



Pileup subtraction at the particle level

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Outline



- Introducing the problem of pileup
- Brief overview of existing methods and a summary of latest developments
- Constituent subtraction
- Performance of the constituent subtraction on jets and jet shapes
- Full Event subtraction
- Conclusions



Introduction

- Pileup (=superposition of multiple pp collisions at each bunch spacing) in **pp collisions**. It affects:
 - jets and jet shapes variables, e.g. used to tag boosted objects
 - missing-pt and event shapes
 - photons and lepton isolation
- Underlying event in **heavy ion collisions**. It affects:
 - jets and jet internal structure used to study jet quenching
 - photons and lepton isolation
- Backgrounds at LHC influence practically each measurement – starting from Higgs ending with jet quenching => need to develop (or keep improving) methods for subtraction!

	Pileup (μ)	Additional energy
LHC run II	~50	15 GeV
HL LHC	~140	40 GeV
Heavy Ions (central)	~0	>100 GeV

mean number of collisions

energy underneath the $R=0.4$ jet



Basic overview of methods

Method	Works for any experiment?	Can do full event subtraction?	Can work without tracking?	Can work in heavy ions?	Is the code public?
4-momentum subtraction	✓	✗	✓	✓	✓
Shape expansion	✓	✗	✓	✗	✓
Grooming [*]	✓	✗	✓	✗	✓
Charged hadrons	✗ (CMS)	✓	✗	✗	-
Topoclusters	✗ (ATLAS)	✓	✓	✗	-
Cleansing ^{**}	✓	✗	✗	✗	✓
SoftKiller	✓	✓	✓	?	✓
PUPPI	✓	✓	✗	?	✗
Constituent Subtraction	✓	✓	✓	?	✓

main focus of this talk

per-particle methods

* Grooming is a general term including various methods (see backup) for details see e.g. [arXiv:1311.2708](https://arxiv.org/abs/1311.2708).

** Cleansing is not discussed in this presentation, method is described in [arXiv:1309.4777](https://arxiv.org/abs/1309.4777) and further discussed in [arXiv:1404.7353](https://arxiv.org/abs/1404.7353).

Basic four-momentum subtraction



Cacciari, Salam, PLB 659 (2008) 119

- Calculate the **background density** ρ as a median over patches of transverse momentum p_T in the area A

$$\rho = \text{median}_{i \in \text{patches}} \left\{ \frac{p_{Ti}}{A_i} \right\}$$

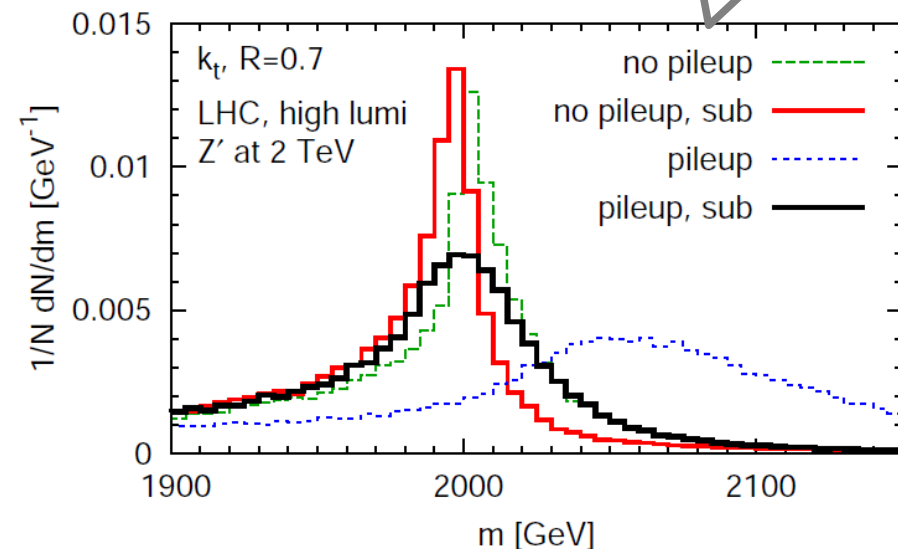
(The use of median excludes a bias from jets)

- Subtract the jet **four-momentum** as follows:

$$p_{\mu}^{(\text{sub})} = p_{\mu} - A_{\mu} \rho$$

- ... the basic scheme for subtraction of background both in pp and HI
- It corrects the kinematics, but it **cannot correct the internal structure** of the jet

Demonstration of performance on Z' mass reconstructed from di-jets



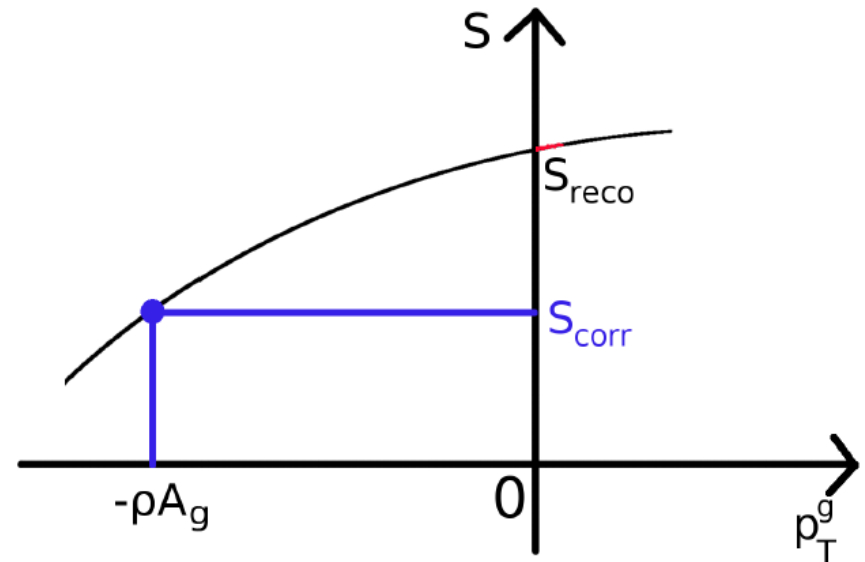


Shape expansion method

Soyez, et al. PRL 110 (2013) 162001

- Basic objects: **ghosts** = infinitesimally soft mass-less particles uniformly covering the η, ϕ space. Each ghost covers certain area A_g .
- Ghosts are clustered by the jet algorithm with real particles with no impact on kinematics of a jet.
- To subtract background, one might set for each ghost $p_{T,g} = -\rho A_g$
- It is cumbersome to work with negative particles => do **extra-polation** using Taylor expansion.
- Jet shape S = arbitrary function of jet constituents. Corrected jet shape:

$$S_{\text{corr}} = \sum_{k=0}^{\infty} (-\rho A_g)^k \cdot \left. \frac{\partial^k S(p_T^g)}{\partial p_T^{g k}} \right|_{p_T^g=0}$$



