The OPERA Experiment

Latest Results

Annika Hollnagel
(annika.hollnagel@desy.de)
on behalf of the OPERA Collaboration

Hamburg University
Institute for Experimental Physics

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Neutrino Oscillations

Neutrino oscillation in disappearance mode:

- **First observation:** SK, MACRO...
- **Further studies:** SNO, MINOS, KamLAND, Borexino...

Neutrino oscillation in appearance mode:

- Observation needed to establish the picture of neutrino oscillations

Solar scale:

- $\nu_e \to \nu_\mu$: Below threshold for μ production

Atmospheric scale:

- $\nu_\mu \to \nu_e$: Sub-leading (T2K, OPERA)
- $\nu_\mu \to \nu_\tau$: ν_μ from cosmic rays (SK: statistical analysis, large BG)
- $\nu_\mu \to \nu_\tau$: ν_μ from long-baseline beams
  OPERA: τ lepton identification on an event-by-event basis

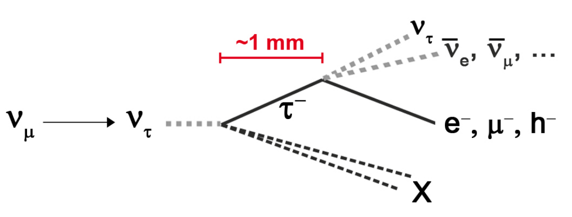
The OPERA Experiment

The OPERA experiment
in the CERN to Gran Sasso neutrino beam,
JINST **4** (2009) P04018

The OPERA Experiment

OPERA: Oscillation Project with Emulsion Tracking Apparatus

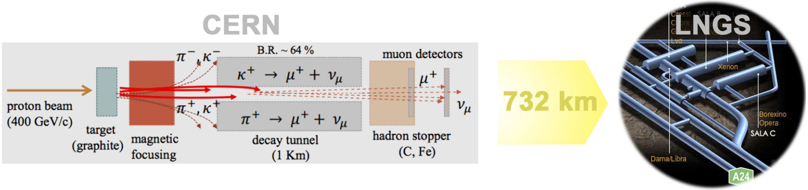
- **Appearance search:** Direct observation of $\nu_\mu \to \nu_\tau$ oscillations
  detection of τ production & decay
- **Characteristic 'kink' topology:**



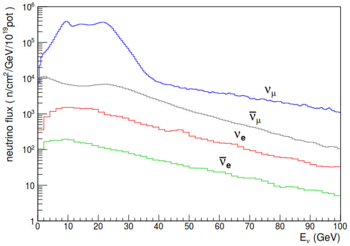
- **ν beam:** High-intensity & high-energy long-baseline ν_μ beam
- **Detector:** Large target mass, high precision $\mathcal{O}(\mu\mathrm{m})$
- **Location:** Laboratori Nazionali del Gran Sasso (LNGS)
  $1\,400\,\mathrm{m}$ rock coverage, $3\,800\,\mathrm{m}$ *w.e.*

The CNGS ν_μ Beam

CNGS: CERN Neutrinos to Gran Sasso (2008 – 2012)

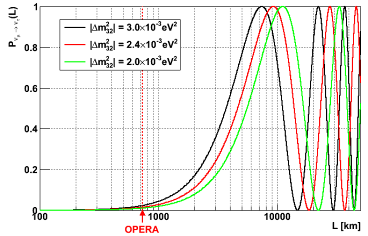


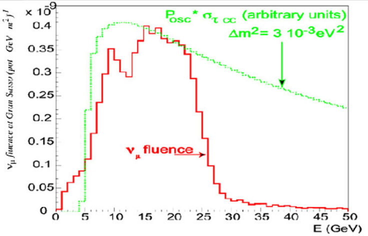
$\langle E_\nu \rangle$		17 GeV
$\overline{\nu}_\mu / \nu_\mu$	CC	2.1 %
ν_e / ν_μ	CC	0.89 %
$\overline{\nu}_e / \nu_\mu$	CC	0.06 %
ν_τ / ν_μ	CC	$< 10^{-4}$ %
p.o.t. (total)		17.97×10^{19}



Beam Characteristics at LNGS

$$P(\nu_\mu \to \nu_\tau) \sim \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2\left(\Delta m^2_{23} \frac{L}{4E}\right)$$



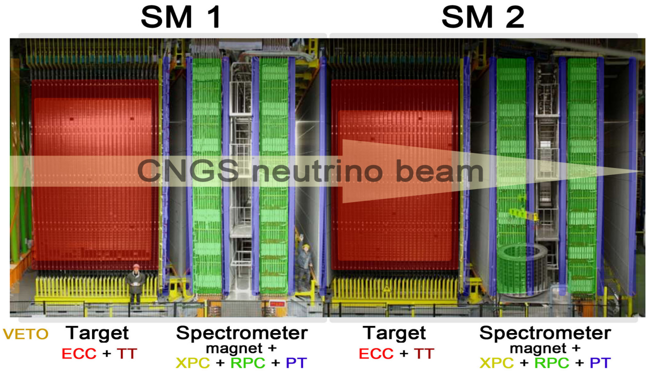


- $\frac{L}{\langle E \rangle} \sim 43\ \frac{\mathrm{km}}{\mathrm{GeV}}$

▷ $P(\nu_\mu \to \nu_\tau)\ \mathcal{O}(1\,\%)$

- ν_μ energy optimised for τ detection
  (CC τ production threshold: $3.5\,\mathrm{GeV}$)

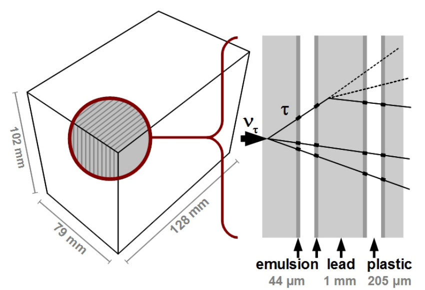
The OPERA Detector

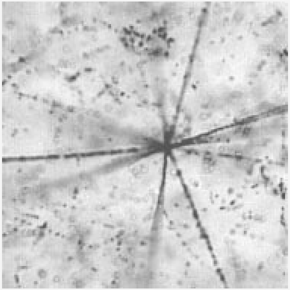


Hybrid detector (ED & ECC):

