# Equation of state for supernova and neutron stars in a relativistic mean field model with density dependent couplings

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# Plan of the Talk

- Introduction
- Relativistic mean field model with Density-dependent couplings(DD2)
- Equation of state (EoS)
- Numerical simulation (GR1D code)
- Results
- Summary & Outlook

# Introduction

The core collapse supernova explosion (Type II) mechanism is being investigated over the last five decades.
Still, the theory of successful supernova explosion is beyond our reach.
Microphysics inputs such as equation of state (EoS) are important for simulations of stellar collapse for a wide range of

- density ($10^4 - 10^{15} \mathrm{g/cm}^3$),
- temperature ($0 - 100$MeV)
- composition (proton fraction $0 - 0.6$).

Mainly two sets of nuclear EoS are used-
a) Lattimer-Swesty(LS) Lattimer and Swesty, 1991
b) Shen Shen, Toki, Oyamatsu and Sumiyoshi, 1998
The constituents here are non-strange particles like neutrons, protons, $\alpha$-particles and heavy nuclei.

# Strangeness in the post-bounce phase of a core-collapse supernova

- Pauli exlusion principle dictates the appearance of strange degrees of freedom in the high density baryonic matter.
  - Hyperons
  - Bose-Einstein condensates of Kaons
  - Quarks
- Recent Observations put limit of $2M_{\odot}$ on neutron star mass.
  Ref:D.J. Champion, et al., Science, 320, 1309 (2008).
  P. B. Demorest et. al., Nature **467**, 1081 (2010).

  J. Antoniadis et. al., Science **340**, 6131 (2013).
- Presence of strange hadrons results in a softer EoS which lowers maximum mass of the neutron star.
- These observations put stringent constraints on the model of neutron star and abandons most of the soft EoS models.

# Equation of State

- We constructed an EoS with strange matter for neutrons stars that yields maximum mass within the observable limit of $2M_\odot$ .
- Our hyperon EoS tables are for densities ($10^3 - 10^{15}\mathrm{g/cm^3}$), temperatures ($0.1 - 158$MeV) and proton fractions ($0.01 - 0.6$).
- We adopt a Density Dependent Relativistic Mean Field (RMF) Model to describe uniform matter including hyperons.
- At low temperature and sub-saturation density, matter is mainly composed of light and heavy nuclei coexisting with unbound nucleons. This is treated in the Nuclear Statistical Equilibrium model (Saha Equation) (Hempel and Schaffner, Nucl. Phys. A837, 210 (2010)).

# Density Dependent Relativistic Model

- The interaction between baryons is mediated by the exchange of scalar ($\sigma$) and vector ($\omega, \phi, \rho$) mesons.

$$
\begin{aligned}
\mathcal{L}_B = & \sum_{B=N,\Lambda,\Sigma,\Xi} \bar{\psi}_B \left(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - g_{\rho B}\gamma_\mu \boldsymbol{\tau}_B \cdot \boldsymbol{\rho}^\mu \right) \\
& + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \\
& + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu .
\end{aligned}
$$

Ref: S. Banik, M. Hempel, D.Bandyopadhyay, Astrophys Suppl J 2014;
P. Char, S.Banik, PRC 90, 015801 (2014)

- Leptons are treated as non-interacting particles and described by the Lagrangian density

$$\mathcal{L}_l = \sum_l \bar{\psi}_l \left(i\gamma_\mu\partial^\mu - m_l\right)\psi_l$$

# Density-Dependent Couplings

- The $g_{\alpha B}(\hat{n})$'s, where $\alpha = \sigma,\ \omega$ and $\rho$ specify the coupling strength of the mesons with baryons and are vector density-dependent.
- The density operator $\hat{n}$ has the form, $\hat{n}=\sqrt{\hat{j}_\mu \hat{j}^\mu}$, where $\hat{j}_\mu = \bar{\psi}\gamma_\mu\psi$.
- The meson-baryon couplings become function of total baryon density $n$ i.e. $< g_{\alpha B}(\hat{n}) >= g_{\alpha B}(< \hat{n} >) = g_{\alpha B}(n)$

[ Ref:P. Char, S.Banik, PRC 90, 015801 (2014) ]

