Medium-heavy nuclei from lattice quantum chromodynamics

Takashi Inoue, Nihon University

for

HAL QCD Collaboration

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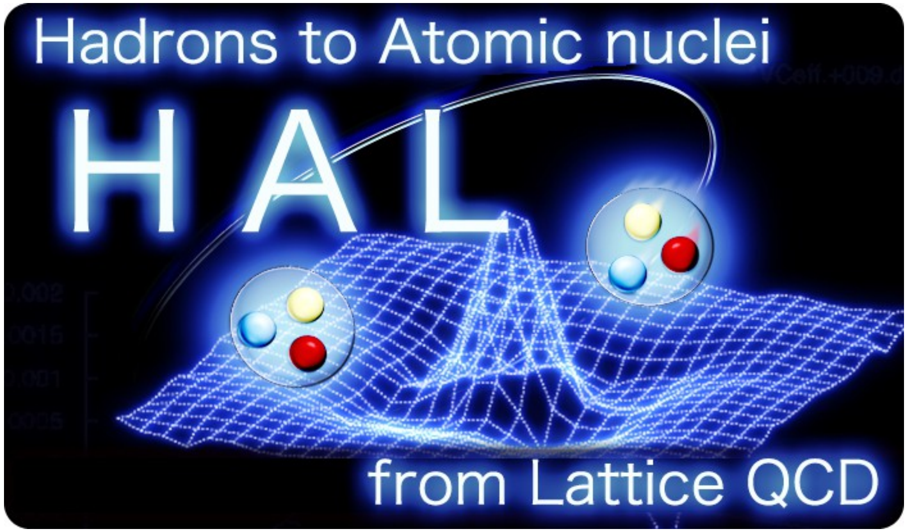

PANIC 2014, Aug 25 to 29, Hamburg

Introduction

✯ Nuclear physics

- Theories have been developed extensively from 1950's
  - Closed nuclei and matters → mean field approximation.
  - Odd nuclei → shell models by fixing core nucleus.
  - Light nuclei → variational methods with exact Hamiltonian.
  - And more sophisticated theories and methods in these days.
- Properties of nuclei have been reproduced and predicted.
- But, need to use a phenomenological nuclear force as an input.

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✯ Quantum Chromodynamics

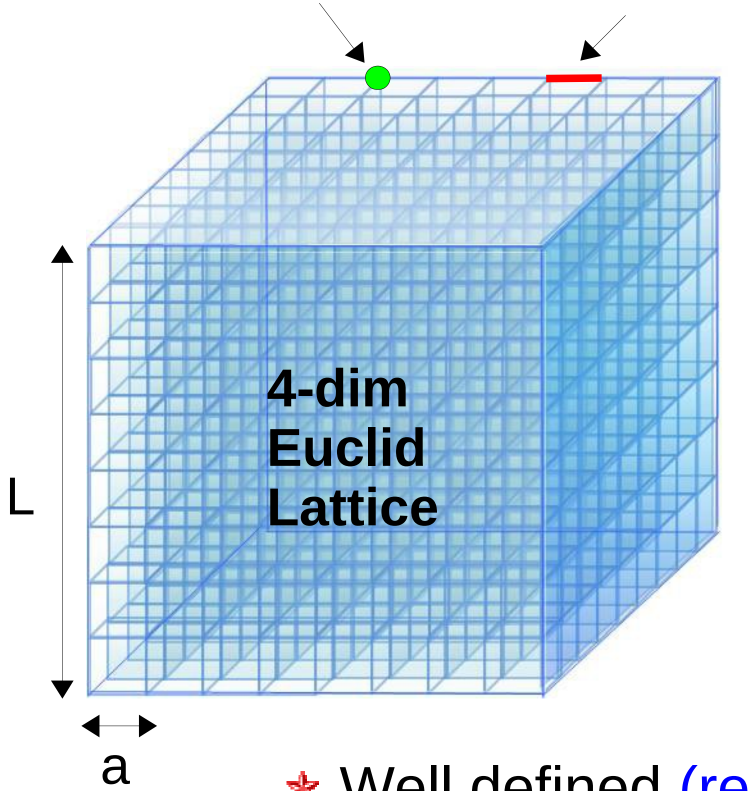
- is the fundamental theory of the strong interaction,
- must govern nuclear properties as well as hadron spectrum.
- But, explaining nuclei from QCD is one of most challenging problem in physics due to the non-perturbative nature of QCD.

Lattice QCD

$$L=-\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}+\bar{q}\gamma^\mu\left(i\partial_\mu-g\,t^a A^a_\mu\right)q-m\bar{q}q$$

