Hadron structure from lattice QCD - outlook and future perspectives

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DESY

Outline

Quantum ChromoDynamics (QCD)

QCD-Gauge theory of the strong interaction
Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right)\psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu$$

Harald Fritzsch

Murray Gell-Mann

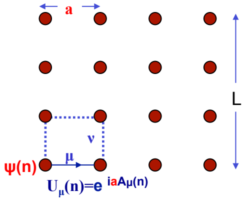
Heinrich Leutwyler

Phys.Lett. B47 (1973) 365

This “simple” Lagrangian produces the amazingly rich structure of strongly interacting matter in the universe.

Numerical simulation of QCD provides essential input for a wide class of complex strong interaction phenomena
→ In this talk: Hadron structure of interest to both the phenomenological and experimental communities.

QCD on the lattice

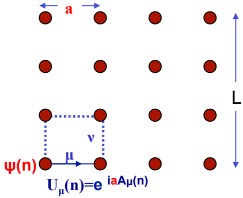


Lattice QCD: K. Wilson, 1974 provided the formulation; M. Creutz, 1980 performed the first numerical simulation

Discretization of space-time with lattice spacing *a* ensuring gauge invariance

- Define quark fields $\psi(x)$ and $\bar{\psi}(x)$ on lattice sites
- Introduce parallel transporter connecting point x and $x + a\hat{\mu}$:
  $U_\mu(x) = e^{iaA_\mu(x)}$ i.e. gauge field $U_\mu(x)$ is defined on links
  - Finite a provides an ultraviolet cutoff at $\pi/a \to$ non-perturbative regularization
  - Finite $L \to$ discrete momenta in units of $2\pi/L$ if periodic b.c.
- Construct an appropriate action S: $S = S_G + S_F$ where
  $S_F = a^4 \sum_x \bar{\psi}(x) D \psi(x)$ i.e. quadratic in the fermions
  $\longrightarrow$ can be integrated out leaving a path integral over gauge fields

QCD on the lattice



Lattice QCD: K. Wilson, 1974 provided the formulation; M. Creutz, 1980 performed the first numerical simulation

Discretization of space-time with lattice spacing *a* ensuring gauge invariance

- Go to imaginary time: $\langle \mathcal{O} \rangle = \frac{1}{Z}\int_U \mathcal{O}(D^{-1}, U)\, det(D[U])\, e^{-S_G[U]}$
  $\to$ Monte Carlo simulation to produce a representative ensemble of $\{U_\mu(x)\}$ using the largest supercomputers
  $\to$ Observables: $\langle \mathcal{O} \rangle = \frac{1}{Z}\sum_{\{U_\mu\}} O(D^{-1}, U_\mu)$

5.0 Pflop/s, second biggest in Europe, 8th in the world - TOP 500 June 2014

Fermion action

Observables: $\langle \mathcal{O} \rangle = \frac{1}{Z} \sum_{\{U_\mu\}} O(D^{-1}, U_\mu)$

Several $\mathcal{O}(a)$-improved fermion actions, K. Jansen, Lattice 2008
$\langle O \rangle_{\rm cont} = \langle O \rangle_{\rm latt} + \mathcal{O}(a^2)$

Action	Advantages	Disadvantages
Clover improved Wilson	computationally fast	breaks chiral symmetry needs operator improvement
Twisted mass (TM)	computationally fast automatic improvement	breaks chiral symmetry violation of isospin
Staggered	computational fast	four doublers (fourth root issue) complicated contractions
Domain wall (DW)	improved chiral symmetry	computationally demanding needs tuning
Overlap	exact chiral symmetry	computationally expensive

Several collaborations:

Clover	QCDSF, BMW, ALPHA, CLS, PACS-CS, NPQCD
Twisted mass	ETMC
Staggered	MILC
Domain wall	RBC-UKQCD
Overlap	JLQCD

Systematic uncertainties

- Finite lattice spacing a - take the continuum limit $a \to 0$
- Finite volume L - take infinite volume limit $L \to \infty$
- Identification of hadron state of interest

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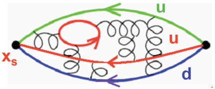
Creation operator for zero momentum: $J_p^\dagger(t_s) = \sum_{\vec{x}_s} J_p^\dagger(\vec{x}_s, t_s)$
Proton propagator:



$$\langle J_p(t_s) J_p^\dagger(0)\rangle = \sum_n \langle 0|J_p\, e^{-H_{QCD} t_s}|n><n|J_p^\dagger|0\rangle$$
$$= \sum_n |\langle 0|J_p|n\rangle|^2 e^{-E_n t_s} \xrightarrow{t_s \to \infty} |\langle 0|J_p|p\rangle|^2\, e^{-m_p t_s}$$

Noise to signal increases with t_s:
$\sim e^{(m_p - \frac{3}{2} m_\pi) t_s}$

Systematic uncertainties

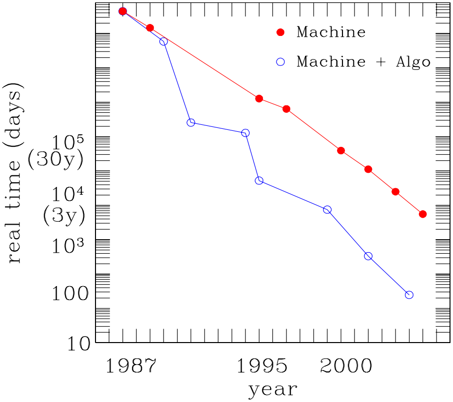
- Finite lattice spacing a - take the continuum limit $a \to 0$
- Finite volume L - take infinite volume limit $L \to \infty$
- Identification of hadron state of interest
- Simulation at physical quark masses - now feasible
- Computation of valence quark loops - now feasible

Recent achievements

Simulations with physical quark masses

A number of collaborations are producing simulations with physical values of the quark mass: MILC, BMW, PACS-CS, ETM

Algorithm development has been decisive

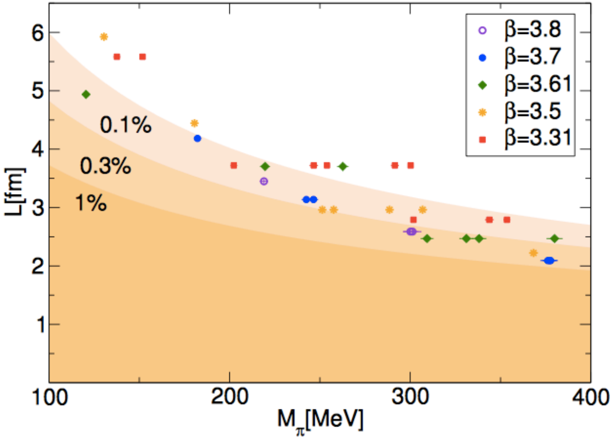


Simulation on a $32^3 \times 64$ lattice, 5000 configurations

Simulations with physical quark masses

A number of collaborations are producing simulations with physical values of the quark mass: MILC, BMW, PACS-CS, ETM

Budapest-Marseille-Wuppertal (BMW) Collaboration: $N_f = 2 + 1$ Clover improved Wilson fermions with HEX smearing



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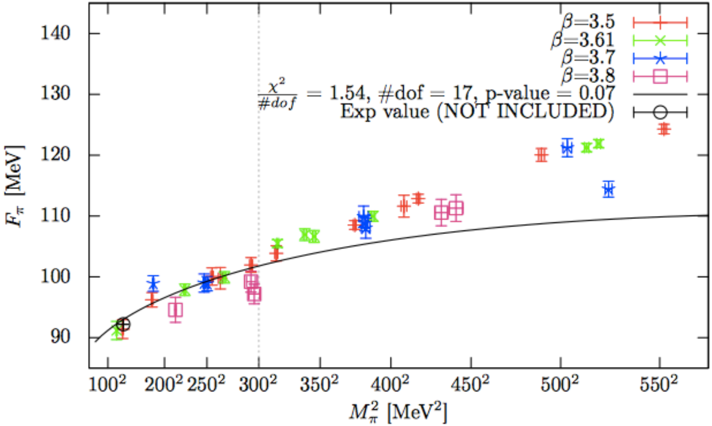

NLO SU(2) chiral perturbation theory for $m_\pi < 300$ MeV
S. Durr *et al.*, 1310.3626

