

Kathlynne Tullney

Limits for Spin-Dependent Short-Range Interaction of Axion-Like Particles



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Outline

- Motivation
- Principle of measurements
- Experimental setup
- Results
- Summary

The strong CP-Problem

The non-trivial vacuum structure of QCD predicts violation of CP-symmetry:

$$L_\theta = \bar{\theta} \frac{g^2}{32\pi^2} G_{\mu\nu}^b \tilde{G}_b^{\mu\nu}$$

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$$|d_n| \approx \bar{\theta} \cdot 10^{-16} \text{ ecm} < 3 \cdot 10^{-26} \text{ ecm}$$

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Original proposal for Axion (*R. Peccei, H. Quinn PRL 38(1977), 1440*)

as possible solution to the „strong CP-Problem“ that cancels the CP violating term in the QCD.

$$L_a = \xi \frac{a}{f_a} \frac{g^2}{32\pi^2} G_{\mu\nu}^b \tilde{G}_b^{\mu\nu}$$

$$\langle a \rangle = -f_a \frac{\bar{\theta}}{\xi}$$

with: $m_a \propto \frac{1}{f_a}$

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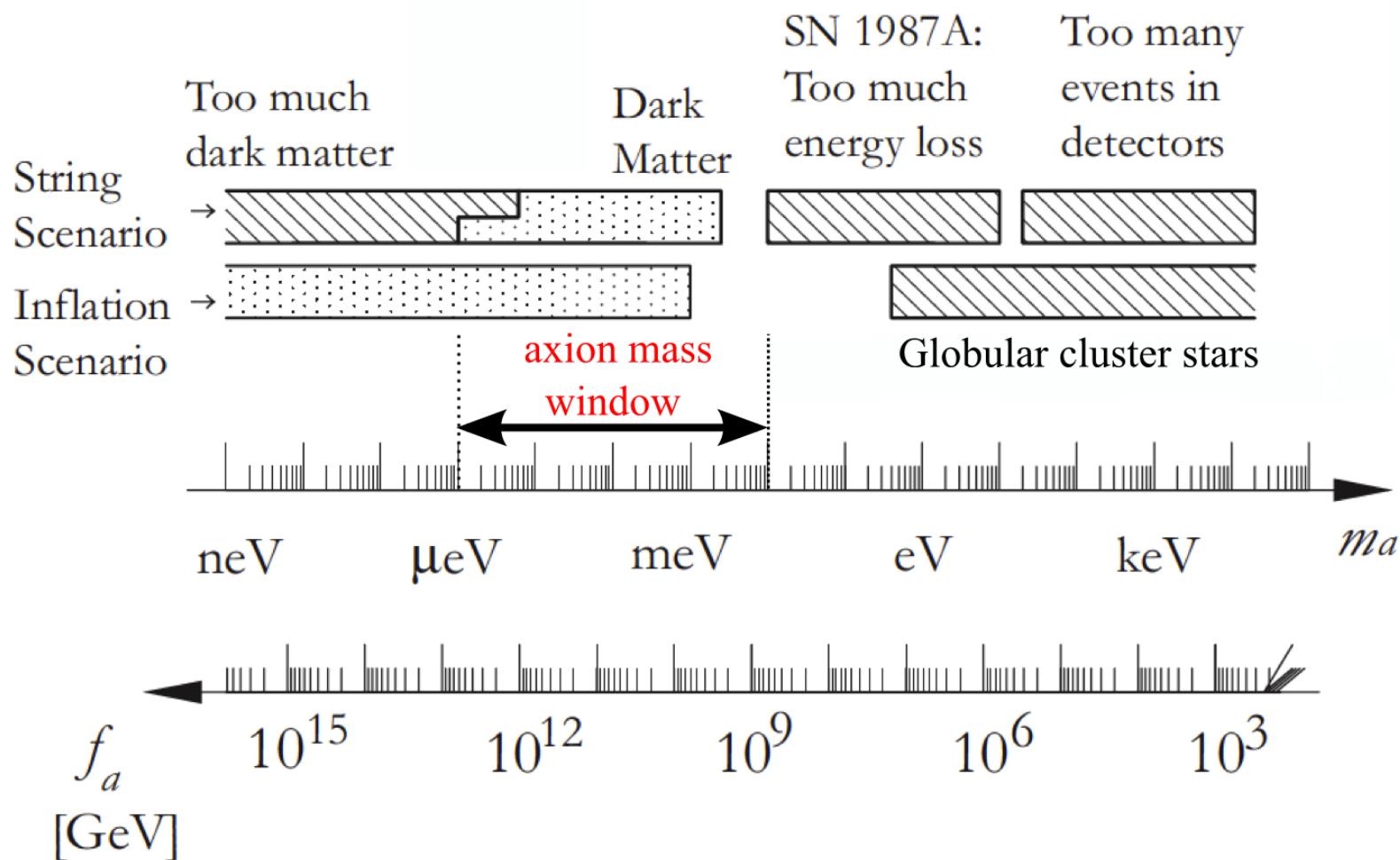
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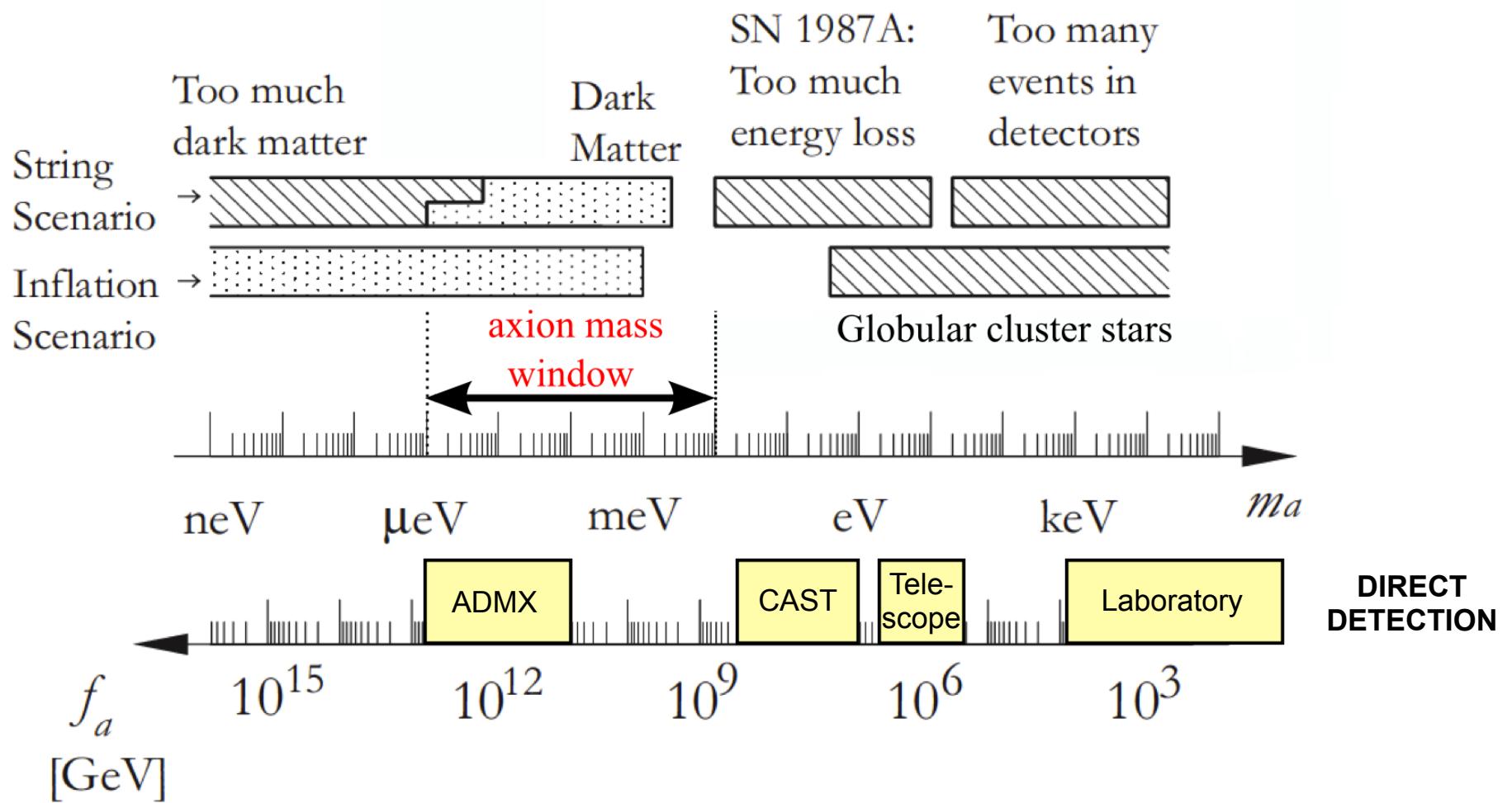
AXIONS: Light and weak interacting particles.

Modern Interest: Dark Matter candidate.

Axion Mass



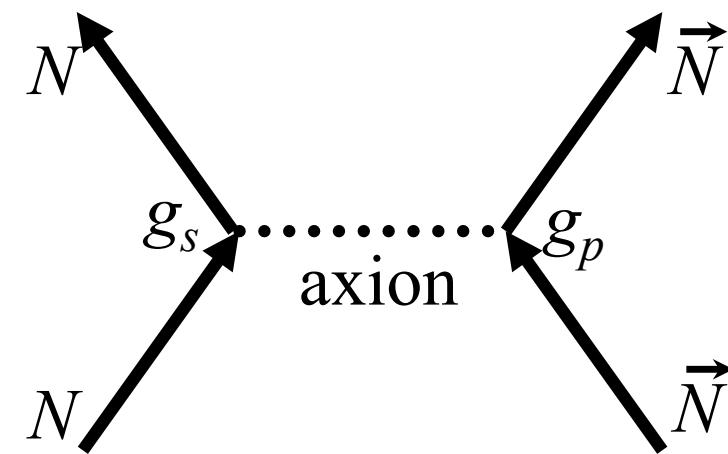
Axion Mass



Axion Potential:

Yukawa type potential with monopole-dipole coupling [1]

$$V(r) = \kappa \hat{n} \cdot \vec{\sigma} \left(\frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}$$

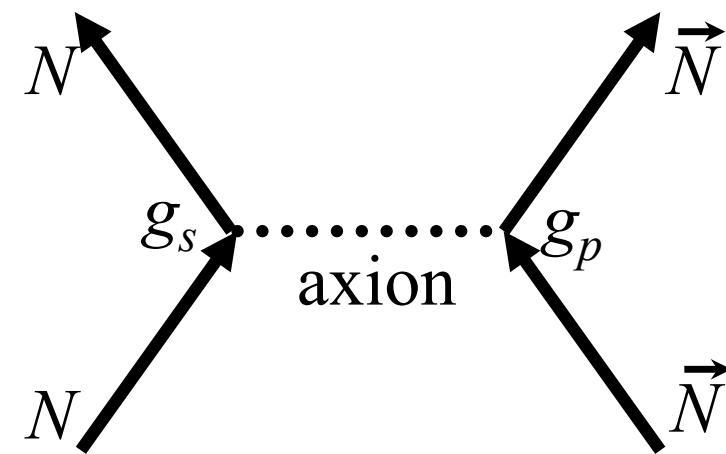


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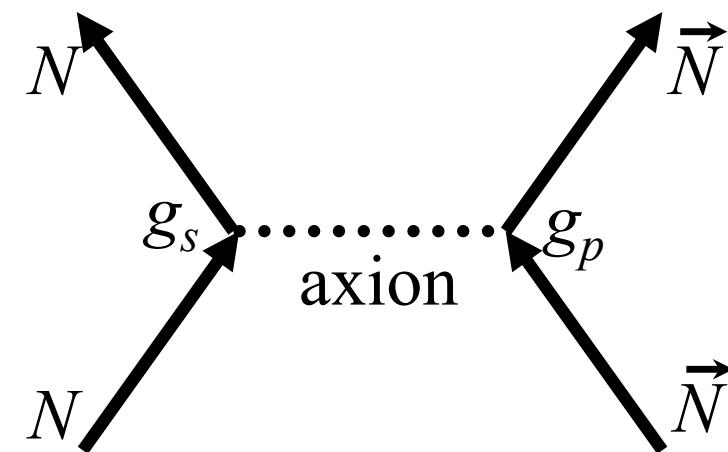


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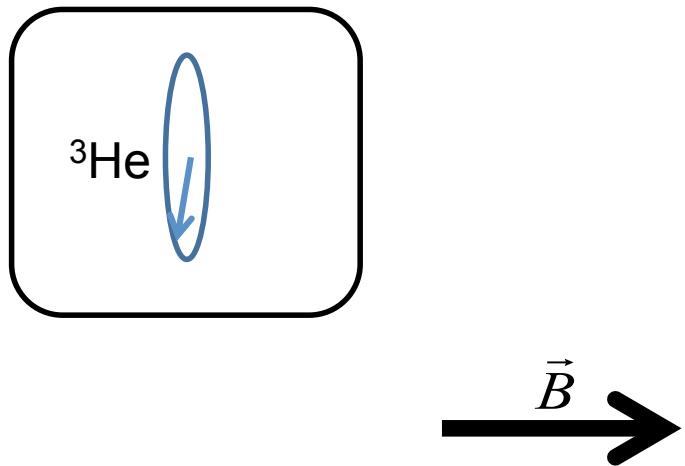
⇒ Indirect search of the axion via the axion potential in the range of the „axion mass window“:

$$10^{-6} \text{ eV} < m_a < 10^{-2} \text{ eV}$$

$$10^{-5} \text{ m} < \lambda < 10^{-1} \text{ m}$$

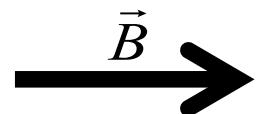
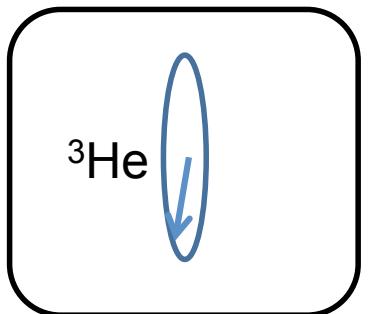
Principle of measurements

How to measure?



Principle of measurements

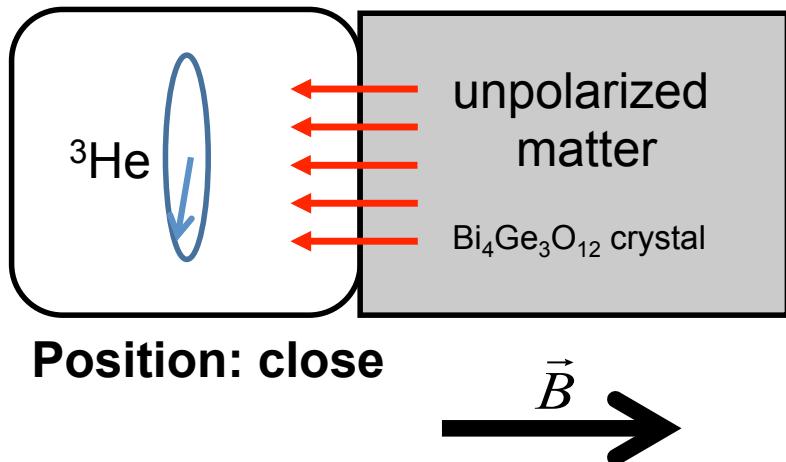
How to measure?



$$\omega_{\text{L,He}}(t) = \gamma_{\text{He}} \cdot B(t)$$

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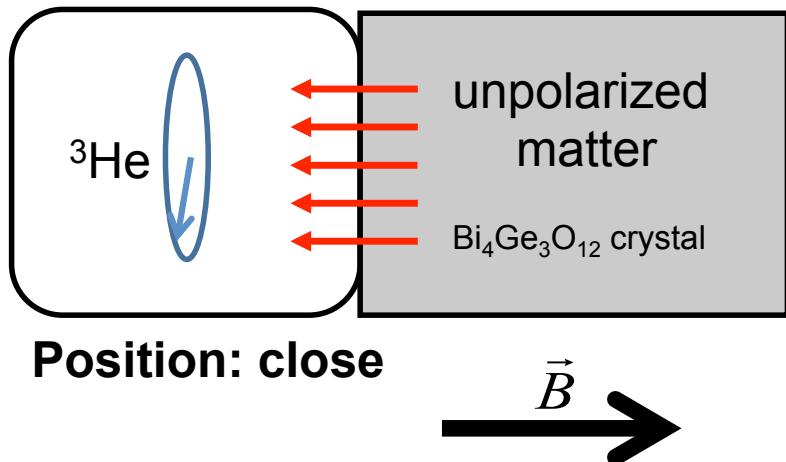
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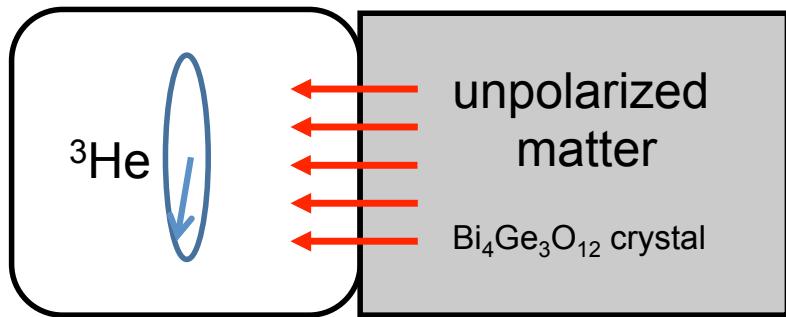
$$\omega_{\text{close}}(t) = \omega_{\text{L,He}}(t) + \omega_{\text{sp}}$$

with: $\omega_{\text{L,He}}(t) = \gamma_{\text{He}} \cdot B(t)$

$$\omega_{\text{sp}} = 2\pi \cdot \nu_{\text{sp}} = 2 \cdot \bar{V} / \hbar$$

Principle of measurements

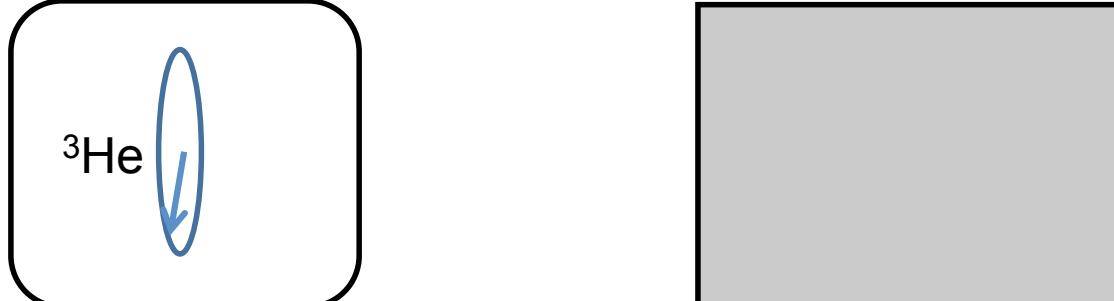
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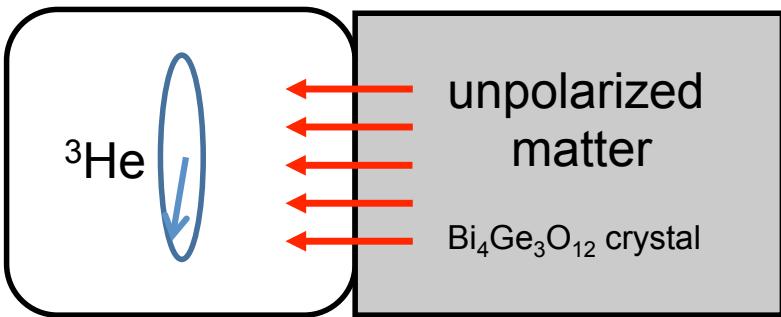
$$\omega_{\text{sp}} = 2\pi \cdot \nu_{\text{sp}} = 2 \cdot \bar{V} / \hbar$$



$$\omega_{\text{distant}}(t) = \omega_{\text{L,He}}(t)$$

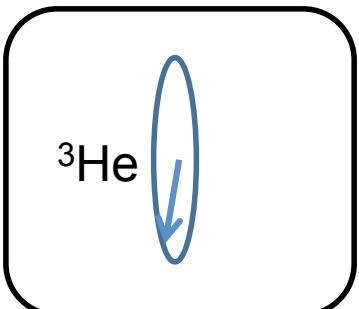
Principle of measurements

How to measure?