- 2 identical Super Modules (SM) + VETO system
- **Spectrometer:** RPC & XPC, PT
- **Target Area:** TT, ECC bricks

The OPERA Detector

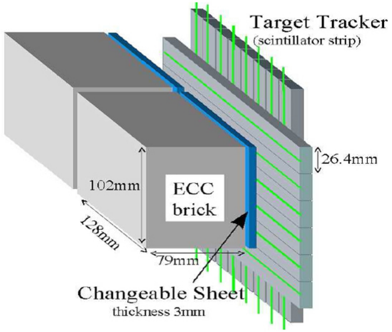




Emulsion Cloud Chamber (ECC) bricks:

- 57 $\times$ 2 AgBr nuclear emulsions on plastic bases, interleaved with 56 lead plates ($\sim 10\,X_0$)
- **Total:** $\sim 150\,000 \times 8.3\,\mathrm{kg}$ $\sim 1.25\,\mathrm{kt}$ total target mass
- **Spatial / angular resolution:** $\sim 1\,\mu\mathrm{m}$ / $\sim 2\,\mathrm{mrad}$

The OPERA Detector



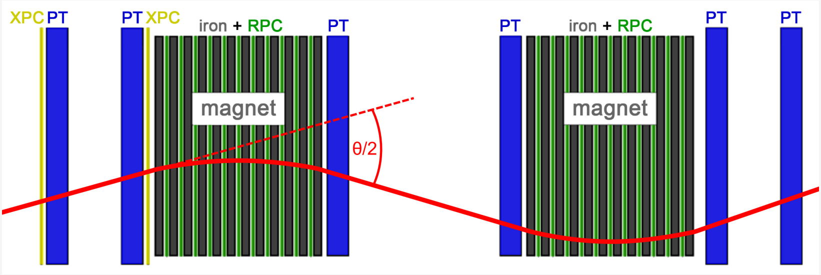
Changeable Sheets (CS):

- 2 extra nuclear emulsion sheets per brick

Target Tracker (TT) detectors:

- Plastic scintillator strips (horizontal & vertical), 31 walls per SM

The OPERA Detector

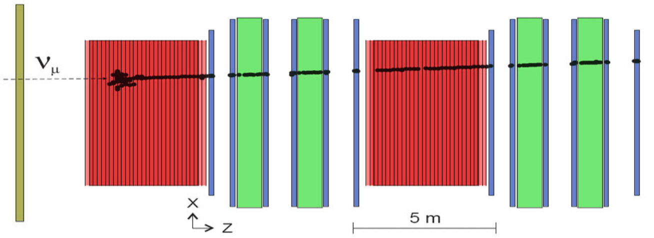


Magnetic Spectrometer:

- Downstream of each target area

- **Magnets**: Iron core dipole, $1.55\,\mathrm{T}$
- **RPC, XPC**: Resistive plate chambers
- **Precision Tracker (PT)**: $\sim 10\,000$ drift tubes

Event Reconstruction

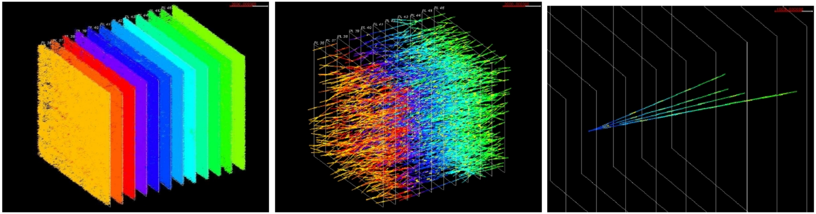


ED event reconstruction:

- **Time resolution:** $\mathcal{O}(\mathrm{ns})$
- μ identification, charge & momentum measurement
- Hadronic shower energy reconstruction
- ν interaction brick localisation

▷ **Trigger:** ECC event reconstruction

Event Reconstruction



ECC event reconstruction:

- **Spatial resolution:** $\mathcal{O}(\mu\text{m})$
- $3D$ track segment & track reconstruction
- ν interaction vertex localisation
- Decay search procedure:
  - ▷ kink angle / IP measurement, parent / daughter search...
- Momentum measurement via MCS

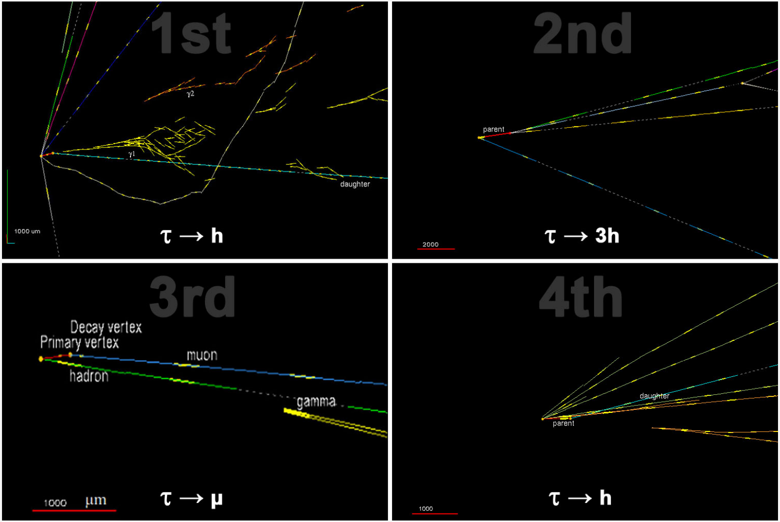
Oscillation Search:

$$\nu_\mu \to \nu_\tau$$

Observation of ν_τ appearance in the CNGS beam with the OPERA experiment,
arXiv:1407.3513 (accepted by PTEP)

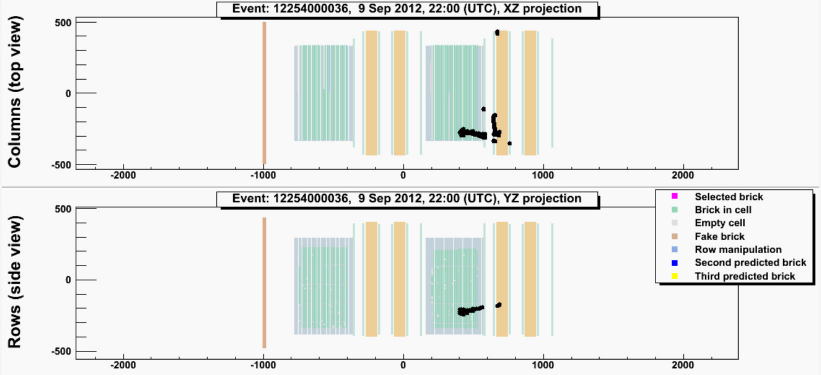
$\nu_\mu \to \nu_\tau$ Oscillation Search