quarks q
on the sites

gluons $U=e^{iaA_\mu}$
on the links



Vacuum expectation value

$$\langle O(\bar{q},q,U)\rangle$$

path integral

$$=\int dU\,d\bar{q}\,dq\,e^{-S(\bar{q},q,U)}\,O(\bar{q},q,U)$$

$$=\int dU\,\det D(U)\,e^{-S_U(U)}\,O(D^{-1}(U))$$

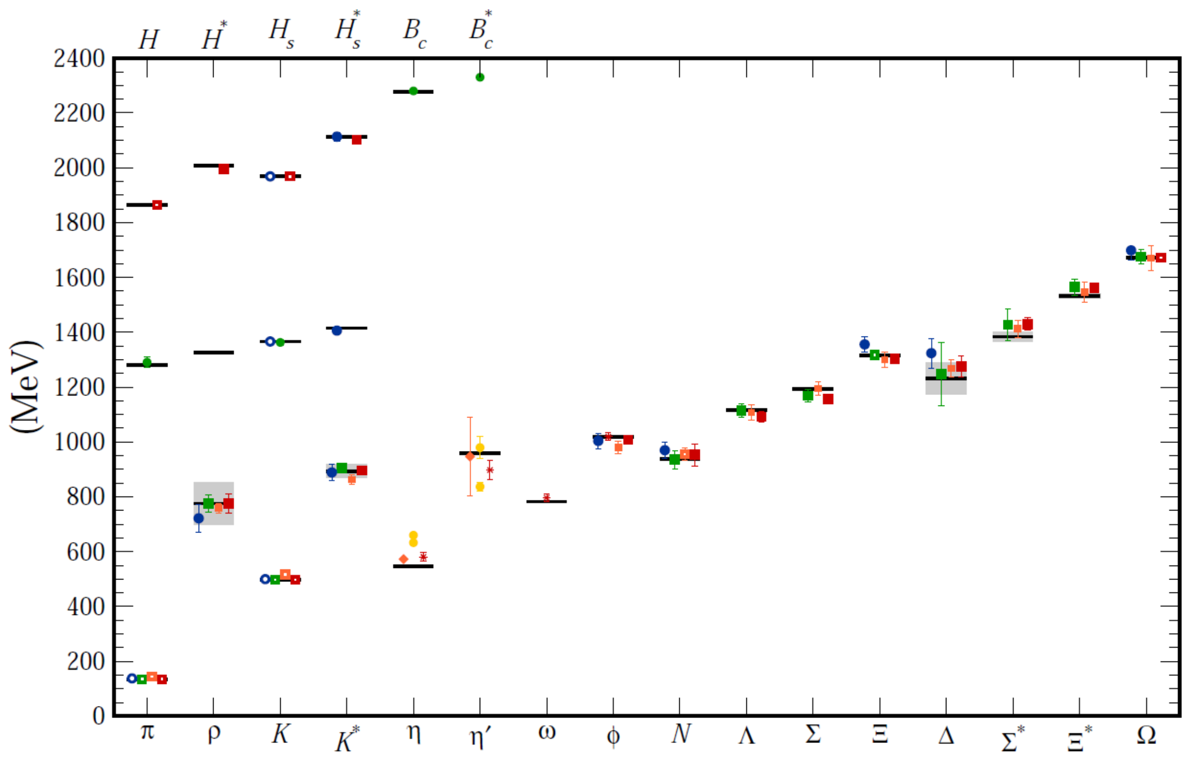
quark propagator

$$=\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}O(D^{-1}(U_i))$$

{ U_i } : ensemble of gauge conf. U
generated w/ probability $\det D(U)\,e^{-S_U(U)}$

- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturvative
- Highly predictive

Lattice QCD



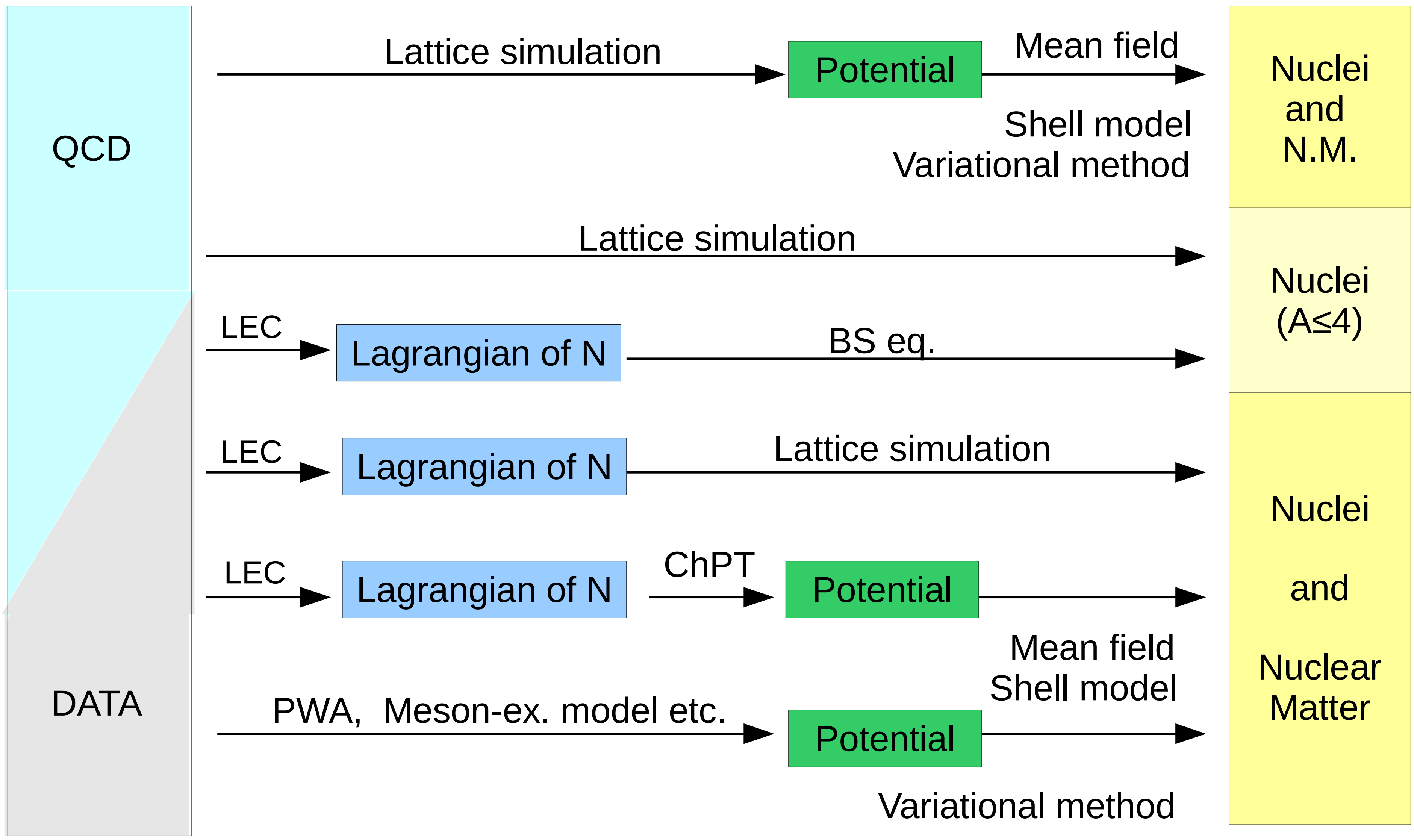
Summary by Kronfeld
arXive 1203.1204

- LQCD simulations at the physical quark mass ware done.

  PACS-CS Colla. Phys. Rev. D81 (2010) 075403
  BMW Colla. JHEP 1108 (2011) 148

- Hadrons are well reproduced! What about nuclei?