Position: close

$$\vec{B} \rightarrow$$



Position: distant

$$\omega_{\text{close}}(t) = \omega_{L,\text{He}}(t) + \omega_{\text{sp}}$$

$$\text{with: } \omega_{L,\text{He}}(t) = \gamma_{\text{He}} \cdot B(t)$$

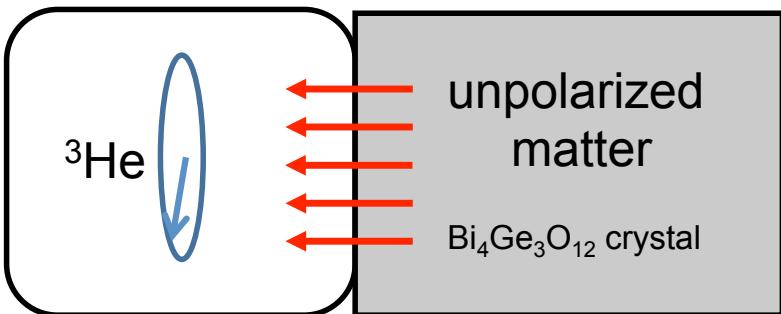
$$\omega_{\text{sp}} = 2\pi \cdot \nu_{\text{sp}} = 2 \cdot \bar{V} / \hbar$$

$$\omega_{\text{distant}}(t) = \omega_{L,\text{He}}(t)$$

$$\Rightarrow \omega_{\text{sp}} = \omega_{\text{close}}(t) - \omega_{\text{distant}}(t)$$

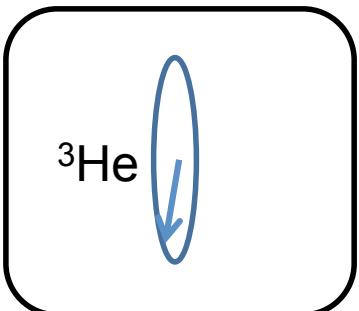
Principle of measurements

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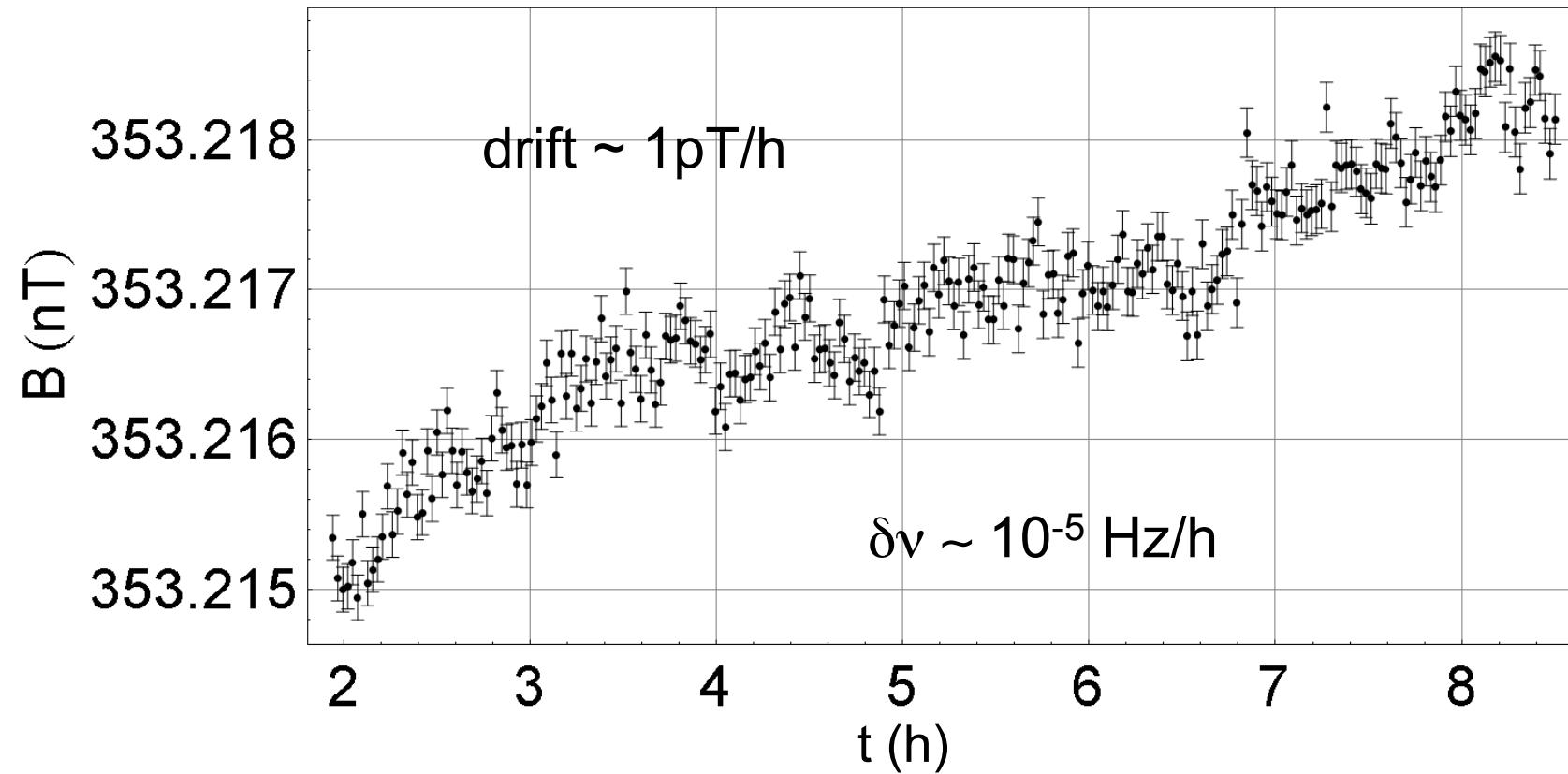
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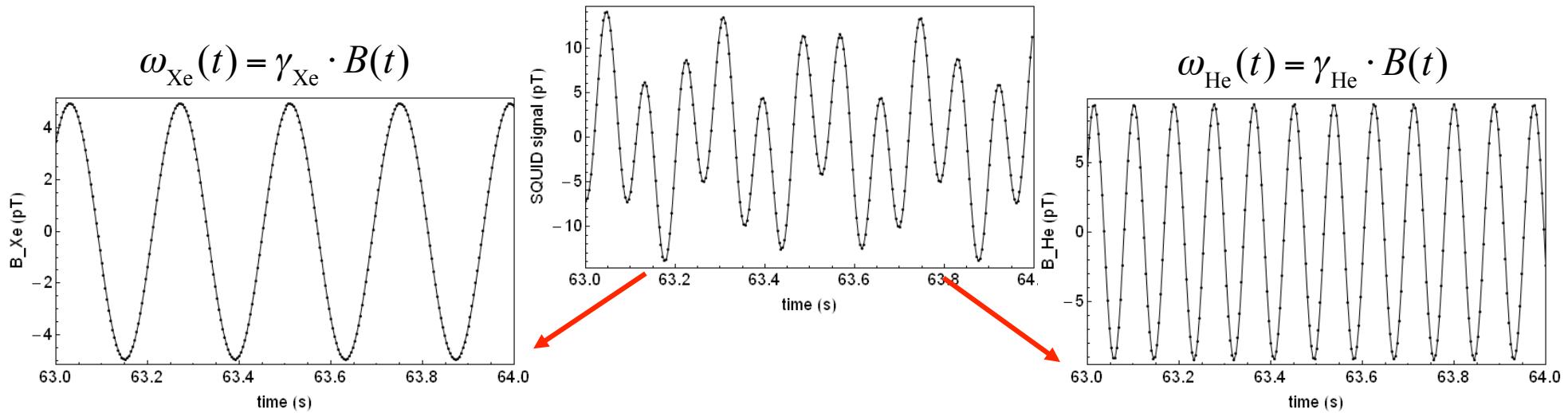
Requirement: $\omega_{L,\text{He}}(t) = \text{const.}$

Principle of measurements



Principle of measurements

${}^3\text{He}/{}^{129}\text{Xe}$ Co-magnetometer

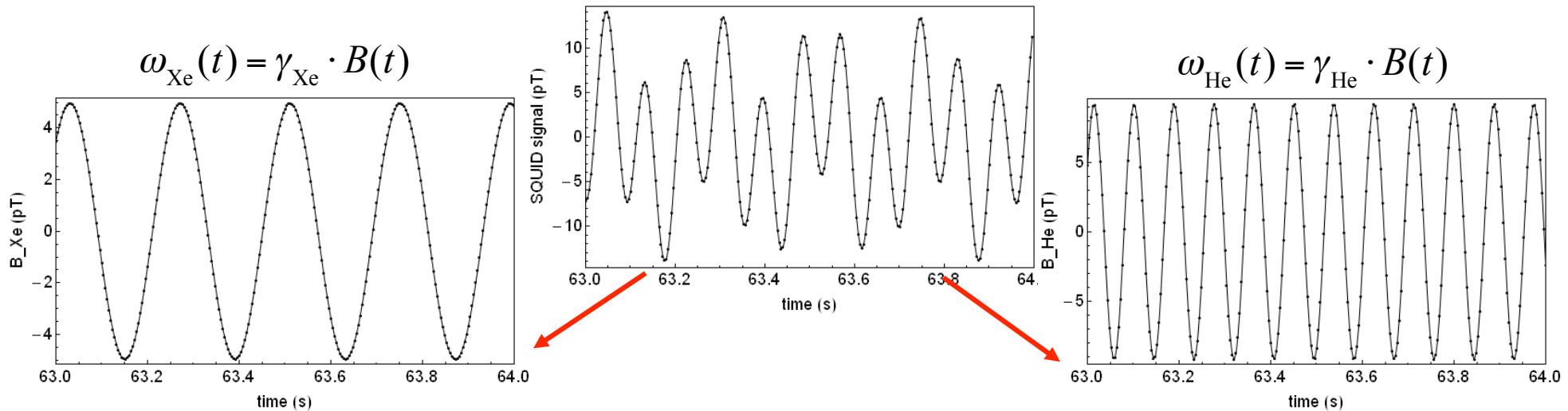


Elimination of magnetic field drifts (Zeeman-Term):

Frequency difference: $\Delta\omega = \omega_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \omega_{\text{Xe}} = \left(\gamma_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \cdot \gamma_{\text{Xe}} \right) \cdot B(t) ! = 0$

Principle of measurements

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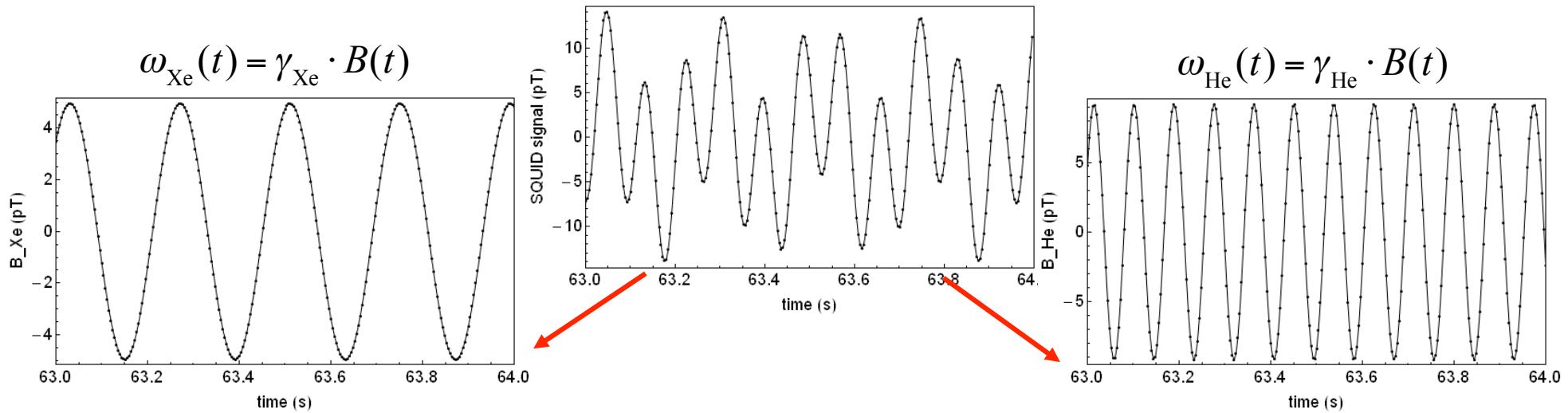
$$\Phi(t) = \int_0^t \omega(t') dt'$$

Phase difference:

$$\Delta\Phi(t) = \Phi_{\text{He}}(t) - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \cdot \Phi_{\text{Xe}}(t) = \Phi_0 \stackrel{!}{=} \text{const.}$$

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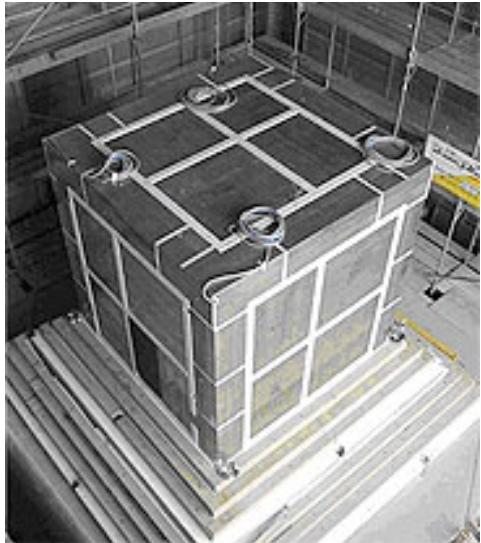
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Phase difference:

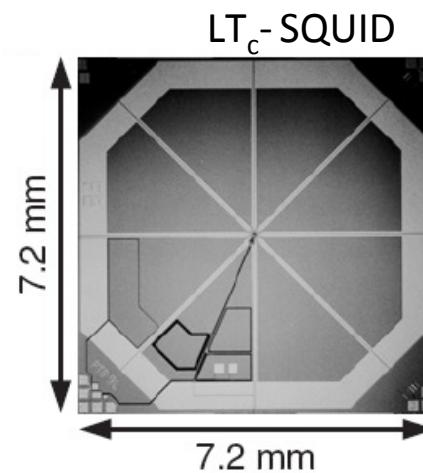
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Experimental Setup



7 layered magnetically
shielded room
(residual field < 1 nT)

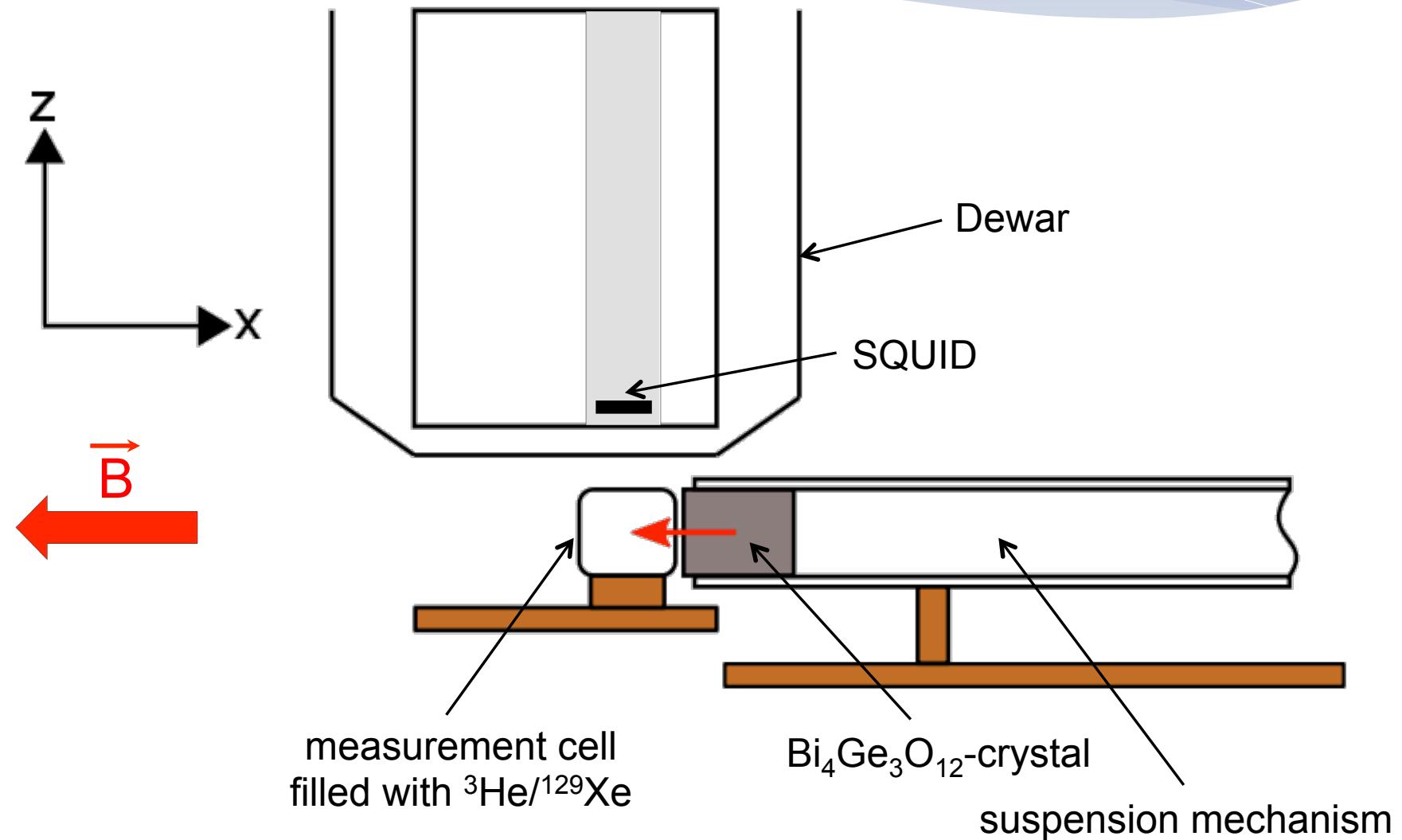
J. Bork, et al., Proc. Biomag 2000, 970 (2000).