4 ν_τ candidate events:



The 4th ν_τ Candidate Event

ED reconstruction:



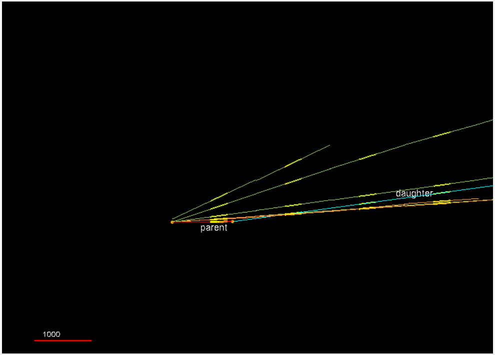
Brick finding information: Super module 2

	BrickId	Wall	Side	Column	Row	Prob	CS x	CS y
brick 1:	1092217	20	-1	5	16	0.77	-1.0	-1.0
brick 2:	1089442	20	-1	5	15	0.12	-1.0	-1.0
brick 3:	1000527	21	-1	5	16	0.07	-1.0	-1.0

Muon track parameters:
Momentum: N/A
Tangent angle XZ: N/A
Tangent angle YZ: N/A

The 4th ν_τ Candidate Event

ECC reconstruction:

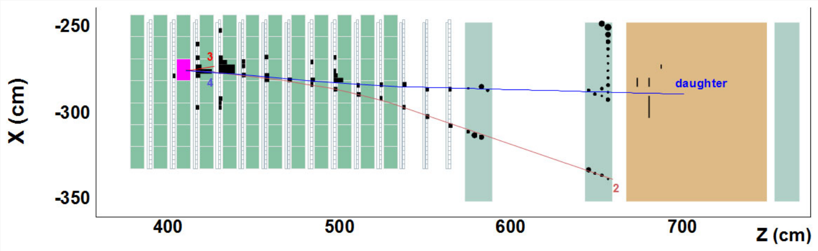


- **1ry vertex:** 4 tracks
- **Red track:** 1-prong decay after 1.09 mm

▷ **Decay channel:** $\tau \to h$

The 4th ν_τ Candidate Event

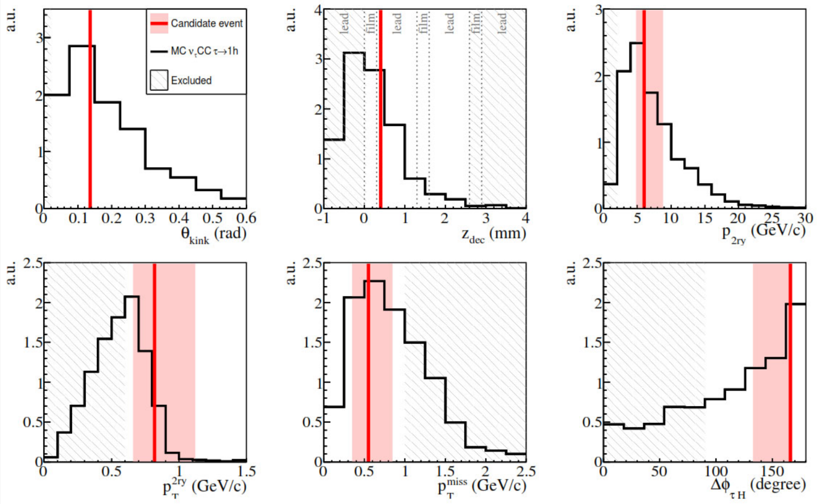
Track follow-down:



- **1ry vertex track 2**: Stopping in 1st magnet iron slab
  - ▷ Hadron ($p = 1.9^{+0.3}_{-0.2}\,\mathrm{GeV/c}$)
- **Daughter track**: Stopping ist 1st magnet
  - ▷ Hadron ($p = 6.0^{+2.2}_{-1.2}\,\mathrm{GeV/c}$)

The 4th ν_τ Candidate Event

Kinematical cuts: $\tau \to h$ decay channel



$\nu_\mu \to \nu_\tau$ Oscillation Search

Fully analysed data sample: 4685 events

- **2008/09:** 1st & 2nd most probable bricks
- **2010/11/12:** 1st most probable brick
- 0μ events & 1μ events with $p_\mu < 15\,\mathrm{GeV/c}$

τ decay channel	Signal (exp.) $\Delta m^2_{23} = 2.32\,\mathrm{meV}^2$	Total BG (exp.)	Data (obs.)
$\tau \to h$	0.41 ± 0.08	0.033 ± 0.006	2
$\tau \to 3h$	0.57 ± 0.11	0.155 ± 0.030	1
$\tau \to \mu$	0.52 ± 0.10	0.018 ± 0.007	1
$\tau \to e$	0.62 ± 0.12	0.027 ± 0.005	0
Total	2.11 ± 0.42	0.233 ± 0.041	4

Observation of ν_τ appearance:

- **p-value:** 1.24×10^{-5} (Fisher) / 1.03×10^{-5} (Likelihood)

▷ **No-oscillation hypothesis excluded @** 4.2σ

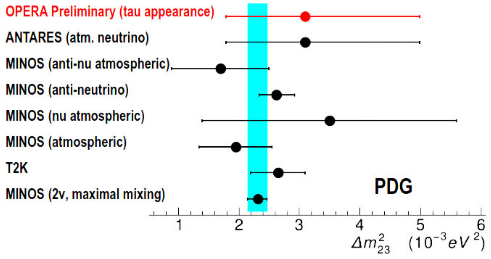
$\nu_\mu \to \nu_\tau$ Oscillation Search

$$N_{\nu_\tau} \propto \int \Phi(E) \sin^2\left(\frac{\Delta m^2_{23} L}{4E}\right) \epsilon(E)\sigma(E) dE \propto (\Delta m^2_{23})^2 L^2 \int \Phi(E)\epsilon(E)\frac{\sigma(E)}{E^2} dE$$

First measurement of Δm^2_{23} in appearance mode:

- $\Delta m^2_{23} = [1.8 - 5.0] \times 10^{-3}\,\mathrm{eV}^2$ **(Feldman&Cousins)**
- $\Delta m^2_{23} = [1.9 - 5.0] \times 10^{-3}\,\mathrm{eV}^2$ **(Bayes)**

(for $\sin^2(2\theta_{23}) = 1$ at 90% C.L.)



