Detecting the long distance structure of the $X(3872)$.

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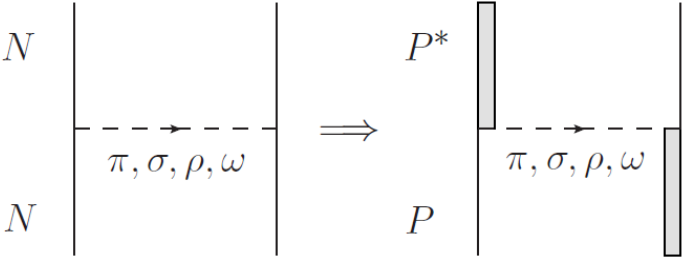
Outline

Outline

Introduction

- Molecular meson-antimeson systems have been predicted inspired by the existing similarities with the nucleon-nucleon system (Voloshin, Okun; 1976). So, as the deuteron is a nucleon-nucleon bound state, it would be natural to assume the existence of meson-antimeson bound states.



Introduction

- This supposition has been tested several times:

1. Jaffe in 1977: $f_0(980)$ y $a_0(980)$ as a $K\bar{K}$ bound state.
2. ...
3. Belle collaboration in 2003: $X(3872)$ as a $D\bar{D}^*$ resonance. This is a clear candidate to be a meson-antimeson molecule.

. . .

- Many new resonances X, Y, Z... have been detected which can also become candidates to be meson-antimeson molecules.

Outline

Symmetries on heavy meson molecules

Our approach for the study of heavy mesonic molecular systems will be based on their symmetries.

Symmetries based on the presence of heavy quarks.

- Heavy Quark Spin Symmetry (HQSS): the dynamics is invariant under separate spin rotations of the heavy quark and antiquark.
- Heavy Flavour Symmetry (HFS). Spectrum in the charm sector must be similar to the spectrum in the bottom sector.
- Heavy Antiquark-Diquark Symmetry (HADS). Heavy diquark behaves as a heavy antiquark

Symmetries based on the presence of light quarks.

- Pion exchange interactions are constrained by chiral symmetry.
- Heavy molecules also come in SU(3)-light flavour multiplets.

HQET

The presence of a heavy quark induces a series of important simplifications in the QCD Lagrangian, which are consequence of a new approximate symmetry: HQSFS.

$$\mathcal{L}_Q^{eff} = \bar{Q}_v \left(iv \cdot D \right) Q_v + \mathcal{O}\left(\frac{1}{m_Q} \right)$$

HQET and its symmetries

So, at lowest order:

- Lagrangian doesn't depend on the heavy quark flavour.
- Lagrangian doesn't depend on the heavy quark spin.

$\Rightarrow$ HQET has a spin-flavour $SU(2N_h)$ symmetry.

HQET eigenstates are "would-be" hadrons composed by a heavy quark with light quarks, light antiquarks and gluons, which, assuming SU(3) flavour symmetry, will be described into triplets, e.g. $D = (D^0, D^+, D_s)$.

Effective field theory for meson-antimeson molecules.

At Leading Order, the most general potential that respects HQSS takes the form:

$$
\begin{aligned}
V_4 \quad = \quad & +\frac{C_A}{4}\; Tr\left[\bar{H}^{(Q)} H^{(Q)} \gamma_\mu\right] Tr\left[H^{(\bar{Q})} \bar{H}^{(\bar{Q})} \gamma^\mu\right] + \\
& +\frac{C_A^\lambda}{4}\; Tr\left[\bar{H}_a^{(Q)} \lambda^i_{ab} H_b^{(Q)} \gamma_\mu\right] Tr\left[H_c^{(\bar{Q})} \lambda^i_{cd} \bar{H}_d^{(\bar{Q})} \gamma^\mu\right] + \\
& +\frac{C_B}{4}\; Tr\left[\bar{H}^{(Q)} H^{(Q)} \gamma_\mu \gamma_5\right] Tr\left[H^{(\bar{Q})} \bar{H}^{(\bar{Q})} \gamma^\mu \gamma_5\right] + \\
& +\frac{C_B^\lambda}{4}\; Tr\left[\bar{H}_a^{(Q)} \lambda^j_{ab} H_b^{(Q)} \gamma_\mu \gamma_5\right] Tr\left[H_c^{(\bar{Q})} \lambda^j_{cd} \bar{H}_d^{(\bar{Q})} \gamma^\mu \gamma_5\right]
\end{aligned}
$$

being λ Gell-Mann matrices.