Simulations with physical quark masses

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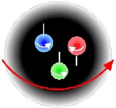
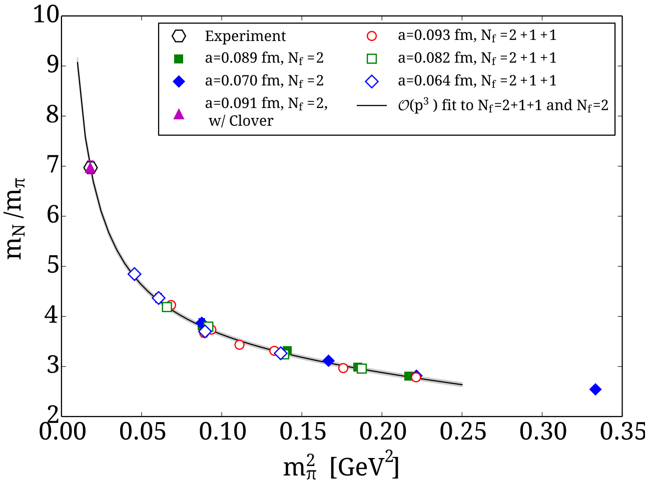
European Twisted Mass (ETM) Collaboration: The nucleon



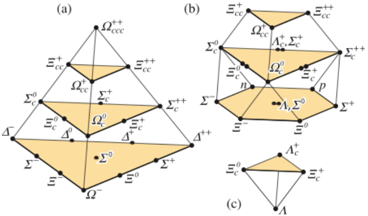


$L \sim 3$ fm and $a \sim 0.1$ fm; Lowest order heavy baryon chiral perturbation theory with experimental value excluded

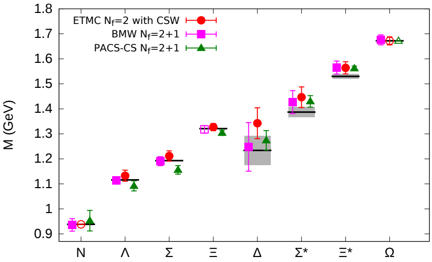
Hyperons and Charmed baryons

SU(4) representations:

$$4 \otimes 4 \otimes 4 \;=\; 20 \oplus 20 \oplus 20 \oplus \overline{4}$$



First goal: reproduce the low-lying masses



Results by ETM Collaboration using $N_f = 2$ simulations with physical pion mass for one lattice volume and lattice spacing $a = 0.091$ fm

Also $N_f = 2 + 1 + 1$ results: C.A., V. Drach, K. Jansen, Ch. Kallidonis, G. Koutsou, arXiv:1406.4310

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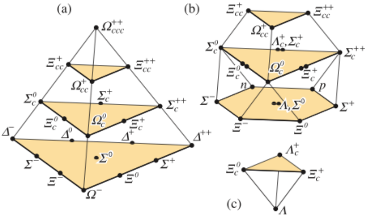
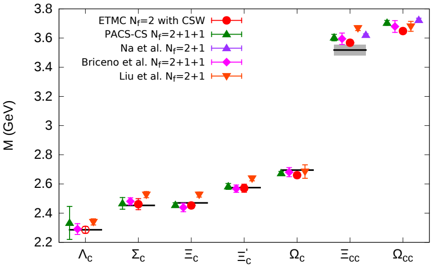
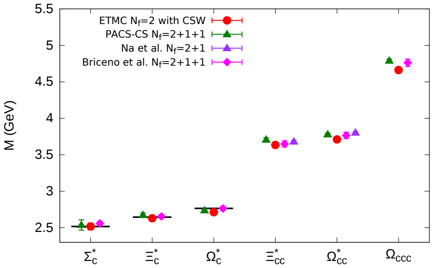


First goal: reproduce the low-lying masses and make predictions





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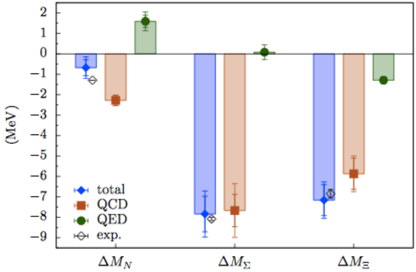
Isospin and electromagnetic mass splitting

RBC and BMW collaborations: Treat isospin and electromagnetic effects to LO



Baryon spectrum with mass splitting from BMW, Sz. Borsanyi *et al.*, Phys. Rev. Lett. 111 (2013) 252001

- Nucleon mass: isospin and electromagnetic effects with opposite signs
- Physical splitting reproduced

Challenges and future perspectives

Challenges: I. Excited states, multi-quark states & exotics

Variational approach: Enlarge basis of interpolating fields $\rightarrow$ correlation matrix

$G_{jk}(t_s) = \langle J_j(t_s) J_k^\dagger(0) \rangle$, $j, k = 1, \ldots N$

Solve the generalized eigenvalue problem (GEVP):

$G(t)v_k(t; t_0) = \lambda_k(t; t_0) G(t_0) v_k(t; t_0)$
$\rightarrow \lambda_k(t; t_0) = e^{-E_k(t-t_0)}$ yields N lowest eigenstates, M. Lüscher & U. Wolff (1990)

Large effort to construct the appropriate basis using lattice symmetries, Hadron Spectrum Collaboration

- must extract all states lying below the state of interest
- as $m_\pi \rightarrow m_{\rm physical}$ need to consider multi-hadron states
- must include disconnected diagrams
- most excited states are unstable (resonances)

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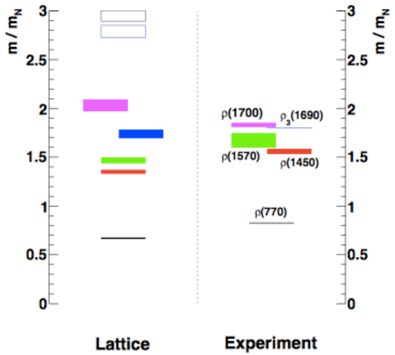
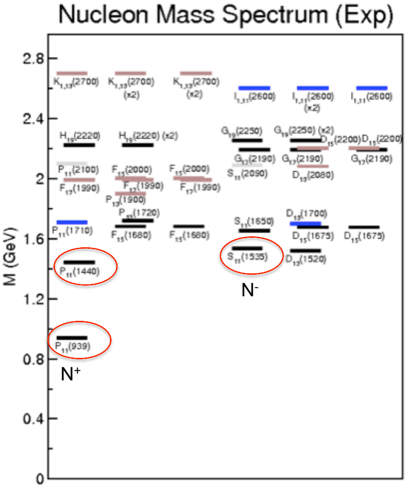


$m_\pi \sim 400$ MeV at one lattice spacing and one lattice volume, J. Bulava *et al.*, 1310.7887

Where is the Roper?

Can we obtain the known low-lying nucleon resonances, $P_{11}(1440)$ and $S_{11}(1535)$?



- R.G. Edwards, J. J. Dudek, D. G. Richards, S. J. Wallace. Phys. Rev. D 84 (2011) 074508
- M. Mahbub *et al.*, Phys. Lett. B679 (2009) 418
- G. P. Engel, C. Lang, D. Mohler, A. Schäfer, 1301.4318

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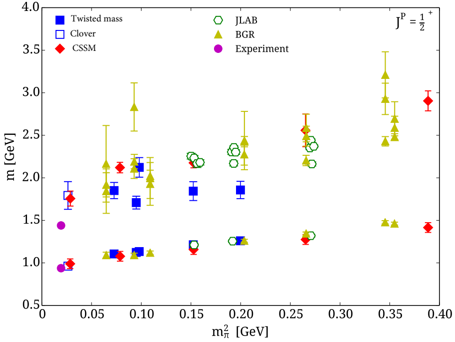
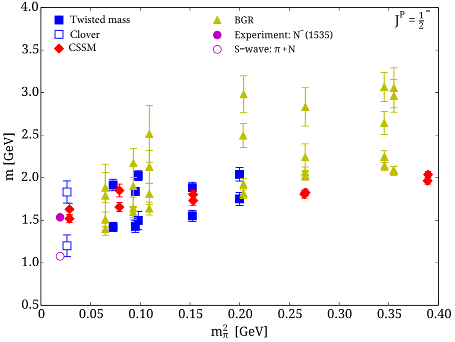


Positive parity



Negative parity

- R.G. Edwards, J. J. Dudek, D. G. Richards, S. J. Wallace. Phys. Rev. D 84 (2011) 074508
- M. Mahbub *et al.*, Phys. Lett. B679 (2009) 418
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- $N_f = 2$ twisted mass and clover fermions, C.A., T. Korzec, G. Koutsou, T. Leontiou, arXiv:1302.4410

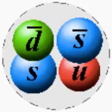
Scalar mesons

Start with $a_0(980)$ and $\kappa(800)$

In our study: 4(+2) operators with the quantum numbers of $a_0(980)$.