SoftKiller

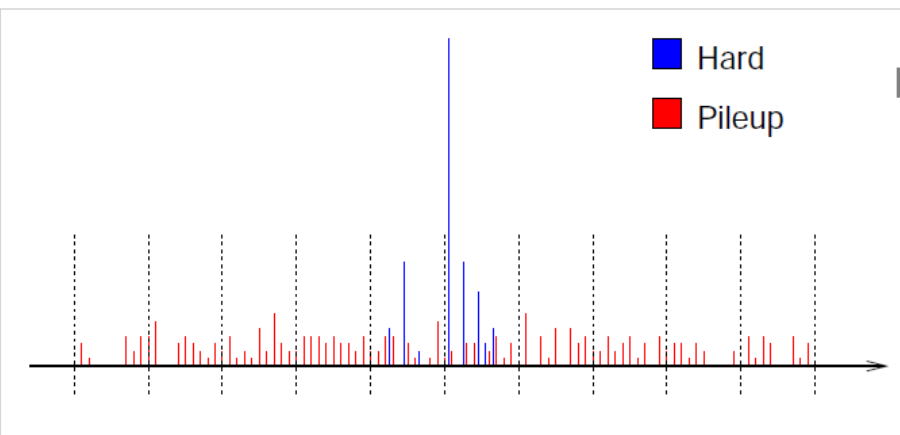


Cacciari et al. arXiv:1407.0408

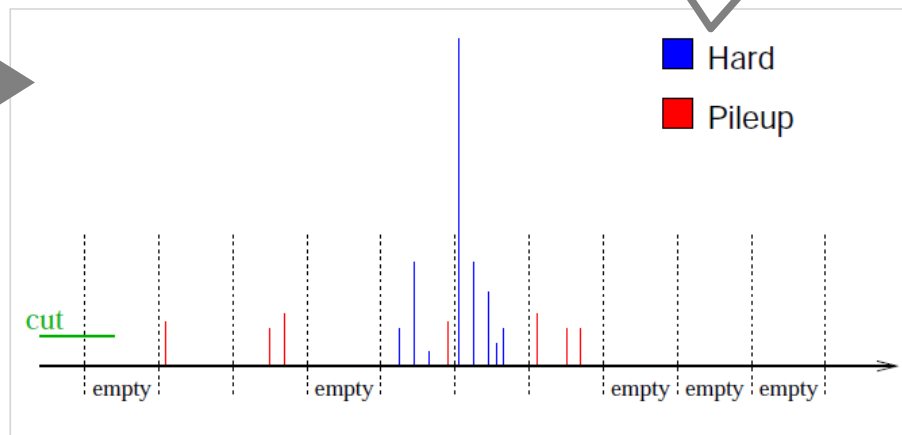
- Discards particles that fall below a certain transverse momentum threshold.
- Estimate the threshold event-by-event such that p estimated from the median-area method is evaluated to be zero.
- Technically very simple – just need to find a median from set of maximal p_T values from each patch.

Half of the patches are empty after the correction

Original event



After SoftKiller

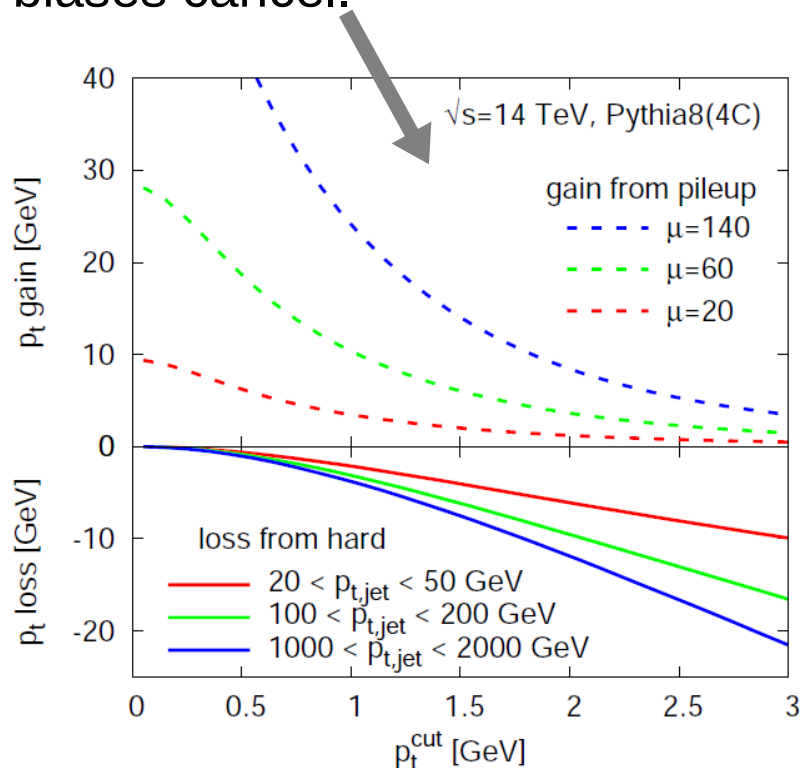


SoftKiller



Cacciari et al. arXiv:1407.0408

- Biases on jet energy scale: net positive bias from background particles that are not subtracted, net negative bias from particles lost from jets.
- For the typical p_T cuts and grid spacing (=size of patches) of $a \sim 0.4$ the biases cancel.