At first order, this model only depends on four undetermined low energy constants (counter-terms)!!

From now on, the four independent counter-terms will be rewritten, through a linear combination without loss of generality, into C_{0A}, C_{0B}, C_{1A} and C_{1B}.

Determination of the counter-terms.

To determine the different counter-terms, the most promising experimental data which are likely to be meson-antimeson molecules are the $X(3872)$ and the $Z_b(10610)$, which, in our model, will correspond to an isoscalar $D\bar{D}^*(J^{PC} = 1^{++})$ and isovector $B\bar{B}^*(J^{PC} = 1^{+-})$ molecules, respectively.

The linear combination of the different counter-terms we can fix with these two assumptions are:

$$X(3872) \Rightarrow \begin{cases} C_{0X} = C_{0A} + C_{0B} & (I = 0) \\ C_{1X} = C_{1A} + C_{1B} & (I = 1) \end{cases}$$

$$Z_b(10610) \Rightarrow \{ C_{1Z} = C_{1A} - C_{1B} \qquad (I = 1)$$

“Isospin violation” in X(3872) decays

It has been difficult to understand the X(3872) decay widths in different isospin channels. In fact, if X(3872) had a well defined isospin, it would be hard to accomodate the following experimental ratio:

$$\frac{\mathcal{B}(X(3872) \to J/\psi\ \overbrace{\pi^+\pi^-\pi^0}^{\omega})}{\mathcal{B}(X(3872) \to J/\psi\ \underbrace{\pi^+\pi^-}_{\rho})} \simeq 0.8 \pm 0.3.$$

"Isospin violation" in X(3872) decays

To explain this ratio there are two different scenarios:

- X(3872) isospin is well defined but then isospin is not conserved in these strong decays.
- X(3872) isospin is not well defined and strong interactions conserve isospin.

We'll assume this latter scenario. Despite our potential conserves isospin, the kinetic term of the whole Lagrangian doesn't because of the mass difference between the charged components ($D^+ D^{-*}$) and the neutral component ($D^0 \bar{D}^{0*}$) which is around 8 MeV. As, the X(3872) is very close to $D^0 \bar{D}^{0*}$ threshold, this mass difference cannot be neglected and, then, the isospin operator doesn't commute with the whole Hamiltonian and, thus, it is not a good quantum number of the system.

more details in *PRD 87 076006*

Lippman-Schwinger Equation.

Once we have determined V, we find bound states by solving the Lippmann-Schwinger equation for each spin, isospin and charge-conjugation sector:

$$T = V + V\ G\ T$$

$$\langle \vec{p}\ |T|\vec{p}\ '\rangle = \langle \vec{p}\ |V|\vec{p}\ '\rangle + \int d^3\vec{k}\, \frac{\left\langle \vec{p}\ |V|\vec{k}\right\rangle \left\langle \vec{k}|T|\vec{p}\ '\right\rangle}{E - m_1 - m_2 - \frac{k^2}{2\mu}}$$

- Bound states of this model will appear as poles in the T matrix.
- Ultraviolet divergences are treated introducing a Gaussian regulator Λ:

$$\langle \vec{p}|V|\vec{p}\ '\rangle = V(\vec{p}, \vec{p}\ ') = v\ e^{-\vec{p}^2/\Lambda^2}\ e^{-\vec{p}'^2/\Lambda^2} \qquad \Rightarrow \qquad G = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{e^{-2\vec{k}^2/\Lambda^2}}{E - m_1 - m_2 - \frac{k^2}{2\mu}}$$

Hadronic Molecules

We can now predict a whole series of resonances which are symmetric partners of the $X(3872)$ and the $Z_b s$.