# Hyperons

- Hyperons produced at the cost of the nucleons.
$n + p \longrightarrow p + \Lambda + K^0, \quad n + n \longrightarrow n + \Sigma^+ + K^-$
- $\Lambda$s, being the lightest hyperons with an attractive potential of $\sim -30$ MeV in nuclear matter, are believed to populate the dense matter among all strange baryons.
- Other hyperons, $\Xi$ & $\Sigma$ are excluded due to their relatively higher threshold and lack of experimental data.
- Interaction among hyperons can be represented by the Lagrangian density

$$\begin{aligned}\mathcal{L}_{YY} &= \sum_B \bar{\psi}_B \left(g_{\sigma^* B}\sigma^* - g_{\phi B}\gamma_\mu \phi^\mu\right)\psi_B \\ &+\frac{1}{2}\left(\partial_\mu \sigma^* \partial^\mu \sigma^* m_{\sigma^*}^2 \sigma^{*2}\right) - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi^2 \phi_\mu \phi^\mu .\end{aligned}$$

# Equation of State

The chemical potential for the baryon B is

$$\mu_B = \sqrt{k_B^2 + m_B^{*2}} + g_{\omega B}\omega_0 + g_{\rho B}\tau_{3B}\rho_{03} + g_{\phi B}\phi_0 + \Sigma_B^{(r)}$$

The rearrangement term,

$$\Sigma_B^{(r)} = \sum_B [-g'_{\sigma B}\sigma n_B^s + g'_{\omega B}\omega_0 n_B + g'_{\rho B}\tau_{3B}\rho_{03} n_B + g'_{\phi B}\phi_0 n_B] \ ,$$

where $g'_{\alpha B} = \frac{\partial g_{\alpha B}}{\partial \rho_B}$, $\alpha = \sigma,\ \omega,\ \rho,\ \phi$

$$\begin{aligned} P_B &= -\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 + \Sigma_B^{(r)}\sum_B n_B \\ &+ \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}} \frac{k^4\,dk}{(k^2+m_B^{*2})^{1/2}} \\ &+ \frac{1}{3}\sum_l \frac{1}{\pi^2}\int_0^{K_{F_l}} \frac{k^4\,dk}{(k^2+m_l^2)^{1/2}} \ . \end{aligned}$$

# Parameters of the Model

- The density dependent couplings $g_{\sigma N}$ and $g\omega N$ are given by

$$g_{\alpha N} = g_{\alpha N}(n_0) f_\alpha(x)$$
$$f_\alpha(n_b/n_0) = a_\alpha \frac{1 + b_\alpha(x + d_\alpha)^2}{1 + c_\alpha(x + d_\alpha)^2}$$

  Here $n_0$ is the saturation density, $\alpha = \sigma, \omega$ and $x = n_b/n_0$.
- For $\rho$ mesons, $g_{\rho N} = g_{\rho N}(n_0) exp[-a_\rho(x-1)]$. S. Typel et. al. Phys. Rev.**C 81** 015803,(2010).
- The scaling factors for vector and isovector mesons from the SU(6) symmetry relations of the quark model
  $\frac{1}{2} g_{\omega\Lambda} = \frac{1}{3} g_{\omega N}$; $g_{\rho\Lambda} = 0$; $2g_{\phi\Lambda} = -\frac{2\sqrt{2}}{3} g_{\omega N}$
- Scalar-$\Lambda$ hyperon is obtained from the potential depth of $\Lambda$ hyperon in saturated nuclear matter: $U_\Lambda^N(n_0) = \Sigma_\Lambda^v - \Sigma_\Lambda^s$
- The potential depth $U_\Lambda^N(n_0) = -30$ MeV from $\Lambda$ hypernuclei data.

# Extended NSE model

Internal excitations, Coulomb screening and excluded volume effects are included.

The total canonical partition function is given by,

$$Z(T, V, \{N_i\}) = Z_{nuc} \prod_{A,Z} Z_{A,Z} \; Z_{Coul} .$$

The free energy density is defined as

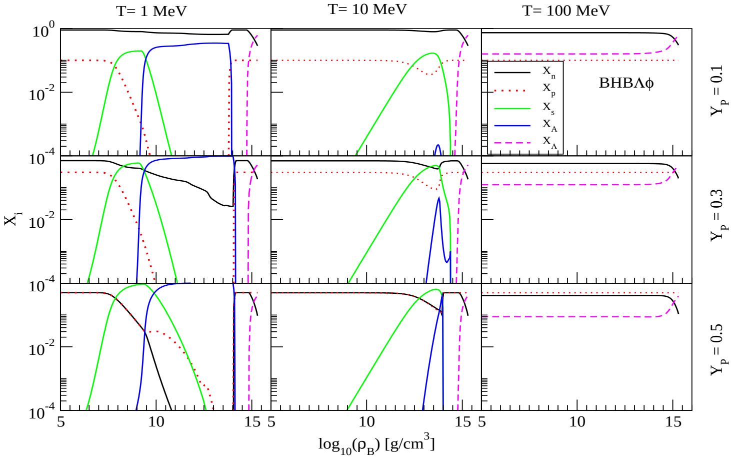
$$f \;=\; \sum_{A,Z} f^0_{A,Z}(T, n_{A,Z}) + f_{Coul}(n_e, n_{A,Z}) + \xi f^0_{nuc}(T, n'_n, n'_p) - T \sum_{A,Z} n_{A,Z} \ln(\kappa)$$

where the last term goes to infinity when available volume fraction of nuclei ($\kappa$) is zero near saturation density.