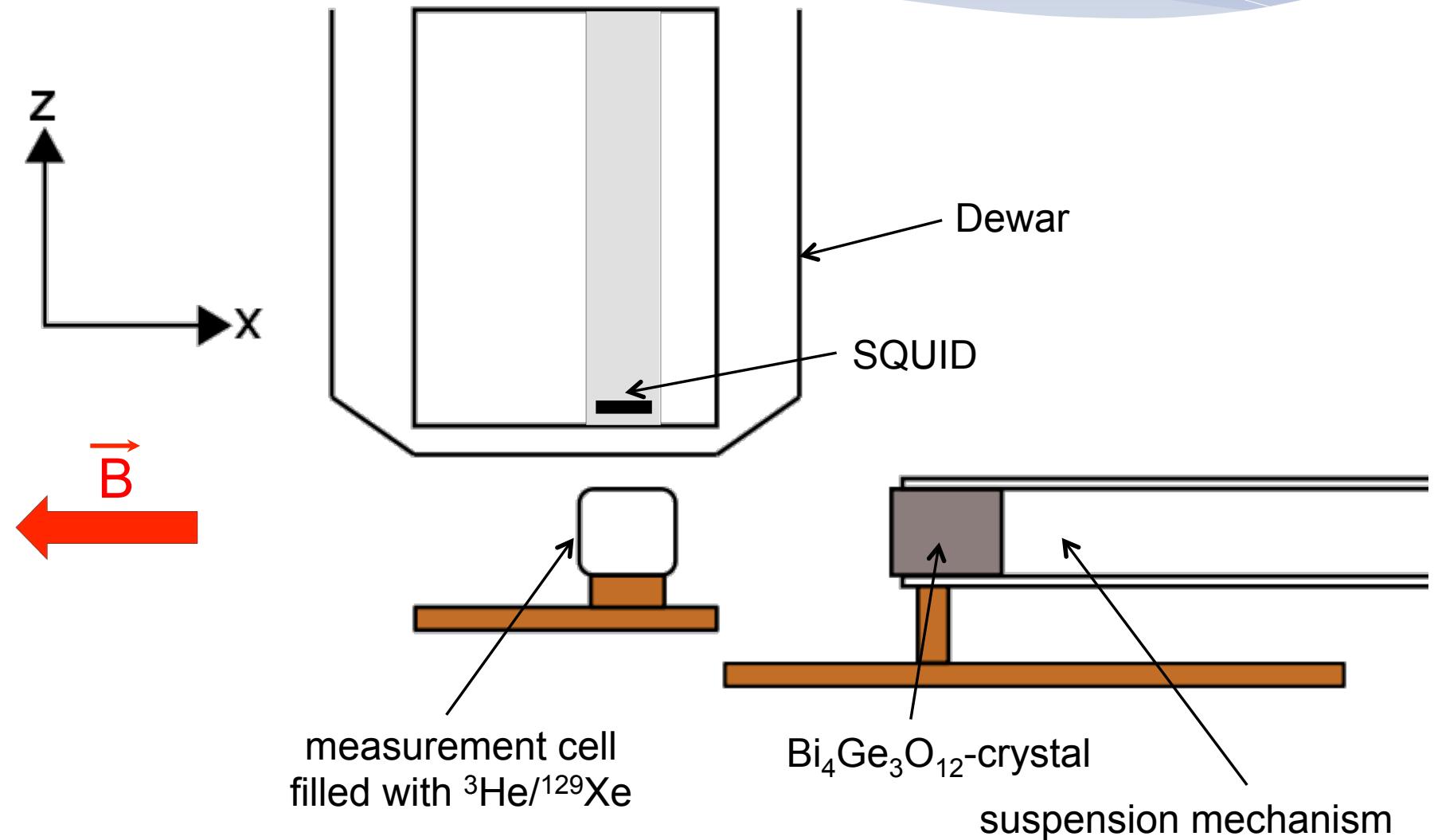
magn. guiding field ≈ 350 nT (Helmholtz-coils)

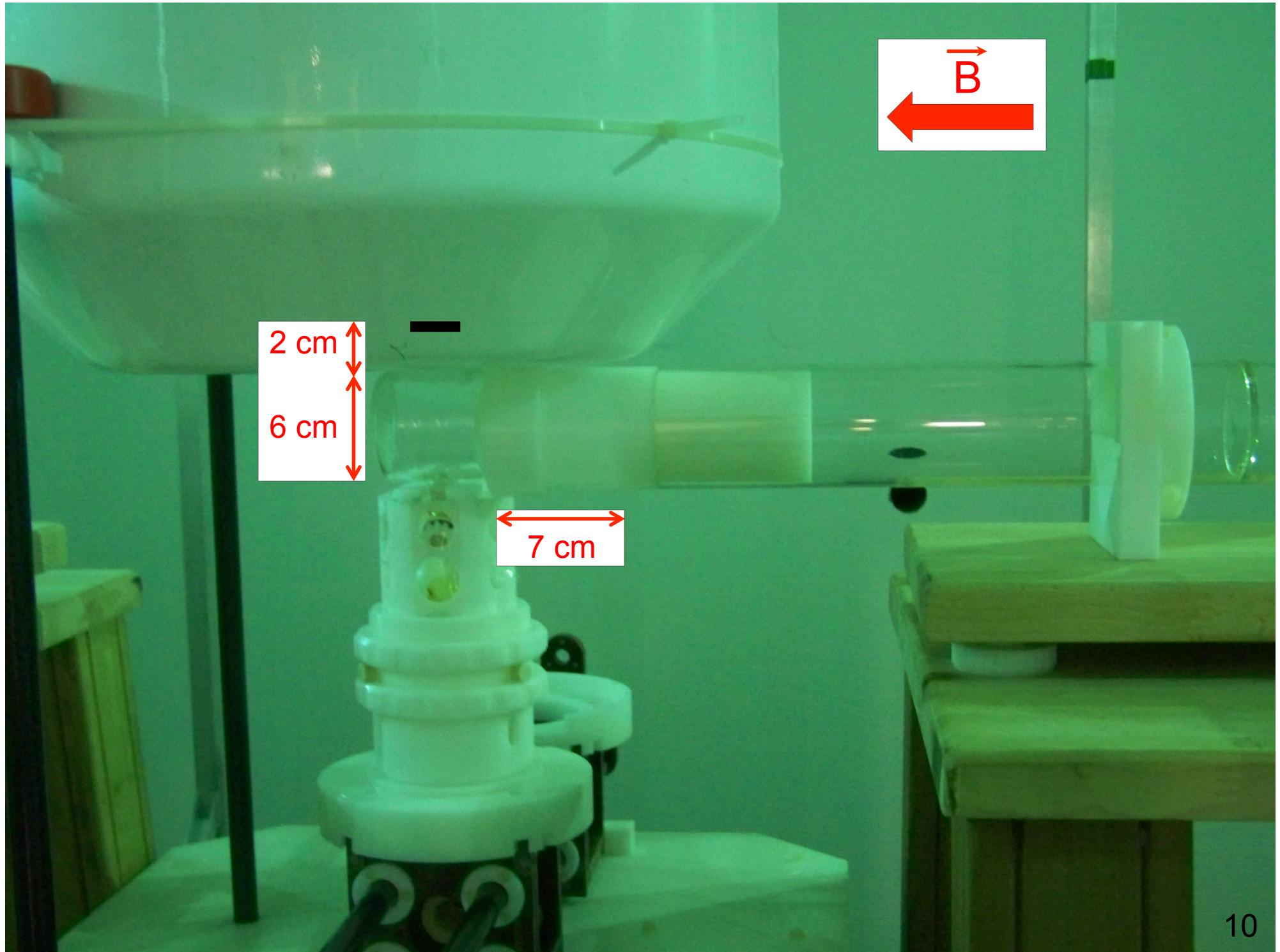
$$|\vec{\nabla}B_{x,y,z}| \approx 20 \text{ pT/cm}$$

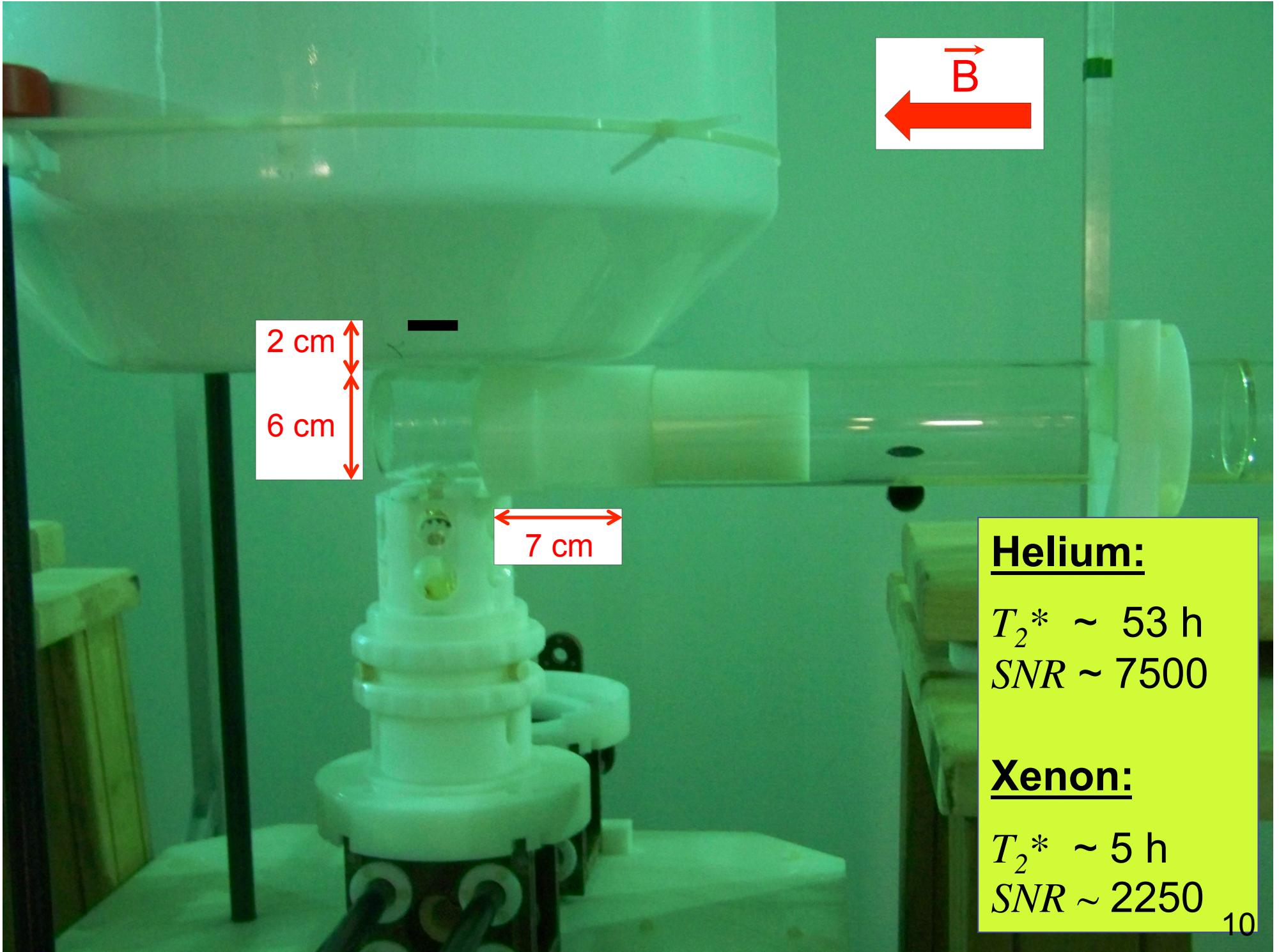
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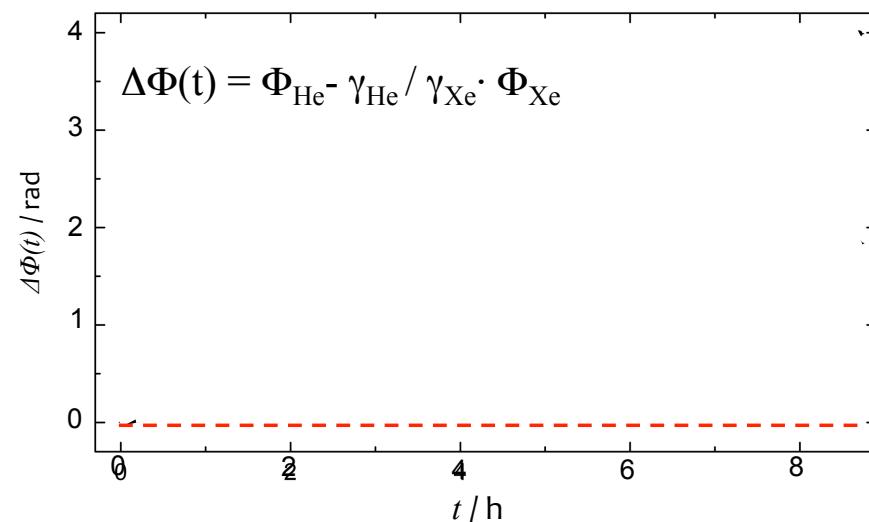




Data Analysis

1. To cancel magnetic field influence we calculate the weighted phase difference:

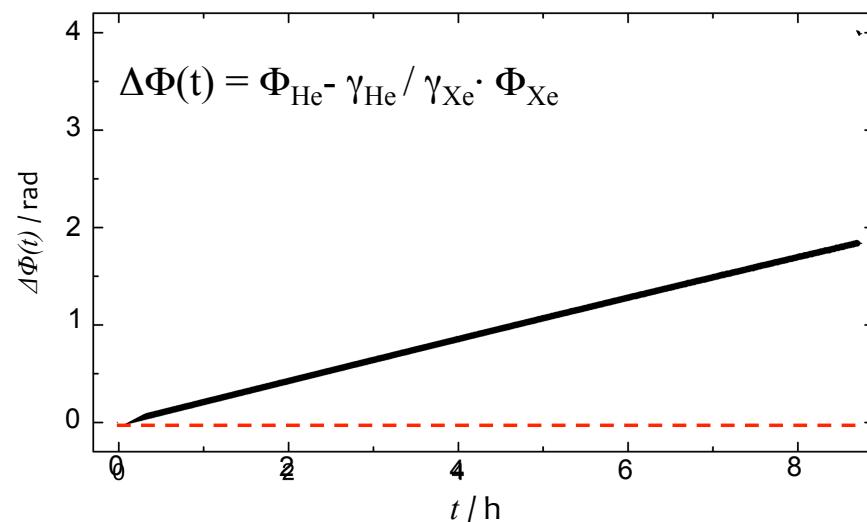
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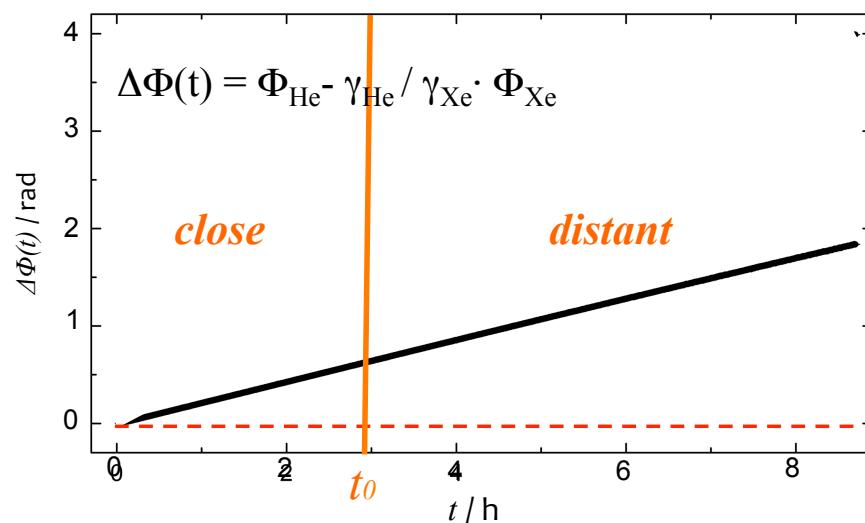
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2. Temporal dependence can be described by:

$$f(t) = c + a_{\text{lin}} \cdot t + a_{\text{He}} \cdot e^{-t/T_{2,\text{He}}^*} + a_{\text{Xe}} \cdot e^{-t/T_{2,\text{Xe}}^*} + \Delta\omega_{\text{sp}} \cdot t \cdot H(t - t_0)$$



Data Analysis

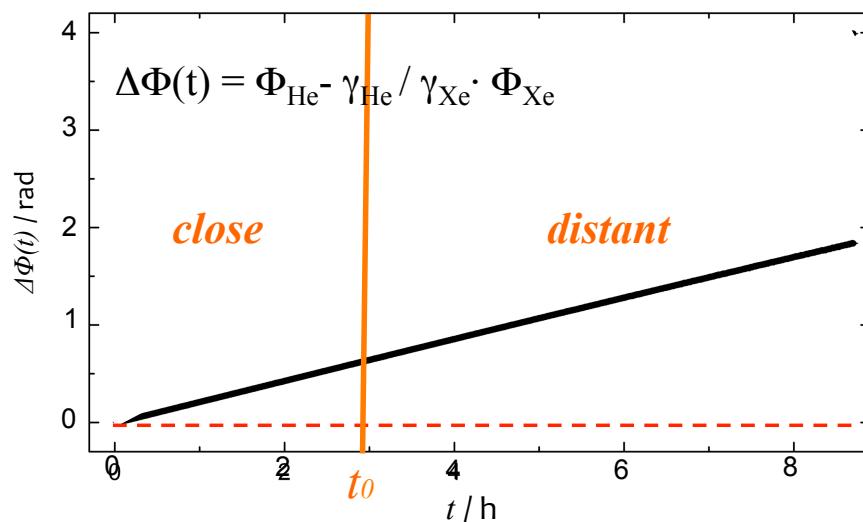
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$$\Rightarrow \Delta\nu_{sp} = \frac{\Delta\omega_{sp}}{2\pi(1 - \gamma_{He}/\gamma_{Xe})}$$



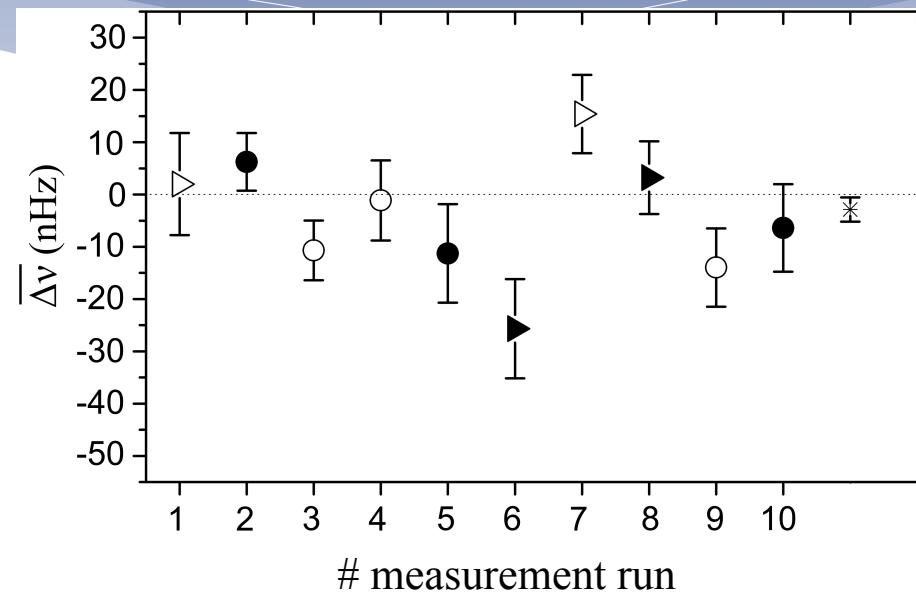
Summary Results

September 2010:

10 measurements (~9 hours)

gap = 2.2 mm

sample: $\text{Bi}_4\text{Ge}_3\text{O}_{12}$ with: $\rho = 7.13 \text{ g/cm}^3$



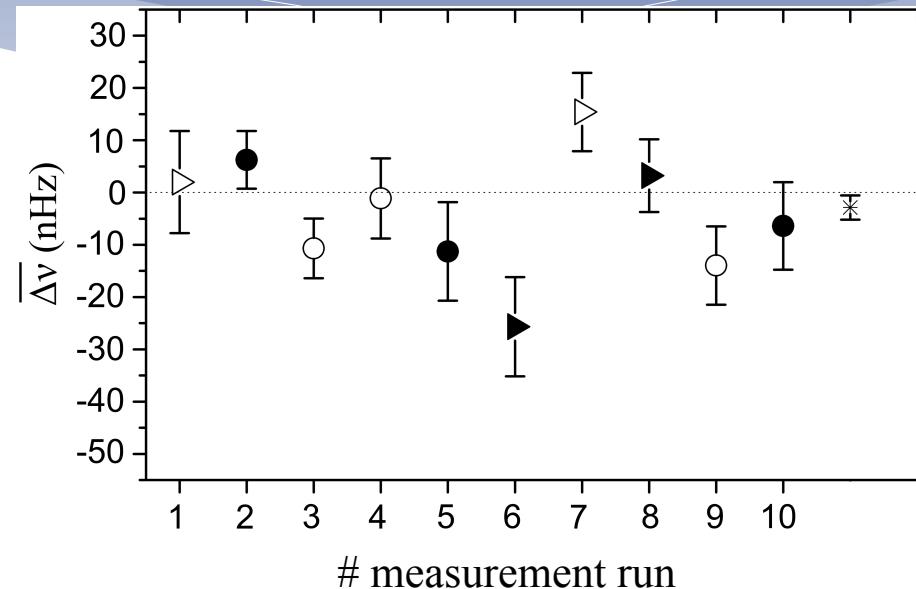
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$$\Rightarrow \Delta v_{\text{sp}} = (-2.9 \pm 3.5) \text{ nHz} \rightarrow \delta(\Delta v_{\text{sp}})_{\text{corr}} = 7.1 \text{ nHz (95\% CL)}$$

K. Tullney et al.
PRL 111, 100801 (2013)

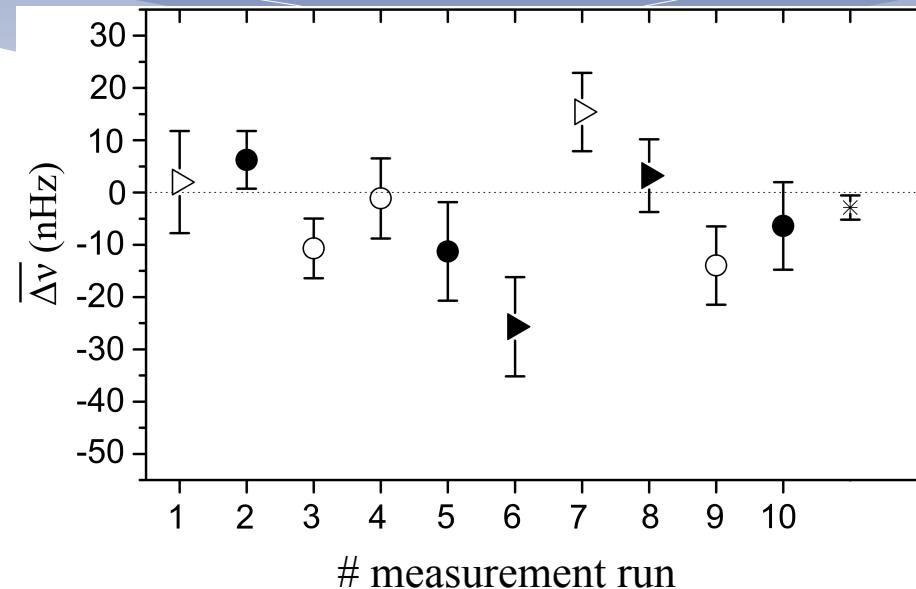
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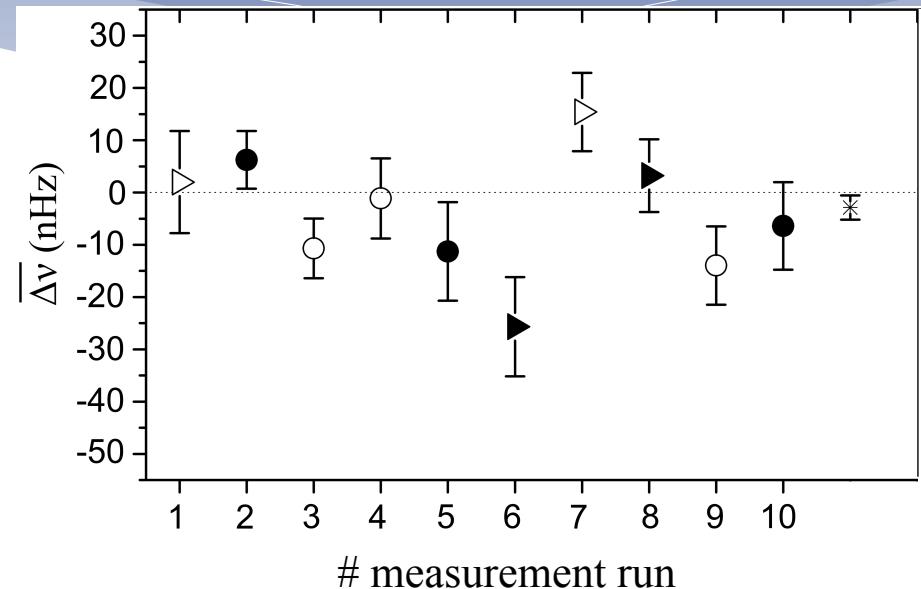
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Potential: $V_{\text{sp}}(\vec{r}) = \frac{\hbar^2 g_s g_p}{8\pi m_n} \left(\frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda}$

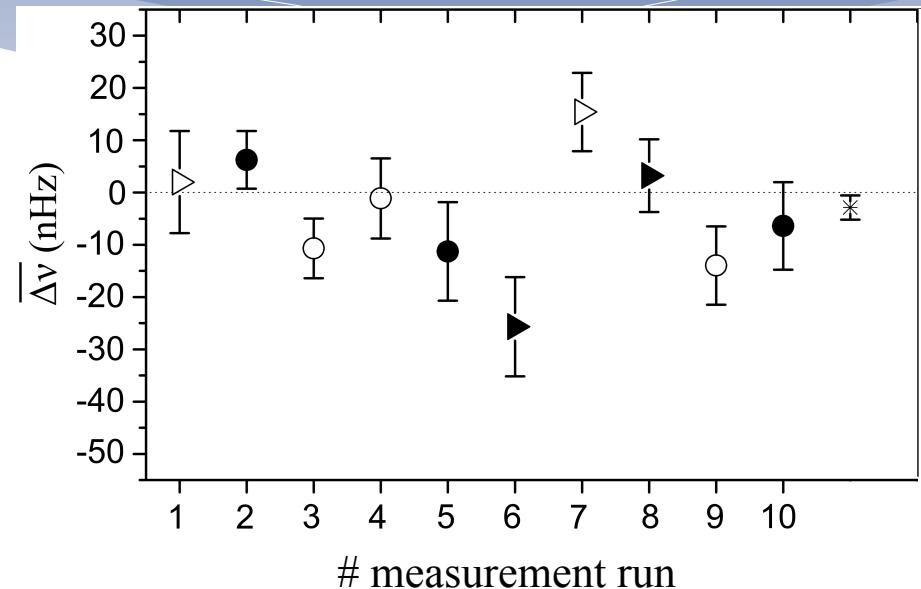
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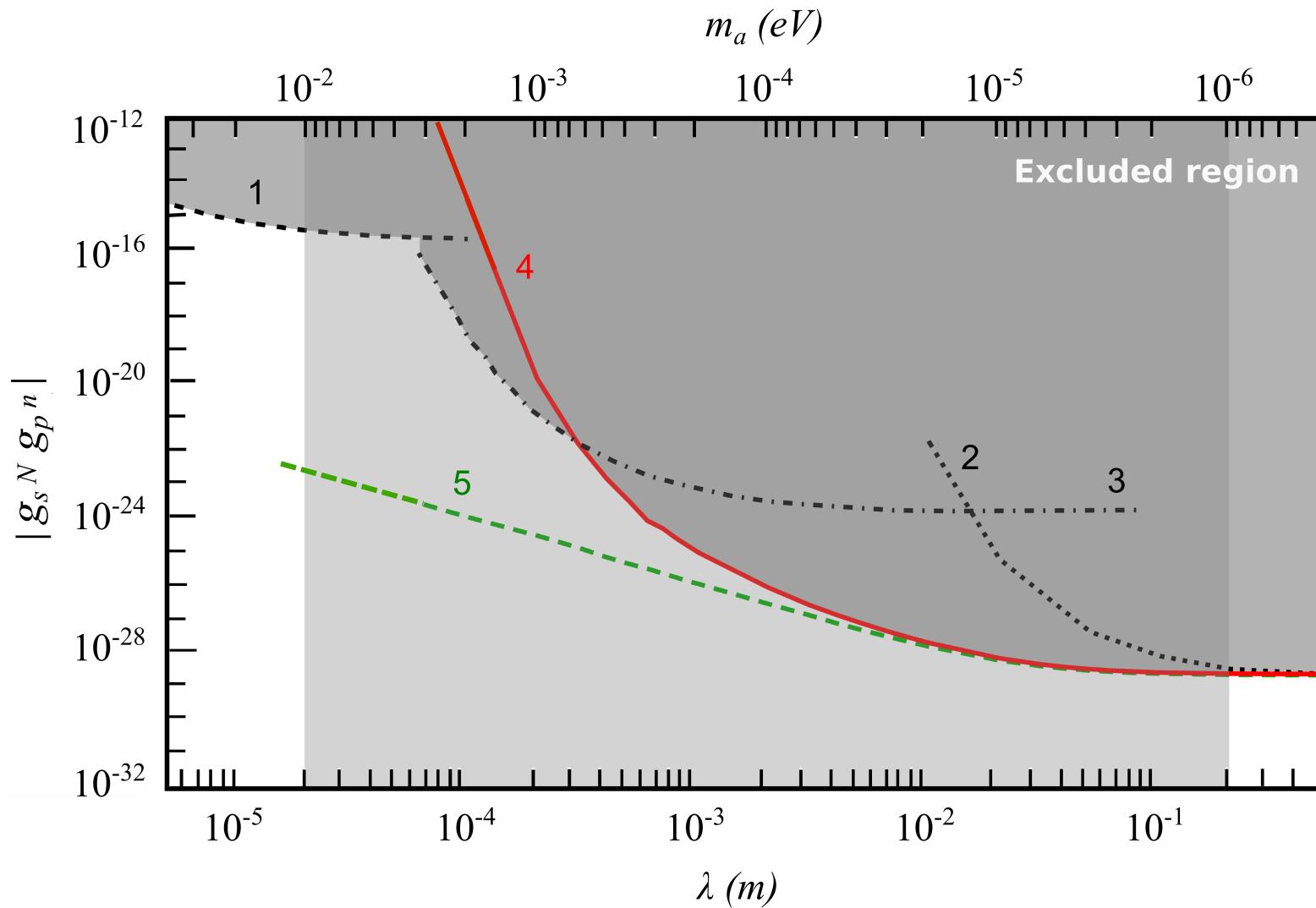
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$$\delta(\Delta v_{\text{sp}}) > 2 \langle V_{\text{sp}} \rangle / h \Rightarrow g_s g_p(\lambda) < \frac{8\pi^2 \cdot m_n \cdot V_{\text{cell}} \cdot \delta(\Delta v_{\text{sp}})}{\hbar \cdot N \cdot \langle V^*(\lambda) \rangle}$$

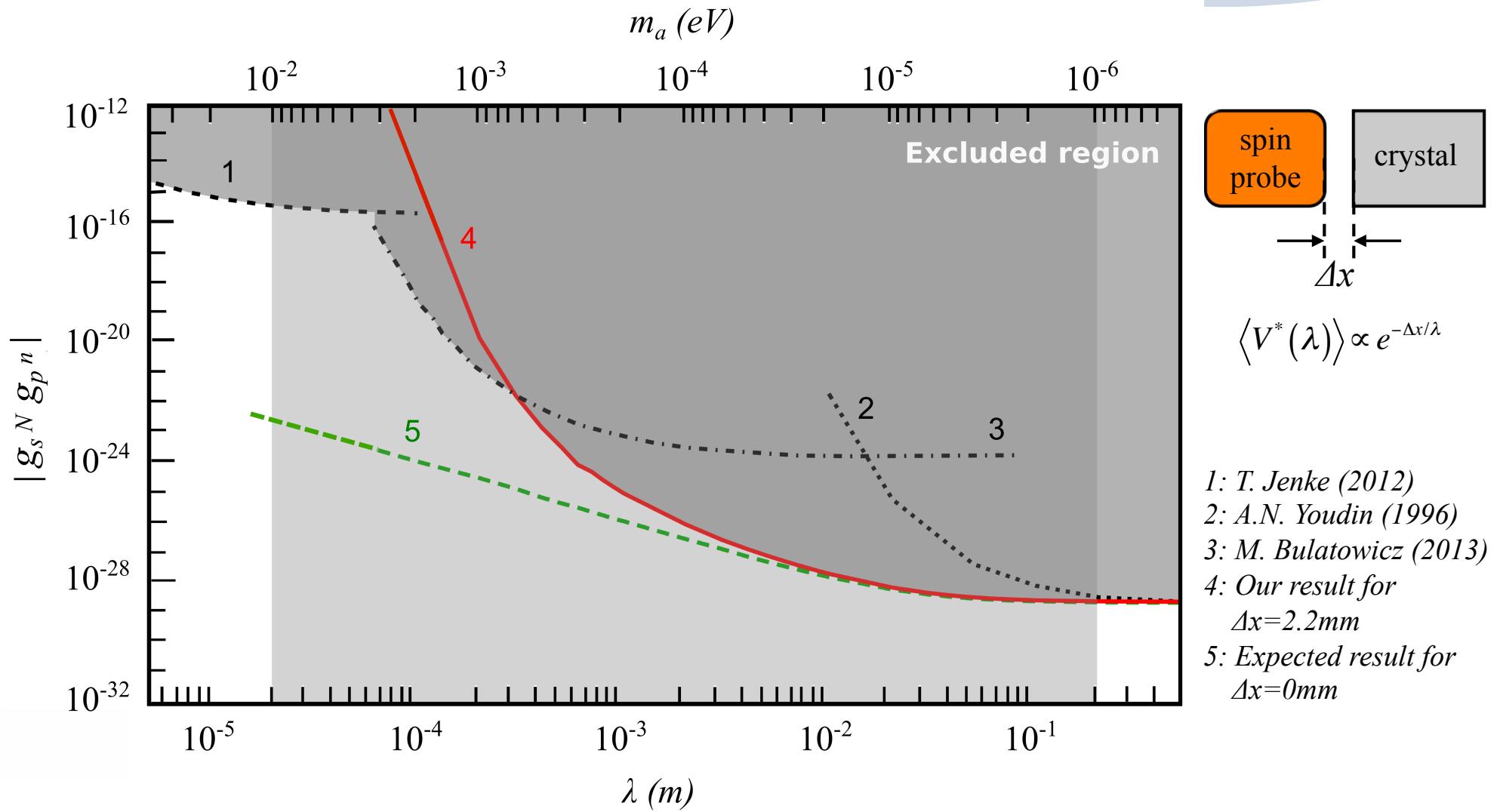
Potential: $V_{\text{sp}}(\vec{r}) = \frac{\hbar^2 g_s g_p}{8\pi m_n} \left(\frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda}$

Exclusion Plot



- 1: T. Jenke (2012)
- 2: A.N. Youdin (1996)
- 3: M. Bulatowicz (2013)
- 4: Our result for
 $\Delta x = 2.2\text{mm}$
- 5: Expected result for
 $\Delta x = 0\text{mm}$

Exclusion Plot



Summary

- Free spin precession of ^3He and ^{129}Xe with long spin coherence times:

$$\overline{T}_{2,\text{He}}^* \approx 53 \text{ h} \quad \overline{T}_{2,\text{Xe}}^* \approx 5 \text{ h}$$

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- **Clock comparison experiment**

Spin-dependent short-range interaction:

$$V(r) = \frac{g_s g_p}{8\pi} \frac{(\hbar)^2}{m_n} (\boldsymbol{\sigma}_n \cdot \vec{n}) \left[\frac{1}{r\lambda} + \frac{1}{r^2} \right] e^{-r/\lambda} \quad J.E. Moody, F.Wilczek PRD 30 (1984), 130$$

$$\Delta\nu_{\text{sp}} = (-2.9 \pm 3.6) \text{ nHz} \rightarrow \delta(\Delta\nu_{\text{sp}})_{\text{corr}} = 7.1 \text{ nHz (95% CL)}$$

Result: New upper limit for $g_s g_p$ in the range $3 \cdot 10^{-4} \text{ m} < \lambda < 10^{-1} \text{ m}$
Improvement up to **4 orders of magnitude!**

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Spin-dependent short-range interaction:

$$V(r) = \frac{g_s g_p}{8\pi} \frac{(\hbar)^2}{m_n} (\boldsymbol{\sigma}_n \cdot \vec{n}) \left[\frac{1}{r\lambda} + \frac{1}{r^2} \right] e^{-r/\lambda} \quad J.E. Moody, F.Wilczek PRD 30 (1984), 130$$

$$\Delta\nu_{\text{sp}} = (-2.9 \pm 3.6) \text{ nHz} \rightarrow \delta(\Delta\nu_{\text{sp}})_{\text{corr}} = 7.1 \text{ nHz (95% CL)}$$

Result: New upper limit for $g_s g_p$ in the range $3 \cdot 10^{-4} \text{ m} < \lambda < 10^{-1} \text{ m}$
Improvement up to **4 orders of magnitude!**

- **Improvement:**

Direct contact of spin probe and crystal → more sensitive to $\lambda < 1 \text{ mm}$

Summary

- Free spin precession of ^3He and ^{129}Xe with long spin coherence times:

$$\overline{T}_{2,\text{He}}^* \approx 53 \text{ h} \quad \overline{T}_{2,\text{Xe}}^* \approx 5 \text{ h}$$

longer $T_{2,\text{Xe}}^*$ → increase of sensitivity ($\sigma_v \sim 1/T^{3/2}$)

- **Clock comparison experiment**

Spin-dependent short-range interaction:

$$V(r) = \frac{g_s g_p}{8\pi} \frac{(\hbar)^2}{m_n} (\sigma_n \cdot \vec{n}) \left[\frac{1}{r\lambda} + \frac{1}{r^2} \right] e^{-r/\lambda} \quad J.E. Moody, F.Wilczek PRD 30 (1984), 130$$

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Improvement up to **4 orders of magnitude!**

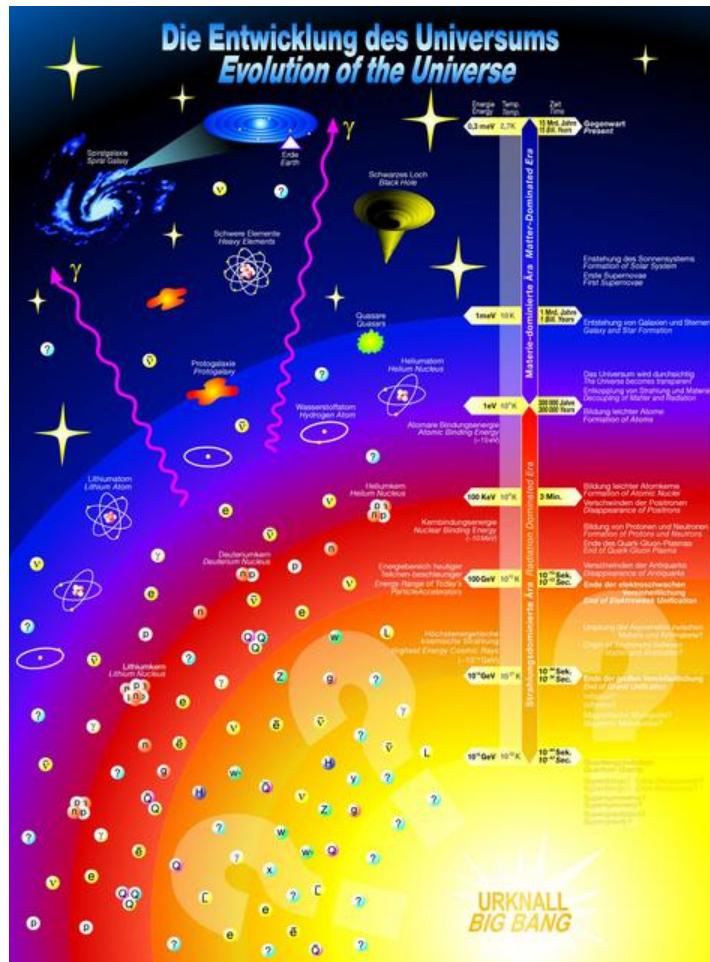
- **Improvement:**

Direct contact of spin probe and crystal → more sensitive to $\lambda < 1 \text{ mm}$

THANK YOU!