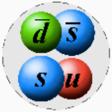
$$\mathcal{O}^{q\bar{q}} = \sum_{\mathbf{x}} \left(\bar{d}_{\mathbf{x}} u_{\mathbf{x}}\right)$$

$$\mathcal{O}^{K\bar{K},\,\text{point}} = \sum_{\mathbf{x}} \left(\bar{s}_{\mathbf{x}}\gamma_5 u_{\mathbf{x}}\right)\left(\bar{d}_{\mathbf{x}}\gamma_5 s_{\mathbf{x}}\right)$$

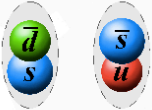


$$\mathcal{O}^{\eta_s\pi,\,\text{point}} = \sum_{\mathbf{x}} \left(\bar{s}_{\mathbf{x}}\gamma_5 s_{\mathbf{x}}\right)\left(\bar{d}_{\mathbf{x}}\gamma_5 u_{\mathbf{x}}\right)$$

$$\mathcal{O}^{Q\bar{Q}} = \sum_{\mathbf{x}} \epsilon_{abc}\left(\bar{s}_{\mathbf{x},b}(C\gamma_5)\bar{d}^T_{\mathbf{x},c}\right)\epsilon_{ade}\left(u^T_{\mathbf{x},d}(C\gamma_5)s_{\mathbf{x},e}\right)$$



$$\mathcal{O}^{K\bar{K},\,\text{2-part}} = \sum_{\mathbf{x},\mathbf{y}} \left(\bar{s}_{\mathbf{x}}\gamma_5 u_{\mathbf{x}}\right)\left(\bar{d}_{\mathbf{y}}\gamma_5 s_{\mathbf{y}}\right)$$



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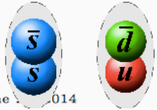
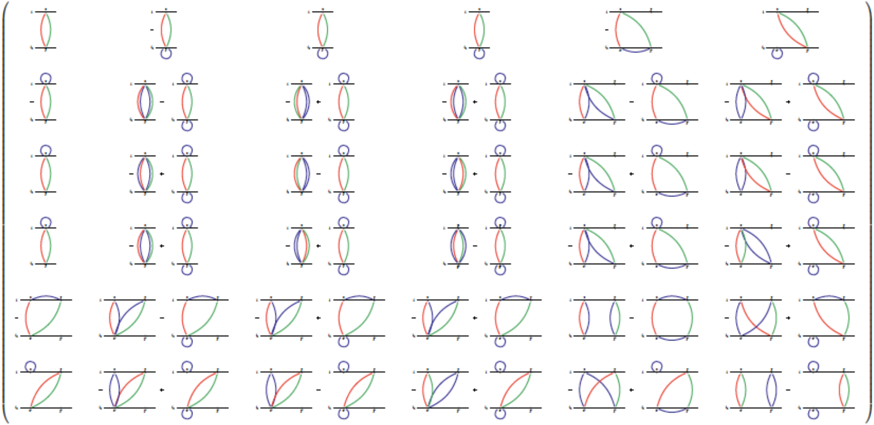


Investigation of the tetraquark candidate $a_0(980)$: technical aspects - Joshua Berlin, June [illegible]014

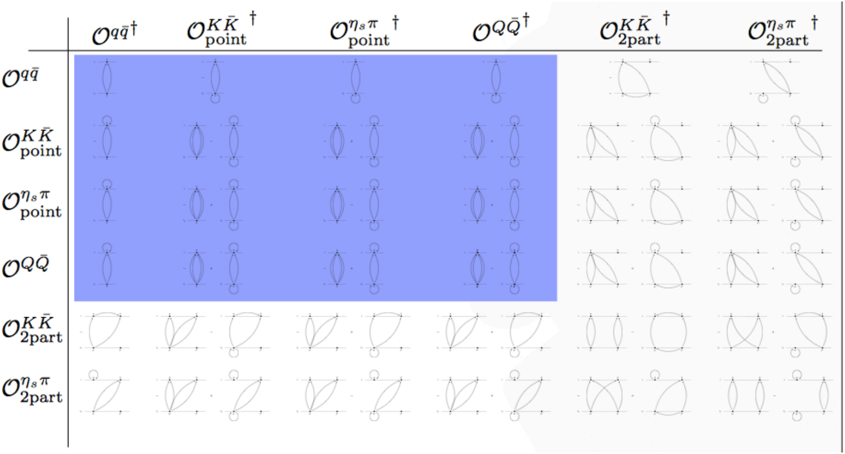
Scalar mesons



Need to compute disconnected diagrams - very challenging!

Scalar mesons

Start with $a_0(980)$ and $\kappa(800)$

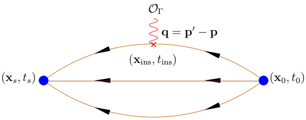


Preliminary results using 4×4 correlation matrix show weak overlap with a $q\bar{q}$ state

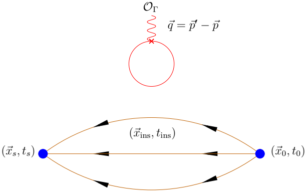
Challenges: II. Nucleon structure

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$



connected contribution



disconnected contribution

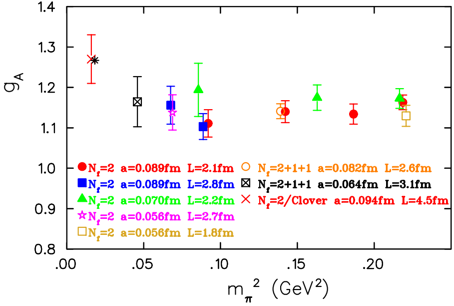
- $\mathcal{M}$ the desired matrix element; t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices; $\Delta(\mathbf{p})$ the energy gap with the first excited state
- Identification of hadron state of interest - dependent on $\mathcal{O}_\Gamma$ i.e. different for g_A, σ-terms, EM form factors
- Connect lattice results to measurements:
  $\mathcal{O}_{\overline{\rm MS}}(\mu) = Z(\mu, a)\mathcal{O}_{\rm latt}(a) \Longrightarrow$ evaluate $Z(\mu, a)$ non-perturbatively

Axial charge g_A

The good news:

Axial-vector FFs: $A^3_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x) \Longrightarrow \frac{1}{2}\bar u_N(\vec{p}')\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]u_N(\vec p)|_{q^2=0}$
$\rightarrow$ yields $G_A(0)\equiv g_A$: i) well known experimentally & ii) no quark loop contributions

Isovector



ETM Collaboration

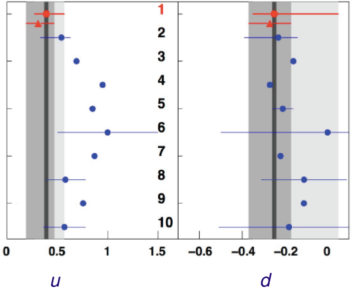
- g_A at the physical point mass indicates agreement with the physical value $\rightarrow$ important to reduce error
- many results from other collaborations, e.g.
  - $N_f = 2+1$ Clover, LHPC, J. R. Green *et al.*, arXiv:1209.1687
  - $N_f = 2$ Clover, QCDSF, R.Hosley *et al.*, arXiv:1302.2233
  - $N_f = 2$ Clover, CLS, S. Capitani *et al.* arXiv:1205.0180
  - $N_f = 2+1$ Clover, CSSM, B. J. Owen *et al.*, arXiv:1212.4668
  - $N_f = 2+1+1$ Mixed action (HISQ/Clover), PNDME, T. Bhattacharya *et al.*, arXiv:1306.5435

Nucleon charges: g_A, g_s, g_T

- scalar operator: $\mathcal{O}^a_S = \bar\psi(x)\frac{\tau^a}{2}\psi(x)$
- axial-vector operator: $\mathcal{O}^a_A = \bar\psi(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}^a_T = \bar\psi(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