V_C	$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)	Masses ($\Lambda = 1$ GeV)	Measurements
C_{0X}	$0(1^{++})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D})$	3875.87	3871.68 (input)	3871.68 (input)	3871.68 ± 0.17 [33]
	$0(2^{++})$	$D^*\bar{D}^*$	4017.3	4012^{+4}_{-5}	4012^{+5}_{-12}	?
	$0(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	10604.4	10580^{+9}_{-8}	10539^{+25}_{-27}	?
	$0(2^{++})$	$B^*\bar{B}^*$	10650.2	10626^{+8}_{-9}	10584^{+25}_{-27}	?
	$0(2^{+})$	D^*B^*	7333.7	7322^{+6}_{-7}	7308^{+16}_{-20}	?
C_{0Z}	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	10604.4	10602.4 ± 2.0 (input)	10602.4 ± 2.0 (input)	10607.2 ± 2.0 [5]
						10597 ± 9 [34]
	$1(1^{+-})$	$B^*\bar{B}^*$	10650.2	10648.1 ± 2.1	$10648.1^{+2.1}_{-2.5}$	10652.2 ± 1.5 [5]
						10649 ± 12 [34]
	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* + D^*\bar{D})$	3875.87	3871^{+4}_{-12} (V)	3837^{+17}_{-35} (V)	$3899.0 \pm 3.6 \pm 4.9$ [24]
						$3894.5 \pm 6.6 \pm 4.5$ [25]
	$1(1^{+-})$	$D^*\bar{D}^*$	4017.3	4013^{+4}_{-11} (V)	3983^{+17}_{-32} (V)	?
	$1(1^{+})$	D^*B^*	7333.7	$7333.6^{\dagger}_{-4.2}$ (V)	7328^{+5}_{-14} (V)	?

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Heavy Antiquark-Diquark Symmetry

Apart from HQSS, there are extra symmetries which appear in the $m_Q \to \infty$ limit of QCD. One of these symmetries is, firstly discussed by Savage and Wise (*Phys.Lett. B248 (1990) 177-180*), the so called Heavy Antiquark-Diquark Symmetry (HADS).

This symmetry states that a heavy diquark behaves as a heavy antiquark up to corrections of the order $\mathcal{O}\left(\frac{1}{m_Q v}\right)$, being v the velocity of the heavy diquark system.

Thanks to this symmetry, our meson-antimeson analysis can also be useful to study heavy meson-doubly heavy baryon molecules.

Since our dynamics only depends on the light degrees of freedom, the potentials can be taken from the previous relations in meson-antimeson system.

Hadronic Molecules

The heavy meson-doubly heavy baryon symmetric partners of the $X(3872)$ and the $Z_b(10610)/Z_b'(10650)$.