Various approaches to nuclei



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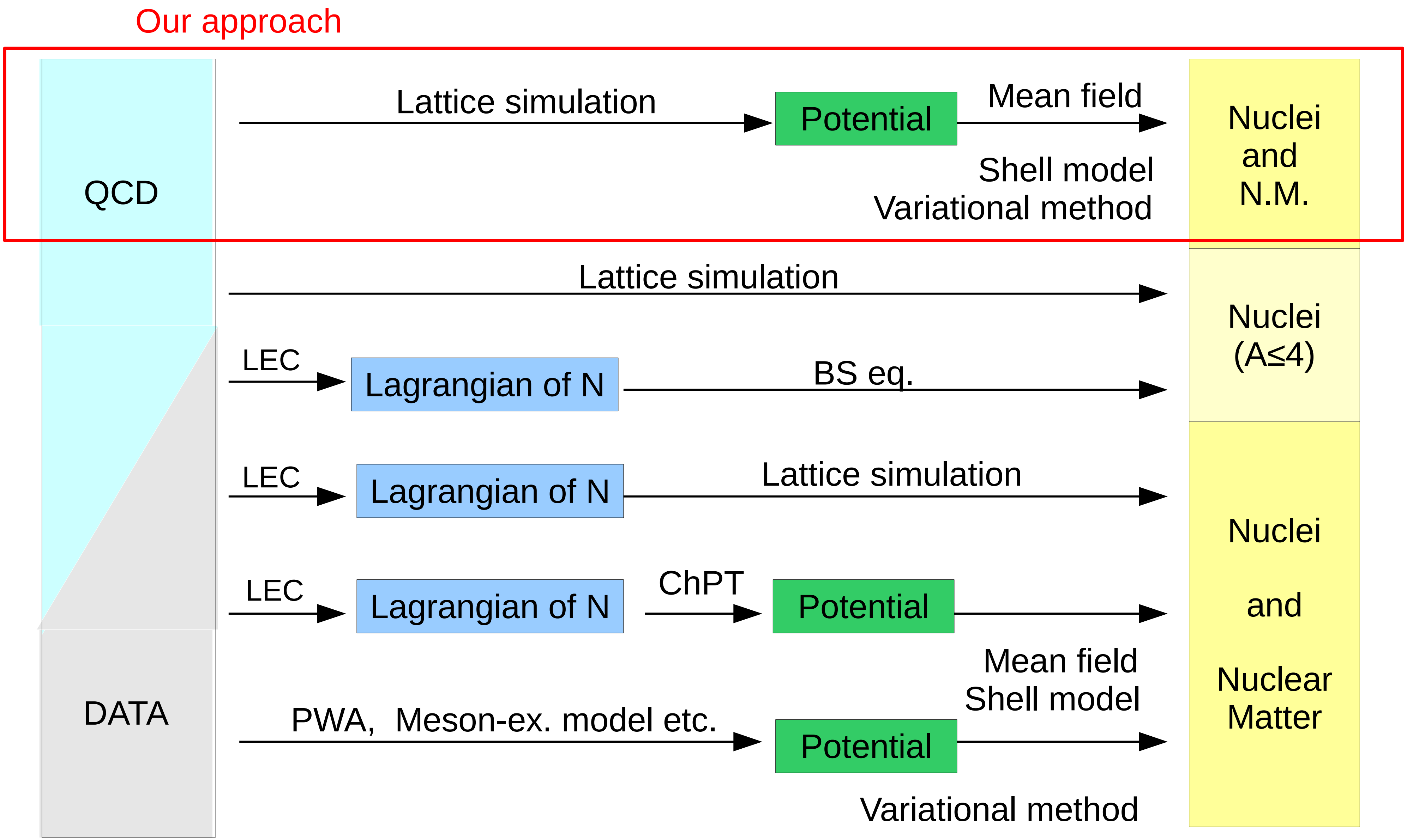

Introduction

- Difficulties in the 2nd approach (LQCD direct).
  - Prohibitively large number of contraction for large A (> 50).
  - Prohibitively expensive to separate energy eigenstates (A>4).
  - Very large volume is required, especially for nuclei with haro.
  - Difficult to access detailed structure of nuclei s.t. the Hoyle state.

    ➔ There are only few works.

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- Our approach overcome these difficulties. In fact, today, we get mass and structure of ^{16}O and ^{40}Ca in QCD.
  - ^{16}O, ^{40}Ca are simple to study (doubly closed, iso-sym. etc)
  - HAL QCD method **+** Brueckner-Hartree-Fock framework
  - We'll show that out approach is quite promising in practice.
  - But, at present, we suffer from many limitations as you will see.
    So, my talk today is just a first one step to our goal.

Outline

1. Introduction
2. HAL QCD method to multi-hadron in LQCD
3. Simulation setup and NN potentials from LQCD
4. Medium-heavy nuclei in LQCD
5. Summary and Plan

HAL QCD method
to
multi-hadron in LQCD

HAL QCD method

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$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

4-point function
$\psi(\vec{r},t) \ni$ NBS W.F.

  - Advantages
    - No need to separate E eigenstate.
      Just need to measure
      Then, potential can be extracted.
    - Demand a minimal lattice volume.
      No need to extrapolate to V=∞.
    - Can output many observables.

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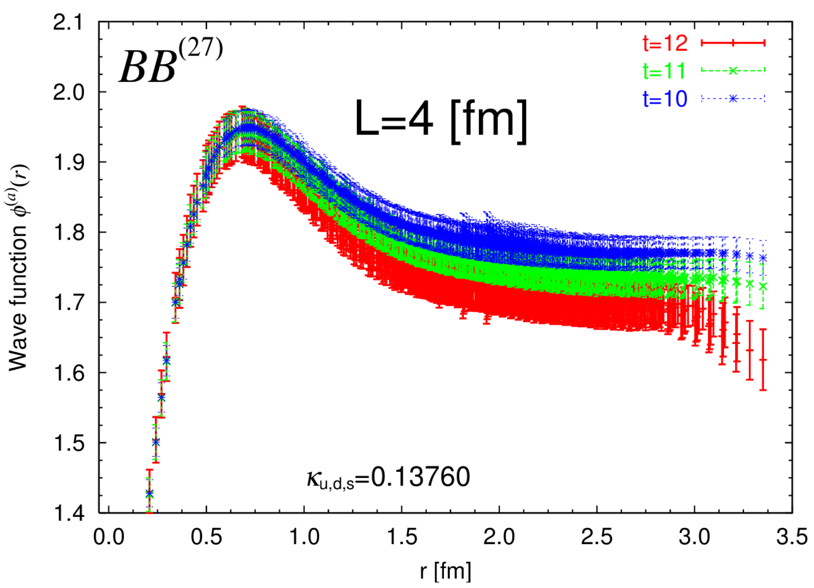

