\eta_8} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma NK} = & (1-2\alpha)f \\
f_{\Sigma\Sigma\pi} = & 2\alpha f & f_{\Lambda\Lambda\eta_8} = & -\frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Xi\Sigma K} = & -f
\end{array}$$

Pseudoscalar-meson exchange

One-pseudoscalar-meson exchange (V^{OBE}) [LO]

$$V^{OBE}_{B_1 B_2 \to B_1' B_2'} \quad = \quad -f_{B_1 B_1' P} f_{B_2 B_2' P} \frac{\left(\vec{\sigma}_1 \cdot \vec{q}\right) \left(\vec{\sigma}_2 \cdot \vec{q}\right)}{\vec{q}^{\,2} + m_P^2}$$

$f_{B_1 B_1' P}$ ... coupling constants
m_P ... mass of the exchanged pseudoscalar meson

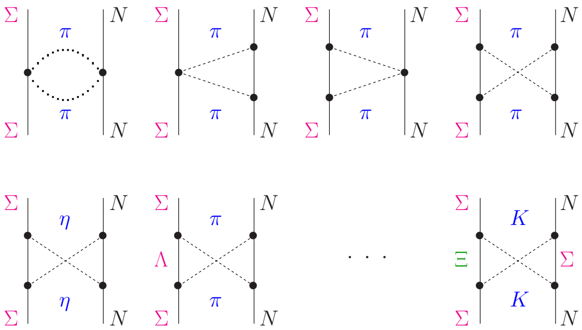
- dynamical breaking of $SU(3)$ symmetry due to the mass splitting of the ps mesons
  (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)
  taken into account already at LO!

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244; PLB 653 (2007) 29)

Two-pseudoscalar-meson exchange diagrams

Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



⇒ J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24

Coupled channels Lippmann-Schwinger Equation

$$
\begin{aligned}
T^{\nu'\nu,J}_{\rho'\rho}(p',p) &= V^{\nu'\nu,J}_{\rho'\rho}(p',p) \\
&+ \sum_{\rho'',\nu''} \int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V^{\nu'\nu'',J}_{\rho'\rho''}(p',p'')\, \frac{2\mu_{\rho''}}{p^2-p''^2+i\eta}\, T^{\nu''\nu,J}_{\rho''\rho}(p'',p)
\end{aligned}
$$

$\rho', \ \rho = \Lambda N, \Sigma N$

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$
V^{\nu'\nu,J}_{\rho'\rho}(p',p) \rightarrow f^\Lambda(p')\, V^{\nu'\nu,J}_{\rho'\rho}(p',p)\, f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}
$$

consider values Λ = 450 - 700 MeV [500 - 650 MeV]

NLO calculation

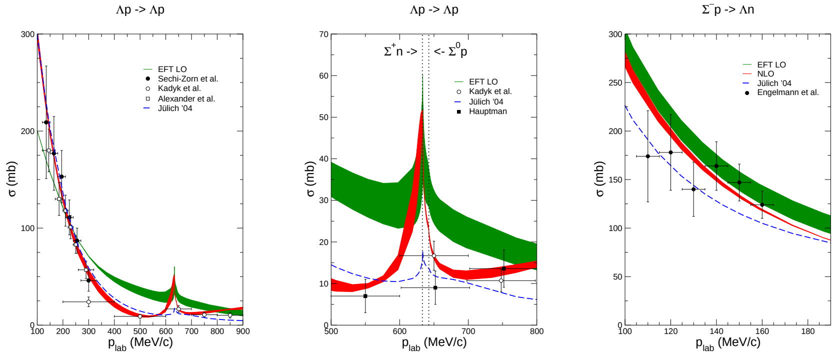
Pseudoscalar-meson exchange

- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$ symmetry is broken by using the physical m_π, m_K, and m_η
- $SU(3)$ breaking in the coupling constants is ignored
  $F_\pi = F_K = F_\eta = F_0 = 93$ MeV; $\alpha = 0.4$
- Correction to V^{OBE} due to baryon mass differences are ignored

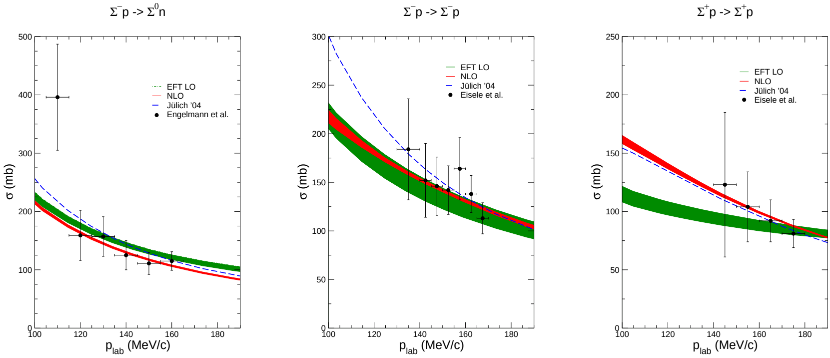
Contact terms

- $SU(3)$ symmetry is assumed
  (at NLO $SU(3)$ breaking corrections to the LO contact terms arise!)
- 10 contact terms in S-waves
  no SU(3) constraints from the NN sector are imposed!
- 12 contact terms in P-waves and in ${}^3S_1 - {}^3D_1$
  SU(3) constraints from the NN sector are imposed!
- 1 contact term in ${}^1P_1 - {}^3P_1$ (singlet-triplet mixing) is set to zero