State	$I(J^P)$	$V^{\rm LO}$	Thresholds	Mass ($\Lambda = 0.5$ GeV)	Mass ($\Lambda = 1$ GeV)
$\Xi_{cc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a}+C_{0b}$	5715	$(M_{\rm th}-10)^{+10}_{-15}$	$(M_{\rm th}-19)^{\dagger}_{-44}$
$\Xi_{cc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a}+C_{0b}$	9031	$(M_{\rm th}-21)^{+16}_{-19}$	$(M_{\rm th}-53)^{+45}_{-59}$
$\Xi_{bb}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a}+C_{0b}$	12160	$(M_{\rm th}-15)^{+9}_{-11}$	$(M_{\rm th}-35)^{+25}_{-31}$
$\Xi_{bb}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a}+C_{0b}$	15476	$(M_{\rm th}-29)^{+12}_{-13}$	$(M_{\rm th}-83)^{+38}_{-40}$
$\Xi_{bc}' D^*$	$0(\frac{3}{2}^-)$	$C_{0a}+C_{0b}$	8967	$(M_{\rm th}-14)^{+11}_{-13}$	$(M_{\rm th}-30)^{+27}_{-40}$
$\Xi_{bc}' \bar{B}^*$	$0(\frac{3}{2}^-)$	$C_{0a}+C_{0b}$	12283	$(M_{\rm th}-27)^{+15}_{-16}$	$(M_{\rm th}-74)^{+45}_{-51}$
$\Xi_{bc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a}+C_{0b}$	9005	$(M_{\rm th}-14)^{+11}_{-13}$	$(M_{\rm th}-30)^{+27}_{-40}$
$\Xi_{bc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a}+C_{0b}$	12321	$(M_{\rm th}-27)^{+15}_{-16}$	$(M_{\rm th}-74)^{+46}_{-51}$
$\Xi_{bb} \bar{B}$	$1(\frac{1}{2}^-)$	C_{1a}	15406	$(M_{\rm th}-0.3)^{\dagger}_{-2.5}$	$(M_{\rm th}-12)^{+11}_{-15}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a}+\frac{2}{3}C_{1b}$	15452	$(M_{\rm th}-0.9)[{\rm V}]^{\rm N/A}_{\dagger\dagger}$	$(M_{\rm th}-16)^{+14}_{-17}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a}-\frac{1}{3}C_{1b}$	15452	$(M_{\rm th}-1.2)^{\dagger}_{-2.9}$	$(M_{\rm th}-10)^{+9}_{-13}$
$\Xi_{bb}^* \bar{B}$	$1(\frac{3}{2}^-)$	C_{1a}	15430	$(M_{\rm th}-0.3)^{\dagger}_{-2.4}$	$(M_{\rm th}-12)^{+11}_{-13}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a}-\frac{5}{3}C_{1b}$	15476	$(M_{\rm th}-8)^{+8}_{-7}$	$(M_{\rm th}-5)^{\dagger}_{-8}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a}-\frac{2}{3}C_{1b}$	15476	$(M_{\rm th}-2.5)^{\dagger}_{-3.6}$	$(M_{\rm th}-9)^{+9}_{-11}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{5}{2}^-)$	$C_{1a}+C_{1b}$	15476	$(M_{\rm th}-4.3)[{\rm V}]^{\rm N/A}_{+3.3}$	$(M_{\rm th}-18)^{+17}_{-19}$

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Sources of error in the analysis.

In this analysis there are a few different sources of error:

- The masses of the $X(3872)$ and $Z_b(10610)$ resonances.
- The ratio of the $X(3872)$ decays done by Hanhart in *PRD* **85** *(2012) 011501*.
- The errors due to the two EFT expansions used in this work: HQSS (of the order $\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_Q}\right)$) and HADS (of the order $\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_Q\ v}\right)$).

Considering $\Lambda_{QCD} \simeq 300$ MeV, $m_c \simeq 1.3$ GeV and $m_b = 4.2$ GeV, then we obtain:

$$\Delta_{HQSS} \simeq \begin{cases} 20\% \ \textit{in the charm sector} \\ 7\% \ \textit{in the bottom sector} \end{cases}$$

$$\Delta_{HADS} \simeq \begin{cases} 40\% \ \textit{in the charm sector} \\ 30\% \ \textit{in the charm} - \textit{bottom sector} \\ 20\% \ \textit{in the bottom sector} \end{cases}$$

Outline

Conclusions I

- Light flavour symmetry and HQSS in heavy meson-antimeson systems, combined with the identification of some resonances, has allowed us to obtain a whole family -including the $Z_c(3900)$- of hidden charm and bottom molecules.
- In this work we have also studied the implications of HADS (plus HQSS and HFS) for heavy hadronic molecules.
- As a consequence of these symmetries, we may expect the existence of heavy meson-doubly heavy baryon partners of heavy meson-heavy antimeson molecules. From the $X(3872)$ and $Z_b(10610)$ resonances we can predict these exotic pentaquark-like states. Unfortunately, the decays of these states are so suppressed that they will be difficult to be found experimentally.
- Eventually, this analysis can also be extended to doubly heavy baryon-doubly heavy antibaryon molecules $\Xi_{QQ}\bar{\Xi}_{\bar{Q}\bar{Q}}$.