$\Longrightarrow \langle N(\vec{p'})\mathcal{O}_\Gamma N(\vec{p})\rangle|_{q^2=0}$ yeilds $g_s,\ g_A,\ g_T$
(i) isovector combination has no disconnect contributions; (ii) g_A well known experimentally, g_T to be measured at JLab

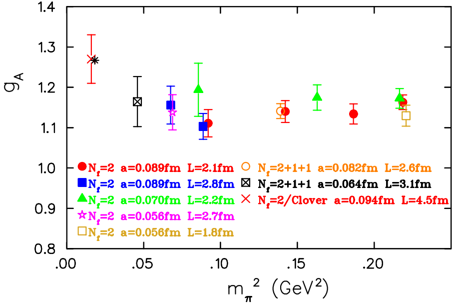
Planned experiment at JLab, SIDIS on ^{3}He/Proton at 11 GeV:

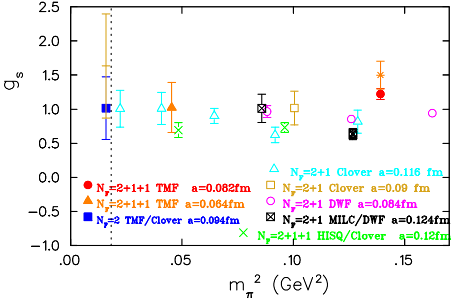


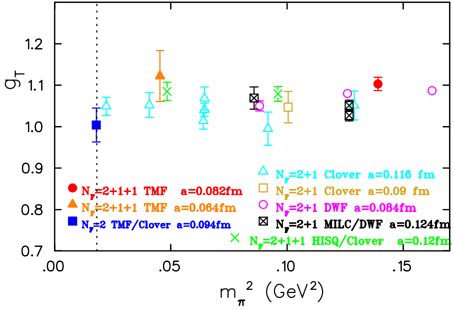
Experimental values: $g_T^u = 0.39^{+0.18}_{-0.12}$ and $g_T^d = -0.25^{+0.3}_{-0.1}$

Nucleon charges: g_A, g_S, g_T

Isovector

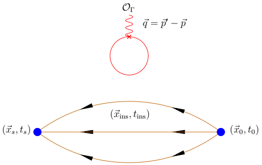






- g_A at the physical point mass indicates agreement with the physical value $\rightarrow$ important to reduce error - many results from other collaborations
- Experimental value of $g_T \sim 0.54^{+0.30}_{-0.13}$ from global analysis of HERMES, COMPASS and Belle e^+e^- data, M. Anselmino *et al.* (2013)
- Large excited state contributions to g_S: increasing the sink-source time separation to ~ 1.5 fm is crucial

Disconnected quark loop contributions

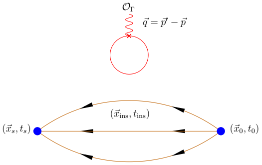


Notoriously difficult

- $L(x_{\rm ins}) = Tr\,[\Gamma G(x_{\rm ins}; x_{\rm ins})]$ $\rightarrow$ need quark propagators from all $\vec{x}_{\rm ins}$ or L^3 more expensive as compared to the calculation of hadron masses
- Large gauge noise $\rightarrow$ large statistics

- Use special techniques that utilize stochastic noise on all spatial lattice sites $\rightarrow$ N_r more expensive that hadron masses with $N_r \ll L^3$
- Reduce noise by increasing statistics
$\Longrightarrow$ take advantage of graphics cards (GPUs) $\rightarrow$ need to develop special multi-GPU codes

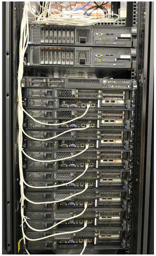
Disconnected quark loop contributions



Notoriously difficult

- $L(x_{\rm ins}) = Tr\,[\Gamma G(x_{\rm ins}; x_{\rm ins})]$ $\rightarrow$ need quark propagators from all $\vec{x}_{\rm ins}$ or L^3 more expensive as compared to the calculation of hadron masses
- Large gauge noise $\rightarrow$ large statistics
- Use special techniques that utilize stochastic noise on all spatial lattice sites $\rightarrow$ N_r more expensive that hadron masses with $N_r \ll L^3$
- Reduce noise by increasing statistics
  $\Longrightarrow$ take advantage of graphics cards (GPUs) $\rightarrow$ need to develop special multi-GPU codes

A Fermi card



Cluster of 8 nodes of Fermi GPUs

C. A., M. Constantinou, S. Dinter, V. Drach, K. Hadjiyiannakou, K. Jansen, G. Koutsou, A. Strelchenko, A. Vaquero arXiv:1211.0126
C.A., K. Hadjiyiannakou, G. Koutsou, A. O'Cais, A. Strelchenko, arXiv:1108.2473

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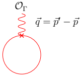
Notoriously difficult



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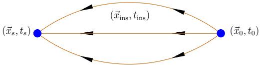

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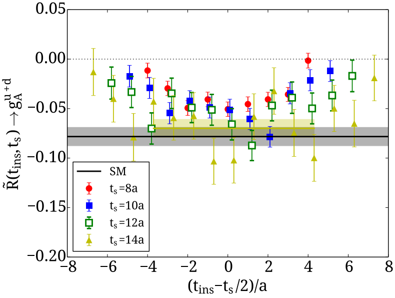
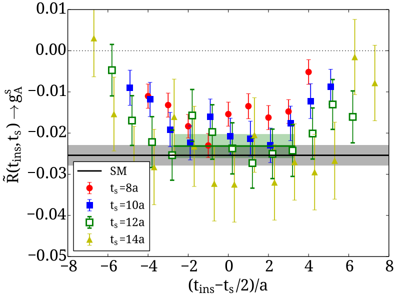
$N_f = 2+1+1$ twisted mass, a = 0.082 fm, m_π = 373 MeV, $\sim$ **150, 000** statistics (on 4700 confs)



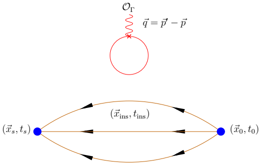
Disconnected isoscalar, agrees with Bali *et al.* (QCDSF), Phys.Rev.Lett. 108 (2012) 222001



Strange quark loop

Disconnected quark loop contributions

Notoriously difficult



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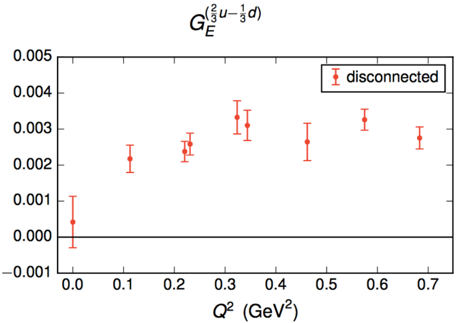
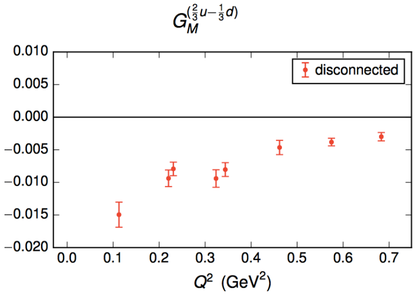




100,000 Statistics using hierarchical probing, $N_f = 2+1$ clover (one level of stout smearing), $V = 32^3 \times 96$, $a \sim 0.114$ fm, $m_\pi \sim 320$ MeV, St. Meinel *et al.*, Lattice 2014, N. York, June, 2014

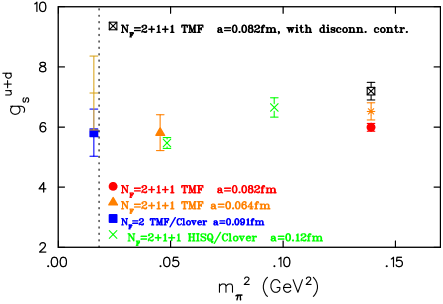
Isoscalar nucleon charges: g_A, g_s, g_T

- scalar operator: $\mathcal{O}_S^a = \bar\psi(x)\frac{\tau^a}{2}\psi(x)$
- axial-vector operator: $\mathcal{O}_A^a = \bar\psi(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar\psi(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