Motivation

Baryon/Anti-Baryon Puzzle

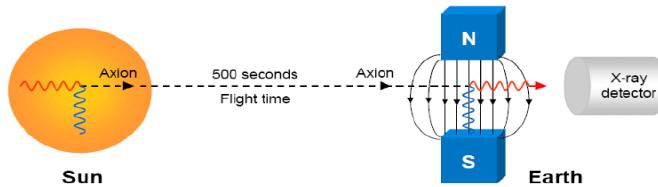


- Baryon/Anti-Baryon Asymmetry developed after 10^{-10} s after the Big Bang
- Observed Asymmetry: $n_B/n_\gamma = 10^{-10}$
- Sacharow criteria:
 - ✓ Violation of baryon number
 - ✓ Thermal disequilibrium
 - ✓ CP-/C-Violation
- Calculated Asymmetry: $n_B/n_\gamma = 10^{-18}$

Motivation

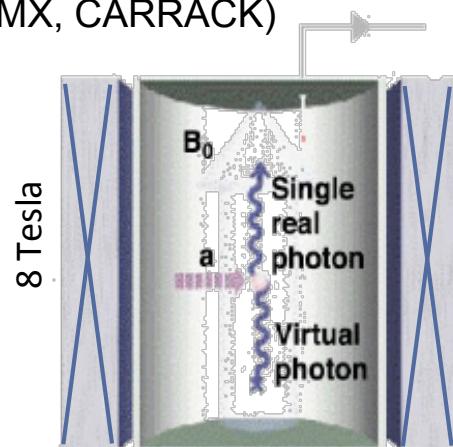
Solar axions

Helioscope experiment (CAST)



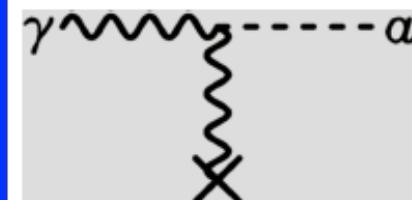
Galactic axions

microwave cavity experiment
(ADMX, CARRACK)



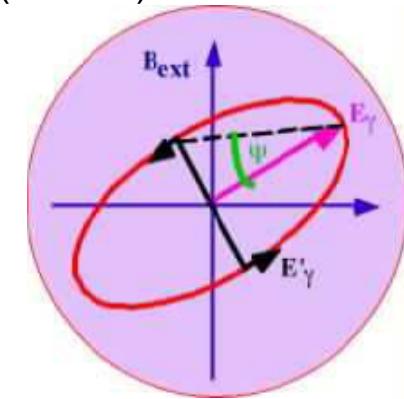
**DIRECT
search of the
axion via the
Primakoff effect**

Primakoff effect
Conversion of an
axion into a photon

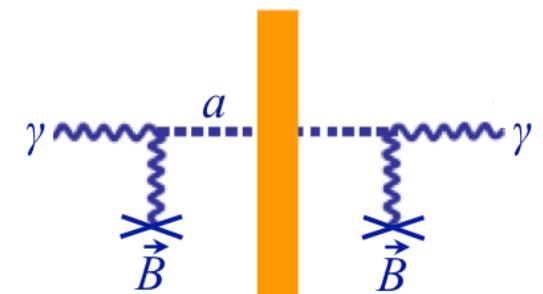


Labor axions

- Polarization experiments
(PVLAS)



- „Light shinning through a wall“ experiment
(BFRT, OSQAR, ALPS,
LIPPS, GammaeV)



Axion Potential:

Yukawa type potential with monopole-dipole coupling [1]

$$V(r) = \kappa \hat{n} \cdot \vec{\sigma} \left(\frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}$$

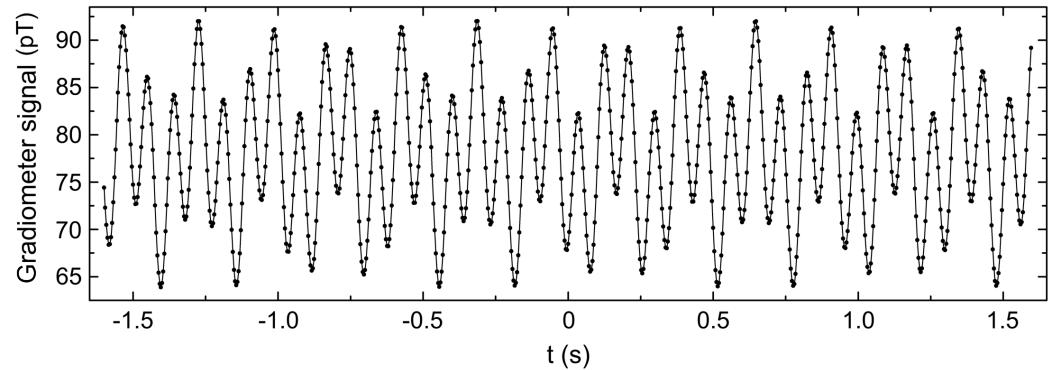
with: $\kappa = \frac{\hbar^2 g_s g_p}{8\pi m_n}$, $\lambda = \frac{\hbar}{m_a c}$

polar vector axial vector
 $\hat{n} \cdot \vec{\sigma} \Rightarrow P(\hat{n} \cdot \vec{\sigma}) = -(\hat{n} \cdot \vec{\sigma}) \Rightarrow P\text{-, T-violation}$
 $T(\hat{n} \cdot \vec{\sigma}) = -(\hat{n} \cdot \vec{\sigma}) \Rightarrow CP\text{-violation}$

Data Analysis

Fit to raw data:

$$f^i(t) = a_{s,\text{He}}^i \sin(\omega_{\text{He}}^i \cdot t) + a_{c,\text{He}}^i \cos(\omega_{\text{He}}^i \cdot t) \\ + a_{s,\text{Xe}}^i \sin(\omega_{\text{Xe}}^i \cdot t) + a_{c,\text{Xe}}^i \cos(\omega_{\text{Xe}}^i \cdot t) \\ + (c_0^i + c_{\text{lin}}^i \cdot t)$$



↓
 ${}^3\text{He}, {}^{129}\text{Xe}$ phase

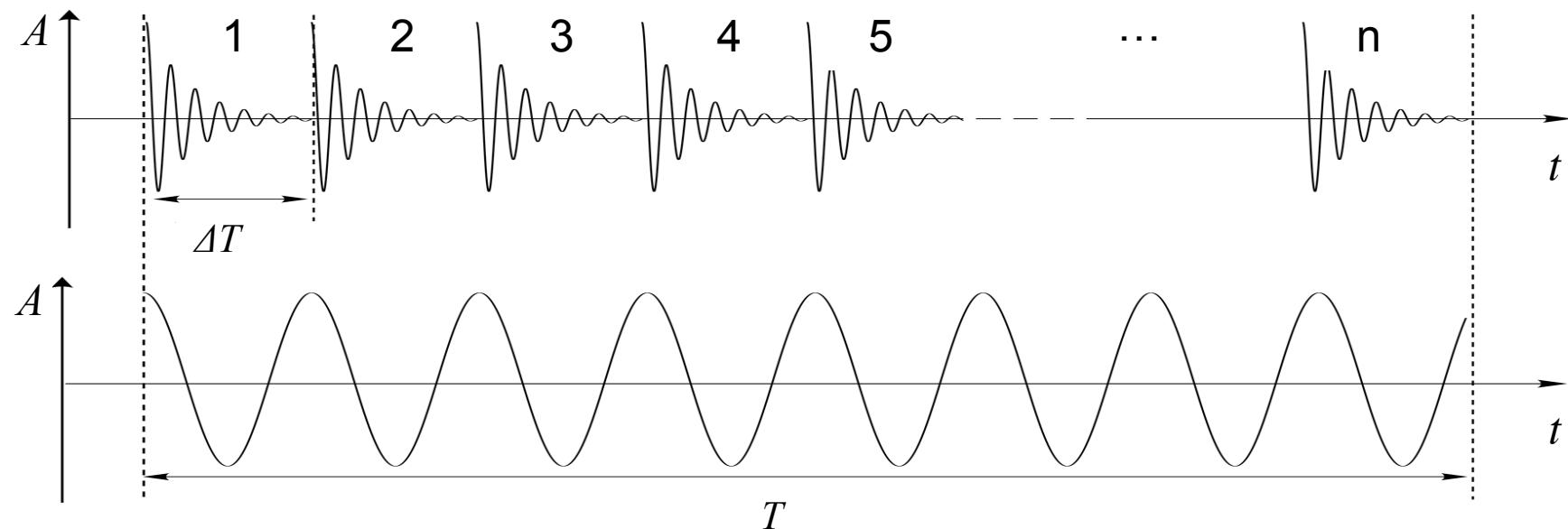
$$\Phi_{\text{He/Xe}}^i = \arctan\left(\frac{a_{s,\text{He}/\text{Xe}}^i}{a_{c,\text{He}/\text{Xe}}^i}\right) + n_{\text{He/Xe}}^i \cdot 2\pi, \quad n_{\text{He/Xe}}^i \approx \bar{\omega}_{\text{He/Xe}} \cdot t \quad (\text{number of periods since begin of measurement})$$

↓
weighted phase difference

$$\Delta\Phi = \Phi_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \cdot \Phi_{\text{Xe}} \stackrel{!}{=} \Phi_0$$

Sensitivity

Spin coherence time: long (T) vs. short (ΔT)



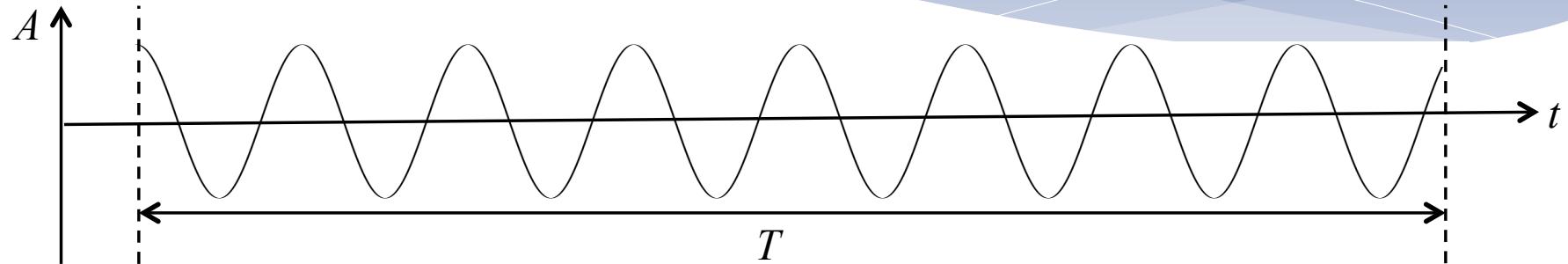
$$\sigma_{v, \text{long}} \propto \frac{1}{T^{3/2}}$$

$$\sigma_{v, \text{short},n} \propto \frac{1}{\sqrt{n}} \frac{1}{\Delta T^{3/2}} = \left(\frac{1}{T^{3/2}} \right) \cdot \frac{T}{\Delta T}$$

$$T = n \cdot \Delta T$$

Example: SNR = 10000:1 , $f_{BW} = 1 \text{ Hz}$, $T = 24 \text{ h}$, $\Delta T = 5 \text{ min} \rightarrow \sigma_{v, \text{short}} = 300 \cdot \sigma_{v, \text{long}}$

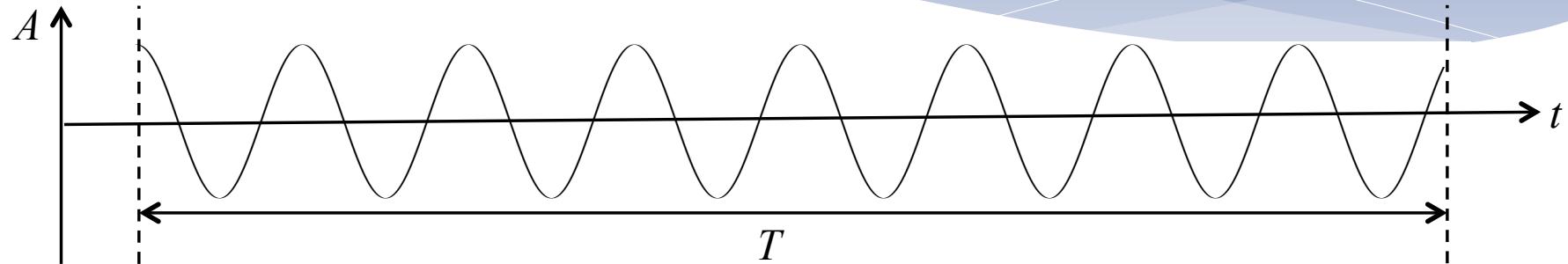
Precision of measurement



If the noise $w[n]$ is **Gaussian distributed**, the “Cramer-Rao-Lower Bound (CRLB)” sets the lower limit on the variance σ_v :

$$\sigma_v \geq \frac{\sqrt{12}}{2\pi \cdot \text{SNR} \cdot \sqrt{v_{\text{BW}}} \cdot T^{3/2}} \cdot \sqrt{C(T_2^*)}$$

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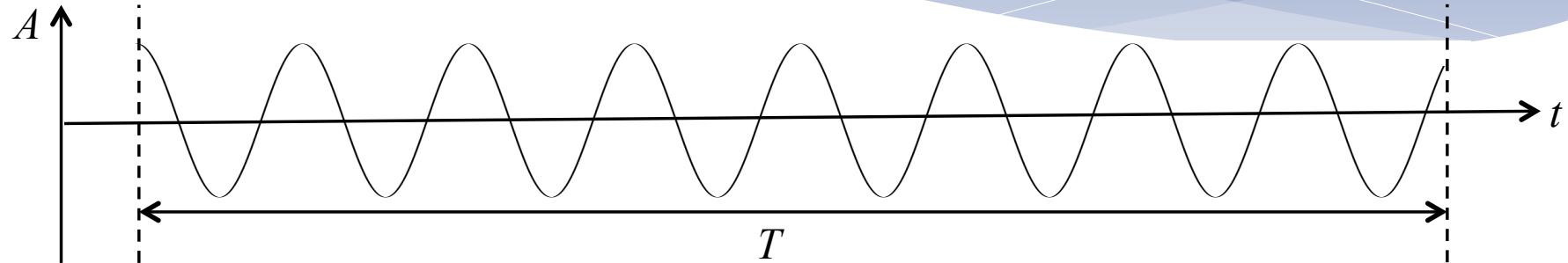
$$\sigma_v \geq \frac{\sqrt{12}}{2\pi \cdot \text{SNR} \cdot \sqrt{v_{\text{BW}}} \cdot T^{3/2}} \cdot \sqrt{C(T_2^*)}$$

Example:

$$\text{SNR} = 2300, v_{\text{BW}} = 1 \text{ Hz}, T = 9 \text{ h}$$

$$\Rightarrow \sigma_v \approx 10^{-10} \text{ Hz}$$

Precision of measurement



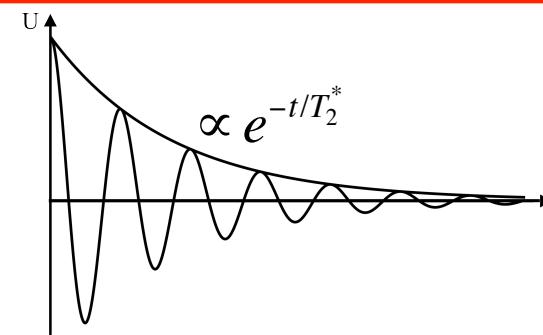
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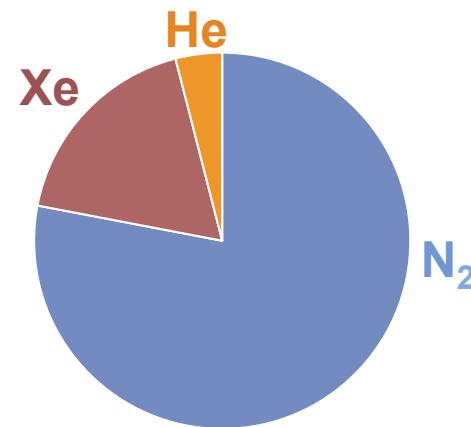


$$(T_2^*)^{-1} \propto V^{4/3} p |\vec{\nabla} B_1|^2 \quad \text{vs.} \quad \text{SNR} \propto pV$$