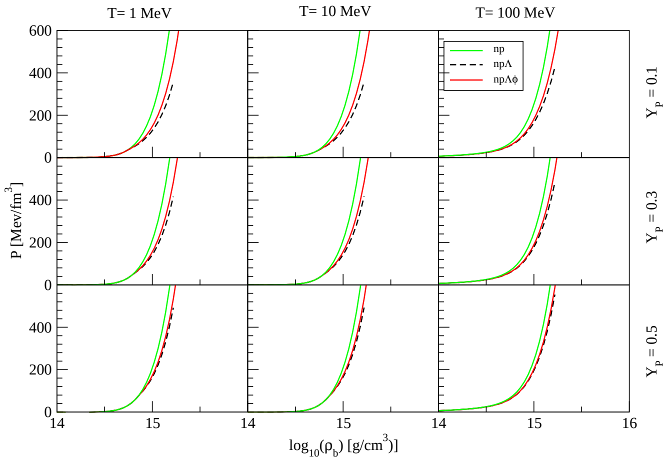
For the merging of the two tables,

i) Free energy/baryon at fixed $T$, $n_B$, and $Y_p$ is to be minimized,

ii) hyperon fraction is small i.e. $10^{-5}$.




S. Banik, M. Hempel, D.Bandyopadhyay, Astrophy.J. Suppl, 2014

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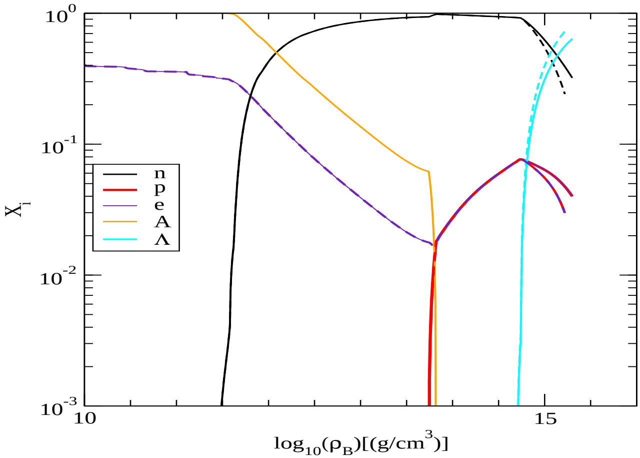
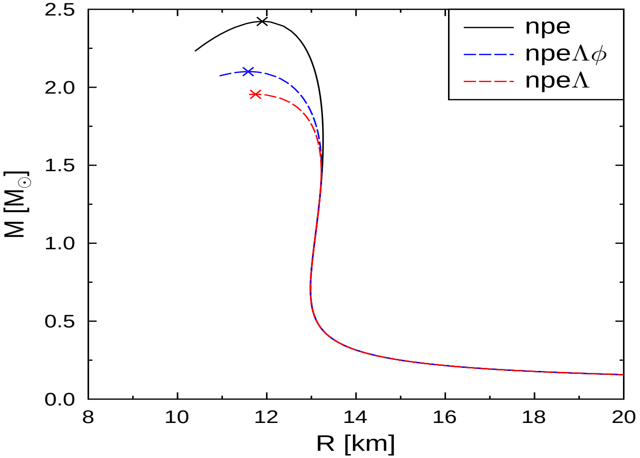
# Composition in Neutron Star Matter



# Mass-Radius Relation of Neutron Stars

Hyperon EoS is compatible with a 2 $M_\odot$ Neutron Star.