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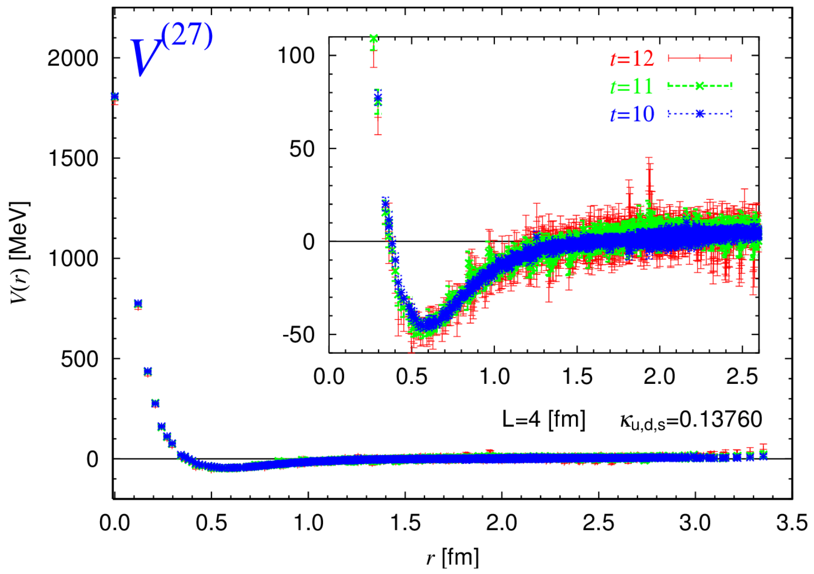

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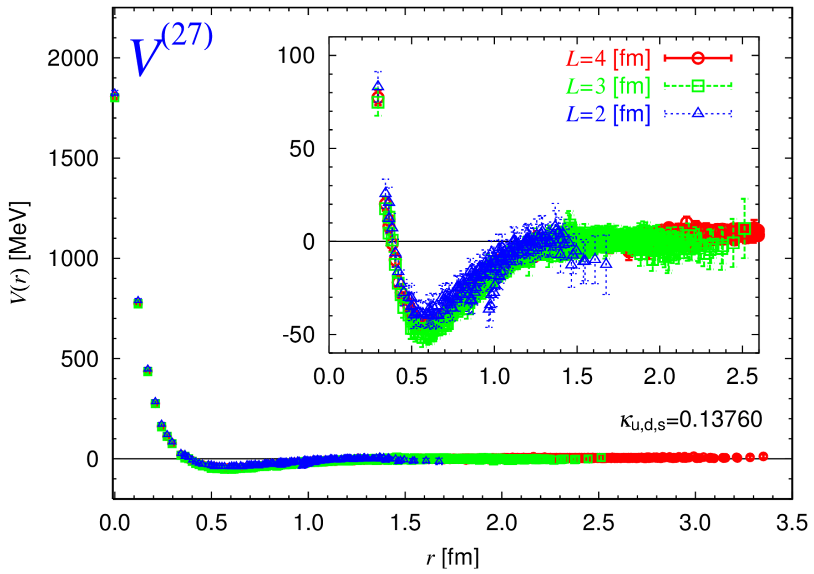

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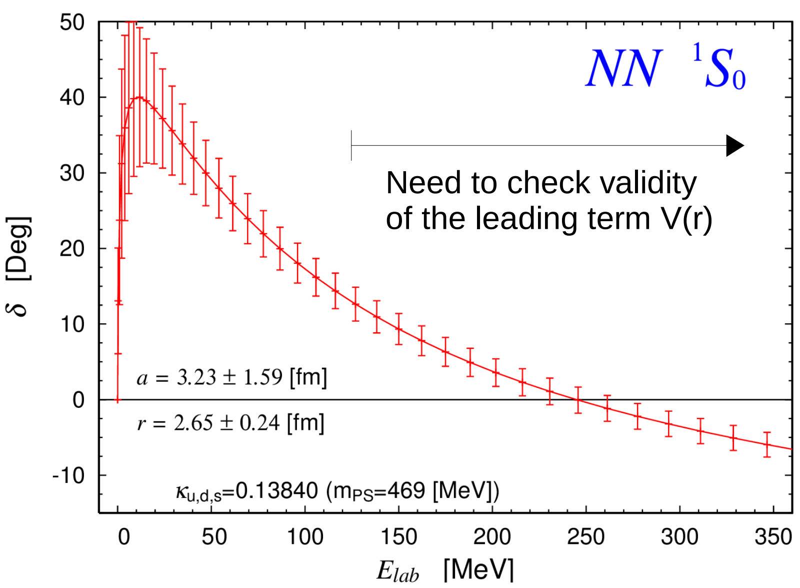

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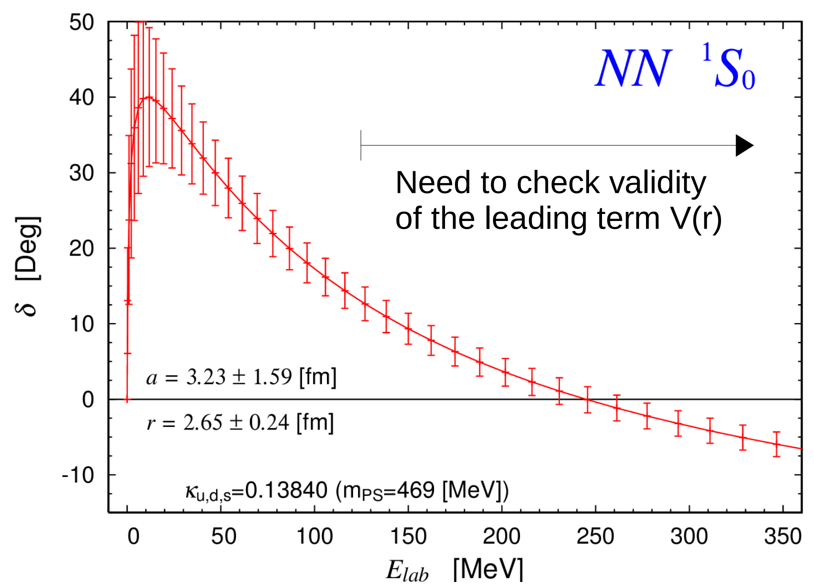


☆ We can attack large nuclei too!!