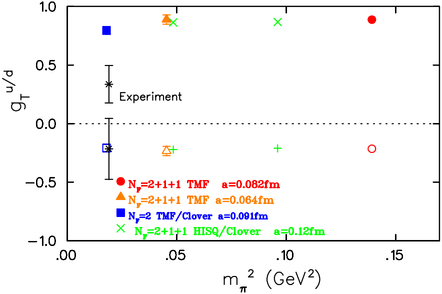
- $N_f = 2+1+1$ twisted mass, a = 0.082 fm, m_π = 373 MeV
- Disconnected part, $\sim$ 150 000 statistics using GPUs

Results shown in $\overline{MS}$ at 4 GeV^2

Analysis at the physical point still preliminary



Large source-sink separation and inclusion of disconnected is required



Experimental values from global analysis of HERMES, COMPASS and Belle e^+e^- data, M. Anselmino *et al.* (2013)

Nucleon momentum fraction and spin

What is the distribution of the nucleon momentum among the nucleon constituents?
$\rightarrow$ needs knowledge of the parton distribution functions (PDFs)
One measures moments of parton distributions,in DIS:

- Unpolarized moments: $\langle x^n \rangle_q = \int_0^1 dx x^n \left[q(x) - (-1)^n \bar{q}(x) \right]$, $q(x) = q(x)_\downarrow + q(x)_\uparrow$
- Helicity moments: $\langle x^n \rangle_{\Delta q} = \int_0^1 dx x^n \left[\Delta q(x) + (-1)^n \Delta \bar{q}(x) \right]$, $\Delta q(x) = q(x)_\downarrow - q(x)_\uparrow$
- Transversity moments: $\langle x^n \rangle_{\delta q} = \int_0^1 dx x^n \left[\delta q(x) - (-1)^n \delta \bar{q}(x) \right]$, $\delta q(x) = q(x)_\perp + q(x)_\top$

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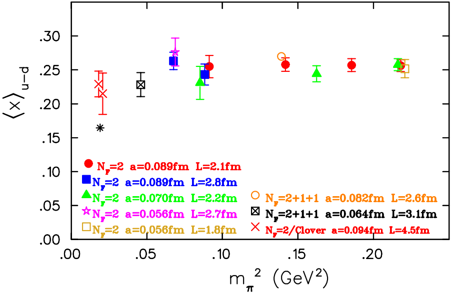
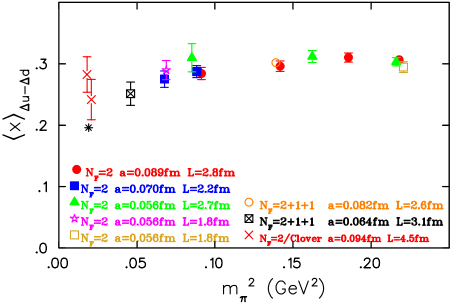
Consider $n = 1$; results obtained in the $\overline{MS}$ scheme at $\mu = 2$ GeV.





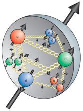
- $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$ approach physical value for bigger source-sink separations $\rightarrow$ need an equivalent high statistics study as at $m_\pi = 373$ MeV
- Can provide a prediction for $\langle x \rangle_{\delta u - \delta d}$

Experimental values:

- $\langle x \rangle_{u-d}$ from S. Alekhin *et al.* arXiv:1202.2281
- $\langle x \rangle_{\Delta u - \Delta d}$ from Blumlein *et al.* arXiv:1005.3113

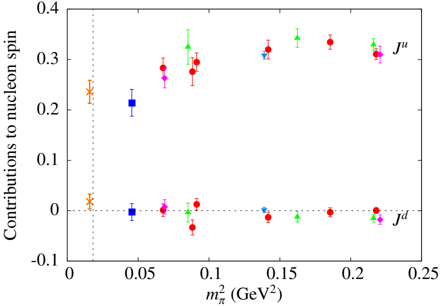
Where is the nucleon spin?

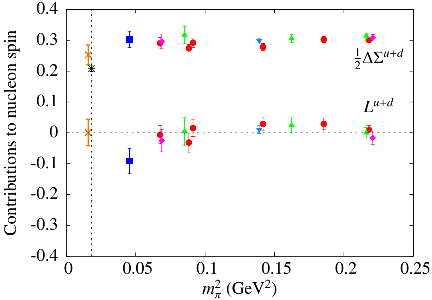
Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$



$J^q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$ and $\Delta\Sigma^q = g^q_A$

Connected contributions





$\Longrightarrow$ Total spin for u-quarks $J^u \lesssim 0.25$ and for d-quark $J^d \sim 0$

- $L^{u+d} \sim 0$ at physical point
- $\Delta\Sigma^{u+d}$ in agreement with experimental value at physical point
- The total spin $J^{u+d} \sim 0.25 \Longrightarrow$ Where is the other half?

However, more statistics and checks of systematics are needed for final results at the physical point

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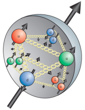
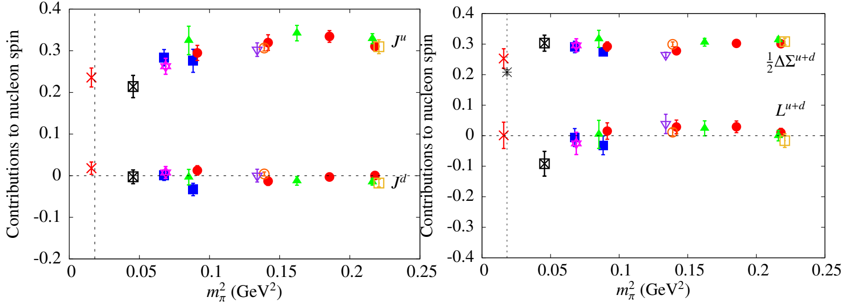


For one ensemble at $m_\pi = 373$ MeV we have the disconnected contribution $\rightarrow$ we can check the effect on the observables, $\mathcal{O}(150,000)$ statistics



- Disconnected quark loop contributions non-zero for $\Delta\Sigma^{u,d,s}$
- $L^d \sim -L^u$
- The total spin $J^{u+d} \sim 0.25 \implies$ Where is the other half?
- Contributions from J^G? $\rightarrow$ on-going efforts to compute them, K.-F. Liu *et al.* (χQCD), arXiv:1203.6388; C.A. *et al.*, arXiv:1311.3174

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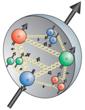

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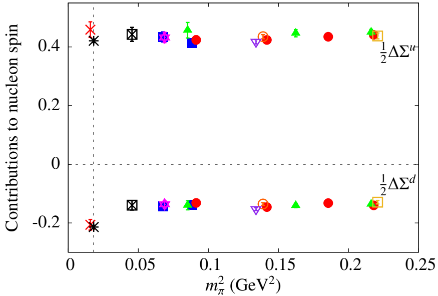

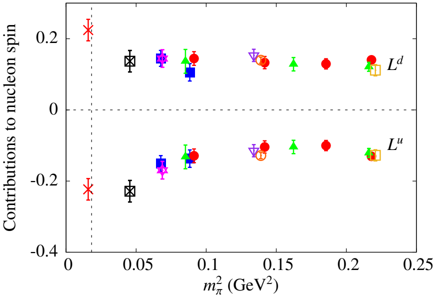

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The quark content of the nucleon

- $\sigma_l \equiv m_l \langle N|\bar{u}u + \bar{d}d|N\rangle$: measures the explicit breaking of chiral symmetry
  Extracted from analysis of low-energy pion-proton scattering data
  Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on σ_l, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD it can be obtained via the Feynman-Hellman theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
- Similarly $\sigma_s \equiv m_s \langle N|\bar{s}s|N\rangle >= m_s \frac{\partial m_N}{\partial m_s}$
- The strange quark content of the nucleon: $y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} = 1 - \frac{\sigma_0}{\sigma_l}$, where σ_0 is the flavor non-singlet
- A number of groups have used the spectral method to extract the σ-terms, R. Young, Lattice 2012
  But they can be also calculated directly

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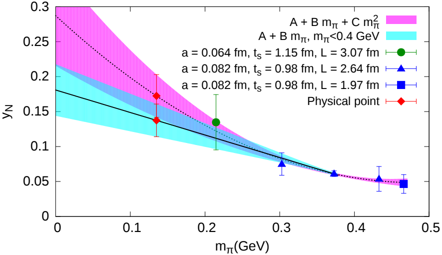
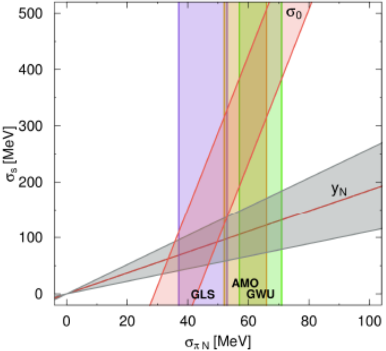