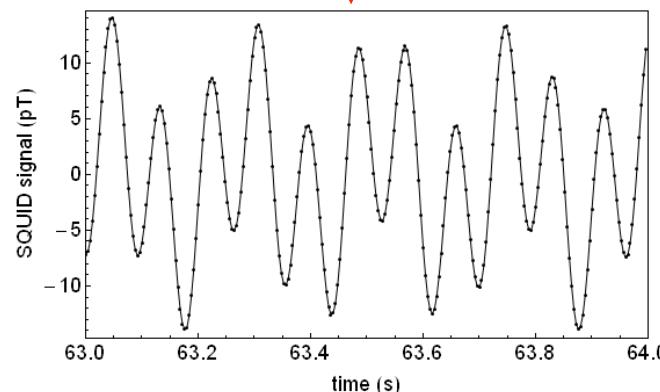
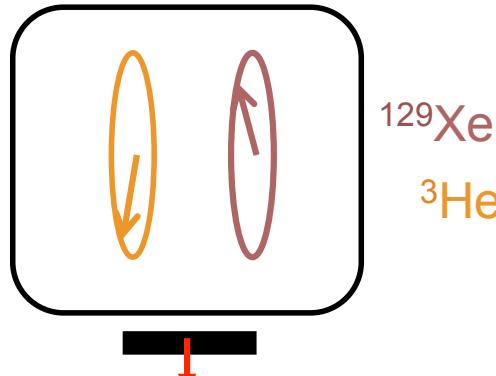
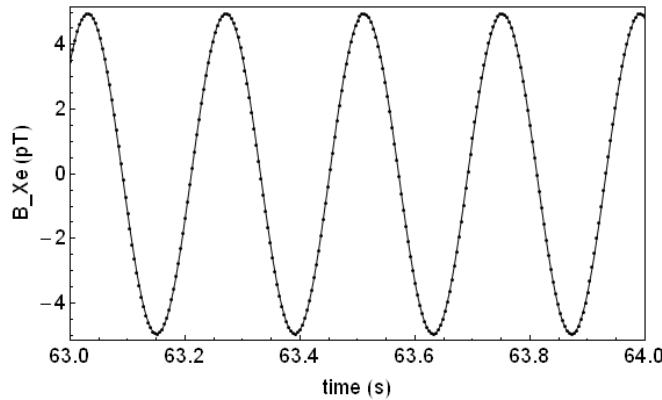
$$p \approx \text{mbar}, V \approx 200 \text{ cm}^3, B_1 \approx \mu\text{T}, |\vec{\nabla} B_1|^2 \approx \frac{pT}{\text{cm}}$$

Data Acquisition

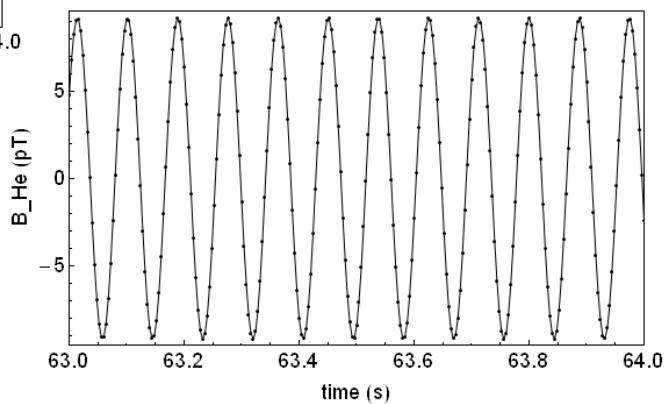
${}^3\text{He}$, ${}^{129}\text{Xe} + \text{N}_2$ (buffer gas)
@ $p_{\text{tot}} = 31 \dots 37 \text{ mbar}$



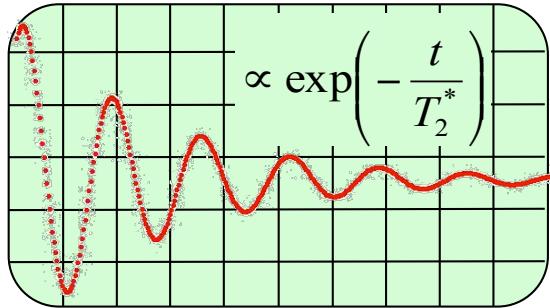
$$\omega_{\text{Xe}}(t) = \gamma_{\text{Xe}} \cdot B(t)$$



$$\omega_{\text{He}}(t) = \gamma_{\text{He}} \cdot B(t)$$



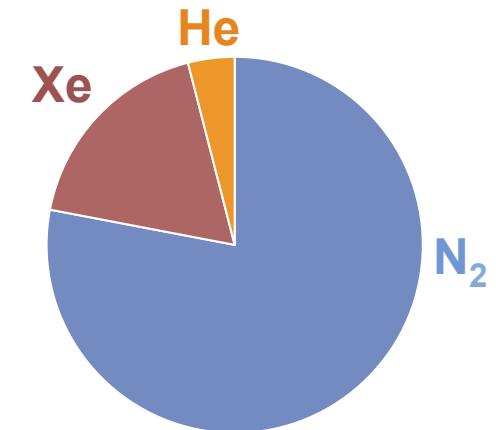
Transversal Relaxation time



Free spin precession:

$$\text{signal} \propto \exp\left(-t / T_2^*\right)$$

T_2^* : ms \rightarrow h



wall collisions [1]

$$T_{1,\text{wand}} \approx 8 \text{ h} - 100 \text{ h}$$

$$\frac{1}{T_2^*} = \frac{1}{T_{1,\text{wand}}} + \frac{1}{T_{1,\text{vdW}}} + \frac{1}{T_{2,\text{grad}}}$$

van der Waals molecules [2]

$$T_{1,\text{vdW}}^{\text{Xe}} \approx 13 \text{ h}$$

field gradients [3]

$$T_{2,\text{grad}} \propto \frac{1}{V^{4/3} p |\vec{\nabla} B_1|^2}$$

CRLB: $\sigma_v \propto \frac{1}{\text{SNR} \cdot T^{3/2}}$

signal $\propto p \cdot V$

$\Rightarrow p \sim \text{mbar}, V \sim 100 \text{ cm}^3, B_1 \sim \mu\text{T}$ [4]

[1] J. Schmiedeskamp et al., Eur. Phys. J. D 38, 2006.

[3] G. D. Cates, S. R. Schaefer, W. Happer, Phys. Rev. A 37, 1988.

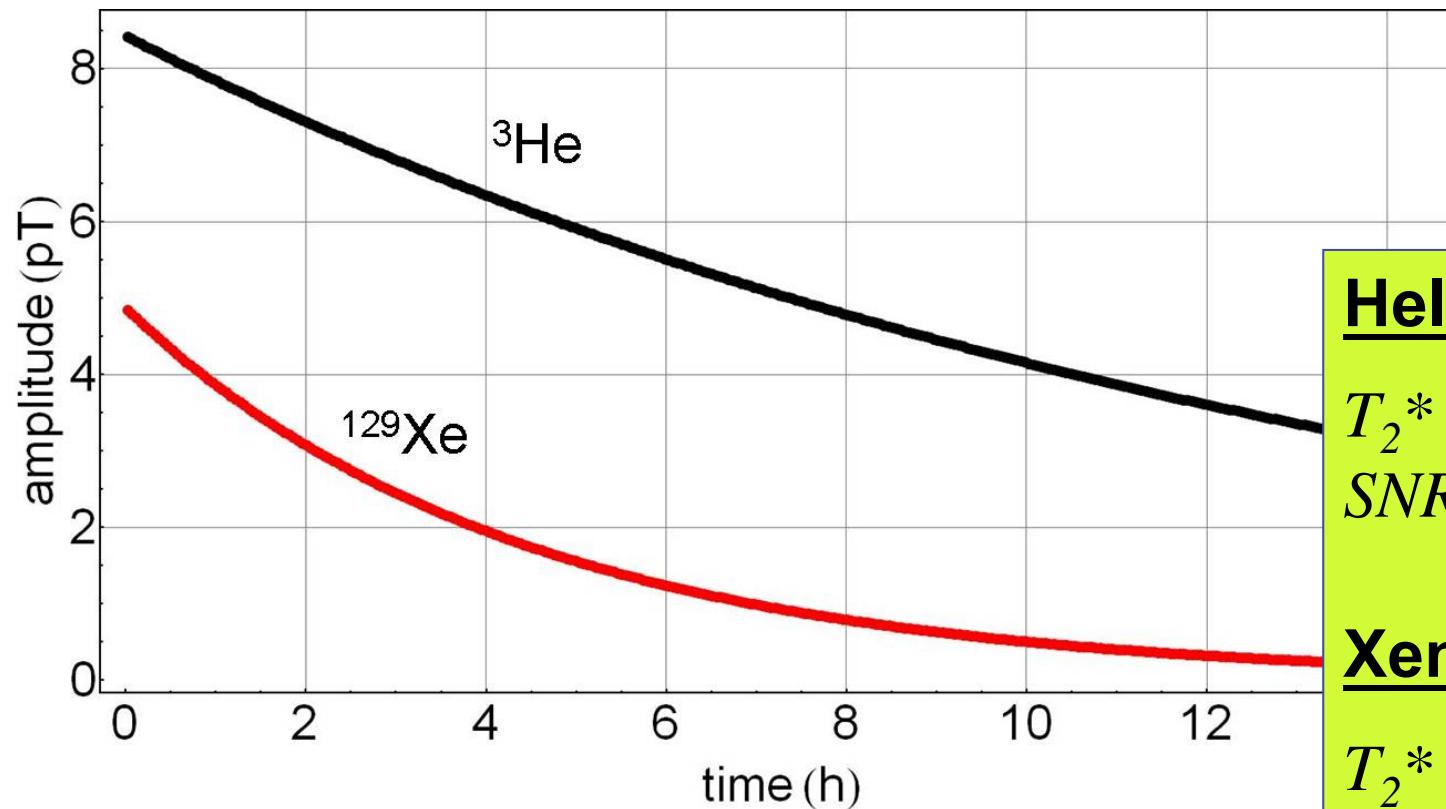
[2] B. Chann et al., PRL. 88(11), 2002.

[4] C. Gemmel et al., Eur. Phys. J. D 57, 2010.

Transversal Relaxationtime

$R \approx 3 \text{ cm}$

$p_{\text{He}} \approx 2 \text{ mbar}$, $p_{\text{Xe}} \approx 8 \text{ mbar}$, $p_{\text{N}_2} \approx 35 \text{ mbar}$



Helium:

$T_2^* \sim 53 \text{ h}$
 $\text{SNR} \sim 7500$

Xenon:

$T_2^* \sim 5 \text{ h}$
 $\text{SNR} \sim 2250$

Summary Results

September 2010:

10 measurements (~9 hours)

gap = 2.2 mm

sample: $\text{Bi}_4\text{Ge}_3\text{O}_{12}$ crystal with density $\rho = 7.13 \text{ g/cm}^3$

*K. Tullney et al.
PRL 111, 100801 (2013)*

$$\Rightarrow \Delta v_{sp} = (-2.9 \pm 3.5) \text{ nHz} \rightarrow \delta(\Delta v_{sp})_{corr} = 7.1 \text{ nHz (95% CL)}$$

Analysis: $V_{sp}(\vec{r}) = \frac{\hbar^2 g_s g_p}{8\pi m_n} \left(\frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda}$

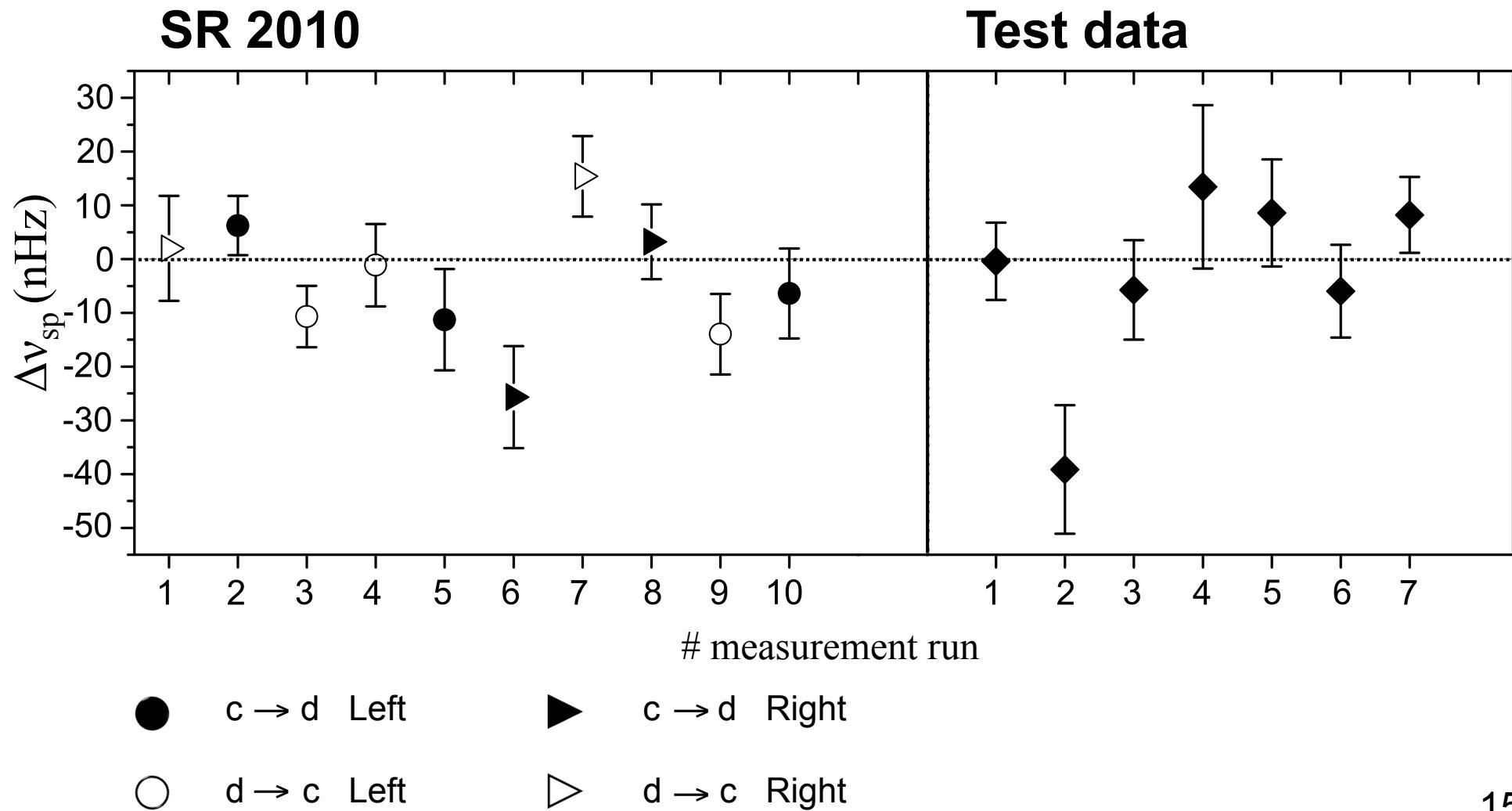
$$\langle V_{sp} \rangle = \frac{\hbar^2 g_s g_p}{8\pi m_n} \int_{VBGO} \int_{V_{cell}} \left(\frac{\vec{r}}{r} \cdot \vec{\sigma} \right) \left(\frac{1}{r\lambda} + \frac{1}{r^2} \right) \cdot e^{-r/\lambda} dV_{cell} dV_{BGO} = \frac{\hbar^2 g_s g_p}{8\pi m_n} \langle V^*(\lambda) \rangle$$

The mean potential $\langle V_{sp} \rangle$ was calculated numerically for our measurement cells ($\varnothing = 58 \text{ mm}$, $l = 58 \text{ mm}$). The gap between the inner volume of the measurement cell and the $\text{Bi}_4\text{Ge}_3\text{O}_{12}$ crystal ($\varnothing = 60 \text{ mm}$, $l = 70 \text{ mm}$) was 2.2 mm.

$$\Delta(\Delta v_{sp}) > 2 \langle V_{sp} \rangle / h \Rightarrow g_s g_p < \frac{8\pi^2 m_n V_{cell} \Delta(\Delta v_{sp})}{\hbar N \langle V^*(\lambda) \rangle}$$

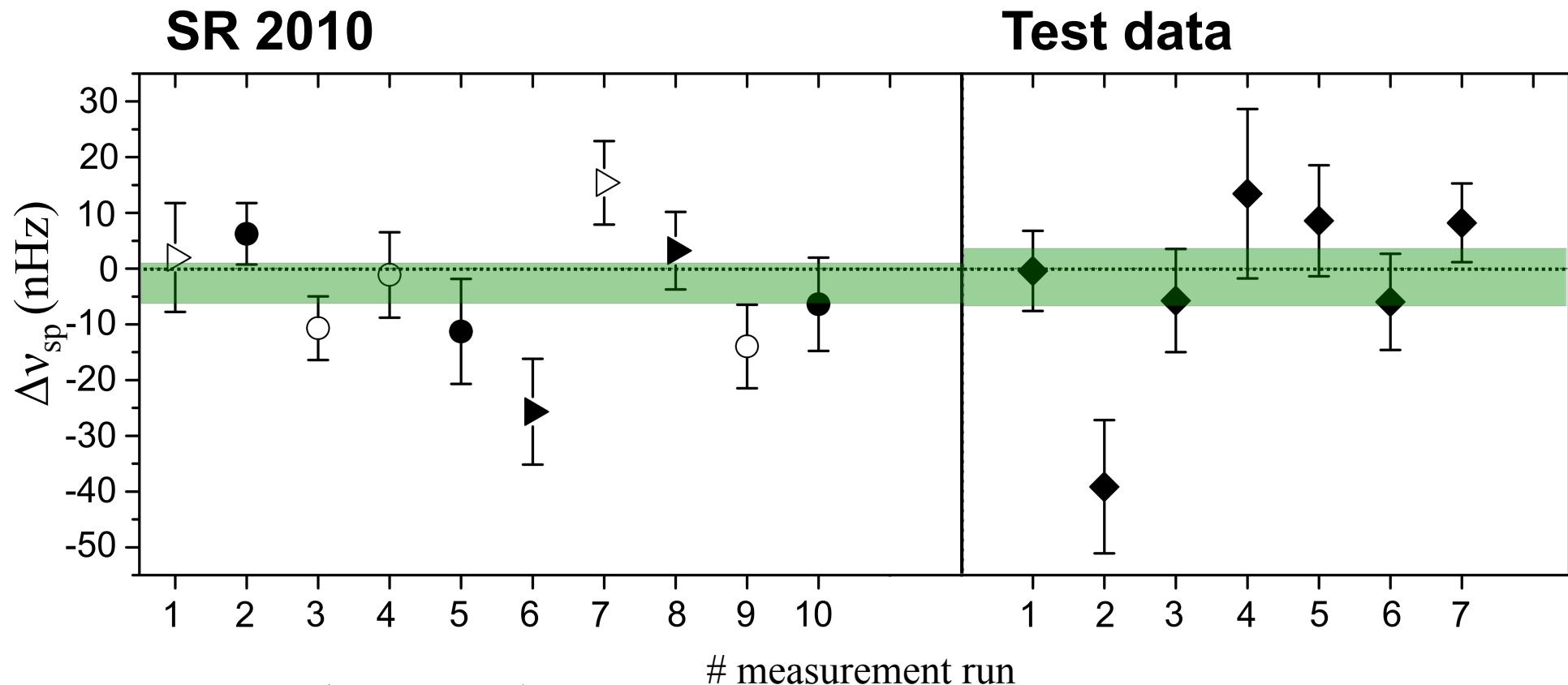
Results

$$\Delta\nu_{sp} = \frac{\Delta\omega_{sp}^w}{2\pi \cdot \left(1 - \gamma_{He}/\gamma_{Xe}\right)}$$