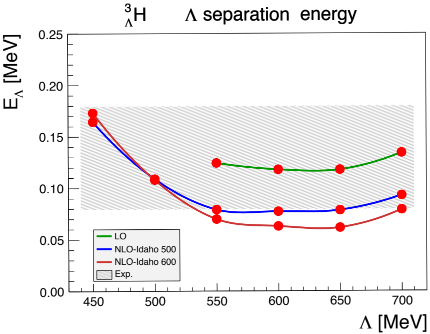
YN integrated cross sections



YN integrated cross sections

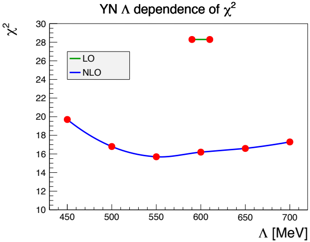


Hypertriton



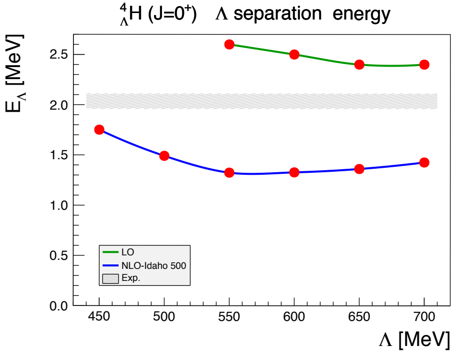
separation energies:

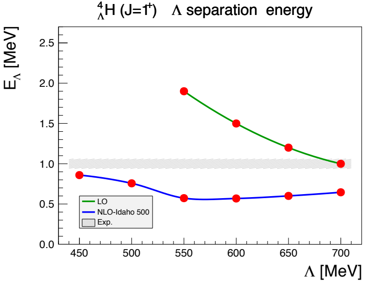
$$E_\Lambda = E(core) - E(hypernucleus)$$



- singlet scattering length for one cutoff chosen so that hypertriton binding energy is OK
- cutoff variation
  - is **lower bound** for magnitude of higher order contributions
  - correlation with χ^2 of YN interaction ?
- long range 3BFs need to be explicitly estimated

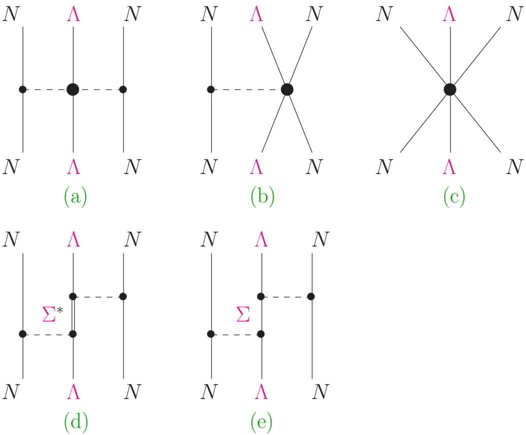
$^{4}_{\Lambda}$H





- LO/NLO results: LO uncertainty in 0^{+} is underestimated by cutoff variation
- NLO results in line with model results, implies underbinding
- long range 3BFs need to be explicitly estimated

Three-body forces



(a) - (c) appear at N2LO

(d) appears at NLO – in EFT that includes decuplet baryons

(e) is already included by solving coupled-channel Faddeev equations

Nuclear matter properties

conventional first-order Brueckner calculation:

Partial-wave contributions to $-U_\Lambda(p_\Lambda = 0)$ (in MeV) at $k_F = 1.35\ \mathrm{fm}^{-1}$ (for Λ = 600 MeV)

	$^1S_0$	$^3S_1 + ^3D_1$	$^3P_0$	$^1P_1 + ^3P_1$	$^3P_2 + ^3F_2$	Total
EFT LO	12.0	25.5	1.7	−3.3	0.4	36.5
EFT NLO	12.5	12.0	−0.9	−2.1	1.1	22.9
Jülich '04	9.9	35.0	0.7	0.2	3.3	49.7
Jülich '94	3.6	27.2	−0.6	−2.0	0.8	29.8
NSC97f	14.4	22.9	−0.5	−6.4	0.7	31.1

"Empirical" value for the Λ binding energy in nuclear matter:

$\approx$ 30 MeV

Nuclear matter properties

	EFT LO	EFT NLO	Jülich '04	Jülich '94	NSC97f
Λ [MeV]	$550 \cdots 700$	$500 \cdots 650$			
$-U_\Lambda(0)$	$38.0 \cdots 34.4$	$29.3 \cdots 22.9$	49.7	29.8	31.1
$-U_\Sigma(0)$	$-28.0 \cdots -11.1$	$-17.4 \cdots -12.1$	22.2	71.45	16.1

ΣN (I=3/2): ${}^3S_1$–${}^3D_1$: decisive for Σ properties in nuclear matter

- A description of YN data is possible with an attractive as well as a repulsive ${}^3S_1$–${}^3D_1$ interaction
- adopt the repulsive solution in accordance with evidence from
  - level shifts and widths of Σ^- atoms
  - (π^-, K^+) inclusive spectra related to Σ^- formation in heavy nuclei

Summary

YN interaction based on chiral EFT

- approach is based on a modified Weinberg power counting, analogous to the NN case
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- Excellent results at next-to-leading order (NLO)
  YN data are reproduced with a quality comparable to phenomenological models
- Hypertriton binding energy is reproduced
  Some underbinding in ${}^{4}_{\Lambda}\mathrm{H}$ – Is that a signal for missing 3BFs?
- Λ and Σ in nuclear matter:
  Λ single-particle potential at nuclear matter saturation density is in line with “empirical” value
  a repulsive Σ single-particle potential is achieved