- + very fast and simple => e.g. potential for triggering
- ? may induce a flavor dependence of jet energy scale
- ? performance for larger jet radii and jet shapes



PUPPI

(PileUp Per Particle Identification)

Bertolini et al., arXiv:1407.6013

- For each particle define a local shape variable α_i using other particles

$$\alpha_i = \log \sum_{j \in \text{event}} \xi_{ij} \times \Theta(R_{\min} \leq \Delta R_{ij} \leq R_0)$$

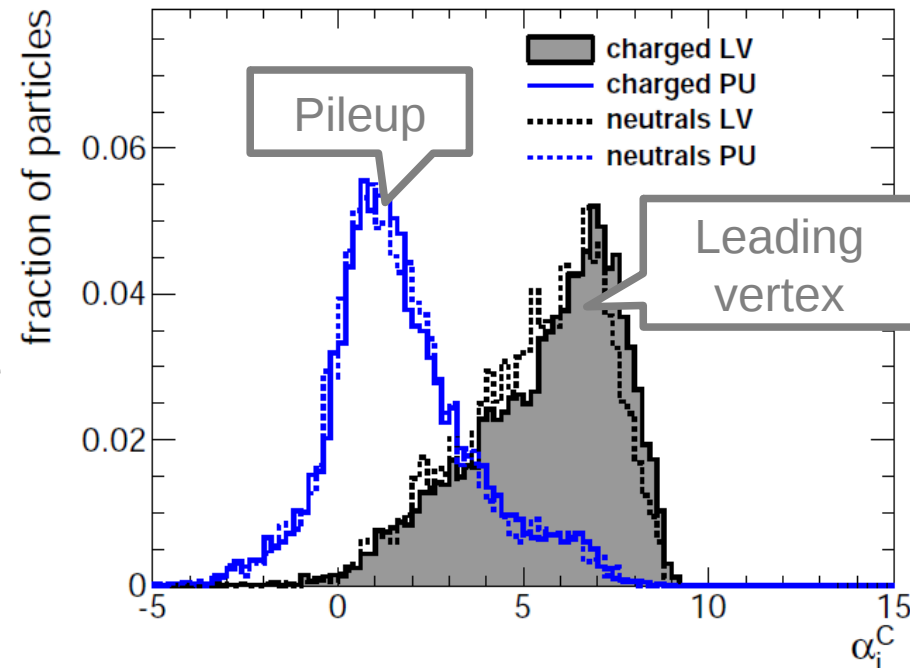
do it within a cone
of certain size

$$\xi_{ij} = \frac{p_{Tj}}{\Delta R_{ij}}$$

High-pt particles
are likely from jets

Parton radiation is
predominantly
collinear

- Then translate the local shape variable to particle weights that multiply the track momentum.





PUPPI

(PileUp Per Particle Identification)

Bertolini et al., arXiv:1407.6013

- Translate the local shape variable to particle weights using cumulative distribution function of χ^2 distribution

$$w_i = F_{\chi^2, \text{NDF}=1}(\chi_i^2)$$

$$\chi_i^2 = \Theta(\alpha_i - \bar{\alpha}_{\text{PU}}) \times \frac{(\alpha_i - \bar{\alpha}_{\text{PU}})^2}{\sigma_{\text{PU}}^2}$$

pileup median

=1 if $\alpha_i > \text{median}$
=0 otherwise

- Weights with values between 0 and 1 used to multiply the p_T of particle.
- Zero weight discards the particle.
- Other discriminants can be easily included.



Constituent subtraction

- Two main ingredients of constituent subtraction:

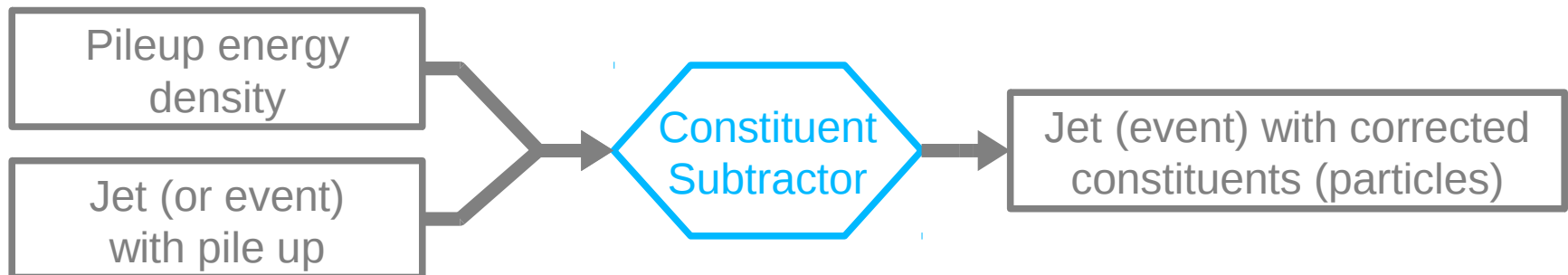
- background (pileup) p_T density

$$\rho = \text{median}_{\text{patches}} \left\{ \frac{p_{T,\text{patch}}}{A_{\text{patch}}} \right\}$$

- ghosts

$$p_\mu^g = p_T^g \cdot [\cos \phi^g, \sin \phi^g, \sinh y^g, \cosh y^g]$$

- Constituent subtraction provides a rule for associating the background p_T density with a given constituent, independent of any tracking information.