HAL method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)
N. Ishii etal. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function $\varphi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}, t) B_j(\vec{x}, t) | B=2, \vec{k} \rangle$

DEFINE a potential U for all E eigenstates through a "Schrödinger eq."

$$\left[-\frac{\nabla^2}{2\mu} \right] \varphi_{\vec{k}}(\vec{r}) + \int d^3\vec{r}\,' U(\vec{r}, \vec{r}\,') \varphi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}} \varphi_{\vec{k}}(\vec{r})$$

Non-local but energy independent

4-point function $G(\vec{x}, \vec{y}, t-t_0) = \langle 0 | B_i(\vec{x}, t) B_j(\vec{y}, t)\ J(t_0) | 0 \rangle$

We measure $\psi(\vec{r}, t) = \sum_{\vec{x}} G(\vec{x}+\vec{r}, \vec{x}, t-t_0) = \sum_{\vec{k}} A_{\vec{k}} \varphi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$

$$\left[2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r}, t) + \int d^3\vec{r}\,' U(\vec{r}, \vec{r}\,') \psi(\vec{r}\,', t) = -\frac{\partial}{\partial t} \psi(\vec{r}, t)$$

∇ expansion & truncation $\quad U(\vec{r}, \vec{r}\,') = \delta(\vec{r}-\vec{r}\,') V(\vec{r}, \nabla) = \delta(\vec{r}-\vec{r}\,') [V(\vec{r}) + \cancel{\nabla + \nabla^2} \ldots]$

Therefor, in the leading

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\frac{\partial}{\partial t} \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2M_B$$

Simulation setup
and
NN potentials from QCD

Simulation setup

T. Inoue etal, Nucl. Phys. A881, 28 (2012)

- Gauge confs. of PACS-CS, BMW at physical quark mass, lattice volume ware small even for NN system. $L < 2\ fm$

- We've generated gauge confs. at five flavor SU(3) limits

size	β	C_{SW}	a [fm]	L [fm]
32^3 x 32	1.83	1.761	0.121(2)	3.87

K_uds	M_P.S. [MeV]	M_B8 [MeV]
0.13660	1170.9(7)	2274(2)
0.13710	1015.2(6)	2031(2)
0.13760	836.8(5)	1749(1)
0.13800	672.3(6)	1484(2)
0.13840	468.9(8)	1161(2)

limitation

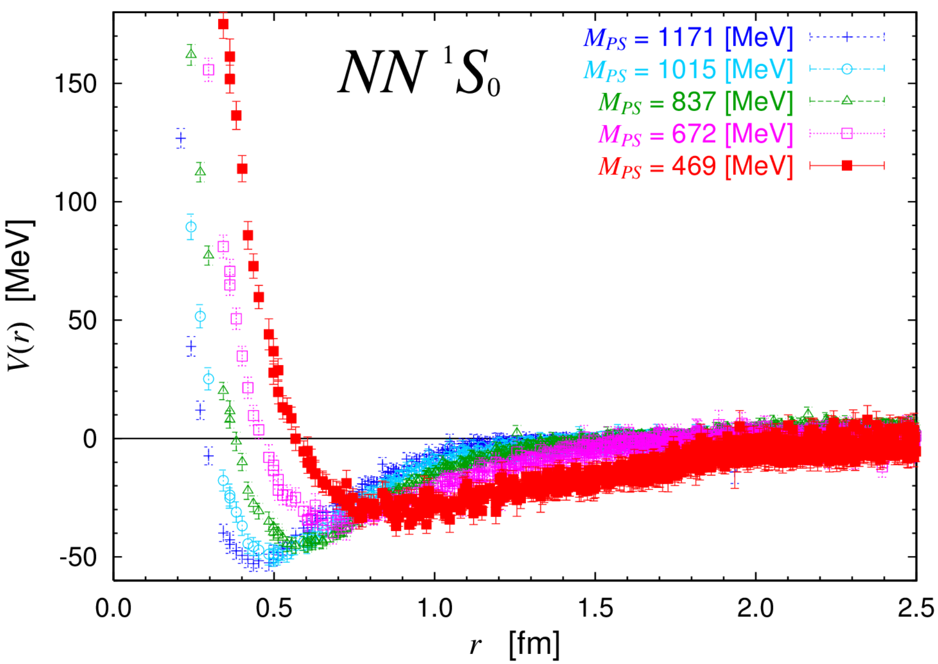
  - Iwasaki gauge & Wilson quark.
  - Thanks to PACS-CS collaboration for their DDHMC/PHMC code.
  - SU(3)F limit is useful to capture essential feature of BB interaction

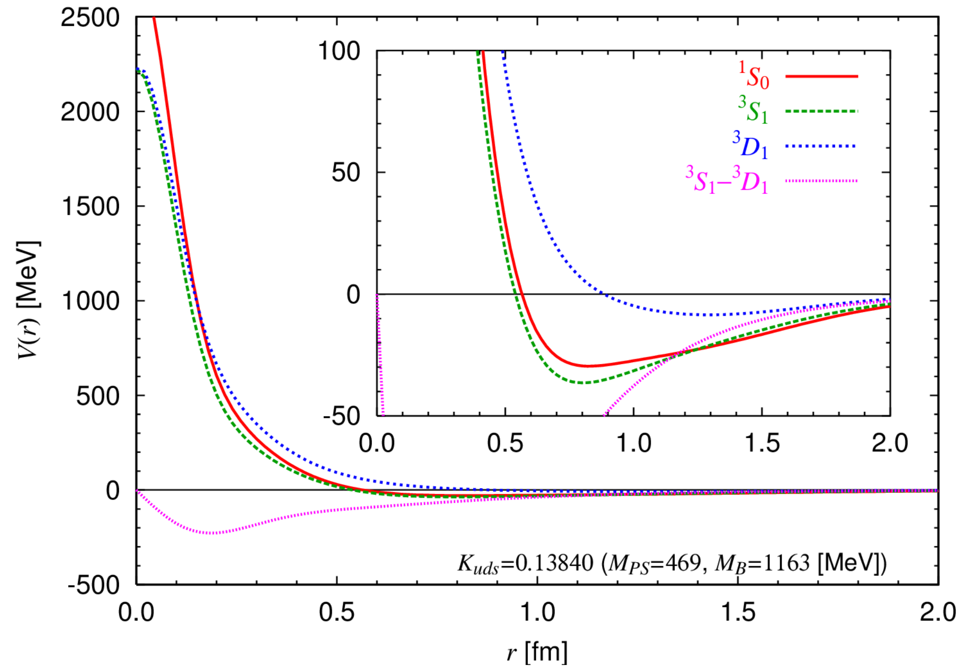
$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

$L > 8\ fm$

- Gauge confs. at physical quark mass with large volume being generated now at RIKEN AICS in Japan.