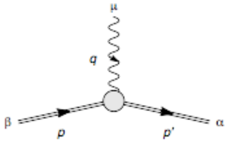
We find $y_N = 0.135 \underbrace{(22)}_{\text{stat.}} \underbrace{(33)}_{\text{ch.extr.}} \underbrace{(22)}_{\text{exc.stat.}} \underbrace{(9)}_{\text{fin.a}}$



Using $\sigma_s = \frac{1}{2}\frac{m_s}{m_l} y_N \sigma_l$ we find σ_s to be less ~ 150 MeV

Electromagnetic form factors

$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$



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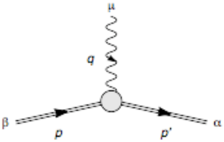
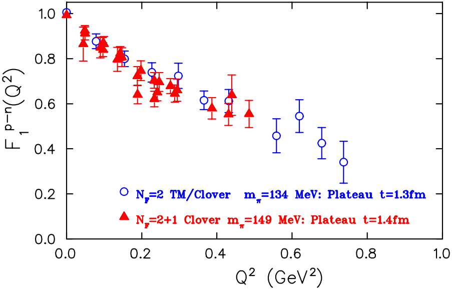
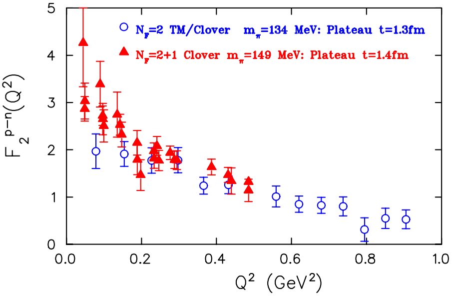


The good news

Two studies at near physical pion mass:

- ETMC: $N_f = 2$ twisted mass with clover, a = 0.091 fm, m_π = 134 MeV, 1020 statistics
- MIT: $N_f = 2+1$ clover produced by the BMW collaboration, $a = 0.116$ MeV, $m_\pi = 149$ MeV, ∼7750 statistics, J.M. Green *et al.* 1404.4029



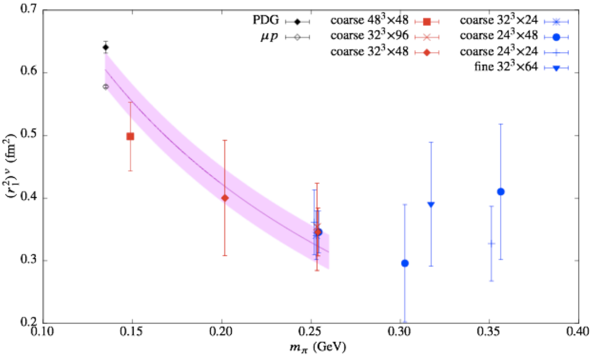


Agreement even before taking the continuum limit

Dirac and Pauli radii

Dipole fits: $\frac{G_0}{(1+Q^2/M^2)^2} \Rightarrow \langle r_i^2 \rangle = -\frac{6}{F_i}\frac{dF_i}{dQ^2}|_{Q^2=0} = \frac{12}{M_i^2}$

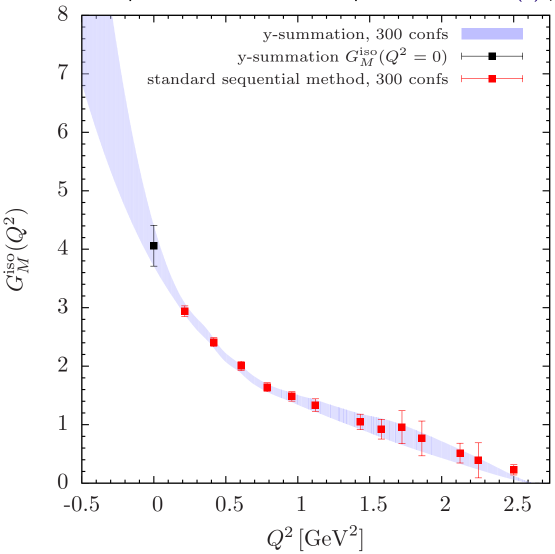
Need better accuracy at the physical point



Using results from summation method, J. M. Green *et al.*, 1404.4029

Momentum dependence of form factors

Avoid model dependence-fits: As a first step we calculated $G_M(0)$ (equivalently $F_2(0)$) at $m_\pi = 373$ MeV



Work in progress, C.A., G. Koutsou, K. Ottnad, M. Petschlies

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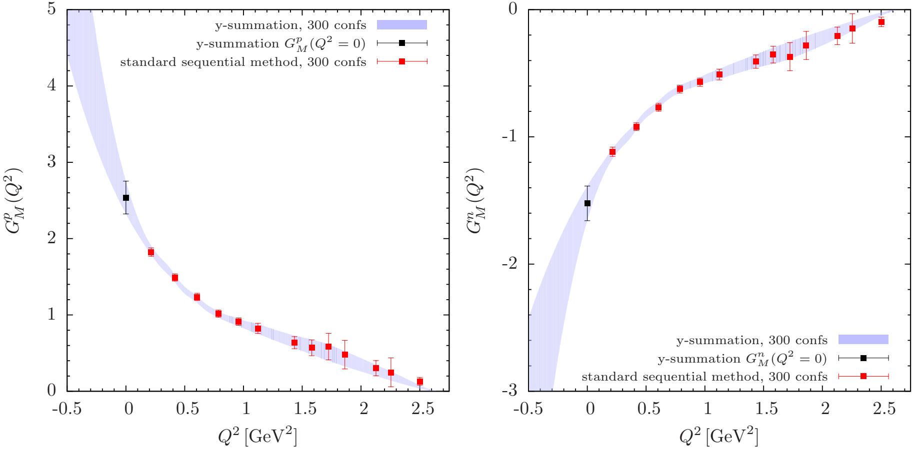
Disconnected contributions small



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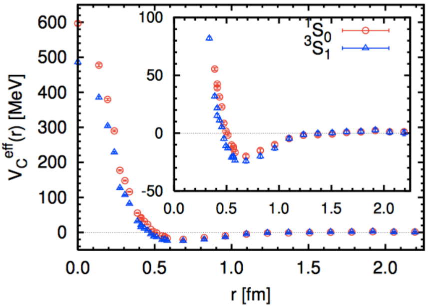
Challenges: III. Nuclear Physics

Going beyond single hadrons
First attempts by HAL QCD and NPLQCD



HAL QCD: Compute the Bethe-Salpeter amplitude and extract the *NN* (effective) central potential for the spin-singlet and spin-triplet channel, $m_\pi = 529$ MeV, S. Aoki, T. Hatsuda and N. Ishii, Prog.Theor.Phys.123 (2010) 89

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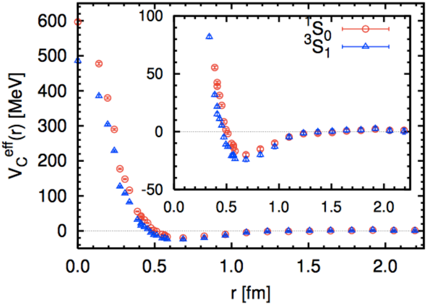

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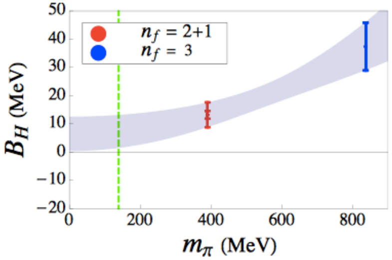

H-dibaryon: a bound system with the quantum numbers of $\Lambda\Lambda$, R. L. Jaffe, Phys. Rev. Lett. 38, 195 (1977)
Still inconclusive:

- NPLQCD $N_f = 2+1$: Not bound, S.R. Beane *et al.*, Phys.Rev. D85 (2012) 054511.
- HAL QCD $N_f = 3$: Bound H-dibaryon with the binding energy of 30-40 MeV for $m_\pi \sim 673 - 1015$ MeV, T. Inoue *et al.*, Phys. Rev. Lett. 106 (2011) 162002.