Constituent subtraction algorithm



- For each event:
 1. estimate the background p_T density, ρ
 2. add ghosts ($p_{T,g} \rightarrow 0$) among particles and run the jet algorithm

- For each jet in the event:

3. for each ghost set $p_{T,g} = \rho A_g$
4. evaluate distance $\Delta R_{i,k}$ between particle i and ghost k for all particle-ghost pairs

$$\Delta R_{i,k} = p_{Ti}^\alpha \cdot \sqrt{(y_i - y_k^g)^2 + (\phi_i - \phi_k^g)^2}$$

5. sort the distances and iteratively change the momenta as follows

$$\text{If } p_{Ti} \geq p_{Tk}^g : \quad \begin{array}{l} p_{Ti} \longrightarrow p_{Ti} - p_{Tk}^g, \\ p_{Tk}^g \longrightarrow 0; \end{array} \quad \left| \quad \begin{array}{l} \text{otherwise:} \quad p_{Ti} \longrightarrow 0, \\ p_{Tk}^g \longrightarrow p_{Tk}^g - p_{Ti}. \end{array} \right.$$

until no more pairs remain or $\Delta R_{i,k} > \Delta R^{\max}$

6. discard all particles with zero transverse momentum



Properties of the algorithm

- Generic method for the background subtraction.
- Generic = independent of jet algorithm, tracking or calorimeter information => can be used across experiments and across backgrounds.
- Output:
 - a) jet with corrected constituents => can estimate unbiased jet shapes
 - b) subtracted event => can improve e.g. missing- p_T reconstruction
- Other properties:
 - accounts for fluctuations of the background
 - is longitudinally invariant
 - is reasonably fast
 - incorporates the subtraction of mass density for massive particles (for simplicity not discussed here)
 - can accommodate any improved energy density estimates
 - can accommodate discriminants helping to identify pileup

Properties of the algorithm



- Free parameters:
 - Ghost area A_g - the smaller the better, but also slower.
 - Parameter α - configuration with $\alpha > 0$ prefers to subtract lower p_T particles.
 - Maximal distance, ΔR_{\max} - restricts subtraction between ghost-particle pairs with large distance.
- Default settings **do not need to be tuned** e.g. for different jet sizes or different level of pile-up.
- Tuning of free parameters may lead to modest improvements.
- Algorithm is publicly available from FastJet Contrib and documented in [Berta et al., JHEP 06 \(2014\) 092](#).



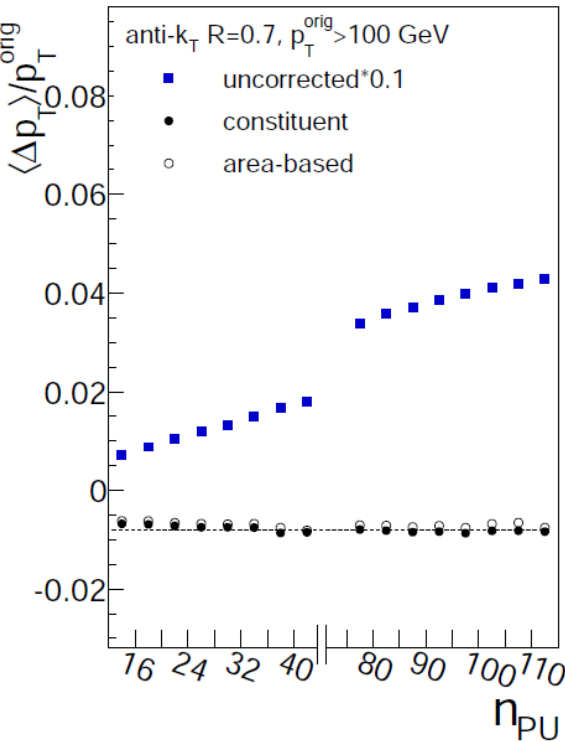
Performance tests

- Pythia8 simulated samples of dijets and $Z' \rightarrow tt$ with $m(Z')=1500$ GeV.
- Simulation at 8 TeV without underlying event.
- Using clustering algorithms anti-kt with $R=0.7$, $R=1.0$ and Cambridge/Aachen with $R = 1.2$.
- Pseudorapidity cut on jets $|\eta|<2$.
- Performance compared to the area 4-vector and shape-expansion methods.