NN potentials from QCD





- Left: NN $^1S_0$ potential at five quark mass. (27-plet)
  - Repulsive core & attractive pocket grow as m_q decrease.
- Right: NN potential in partial waves at the lightest m_q.
  - Least χ^2 fit of data which gives <u>central value of observable</u>.

Medium-heavy nuclei
in LQCD

Brueckner-Hartree-Fock for nuclei

- Brueckner G-matrix in a single-particle-orbit base

$$G(\omega)_{ij,kl} = V^{LQCD}_{ij,kl} + \frac{1}{2}\sum_{m,n}^{>e_F} V^{LQCD}_{ij,mn} \frac{1}{\omega - e_m - e_n + i\epsilon} G(\omega)_{mn,kl}$$

- Hartree-Fock mean field $U_{ab} = \sum_{c,d} G(\tilde{\omega})_{ac,bd}\, \rho_{dc}$

- New single-particle orbit $[K+U]\,\Psi^i = e_i\,\Psi^i$

iterate until converge

- Hartree-Fock ground state energy

$$E_0 = \sum_{a,b}\left(K_{ab} + \frac{1}{2}U_{ab}\right)\rho_{ba} - K_{c.m.}$$

limitation of V^{LQCD}

- P.W. decomposition and truncation ${}^{2S+1}L_J = {}^1S_0,\ {}^3S_1,\ {}^3D_1,\ {}^1P_1,\ {}^3P_J \cdots$

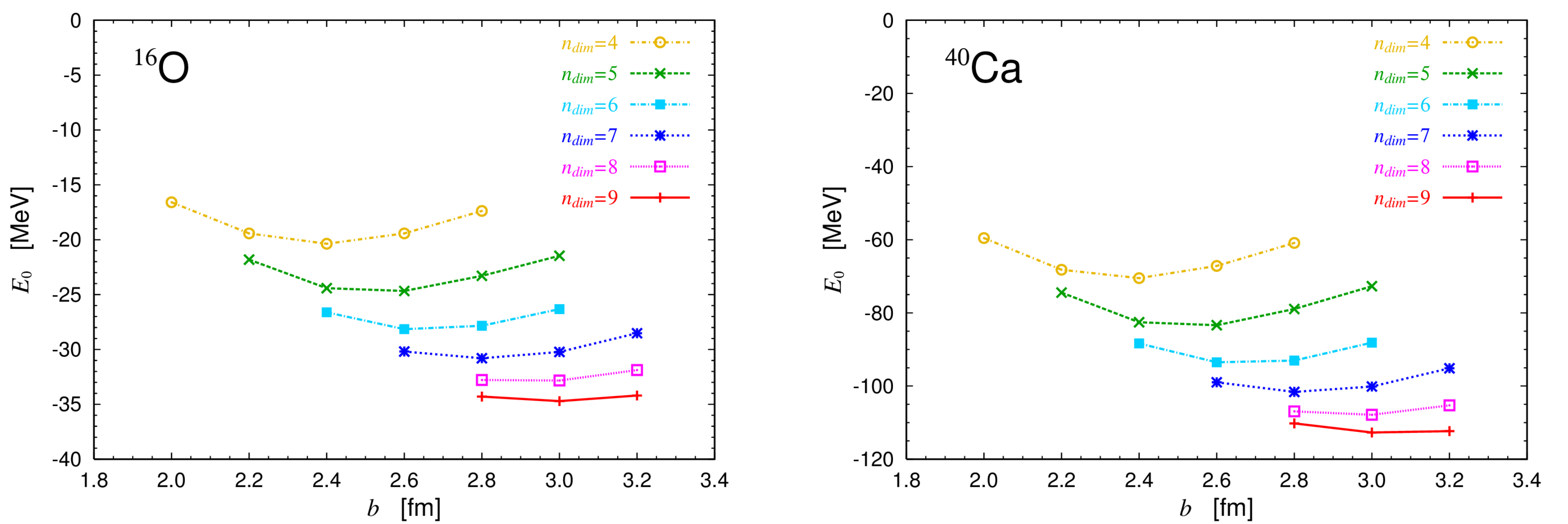
P. Ring and P. Schuck, “The Nuclear Many-Body Problems”, Springer (1980)
K.T.R. Davies, M. Baranger, R.M. Tarbutton T.T.S. Kuo, Phys. Rev. 177 1519 (1969)

Existence of nuclei in LQCD

- We use the Harmonic-Oscillator basis to expand s.p. states.

$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+3/2)}} \left(\frac{r}{b}\right)^l \sum_{m=0}^{n} C_{n-m}^{n+l+1/2} \frac{\left(-r^2/b^2\right)^m}{m!} \quad , \quad n = 0,\ 1\ \ldots\ n_{dim}-1$$

- Ground state energy E_0 at the lightest quark (M_{PS} = 470 MeV)



- There definitely exist nuclei ^{16}O and ^{40}Ca at this quark mass!
- We do not find any negative E_0 at the other four quark mass.

= un-existence of nuclei

Properties of nuclei in LQCD

- At the lightest quark (M_{PS} = 470 MeV) with n_{dim} = 9 and b = 3.0 fm

	Single particle levels [MeV]				Total binding [MeV]		Radius
^{16}O	E_{1S}	E_{1P}	E_{2S}	E_{1D}	E_0	E_0/A	<r> [fm]
	-35.8	-13.8			-34.7	-2.17	2.35

	Single particle levels [MeV]				Total binding [MeV]		Radius
^{40}Ca	E_{1S}	E_{1P}	E_{2S}	E_{1D}	E_0	E_0/A	<r> [fm]
	-59.0	-36.0	-14.7	-14.3	-112.7	-2.82	2.78