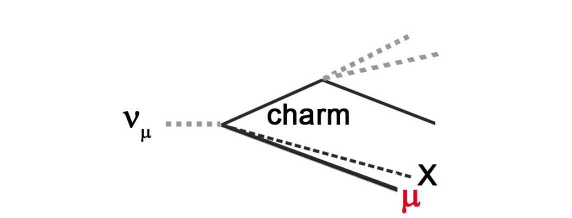
Control Sample: Charmed Particle Decays

Procedure for short-lived particle detection in the OPERA experiment and its application to charm decays, Eur. Phys. J. C **74** (2014) 2986

Control Sample: Charmed Particle Decays

Main BG to τ search:

- ν_μ CC interactions with charm production



- Topology similar to τ decay
- μ at 1ry vertex

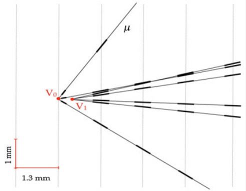
Other BG:

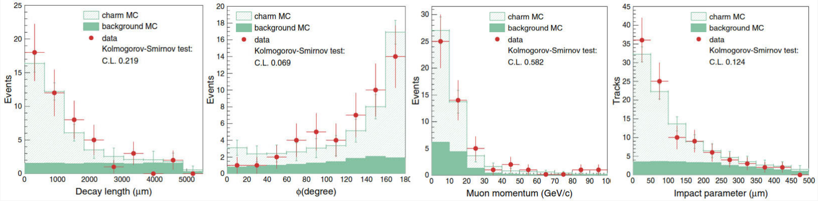
- Hadronic re-interactions in lead
- Large-angle μ scattering

Control Sample: Charmed Particle Decays

2008 – 2010 OPERA data:

	Charm (exp.)	BG (exp.)	Total (exp.)	Data (obs.)
1-prong	21 ± 2	9 ± 3	30 ± 4	19
2-prong	14 ± 1	4 ± 1	18 ± 1	22
3-prong	4 ± 1	1.0 ± 0.3	5 ± 1	5
4-prong	0.9 ± 0.2	–	0.9 ± 0.2	4
Total	40 ± 3	14 ± 3	54 ± 4	50





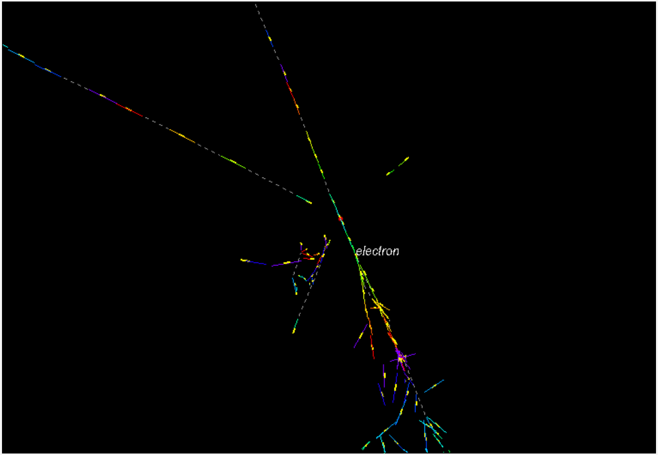
Oscillation Search:

$$\nu_\mu \to \nu_e$$

Search for $\nu_\mu \to \nu_e$ oscillations with the OPERA experiment in the CNGS beam, JHEP **1307** (2013) 004

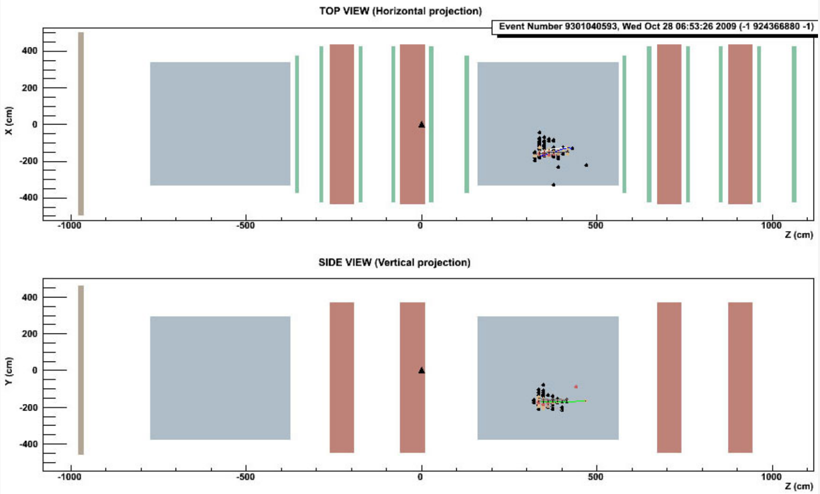
A ν_e Event

ECC reconstruction:



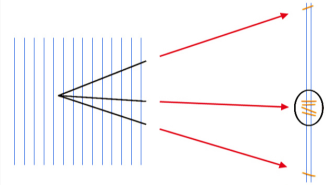
A ν_e Event

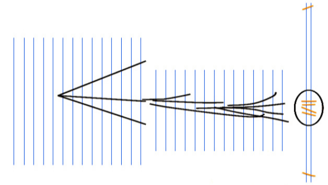
ED reconstruction:



Systematic ν_e Event Selection

CS *em* shower hints:





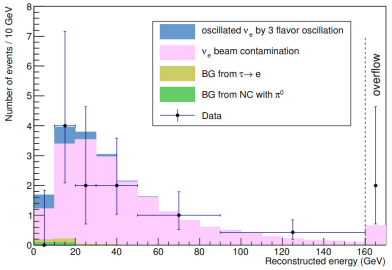
- Interpolation of 1ry vertex tracks to CS
  - ▷ Expanded scan volume
- Analysis of downstream bricks