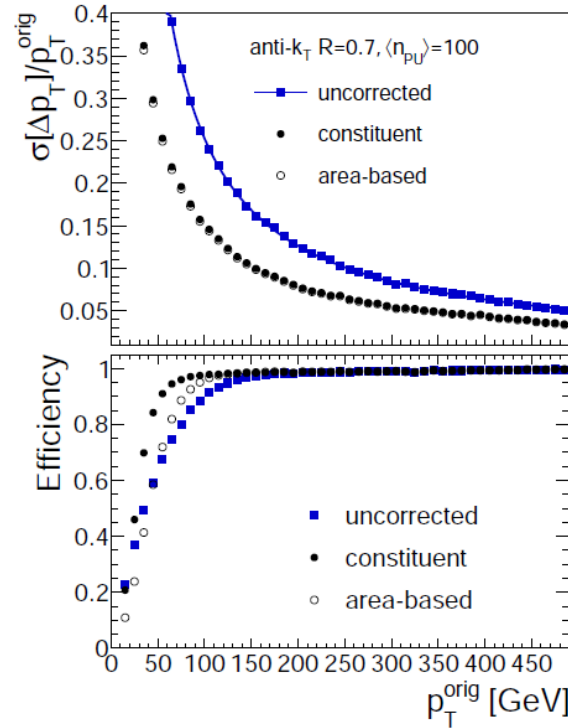


Performance – jet kinematics

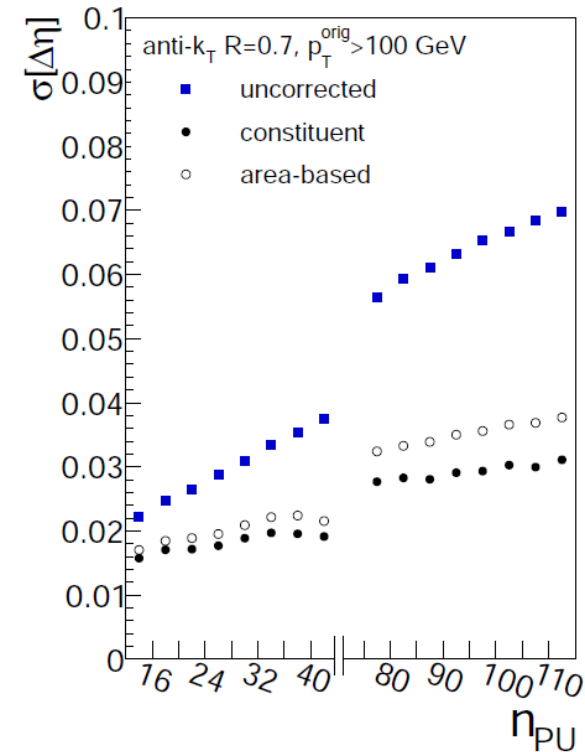
Jet energy scale



Jet energy resolution and efficiency

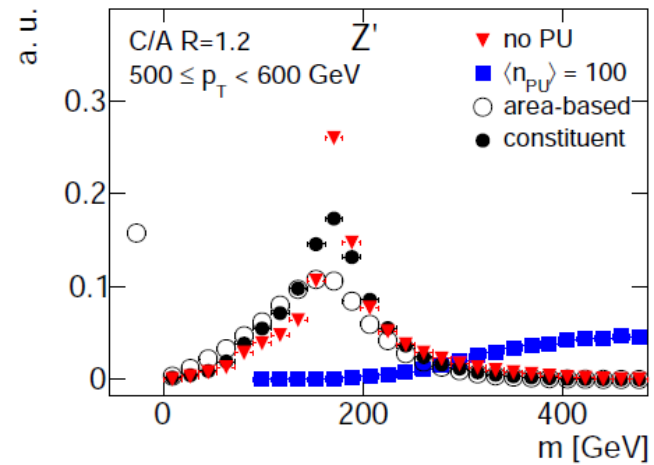
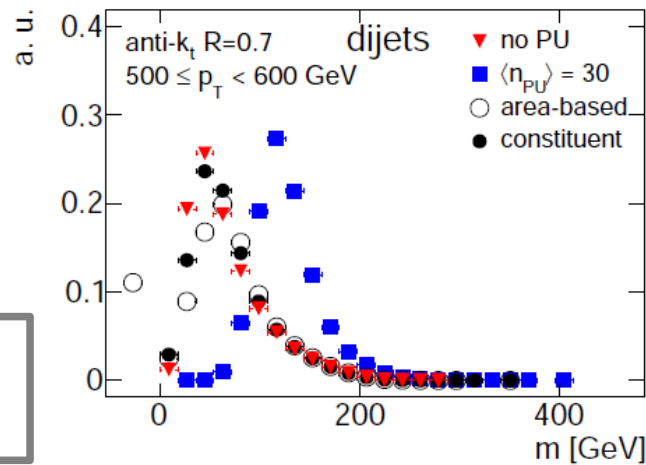


Jet position resolution

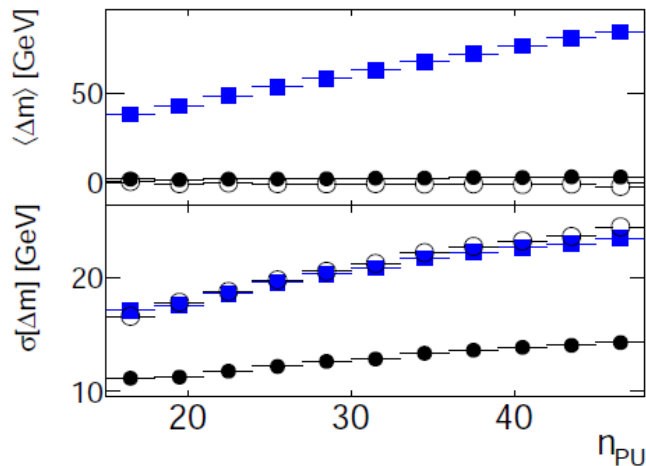


- Perfect stability of jet energy scale with increasing pileup.
- Significant improvement in jet reconstruction efficiency and jet position resolution. Jet energy resolution similar to area-based subtraction.

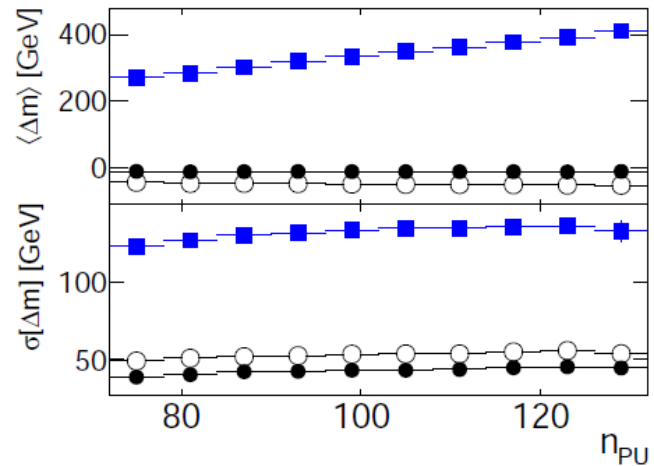
Performance – jet and Z' mass



mean jet mass =
 corrected – truth



mass
 resolution

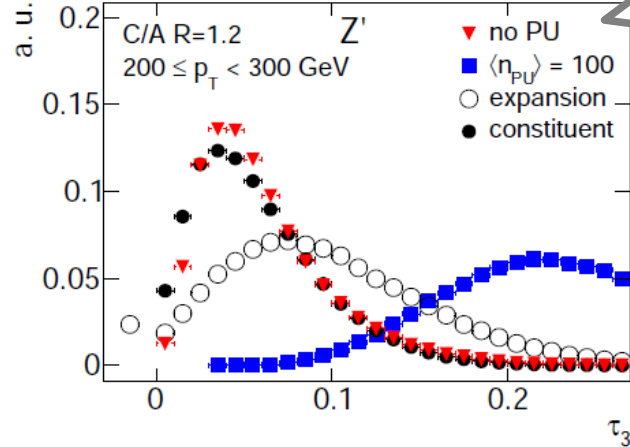
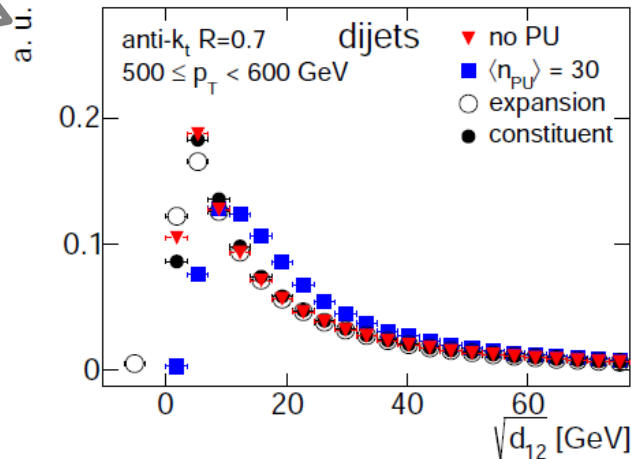