Need a $\Lambda\Lambda - N\Xi - \Sigma\Sigma$ coupled channel analysis

Conclusions

Simulations at the physical point $\rightarrow$ that's where we always wanted to be!

- Results on g_A, $\langle x \rangle_{u-d}$ etc at the physical point are now directly accessible
  But will need high statistics and careful cross-checks $\rightarrow$ noise reduction techniques are crucial e.g. AMA, TSM, smearing etc
- Evaluation of quark loop diagrams has become feasible - need to make our methods work at the physical point
- Predictions for other hadron observables are emerging e.g. axial charge of hyperons and charmed baryons
- Confirmation of experimentally known quantities such as g_A will enable reliable predictions of others $\rightarrow$ provide insight into the structure of hadrons and input that is crucial for new physics such as the nucleon σ-terms, g_s and g_T
- The study of excited states and resonances is under way $\rightarrow$ provide insight into the structure of hadrons and input that is crucial for new physics
- Apply to Nuclear Physics
- Many challenges ahead ...

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As simulations at the physical pion mass and more computer resources are becoming available we expect many physical results on key hadron observables that will impact both experiments and phenomenology

Acknowledgments

European Twisted Mass Collaboration (ETMC)

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
A. Abdel-Rehim, M. Constantinou, V. Drach, K. Hadjiyiannakou, K.Jansen
Ch. Kallidonis, G. Koutsou, A. Strelchenko, A. Vaquero

REPUBLIC OF CYPRUS

EUROPEAN UNION

STRUCTURAL FUNDS of the European Union in Cyprus
our ideas, actions for development

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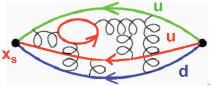
Backup slides

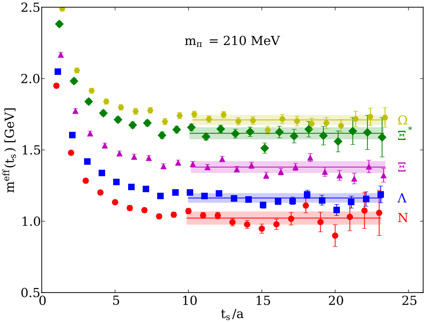
Hadron mass

First goal: reproduce the low-lying masses
As in the meson sector we need:
Use Euclidean correlation functions

$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0)\rangle$$
$$= \sum_{n=0,\cdots,\infty} A_n e^{-E_n(\vec{q})t_s} \xrightarrow{t_s\to\infty} A_0 e^{-E_0(\vec{q})t_s}$$



- Noise to signal increases with t_s
  - use techniques to improve ground state dominance in correlators
  - enough statistics so that the signal extends to large enough t_s at which any remaining contamination from higher states is negligible
- Large Euclidean time evolution gives ground state for given quantum numbers $\Longrightarrow$ enables determination of low-lying hadron properties
  Special techniques to extract excited states
- $aE_{\rm eff}(\vec{q}, t_s) = \ln\left[G(\vec{q}, t_s)/G(\vec{q}, t_s + a)\right]$
  $= aE_0(\vec{q}) + \text{excited states}$
  $\to aE_0(\vec{q}) \stackrel{\vec{q}=0}{\to} am$

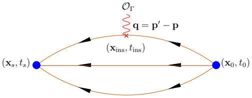


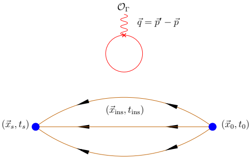
$N_f = 2 + 1 + 1$ TM fermions

Challenges: II. Nucleon structure

Evaluation of three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins})\bar{J}_\beta(\vec{x}_0, t_0)\rangle$$





- Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

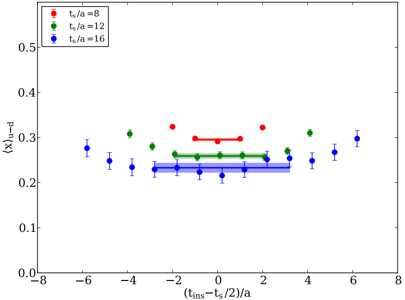
$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + <0|J_N|N><N|\mathcal{O}_\Gamma|N'><N'|J_h^+|0> e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)}$$
$$+ <0|J_N|N'><N'|\mathcal{O}_\Gamma|N><N|J_h^+|0> e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})} + \cdots]$$

- $\mathcal{M}$ the desired matrix element;

$t_s, t_{\rm ins}, t_0$ the sink, insertion and source time-slices;
$\Delta(\mathbf{p})$ the energy gap with the first excited state

- Connect lattice results to measurements:

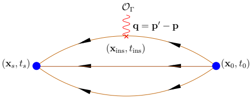
$\mathcal{O}_{\overline{\rm MS}}(\mu) = Z(\mu, a)\mathcal{O}_{\rm latt}(a)$
$\Longrightarrow$ evaluate $Z(\mu, a)$ non-perturbatively

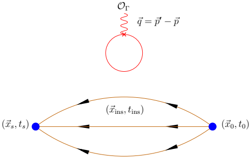


Challenges: II. Nucleon structure

Evaluation of three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{ins}) = \sum_{\vec{x}_s, \vec{x}_{ins}} e^{i\vec{x}_{ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}^{\mu\nu}(\vec{x}_{ins}, t_{ins}) \bar{J}_\beta(\vec{x}_0, t_0)\rangle$$





- Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

$$R(t_s, t_{ins}, t_0) \xrightarrow[(t_s - t_{ins})\Delta \gg 1]{(t_{ins}-t_0)\Delta \gg 1} \mathcal{M}[1 + < 0|J_N|N >< N|\mathcal{O}_\Gamma|N' >< N'|J_h^+|0 > e^{-\Delta(\mathbf{p})(t_{ins}-t_0)}$$

$$+ < 0|J_N|N' >< N'|\mathcal{O}_\Gamma|N >< N|J_h^+|0 > e^{-\Delta(\mathbf{p}')(t_s - t_{ins})} + \cdots]$$

- $\mathcal{M}$ the desired matrix element; t_s, t_{ins}, t_0 the sink, insertion and source time-slices; $\Delta(\mathbf{p})$ the energy gap with the first excited state
- Summing over t_{ins}:

$$\sum_{t_{ins}=t_0}^{t_s} R(t_s, t_{ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

So the excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{ins}$ and/or $t_{ins} - t_0$. However, one needs to fit the slope rather than to a constant.

- Fit $R(t_s, t_{ins}, t_0)$ including the first excited state

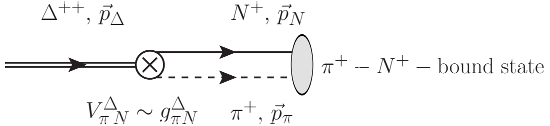
All methods should yield the same result if the ground state is identified

Connect lattice results to measurements: $\mathcal{O}_{\overline{MS}}(\mu) = Z(\mu, a)\mathcal{O}_{latt}(a) \Longrightarrow$ evaluate $Z(\mu, a)$ non-perturbatively

Challenges: IV. Decay width of baryons

Use the transition amplitude method, C. McNeile, C. Michael, P. Pennanen, PRD 65 094505 (2002)

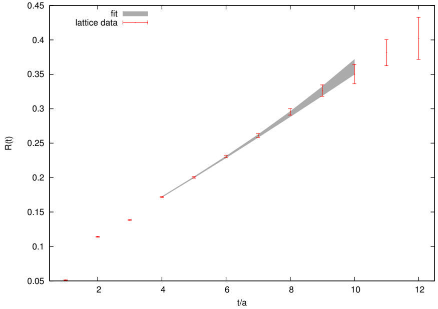
Test the method for the Δ



$\Longrightarrow$ compute transition amplitude $x = \langle \Delta | N\pi \rangle$ from the correlator $G^{\Delta \to N\pi}$