Backgrounds:

- ν_e from intrinsic beam contamination
- e^+e^- from π^0 decays misidentified as single-e
- ν_τ CC interactions with $\tau \to e$

ν_e Energy Reconstruction

2008 + 2009 ν_e candidate events (reconstructed energy)



Cuts on $E_{\nu,rec}$: Separation of signal & BG

Energy cut		20 GeV	30 GeV	No cut
BG common to both analyses	BG (a) from π^0	0.2	0.2	0.2
	BG (b) from $\tau \to e$	0.2	0.3	0.3
	ν_e beam contamination	4.2	7.7	19.4
Total expected BG in 3-flavour oscillation analysis		4.6	8.2	19.8
BG to non-standard oscillation analysis only	ν_e via 3-flavour oscillation	1.0	1.3	1.4
Total expected BG in non-standard oscillation analysis		5.6	9.4	21.3
Data		4	6	19

Assumptions:

- $\sin^2(2\theta_{13}) = 0.098$
- $\sin^2(2\theta_{23}) = 1$
- $\Delta m^2_{23} = \Delta m^2_{31} = 2.32 \times 10^{-3}\,\mathrm{eV}^2$
- $\delta_{CP} = 0$
- No matter effects

Oscillation Analysis: 3-Flavour

2008 + 2009 data sample:

- 5255 ν CC interactions $(5.25 \times 10^{19}\,\mathrm{p.o.t.})$
  - ▷ ν_e **candidates:** 19 events

Separation: Beam contamination and oscillated ν_e

- ν **energy cut:** $E_{\nu,rec} < 20\,\mathrm{GeV}$
  - ▷ **Expected BG:** 4.6 events
  - ▷ **Expected signal:** 1.0 events
  - ▷ **Remaining ν_e candidates:** 4 events

Compatible with no-oscillation hypothesis:

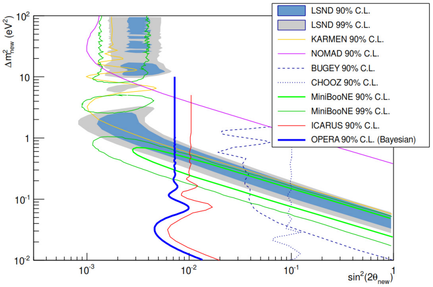
- $\sin^2(2\theta_{13}) < 0.44$ (90% C.L.)

Oscillation Analysis: Non-Standard

Separation: BG and oscillated ν_e

- **ν energy cut:** $E_{\nu,rec} < 30\,\mathrm{GeV}$
  - **Expected BG:** 9.4 events
  - **Remaining ν_e candidates:** 6 events

$$P_{\nu_\mu \to \nu_e} = \sin^2(2\theta_{new}) \cdot \sin^2(1.27 \cdot \Delta m^2_{new} L[\mathrm{km}]/E[\mathrm{GeV}])$$



$\sin^2(2\theta_{new}) < 7.2 \times 10^{-3}$ for $\Delta m^2_{new} > 0.1\,\mathrm{eV}^2$ (90% C.L.) **(Bayes)**

Atmospheric μ: TeV-Range

Measurement of TeV atmospheric muon charge ratio with the full OPERA data,
Eur. Phys. J. C **74** (2014) 2933

Atmospheric μ: TeV-Range

Atmospheric μ charge ratio:

$$R_\mu = \frac{N_{\mu^+}}{N_{\mu^-}} \qquad \left(R_\pi = \frac{Z_{N_{\pi^+}}}{Z_{N_{\pi^-}}},\ R_K = \frac{Z_{N_{K^+}}}{Z_{N_{K^-}}}\right)$$

- Study of cosmic ray interactions in the atmosphere
- Constraints on hadronic interaction models
- Main contributions from π (low energies) & K (high energies)

Atmospheric μ @OPERA:

- **LNGS:** $1\mu\,\mathrm{m}^{-2}\mathrm{h}^{-1}$ ($\mathcal{O}(\mathrm{TeV})$ surface energy)

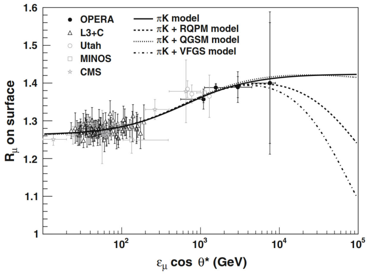
▷ Reduced by $\sim 10^6$ w.r.t. surface

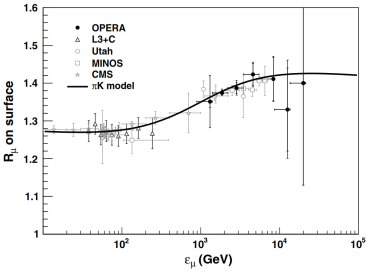
- **Charge-symmetric detector:**
  Measurement at opposite magnetic field polarities

▷ Minimisation of systematic uncertainties

Atmospheric μ: TeV-Range

OPERA 2008 - 2012 data (single μ):





- **Single μ:** $R_\mu(n_\mu = 1) = 1.377 \pm 0.006(stat.)^{+0.007}_{-0.001}(syst.)$
- **Multiple μ:** $R_\mu(n_\mu > 1) = 1.098 \pm 0.023(stat.)^{+0.015}_{-0.013}(syst.)$

▷ R_μ for single μ compatible with simple $\pi - K$ model

▷ No sign. contribution of prompt component for $\epsilon_\mu \cos\theta^* \lesssim 10\,\mathrm{TeV}$

▷ Feynman scaling in the fragmentation region for $\epsilon_\mu \lesssim 20\,\mathrm{TeV}$

Conclusion & Outlook

Conclusion & Outlook

Oscillation Search: $\nu_\mu \to \nu_\tau$

- 4 ν_τ candidate events observed (0.23 BG events expected)
  - ▷ **Observation of ν_τ appearance at 4.2σ**
  - ▷ First measurement of Δm^2_{23} in appearance mode
- **Non-standard analysis:** Limits on Δm^2_{41}, $|U_{\mu 4}|^2$ & $|U_{\tau 4}|^2$...