Significant improvement over area based subtraction

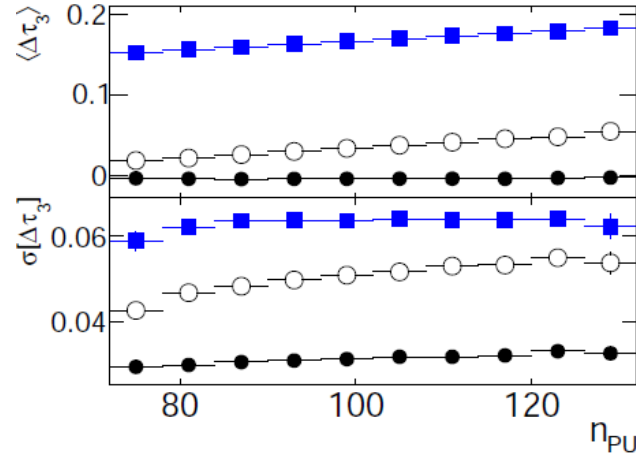
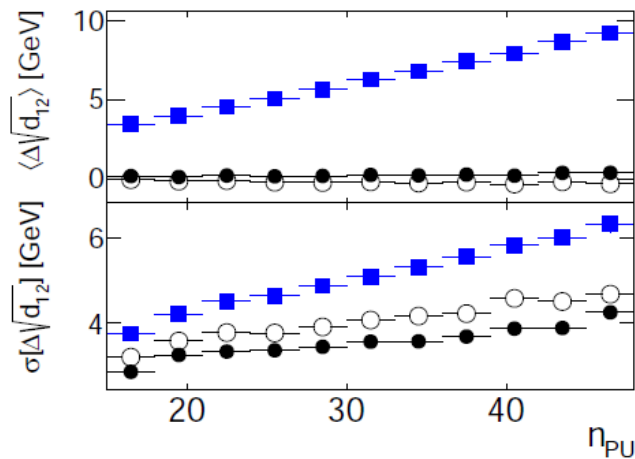
Performance – jet shape variables



splitting
scale



subjettness

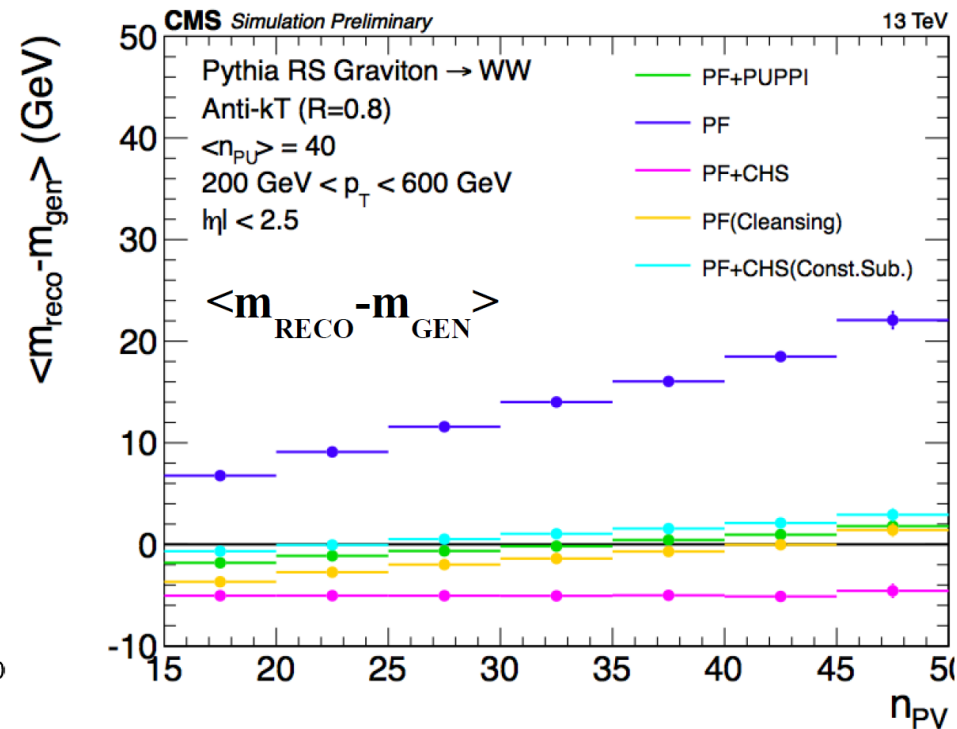
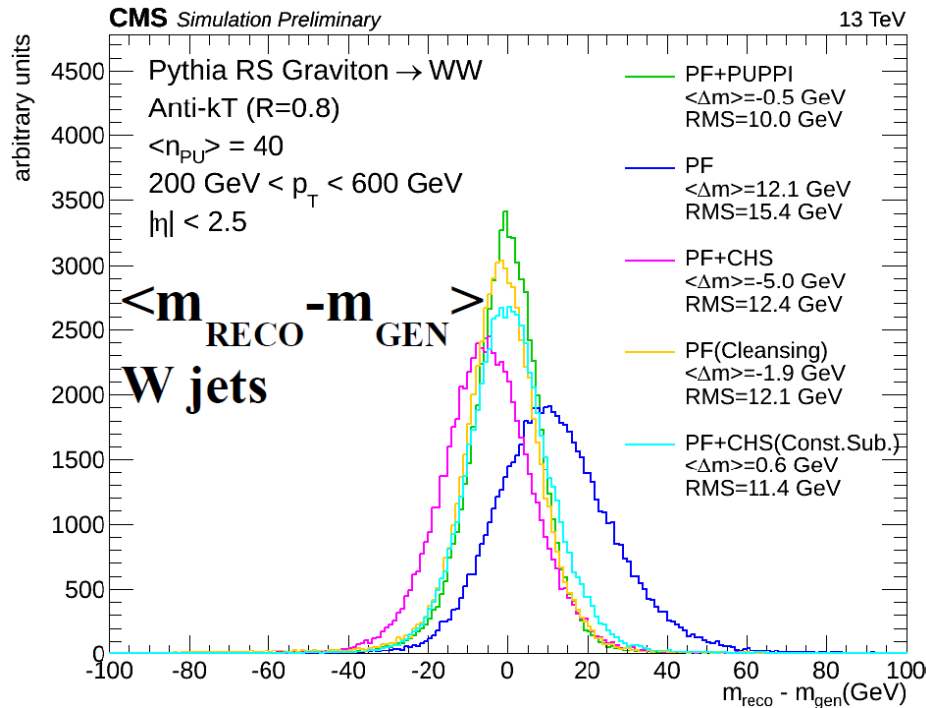


- Examples of variables often used to discriminate highly boosted objects.
- Significant improvement comparing to the shape expansion method.