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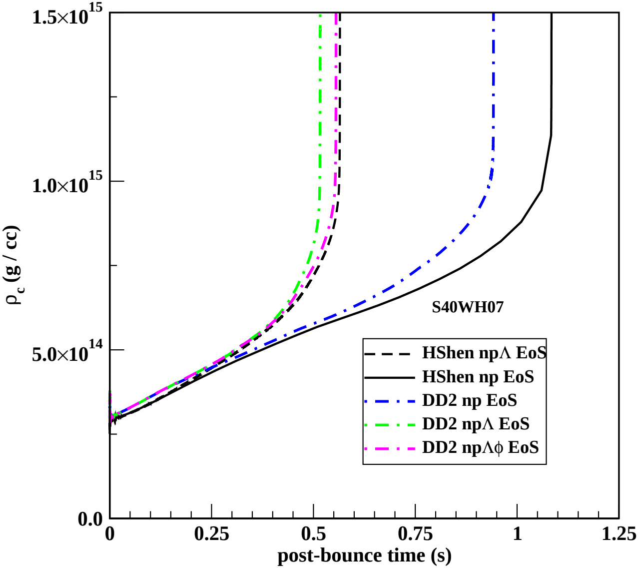
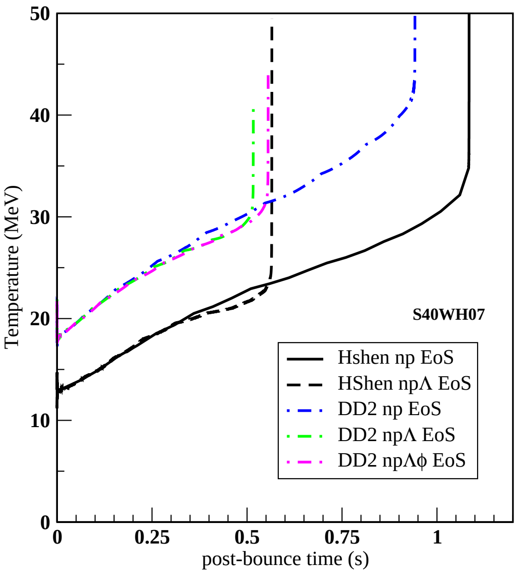
# Results of core collapse supernova simulation

- For various progenitor models of Wooseley et al. we performed the simulations using a spherically symmetric GR hydrodynamics code called *GR1D* for our DD2 EoS.
- GR1D studies dynamical stellar collapse to neutron stars and black hole formation.

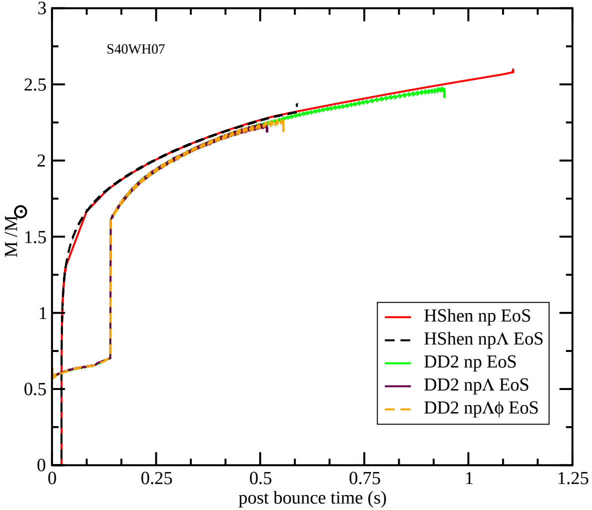
  [ Ref:C. D. Ott and E. O'Connor, Class.Quant.Grav.27:114103, 2010]
- For a $40M_{\odot}$ progenitor models of Wooseley et al. [ Ref: S. E. Woosley, A. Heger, and T. A. Weaver, Rev. Mod. Phys. **74**, 1015 (2002).] we show our simulation results using GR1D for DD2 nucleon and hyperon EoS.


1.5×10^15
1.0×10^15
5.0×10^14
0.0
ρc (g / cc)
0
0.25
0.5
0.75
1
1.25
post-bounce time (s)
S40WH07
HShen npΛ EoS
HShen np EoS
DD2 np EoS
DD2 npΛ EoS
DD2 npΛφ EoS



Temperature (MeV)
post-bounce time (s)
S40WH07
Hshen np EoS
HShen npΛ EoS
DD2 np EoS
DD2 npΛ EoS
DD2 npΛϕ EoS
0
10
20
30
40
50
0
0.25
0.5
0.75
1

# Temporal Evolution of Gravitational Mass



# Summary and Outlook

- New Hyperon EoS is compatible with density dependence of the symmetry energy and 2 $M_\odot$ neutron star.
- Hyperon EoS fails to generate second shock.
- The hadron-hyperon phase transition is a weak phase transition.
- Our aim is to explore the possibility that a hadron-antikaon phase transition during the early post bounce evolution may result in an explosion and its observational consequence in the form of neutrino signatures.

# Collaborators

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