Results

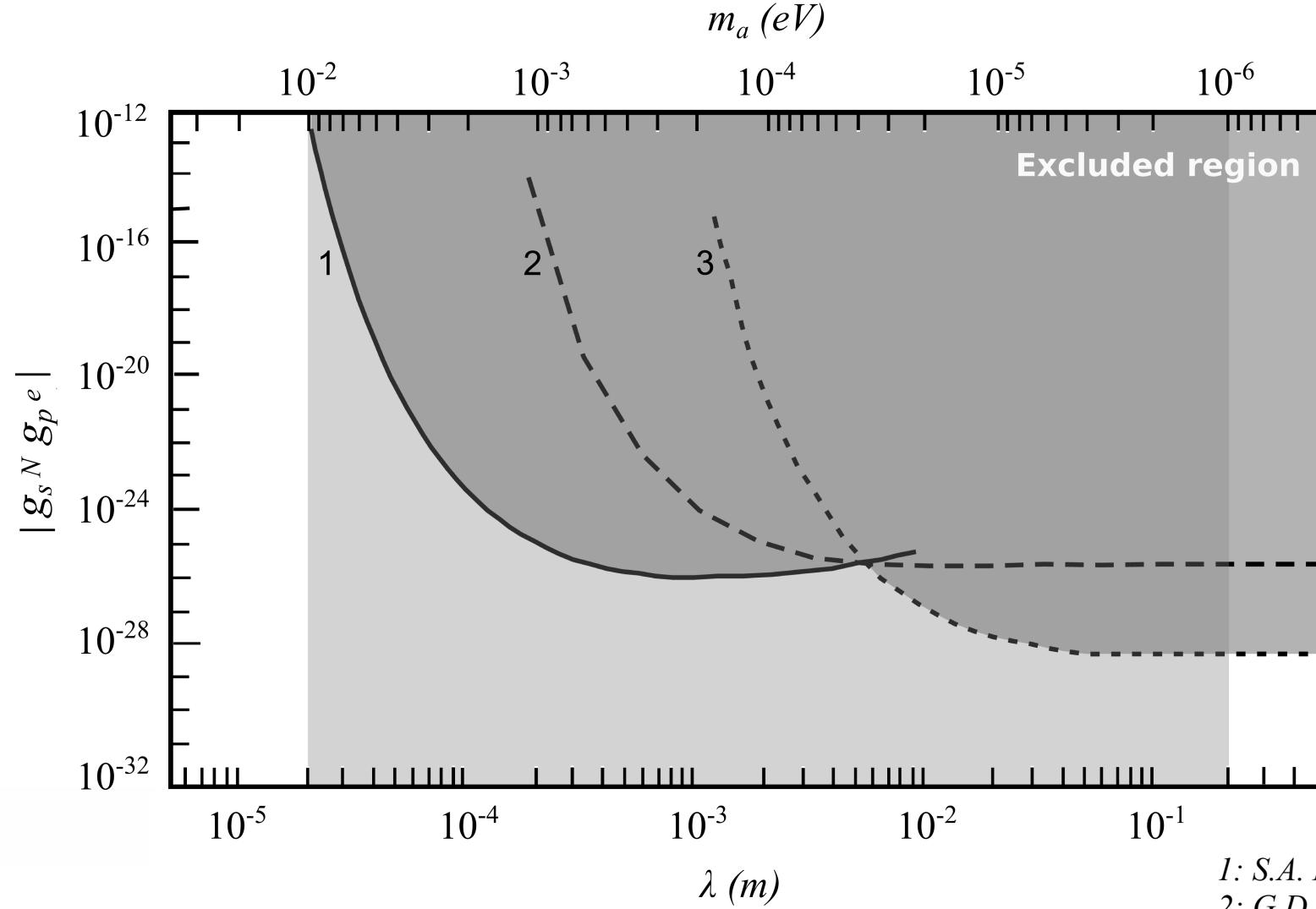
$$\Delta\nu_{sp} = \frac{\Delta\omega_{sp}^w}{2\pi \cdot (1 - \gamma_{He}/\gamma_{Xe})}$$



$$\Delta\nu_{Test} = (-1.4 \pm 5.2) \text{ nHz}$$

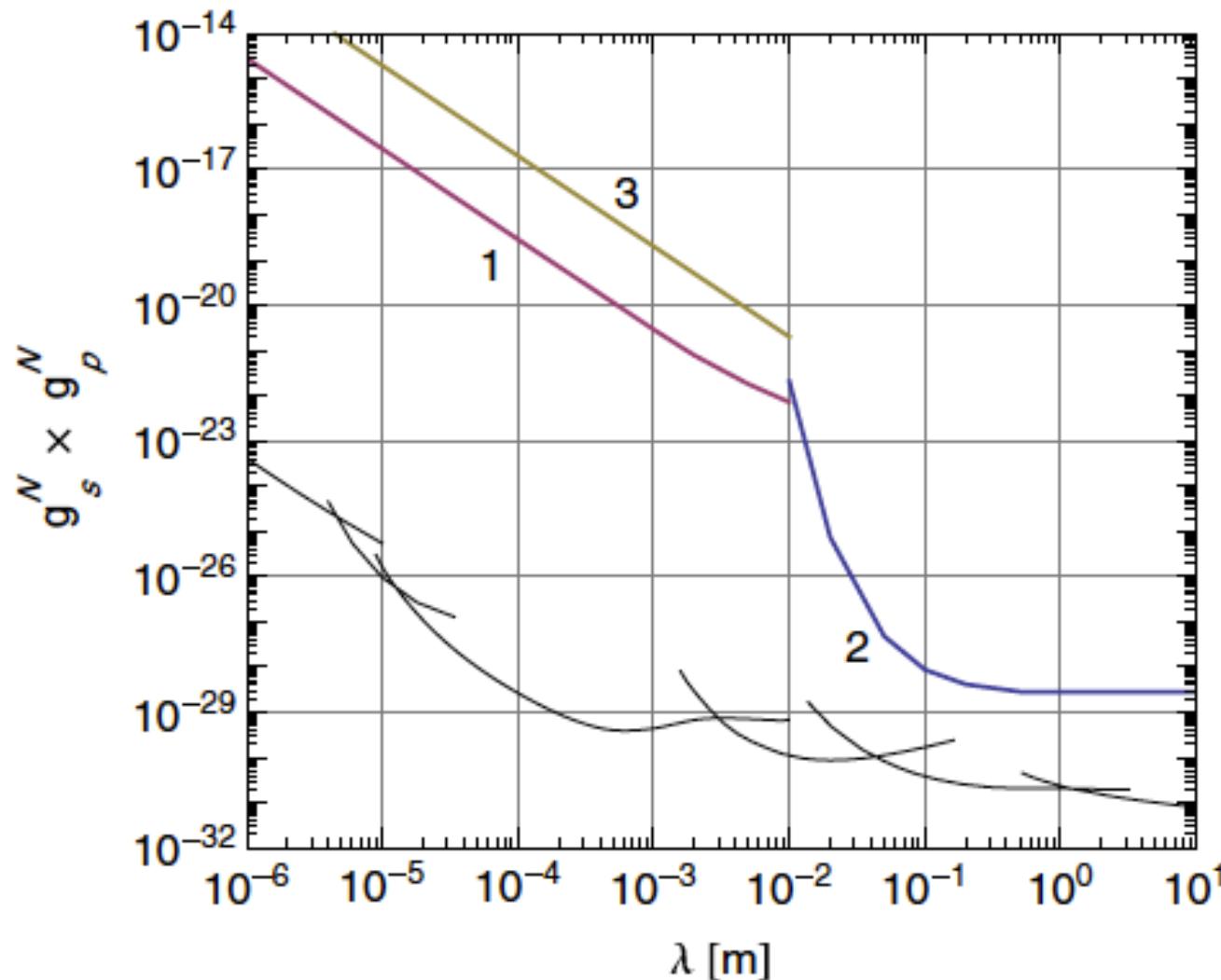
$$\Delta\nu_{sp} = (-2.9 \pm 3.5) \text{ nHz}$$

Exclusion Plot



1: S.A. Hoedel (2011)
2: G.D. Hammond (2007)
3: W.-T. Ni (1998)

Exclusion Plot



1: A.K. Petukhov (2010)
2: A.N. Youdin (1996)
3: A.P. Serebrov (2010)

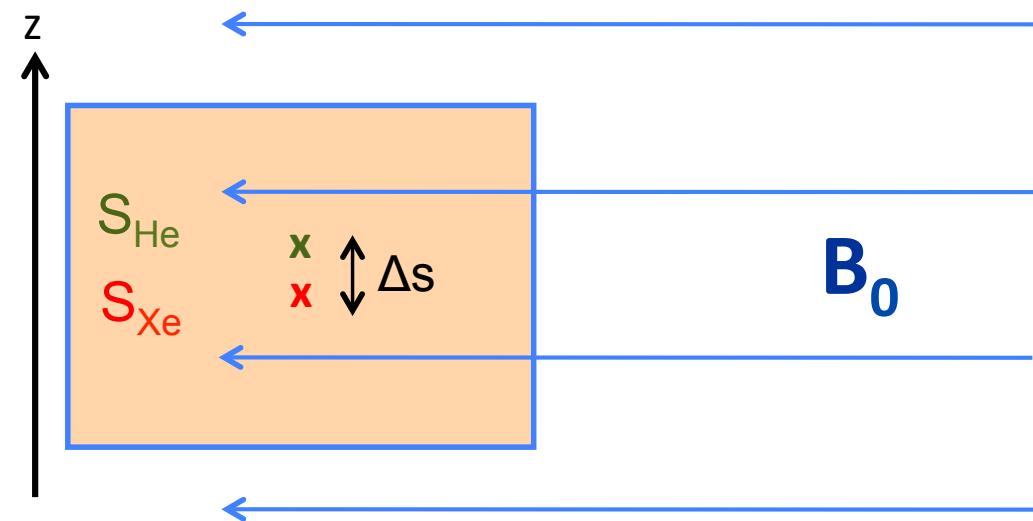
Plot taken from: G. Raffelt, Phys. Rev. D 86, 015001 (2012)

Influence of the $\text{Bi}_4\text{Ge}_3\text{O}_{12}$ crystal on T_2^*

Systematic errors

Transversal field gradients

$$\Delta\omega = \omega_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \omega_{\text{Xe}}$$



Systematic errors

Transversal field gradients

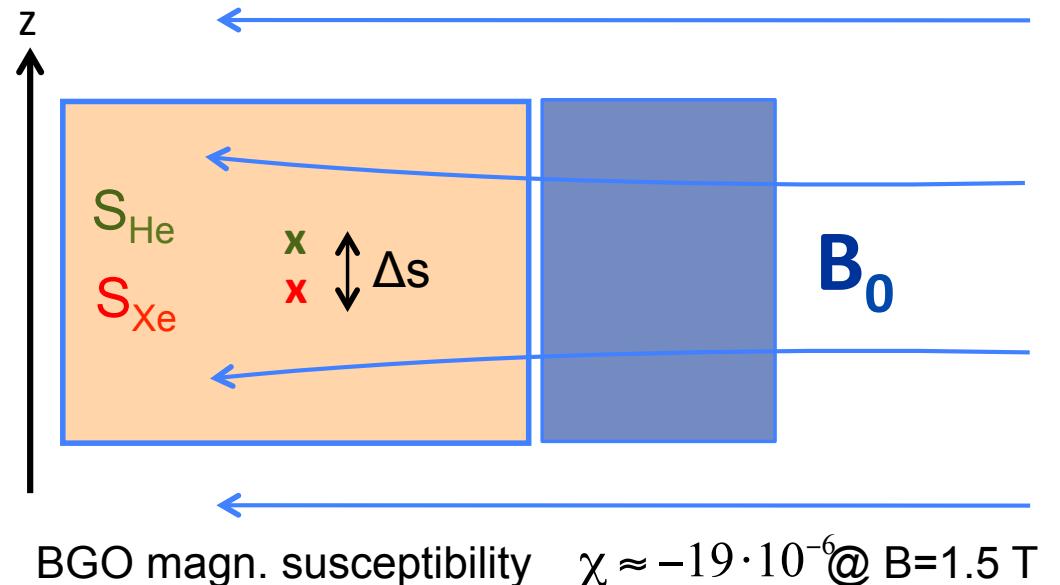
$$\Delta\omega = \omega_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \omega_{\text{Xe}} \Rightarrow \Delta\omega^{\text{CM}} = \gamma_{\text{He}} \cdot (B_{\text{Xe}} + \Delta B) - \gamma_{\text{He}} \cdot B_{\text{Xe}} = \gamma_{\text{He}} \cdot \Delta B \cdot H(t-t_0)$$

simulated: $\frac{\Delta B}{\Delta s} = \overline{\frac{\partial}{\partial z} B_{\text{BGO}}(z)} \leq 0.08 \frac{\text{pT}}{\text{cm}}$

Barometric formula:

$$p(z) = p_0 \cdot \exp\left(-\frac{Mg_0}{R_G T} z\right)$$

$$\Rightarrow \Delta s = S_{\text{He}} - S_{\text{Xe}} \approx 1.2 \cdot 10^{-7} \text{ m}$$



$$\Rightarrow \Delta\nu_{\text{sys}}^{\text{CM}} = \Delta s \cdot \overline{\frac{\partial}{\partial z} B_{\text{BGO}}(z)} \cdot \frac{\gamma_{\text{He}}}{2\pi \left(1 - \gamma_{\text{He}} / \gamma_{\text{Xe}}\right)} \leq 0.03 \text{ nHz}$$

S. Yamamoto, IEEE,
Transactions
on Nuclear Science,
Vol. 50, No. 5, 2003

Systematic errors

Field gradients (general)

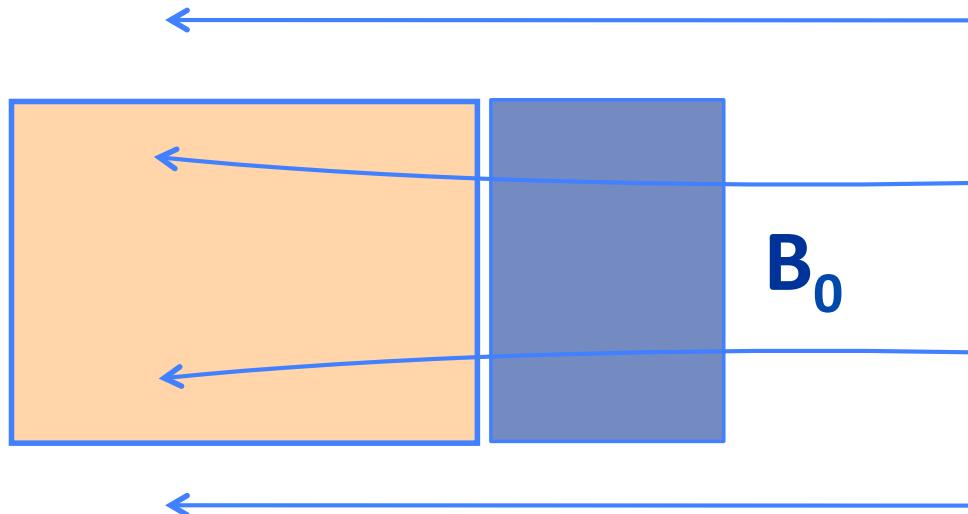
$T_{2,\text{field}}$ in low magnetic fields (G. D. Cates, et al., Phys. Rev. A 37, 2877)

$$\frac{1}{T_2^*} = \frac{1}{T_1} + \frac{1}{T_{2,\text{field}}}$$

$$\frac{1}{T_{2,\text{field}}} \approx \frac{4R^4\gamma^2}{175D} \left(|\vec{\nabla}B_{1,z}|^2 + |\vec{\nabla}B_{1,y}|^2 + 2|\vec{\nabla}B_{1,x}|^2 \right) \propto R^4 \cdot p \cdot |\vec{\nabla}B|^2$$

BGO magn. susceptibility $\chi \approx -19 \cdot 10^{-6}$ @ $B=1.5$ T
⇒ additional field gradients

S. Yamamoto, IEEE, Transactions on Nuclear Science, Vol. 50, No. 5, 2003



Systematic errors

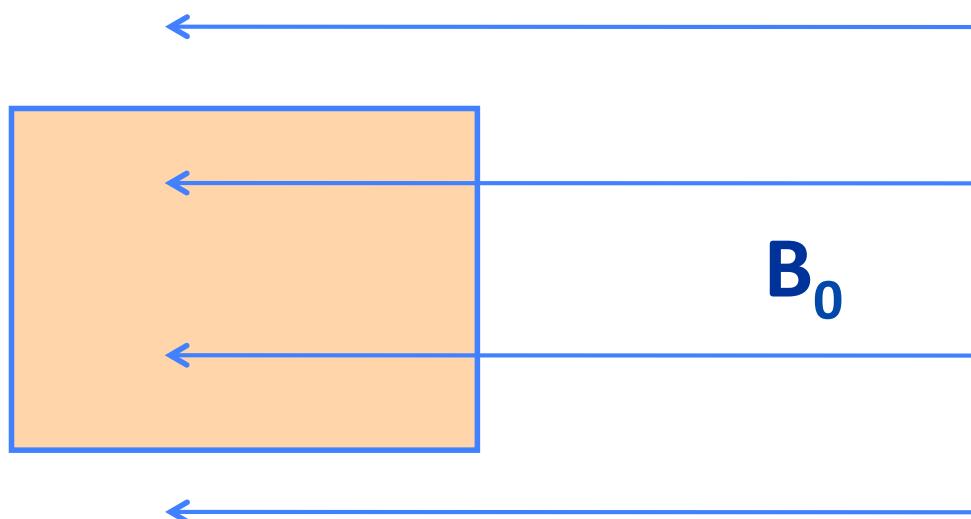
Field gradients (general)

$$\frac{1}{T_2^*} = \frac{1}{T_1} + \frac{1}{T_{2, \text{field}}}$$

$$\frac{1}{T_{2, \text{field}}} \approx \frac{4R^4\gamma^2}{175D} \left(|\vec{\nabla}B_{1,z}|^2 + |\vec{\nabla}B_{1,y}|^2 + 2|\vec{\nabla}B_{1,x}|^2 \right) \propto R^4 \cdot p \cdot |\vec{\nabla}B|^2$$