Performance – full detector simulation



Viola Sordini, BOOST 2014



- Tests using the full detector simulation by CMS.
- Very good performance of both PUPPI and Constituent subtraction.
- PUPPI gives slightly better resolutions than Constituent subtraction.

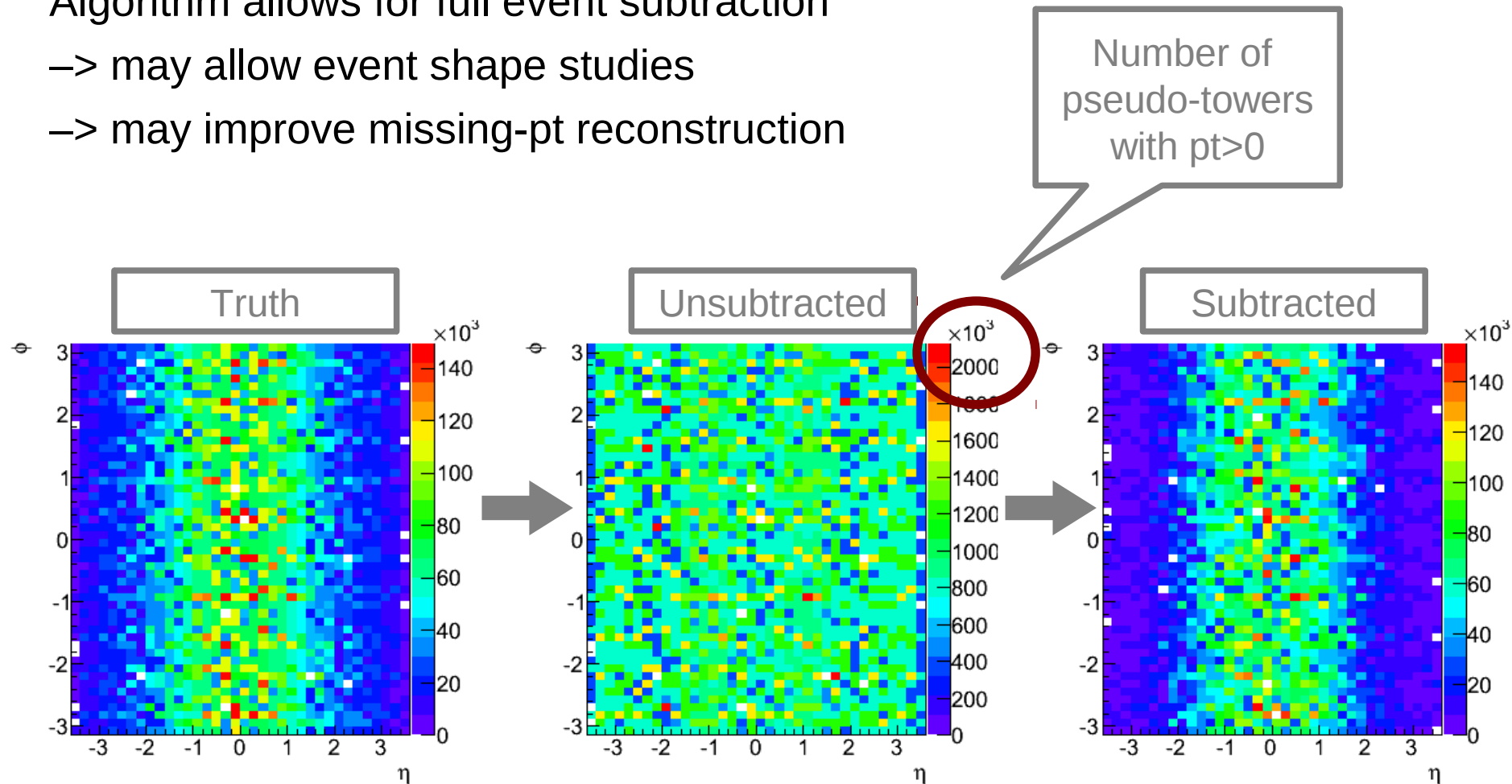


Full event subtraction

Algorithm allows for full event subtraction

→ may allow event shape studies

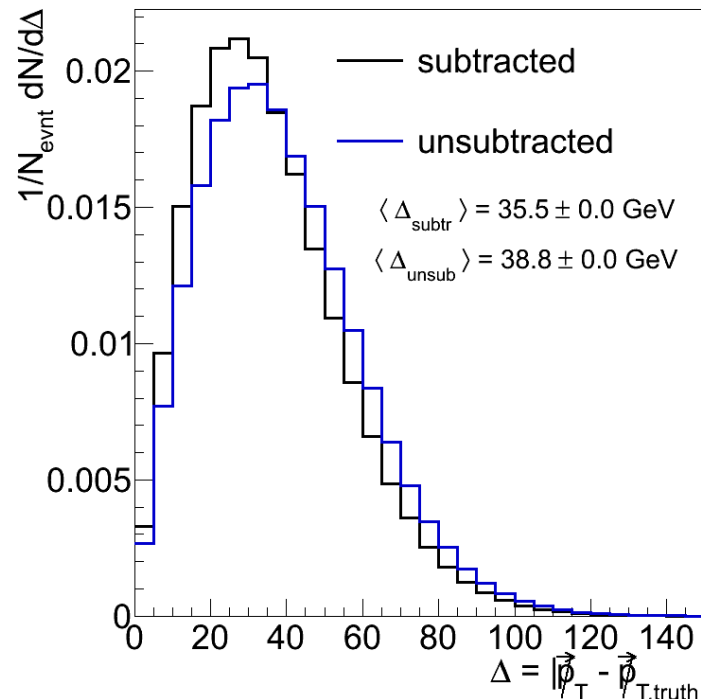
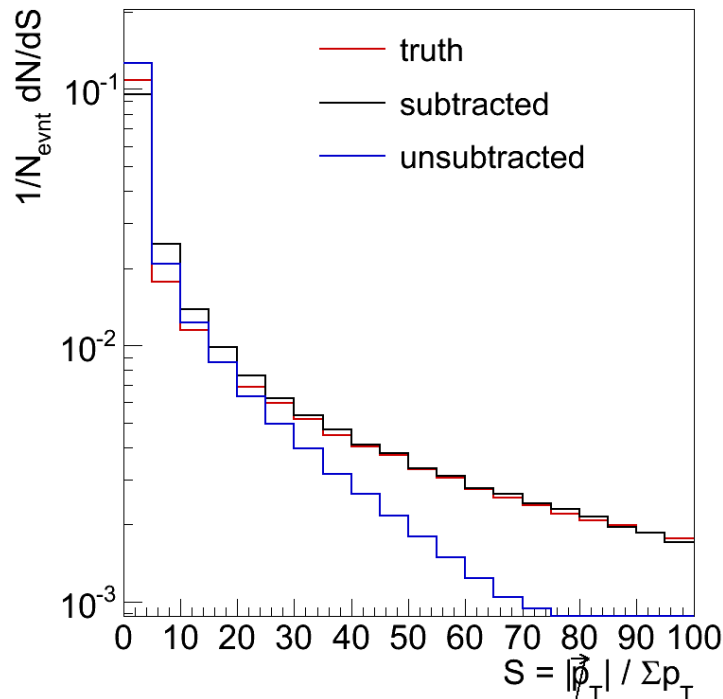
→ may improve missing-pt reconstruction





Missing-pt significance

- Trivial missing-pt reconstruction on subtracted events tested in Z' events with $\mu=100$,
$$\vec{p}_T = (\sum p_T \cos\phi, \sum p_T \sin\phi)$$
- Two kinds of missing-pt significance variables estimated. Subtracted events significantly better agreement with truth than unsubtracted.
- Further improvements possible if subtracted objects enter estimates.



Conclusions



- Pileup/UE subtraction is a very important problem in pp/HI collisions!
- Several new methods for the pileup subtraction on the market. Methods should be tested side by side. Ideas should be shared.
- Constituent subtraction has a very good ability to remove the pileup (or HI underlying event).
- Constituent subtraction is probably the most generic method.
- It is implemented in FastJet Contrib and easy to plug in to the reconstruction chain.

Backup



Two basic methods of grooming



- Trimming

- reconstruct jets with large radii (e.g. $R=1.0$) and recluster to jets with smaller radii (e.g. $R_{\text{trim}}=0.2$)
- accept only if their $f = p_T/p_{T,\text{orig}}$ greater than e.g. 0.03
- recombine surviving sub-jets into a groomed jet

- Pruning

- run Cambridge-Aachen algorithm on jet constituents
- during clustering of old-constituents i and j the softer constituent is removed if