- Need $E_\Delta \sim E_\pi + E_N$
- Applicable for $xt \ll 1$

Hybrid calculation at $m_\pi \sim 360$ MeV with $L = 3.6$ fm



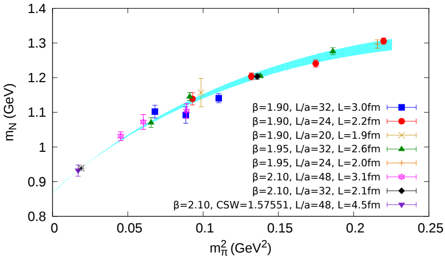
$$R(t) = \frac{G^{\Delta \to \pi N}(t, \vec{Q}, \vec{q})}{\sqrt{G^{\Delta}(t, \vec{q})\, G^{N\pi}(t, \vec{Q}, \vec{q})}} \sim A + B \frac{\sinh(\Delta t/2)}{\sin(\Delta/2)} \sim A + Bt$$

$$g_{\Delta \pi N} = 27.0(0.6)(1.5) \;\to\; \Gamma_\Delta = 99(12)\ \text{MeV}$$

C. A., J. W. Negele, M. Petschlies, A. Tsapalis, PRD 88 031501 (2013)

Setting the scale

- For baryon observables use nucleon mass at physical limit
- Extrapolate using lowest one-loop result: $m_N = m_N^0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2} m_\pi^3$
- Estimate systematic error from next order in HBχPT that includes explicit Δ-degrees of freedom



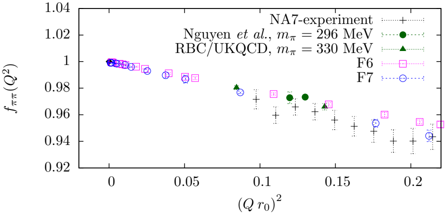
- For $N_f = 2 + 1 + 1$ three different lattice spacings smaller than 0.1 fm → allows continuum extrapolation
- For $N_f = 2$ with clover term $a = 0.0937(2)(2)$ fm and pion mass 130 MeV.

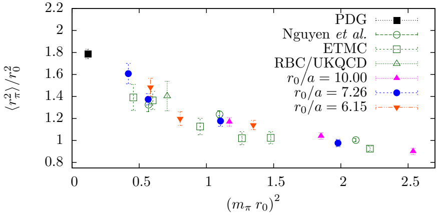
- σ-term from m_N using $\mathcal{O}(p^3)$ and $m_\pi \lesssim 300$ MeV: $\sigma_{\pi N} = 58(8)(7)$ MeV
- Using the nucleon mass we find $r_0 \sim 0.495(5)$ fm in the continuum limit

Pion form factor

Several Collaborations e.g. ETMC, $N_f = 2$, , R. Frezzotti, V. Lubicz and S. Simula, PRD 79, 074506 (2009); B.B. Brandt, A. Juttner, H. Wittig (CLS) arXiv1306.2916, using three lattice spacings smaller than 0.1 fm and pion masses $\sim$ 250 MeV and $\sim$ 600 MeV

- Examine volume and cut-off effects $\Rightarrow$ estimate continuum and infinite volume values
- Twisted boundary conditions to probe small $Q^2 = -q^2$
- All-to-all propagators and 'one-end trick' to obtain accurate results
- Chiral extrapolation using NNLO $\rightarrow$ $\langle r^2 \rangle$ and $F_\pi(Q^2) = \left(1 + \langle r^2 \rangle Q^2/6\right)^{-1}$





B.B. Brandt, A. Juttner, H. Wittig (CLS) arXiv1306.2916

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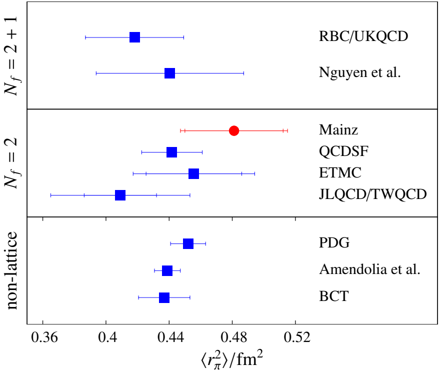

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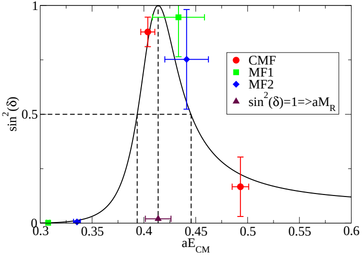
ρ-meson width

- Consider $\pi^+\pi^-$ in the $I=1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{E\left(m_R^2-E^2\right)}$, $k=\sqrt{E^2/4-m_\pi^2}$ $\rightarrow$ determine m_R and $g_{\rho\pi\pi}$ and then extract $\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k_R^3}{m_R^2}$, $k_R=\sqrt{m_R^2/4-m_\pi^2}$

$m_\pi = 309$ MeV, $L=2.8$ fm



$N_F=2$ twisted mass fermions, Xu Feng, K. Jansen and D. Renner, Phys. Rev. D83 (2011) 094505

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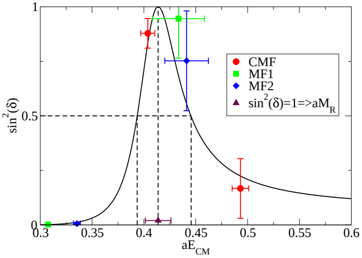

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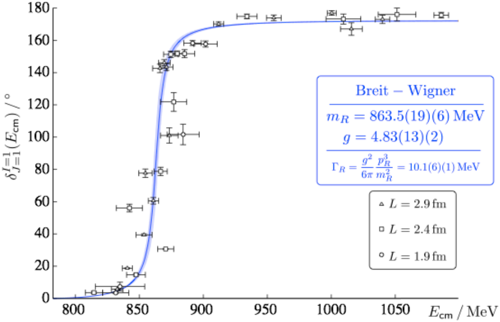
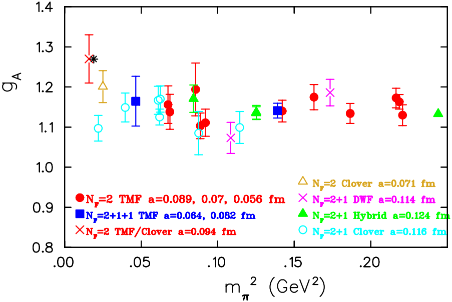


Impressive results using $N_f = 2+1$ clover fermions and 3 asymmetric lattices, J. J. Dudek, R. G. Edwards and C.E. Thomas, Phys. Rev. D 87 (2013) 034505

Volume dependence axial charge g_A

Comparison with other groups



Results obtained using the plateau method with sink-source time separation $\sim (1.0 - 1.2)$ fm

- Results at near physical pion mass are now becoming available → need dedicated study at physical point with high statistics and larger volumes
- A number of collaborations are engaging in systematic studies, e.g.
  - $N_f = 2 + 1$ Clover, J. R. Green *et al.*, arXiv:1209.1687
  - $N_f = 2$ Clover, R.Hosley *et al.*, arXiv:1302.2233
  - $N_f = 2$ Clover, S. Capitani *et al.* arXiv:1205.0180
  - $N_f = 2 + 1$ Clover, B. J. Owen *et al.*, arXiv:1212.4668
  - $N_f = 2 + 1 + 1$ Mixed action (HISQ/Clover), T. Bhattacharya *et al.*, arXiv:1306.5435
  - Also several talks in Lattice 2013 e.g. S. Ohta, M. Lin, RBC-UKQCD

C.A., M. Constantinou, S. Dinter, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, arXiv:1303.5979

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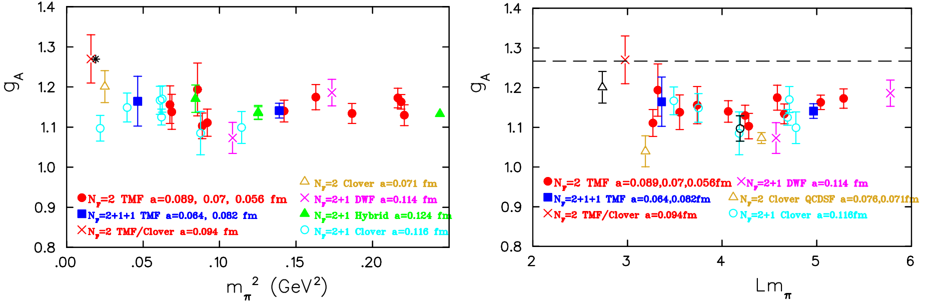

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- Volume effects may not be the full story if we compare the result by QCDSF ($Lm_\pi \sim 2.7$) and LHPC ($Lm_\pi \sim 4.2$) at near physical pion mass

C.A., M. Constantinou, S. Dinter, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, arXiv:1303.5979