BGO magn. susceptibility $\chi \approx -19 \cdot 10^{-6}$ @ B=1.5 T

⇒ additional field gradients



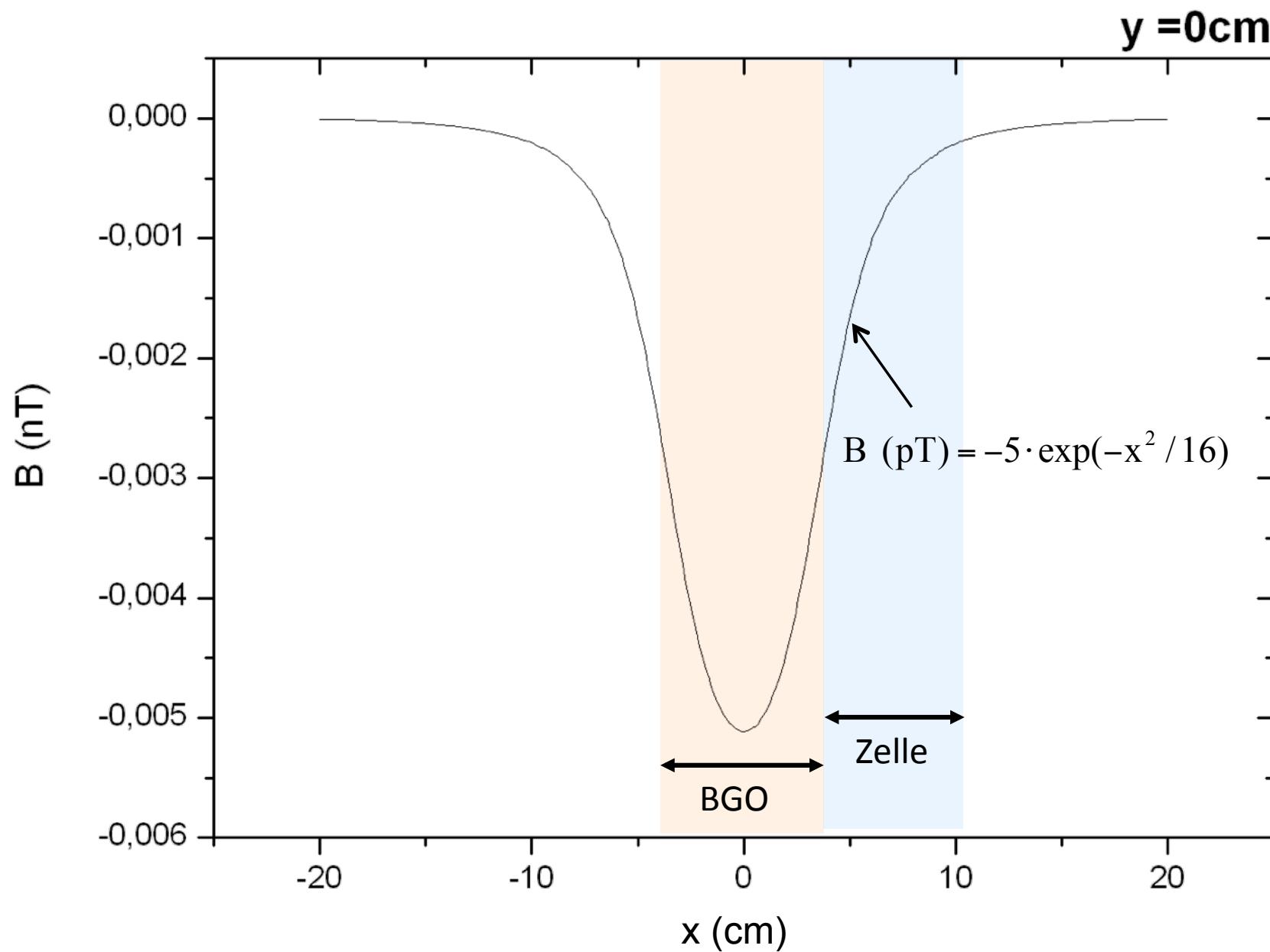
Error due to changes of T_2^* :

$$|\Delta v_{\text{sys}}^{T_2}| \leq \frac{\frac{\Delta T_{2,\text{He}}^*}{(T_{2,\text{He}}^*)^2} \cdot \left(\frac{a_{\text{He}}}{T_{2,\text{He}}^*} - \frac{1}{2} \frac{a_{\text{Xe}}}{T_{2,\text{Xe}}^*} \right) \cdot t_0}{2\pi \left(1 - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \right)}$$

with: $\langle a_{\text{He}} \rangle \approx 11.5 \text{ rad}$, $\langle a_{\text{Xe}} \rangle \approx 0.1 \text{ rad}$, $t_0 = 3 \text{ h}$,
 $T_{2,\text{He}}^* = 53 \text{ h}$, $T_{2,\text{Xe}}^* = 5 \text{ h}$, $|\Delta T_{2,\text{He}}^*| \leq 160 \text{ s}$

$$\Rightarrow |\Delta v_{\text{sys}}^{T_2}| \leq 0.1 \text{ nHz}$$

Comsol Simulation



Comsol Simulation

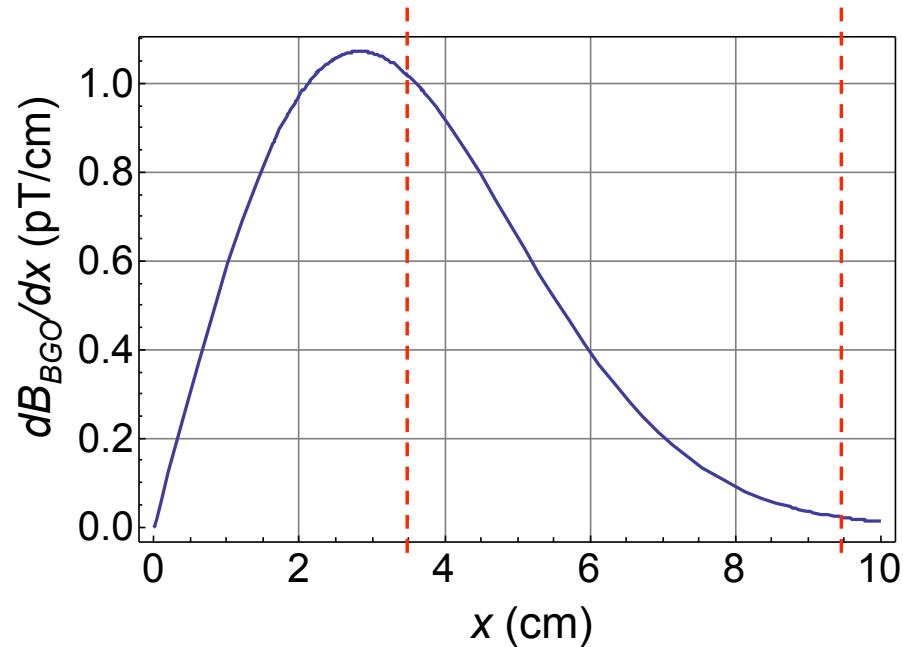
Assumption:

$$B_{\text{BGO}}(\text{pT}) = -5 \cdot \exp(-x^2 / 16) \Rightarrow \partial B_{\text{BGO}} / \partial x (\text{pT} / \text{cm}) = +10 / 16 \cdot x \cdot \exp(-x^2 / 16)$$

Comsol Simulation

Assumption:

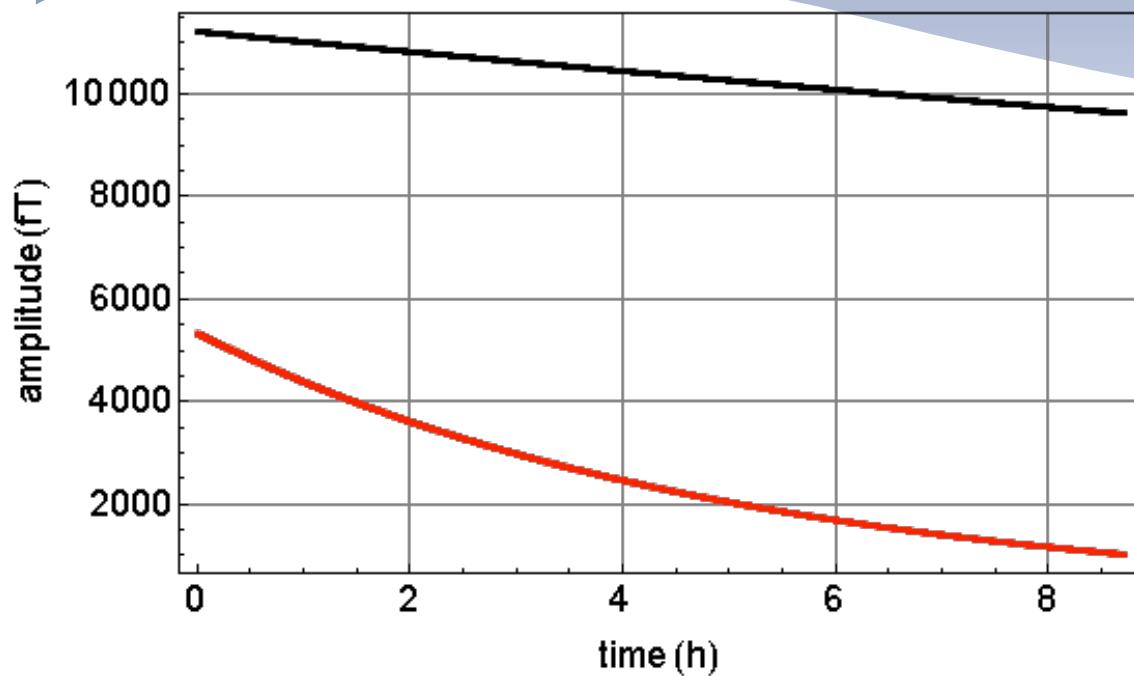
$$B_{BGO}(\text{pT}) = -5 \cdot \exp(-x^2 / 16) \Rightarrow \partial B_{BGO} / \partial x (\text{pT/cm}) = +10 / 16 \cdot x \cdot \exp(-x^2 / 16)$$



$$\overline{\partial B_{BGO} / \partial x} \approx 0.47 \text{ pT/cm}$$

$$\partial B_{BGO} / \partial y, \partial B_{BGO} / \partial z \ll \partial B_{BGO} / \partial x$$

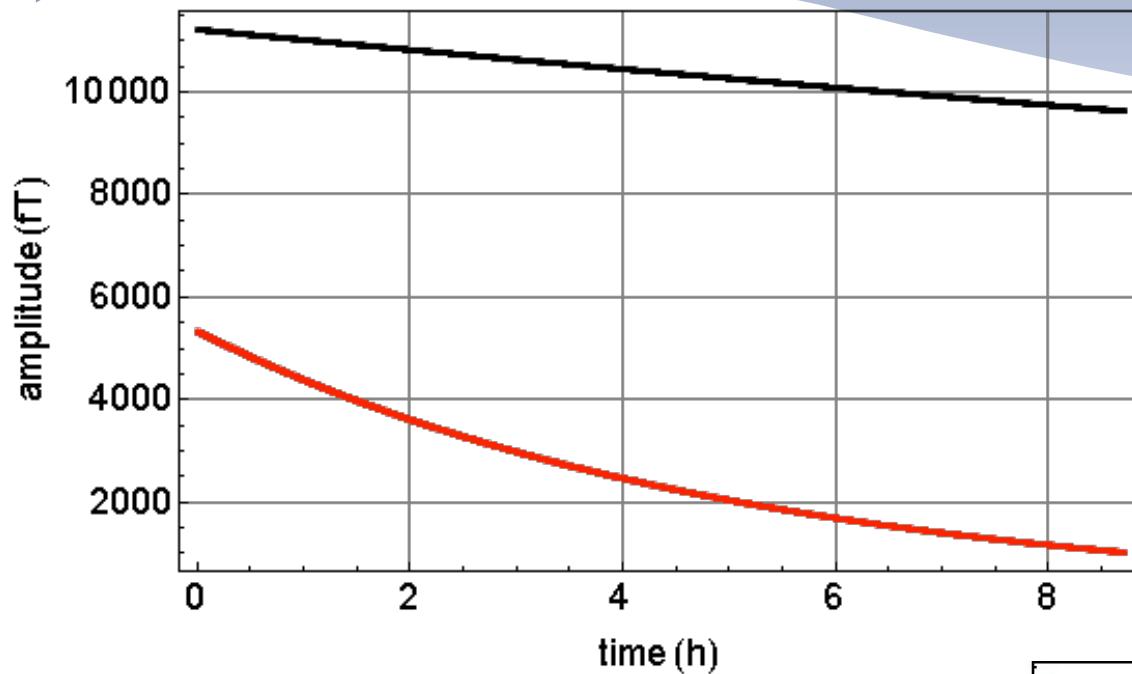
Determination of T_2^*



Exponential fit of amplitudes:

$$fit(t) = a_0 \cdot e^{-t/T_2}$$

Determination of T_2^*

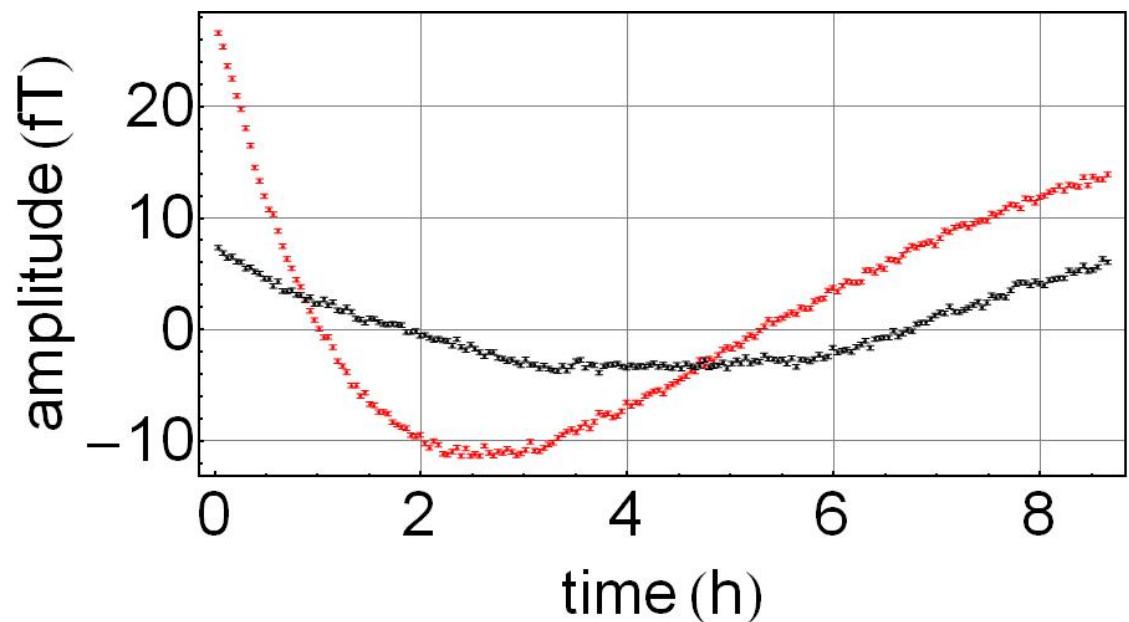


Exponential fit of amplitudes:

$$fit(t) = a_0 \cdot e^{-t/T_2}$$

Residuals:

$$residuals(t) = a(t) - fit(t)$$



Decay of Amplitudes

$$A(t) = f_a(t) \cdot f_b(t) \cdot f_c(t) \cdot \dots \cdot A_0 \cdot e^{-t/T_2}$$

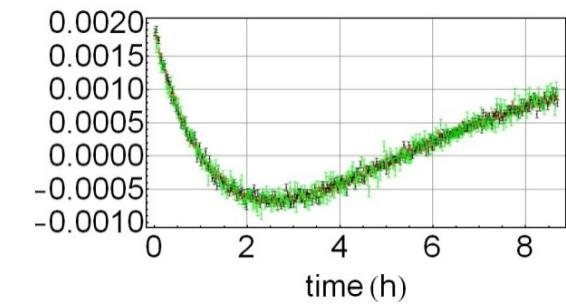
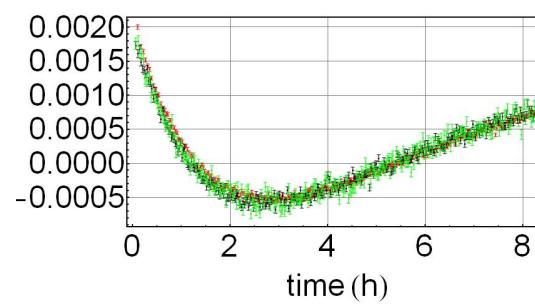
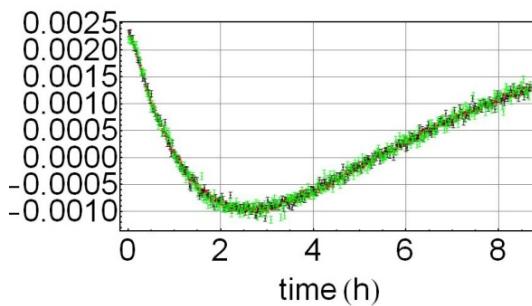
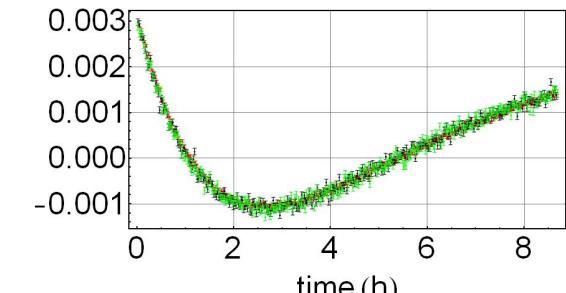
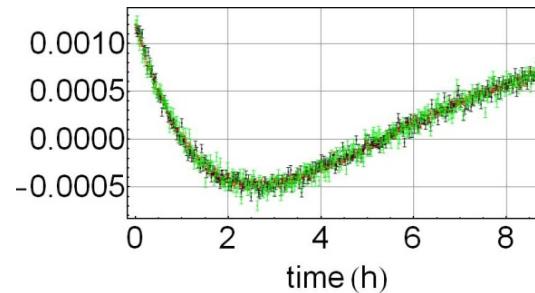
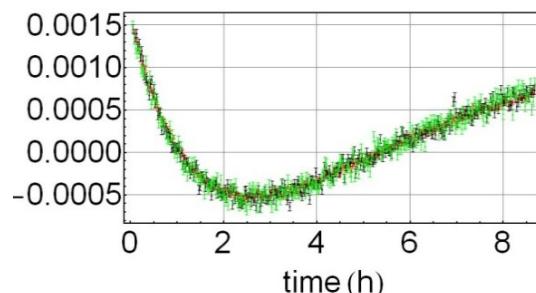
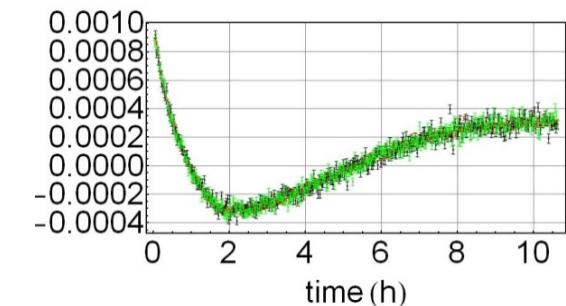
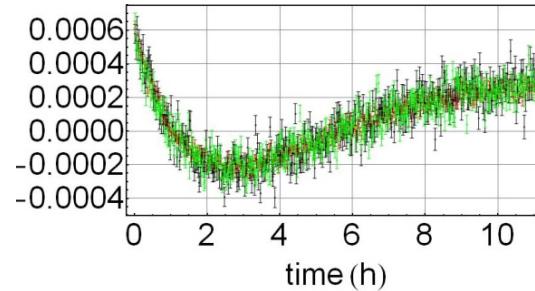
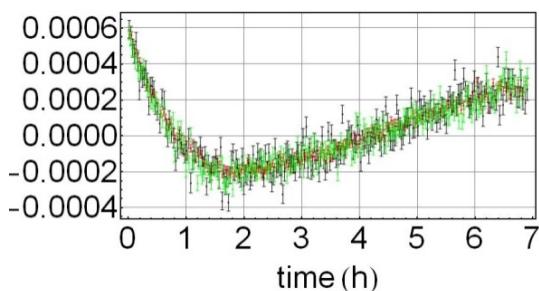
- a.) Drift of SQUIDs and motion of SQUIDs in the magnetic field $f_a(t)$
- b.) Changes of the field gradients

$$Ratio(t) = \frac{f_{b,Xe}(t)}{f_{b,He}(t)} \cdot \frac{f_{c,Xe}(t)}{f_{c,He}(t)} \cdot \frac{A_{0,Xe}}{A_{0,He}} \cdot e^{-t/T} \quad \text{mit: } \frac{1}{T} = \frac{1}{T_{2,Xe}^*} - \frac{1}{T_{2,He}^2}$$

⇒ Drift $f_a(t)$ due to SQUIDs and motion of SQUIDs in the magnetic drops out.

SR Measurements september 2010

green: $Z2E+Z5S$, red: $Z3D-Z2S$, black: $Z3I-Z1S$



LV Measurements march 2009