$$z_{ij} = \frac{\min(p_{T,i}, p_{T,j})}{p_{T,i} + p_{T,j}} < z_{\text{cut}}$$

$$\Delta R_{i,j} > D_{\text{cut}} = \frac{2 r_{\text{cut}} m_{\text{orig}}}{p_{T,\text{orig}}}$$

- typical size of parameters: $z_{\text{cut}} = 0.1$, $r_{\text{cut}} = 0.5$

- Other methods: filtering, mass drop.



Subtracting mass

$$\rho = \text{median}_{\text{patches}} \left\{ \frac{p_{T\text{patch}}}{A_{\text{patch}}} \right\} \quad p_T^g = A_g \cdot \rho,$$

$$\rho_m = \text{median}_{\text{patches}} \left\{ \frac{m_{\delta\text{patch}}}{A_{\text{patch}}} \right\} \quad m_\delta = \sqrt{m^2 + p_T^2} - p_T \quad m_\delta^g = A_g \cdot \rho_m$$

If $p_{Ti} \geq p_{Tk}^g$:

$$p_{Ti} \longrightarrow p_{Ti} - p_{Tk}^g,$$

$$p_{Tk}^g \longrightarrow 0;$$

otherwise:

$$p_{Ti} \longrightarrow 0,$$

$$p_{Tk}^g \longrightarrow p_{Tk}^g - p_{Ti}.$$

If $m_{\delta i} \geq m_{\delta k}^g$:

$$m_{\delta i} \longrightarrow m_{\delta i} - m_{\delta k}^g,$$

$$m_{\delta k}^g \longrightarrow 0;$$

otherwise:

$$m_{\delta i} \longrightarrow 0,$$

$$m_{\delta k}^g \longrightarrow m_{\delta k}^g - m_{\delta i}.$$

Constituent subtraction

$$p_{\text{corr}}^\mu = [p^x - \rho A^x, p^y - \rho A^y, p^z - (\rho + \rho_m) A^y, E - (\rho + \rho_m) A^E]$$

$$A_\mu = \frac{A_g}{p_{T,g}} \sum_{\substack{k \in \text{ghosts} \\ \text{in jet}}} p_{g,k}^\mu$$

4-vector area-median subtraction



Jet shapes

- N-subjettiness:

$$\tau_N = \frac{1}{d_0} \sum_k p_{Tk} \cdot \min(\Delta R_{1k}, \Delta R_{2k}, \dots, \Delta R_{Nk})$$

$$d_0 \equiv \sum_k p_{Tk} \cdot R$$

- reclustering the jet with kt algorithm, requiring to find N subjects
- distances between subjet axes and constituents
- also ratios of t_N can be used

- kt splitting scale

$$\sqrt{d_{12}} = \min(p_T^1, p_T^2) \cdot \Delta R_{12}$$

- transverse momenta of subjects and the distance between them