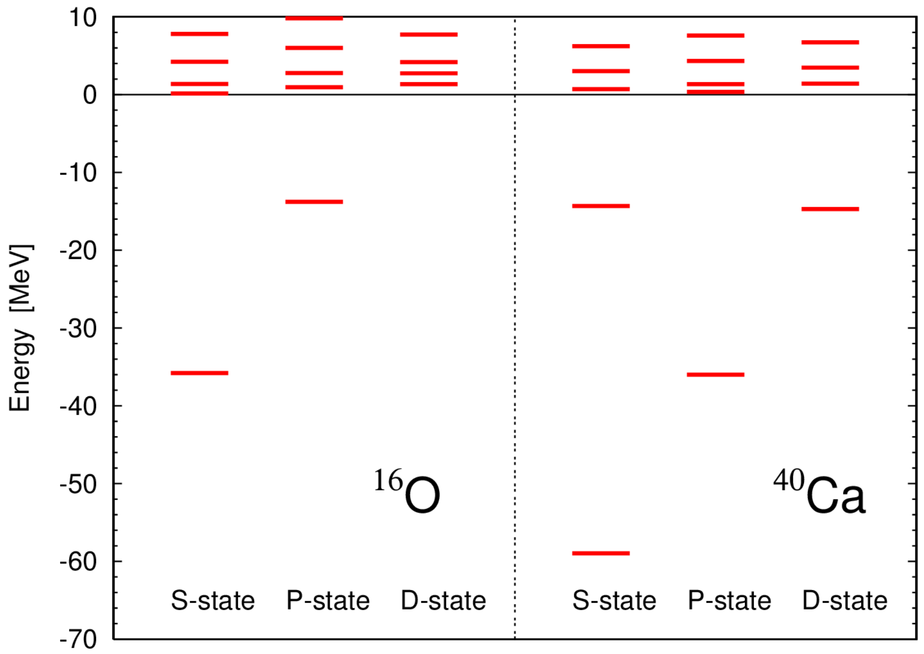
T.I. etal. [HAL QCD Colla.] arXive 1408.4892

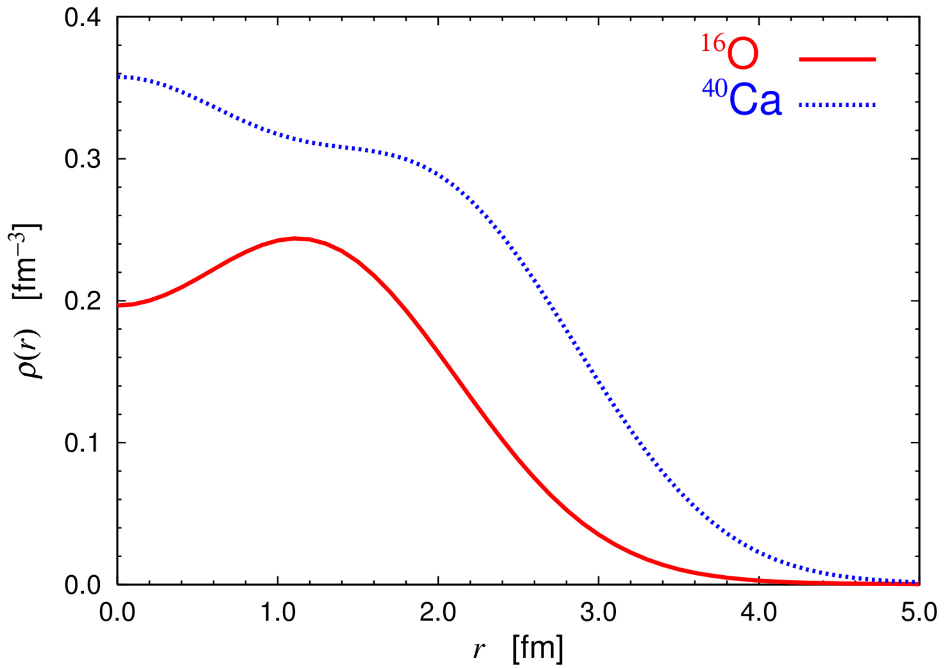
- Obtained binding energies are smaller than the expr. data

$$^{16}\mathrm{O}:\ E_0^{\mathrm{expr.}} = -127.6\ \mathrm{MeV},\quad ^{40}\mathrm{Ca}:\ E_0^{\mathrm{expr.}} = -342.0\ \mathrm{MeV}$$

principally due to the heavy quark mass in our calculation.

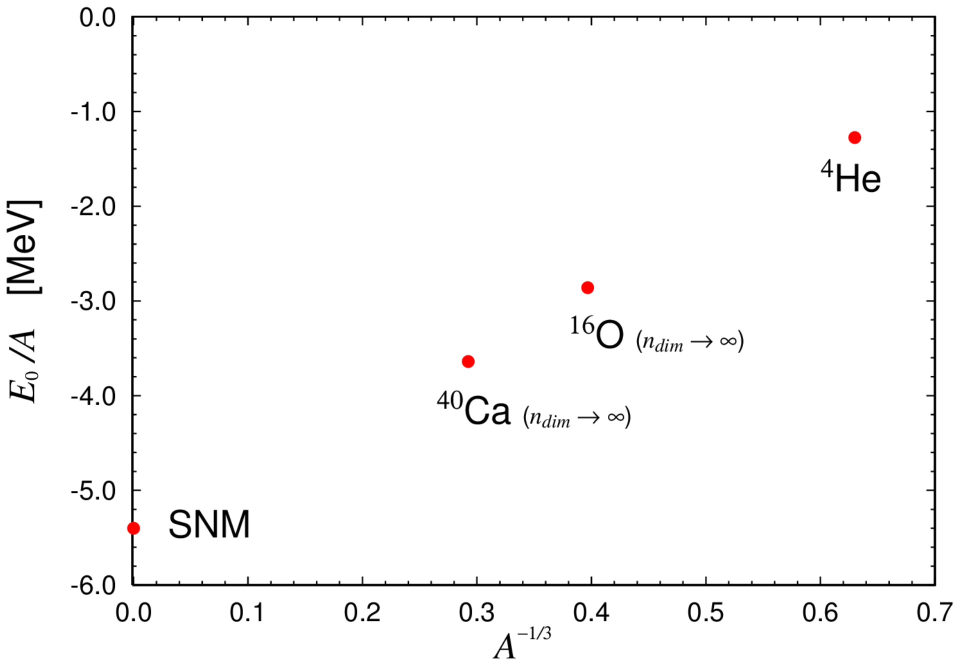
Properties of nuclei in LQCD





- Left: Single particle levels in the nucleus
  - Regular shell structure as seen in experimental data such that neutron/proton separation energy.
- Right: Nucleon number density in the nucleus
  - Distinct shell effect at short distance as seen in expr. data such that charge distribution from electron scattering.