Oscillation Search: $\nu_\mu \to \nu_e$

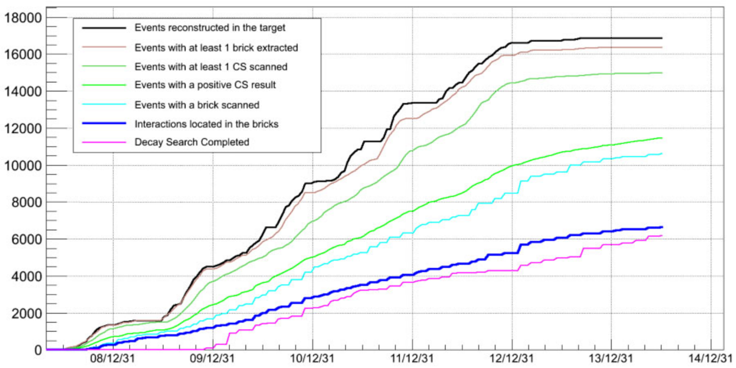
- **3-flavour analysis:** Compatible with no-oscillation hypothesis
- **Non-standard analysis:** New limits on non-standard oscillations

Atmospheric μ:

- Measurement of R_μ at $\mathcal{O}(\mathrm{TeV})$

Conclusion & Outlook

Analysis status:



- **CNGS beam (2008 – 2012):** 1.8×10^{20} p.o.t., 19 505 ν interactions
- **Vertex located:** 6636 interactions
- **Decay search performed:** 6190 interactions

Thank you for your attention!

11 countries, 28 institutes, 140 physicists...

Belgium:
- IIHE-ULB Brussels

Croatia:
- IRB Zagreb

France:
- LAPP Annecy
- IPHC Strasbourg

Germany:
- Hamburg University

Israel:
- Technion Haifa

Italy:
- LNGS Assergi
- Bari
- Bologna
- Frascati
- l'Aquila
- Naples
- Padova
- Rome
- Salerno

Japan:
- Aichi
- Toho
- Kobe
- Nagoya
- Nihon

Korea:
- Jinju

Russia:
- JINR Dubna
- ITEP Moscow
- INR-RAS Moscow
- LPI-RAS Moscow
- SINP-MSU Moscow

Switzerland:
- LHEP Bern

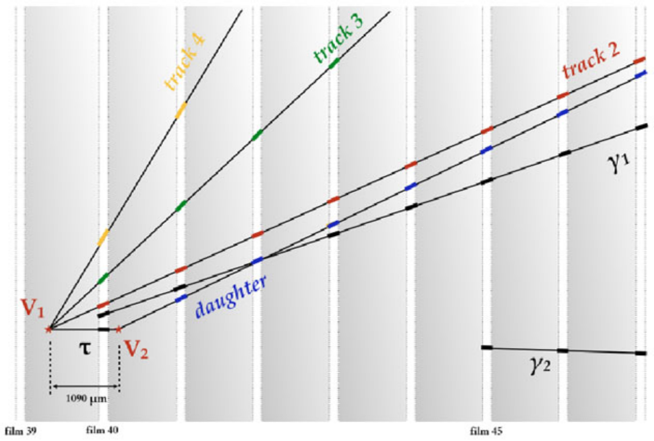
Turkey:
- METU Ankara

Backup Slides

CNGS Roadmap

The 4th ν_τ Candidate Event

Schematic view (ECC reconstruction):



The 4th ν_τ Candidate Event

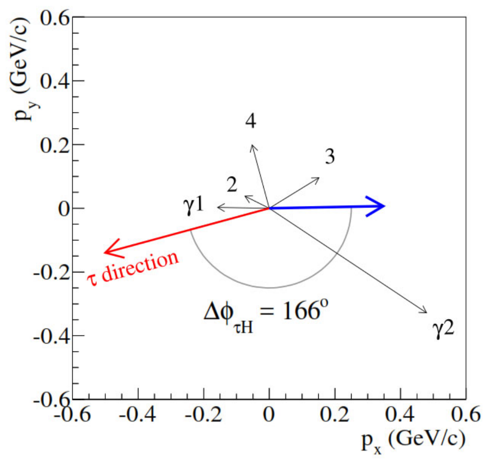
Track features:

			First measurement	Second measurement	Average	
	Track ID	Particle ID	Slopes	Slopes	Slopes	P (GeV/c)
1ry	1 parent	τ	-0.143, 0.026	-0.145, 0.014	-0.144, 0.020	-
	2	Hadron (Range)	-0.044, 0.082	-0.047, 0.073	-0.046, 0.078	1.9 [1.7, 2.2]
	3	Hadron (interact)	0.122, 0.149	0.139, 0.143	0.131, 0.146	1.1 [1.0, 1.2]
	4	proton	-0.083, 0.348	-0.080, 0.355	-0.082, 0.352	0.7 [0.6, 0.8] $p\beta = 0.4$ [0.3, 0.5]
	$\gamma 1$	e-pair	-0.229, 0.068	-0.238, 0.055	-0.234, 0.062	0.7 [0.6, 0.9]
	$\gamma 2$	e-pair	0.111, -0.014	0.115,-0.034	0.113,-0.024	4.0 [2.6, 8.7]
2ry	daughter	Hadron (Range)	-0.084, 0.148	-0.091, 0.145	-0.088, 0.147	6.0 [4.8, 8.2]

		ΔZ (μm)	$\delta\theta_{RM}$ (mrad)	IP (μm)	IP Resolution (μm)	Attachment
$\gamma 1$	To 1ry	676	21.9	2	8	OK
$\gamma 2$	To 1ry	7176	9.2	33	43	OK
	To 2ry	6124	9.2	267	36	Excluded

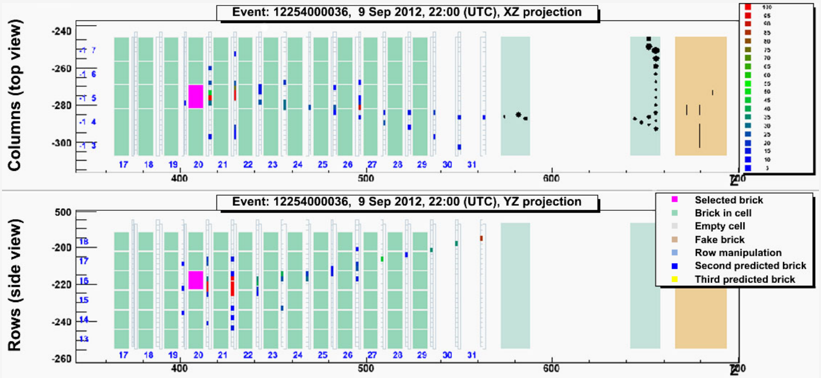
The 4th ν_τ Candidate Event

ϕ:



The 4th ν_τ Candidate Event

ED reconstruction:



Brick finding information: Super module 2

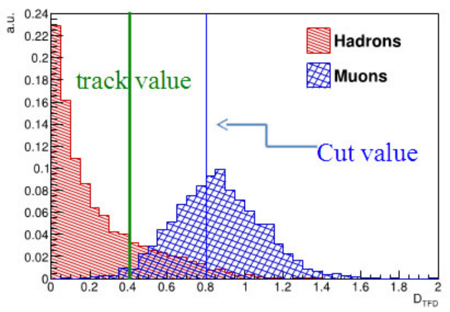
	BrickId	Wall	Side	Column	Row	Prob	CS x	CS y
brick 1:	1092217	20	-1	5	16	0.77	-1.0	-1.0
brick 2:	1089442	20	-1	5	15	0.12	-1.0	-1.0
brick 3:	1000527	21	-1	5	16	0.07	-1.0	-1.0

Muon track parameters:
Momentum: N/A
Tangent angle XZ: N/A
Tangent angle YZ: N/A

The 4th ν_τ Candidate Event

Track length $\times$ density ($L \times \rho$):

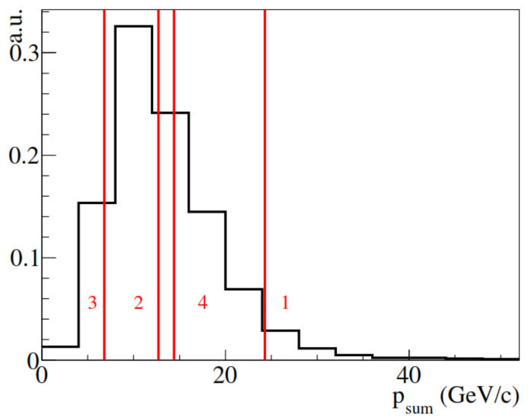
- $L \times \rho = 604\,\mathrm{g/cm^2}$ (cut value: $L \times \rho < 660\,\mathrm{g/cm^2}$)
- $D = \frac{L}{R_{lead}(p)} \times \frac{\rho_{average}}{\rho_{lead}} = 0.40^{+0.04}_{-0.05}$



- Probability for μ to cross ≤ 12 planes: $\sim 0.4\,\%$
- Probability for π to cross ≥ 12 planes: $\sim 9.2\,\%$

$\nu_\mu \rightarrow \nu_\tau$ Oscillation Search

Reconstructed momentum of the ν_τ candidates:

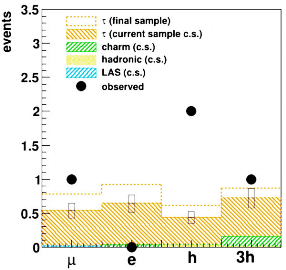


$\nu_\mu \to \nu_\tau$ Oscillation Search

Expected number of events for the analysed sample:

Decay channel	Expected signal	Observed	Expected background			
			Total	Charm decays	Hadronic re-interactions	Large-angle muon scattering
$\tau \to 1h$	0.41 ± 0.08	2	0.033 ± 0.006	0.015 ± 0.003	0.018 ± 0.005	/
$\tau \to 3h$	0.57 ± 0.11	1	0.155 ± 0.030	0.152 ± 0.030	0.002 ± 0.001	/
$\tau \to \mu$	0.52 ± 0.10	1	0.018 ± 0.007	0.003 ± 0.001	/	0.014 ± 0.007
$\tau \to e$	0.62 ± 0.12	0	0.027 ± 0.005	0.027 ± 0.005	/	/
Total	2.11 ± 0.42	4	0.233 ± 0.041	0.198 ± 0.040	0.021 ± 0.006	0.014 ± 0.007

Expected number of events:



$\nu_\mu \to \nu_\tau$ Oscillation Search

τ selection criteria:

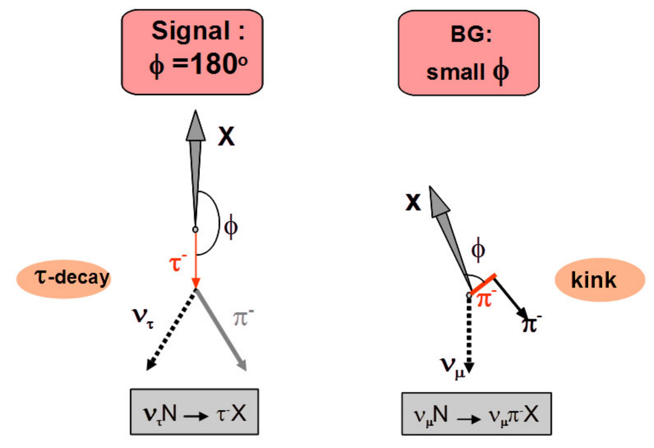
variable	$\tau \to 1h$	$\tau \to 3h$	$\tau \to \mu$	$\tau \to e$
lepton-tag	No μ or e at the primary vertex			
z_{dec} (μm)	$[44, 2600]$	< 2600	$[44, 2600]$	< 2600
p_T^{miss} (GeV/c)	$< 1^\star$	$< 1^\star$	/	/
ϕ_{lH} (rad)	$> \pi/2^\star$	$> \pi/2^\star$	/	/
p_T^{2ry} (GeV/c)	$> 0.6(0.3)^*$	/	> 0.25	> 0.1
p^{2ry} (GeV/c)	> 2	> 3	> 1 and < 15	> 1 and < 15
θ_{kink} (mrad)	> 20	< 500	> 20	> 20
m, m_{min} (GeV/c^2)	/	> 0.5 and < 2	/	/

$\tau \to h$ selection criteria & 4th ν_τ candidate:

Variable	Selection	Measured value
θ_{kink} (mrad)	> 20	137 ± 4
z_{dec} (μm)	< 2600	406 ± 30
p_{2ry} (GeV/c)	> 2	$6.0^{+2.2}_{-1.2}$
p_T^{2ry} (GeV/c)	> 0.6 (0.3^*)	$0.82^{+0.30}_{-0.16}$
p_T^{miss} (GeV/c)	< 1	$0.55^{+0.30}_{-0.20}$
$\Delta\phi_{\tau H}$ (degrees)	> 90	166^{+2}_{-31}

$\nu_\mu \to \nu_\tau$ Oscillation Search

ϕ:

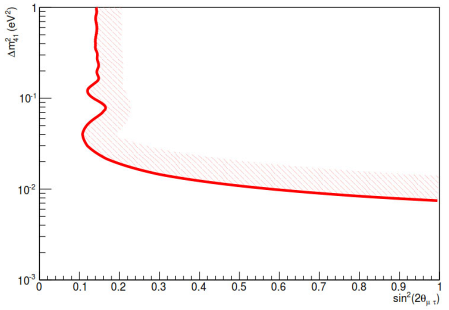


$\nu_\mu \to \nu_\tau$ Non-Standard Oscillations

$3+1$ analysis (approx.):

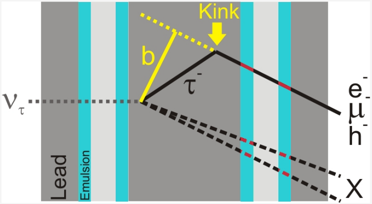
$$\begin{aligned} P(E) &= C^2\ sin^2(1.27\Delta m^2_{32}L/E)\ +\ 0.5\ sin^2(2\theta_{\mu\tau}) \\ &+\ C\ sin(2\theta_{\mu\tau})\ cos(\delta)\ sin^2(1.27\Delta m^2_{32}L/E) \\ &+\ 0.5\ C\ sin(2\theta_{\mu\tau})\ sin(\delta)\ sin(2.54\Delta m^2_{32}L/E) \end{aligned}$$

Δm^2_{41} vs. $\sin^2(2\theta_{\mu\tau})$ (NH): $90\,\%$ C.L. exclusion limits



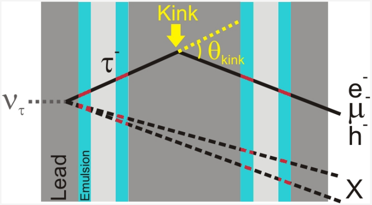
Decay Search Procedure

Short decay:



- Impact parameter b

Long decay:



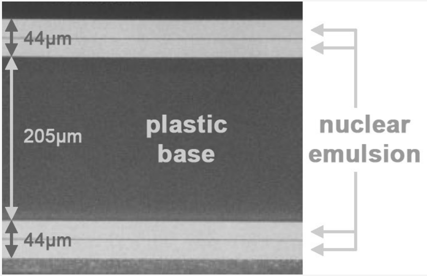
- Kink angle θ_{kink}

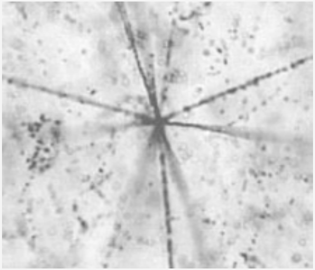
Decay search procedure:

- In-track decay search
- Search for extra tracks (parent / daughters)

▷ Measurement of kink angle θ_{kink} or impact parameter b

The OPERA Detector



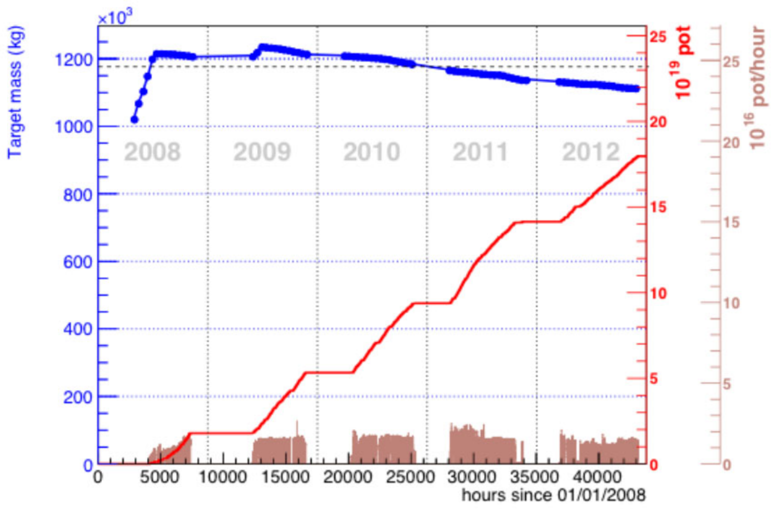


Emulsion Cloud Chamber (ECC) nuclear emulsions:

- **Basic detector elements:** AgBr crystals ($0.2\,\mu\mathrm{m}$)
  - ▷ **Intrinsic resolution:** $50\,\mathrm{nm}$
- **Hadronic momentum measurement:** Via MCS
- π/μ **separation:** Via dE/dx (at low energies)
- e identification, em shower energy estimation

The OPERA Detector

Detector statistics:



3-Flavour Neutrino Oscillations

Mixing of mass & flavour eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \times \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMNS matrix U:

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric terms}} \times \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{Unknown terms}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar terms}} \times \underbrace{\begin{pmatrix} e^{i\epsilon_1/2} & 0 & 0 \\ 0 & e^{i\epsilon_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana terms}}$$

3-Flavour Neutrino Oscillations

Oscillation parameters:

Parameter	Current best fit
Δm^2_{31}	$(2.55^{+0.06}_{-0.09}) \times 10^{-3}\,\mathrm{eV}^2$
Δm^2_{21}	$(7.62 \pm 0.19) \times 10^{-5}\,\mathrm{eV}^2$
$\sin^2(\theta_{23})$	$0.613^{+0.022}_{-0.040}$
$\sin^2(\theta_{12})$	$0.320^{+0.016}_{-0.017}$
$\sin^2(\theta_{13})$	$0.0246^{+0.029}_{-0.028}$
δ	$(0.80^{+1.20}_{-0.80})\pi$

For normal hierarchy

D.V. Forero, M. Tortola, J.W.F. Valle, *Global status of neutrino oscillation parameters after Neutrino-2012*, Phys. Rev. D **86** 073012 (2012)