Mass number A dependence



Bethe-Weizsacker mass formula
for nuclei in the real world

$$E_0(A) = a_V\ A + a_S\ A^{2/3} + \cdots$$

$$a_V = -15.7\ [\mathrm{MeV}]$$
$$a_S = \quad 18.6\ [\mathrm{MeV}]$$

T.I. etal. Nucl. Phys. A881 (2012)
T.I. etal. Phys. Rev. Lett. 111 (2013)

- E_0/A of ^{4}He and SNM ware obtained in our previous studies.
- E_0 of the nuclei with $n_{dim} = 9$ are extrapolated to $n_{dim} = \infty$ as

$$E_0(A;\, n_{dim}) = E_0(A;\, \infty) + c(A)/n_{dim}$$

- E_0/A in LQCD at the m_q has a reasonable A dependence which is well described by a BW type mass formula.

$$a_V = -5.46\ [\mathrm{MeV}],\ \ a_S = 6.56\ [\mathrm{MeV}]$$

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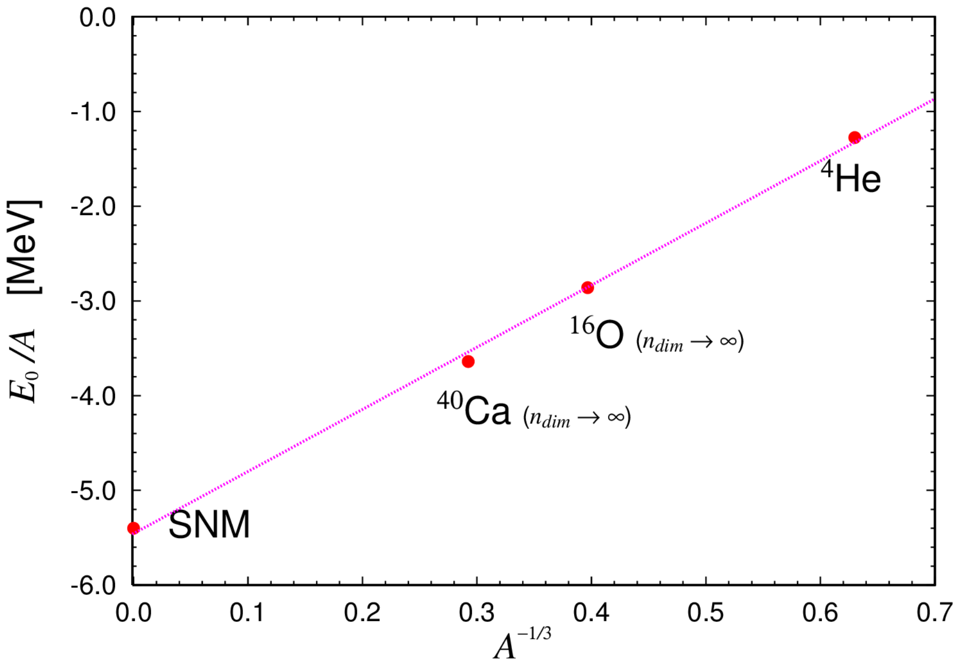

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Summary and Plan

- We introduced our purpose and strategy.
  - Explain properties of nuclei starting from QCD.
  - Extract potential of force in lattice QCD numerical simulation.
  - Apply potentials to a many-body theory, eg BHF framework.
- Results
  - We confirmed existence of large nuclei at one quark mass.
  - We could deduce mass and structure of ^{16}O and ^{40}Ca.
  - We found a reasonable A dependence of binding energy.
- Plan
  - Higher P.W. NN forces and NNN forces form LQCD.
  - LQCD simulation of nuclear force at the physical point.
  - It will open new connection from QCD to nuclear physics.

Thank you for your attention!!

Existence of nuclei in LQCD

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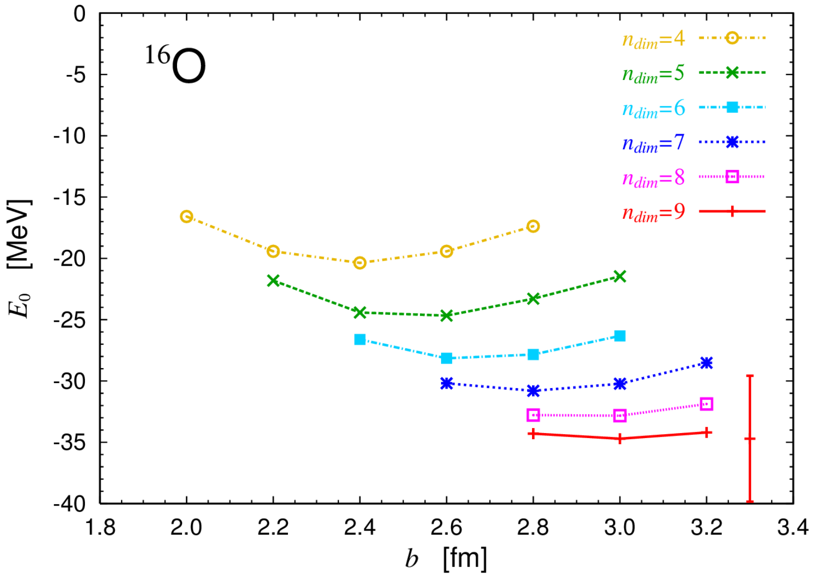

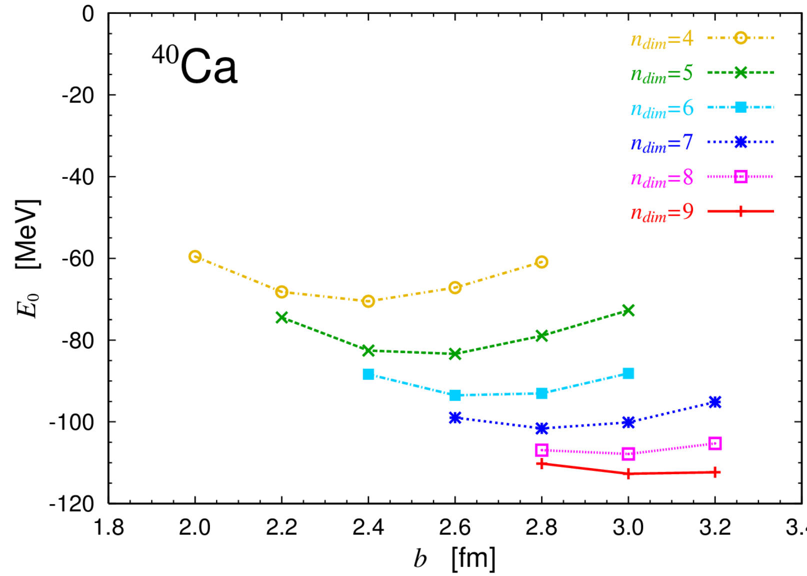

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