Conclusions I

- Uncertainties quoted in tables only account for the approximate nature of HQSS and those induced by the errors of R_X.
- Pion exchanges and coupled channels should be considered. However, according to previous bibliography (*PRD86(2012)056004*), these effects have a smaller contribution than expected and smaller than those expected from HQSS breaking terms.

Outline

Short-distance decays of the $X(3872)$.

In these analysis, only short-distance decays (described as the Feynman diagram in the figure) have been taken into account:

- $X(3872) \to J/\psi\rho \to J/\psi \;\; 2\pi$
- $X(3872) \to J/\psi\omega \to J/\psi \;\; 3\pi$

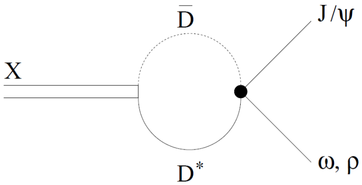

Figure: X(3872) decay mechanism in this model.

so important information might be missed.

Long-distance decays of the $X(3872)$.

For this reason, we will study the long-distance decay $X(3872) \to D^0 \bar{D}^0 \pi^0$.

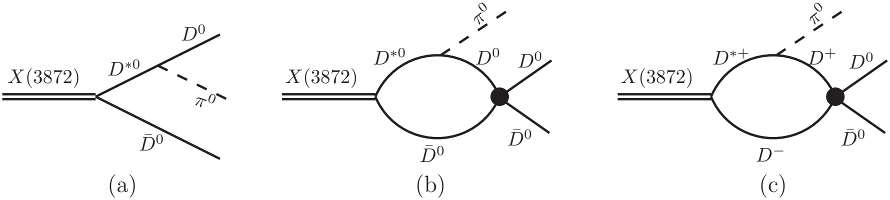

Figure: X(3872) long-distance decay mechanism in this model.

- The tree level amplitude depends on $C_{0A} + C_{0B}$ and $C_{1A} + C_{1B}$ (already determined from the experimental fit to the resonances.
- The NLO diagrams depends on the four LECs: C_{0A} (still undetermined), C_{0B}, C_{1A} and C_{1B}.

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Short-distance decays of the $X(3872)$.

The result for the tree level amplitude is

$$\Gamma\left[X(3872) \to D^0 \bar{D}^0 \pi^0\right]_{tree} = 44.0^{+2.4}_{-7.2} \quad \left(42.0^{+3.6}_{-7.3}\right) \text{ keV for } \Lambda = 500 \quad (1000) \text{ MeV}$$

Including the NLO diagrams the result, as a function of the C_{0A} LEC is:

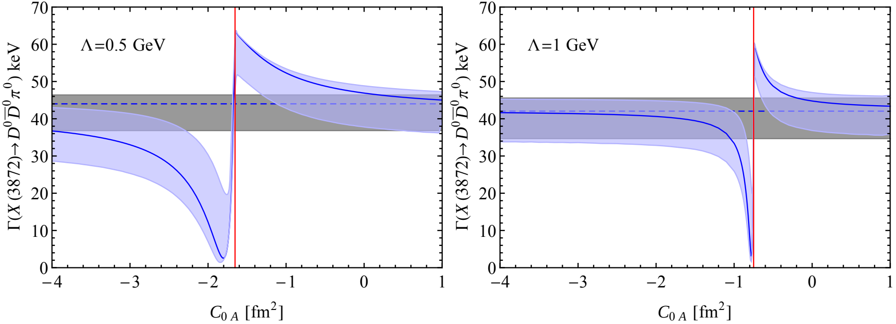

Figure: $X(3872) \to D^0 \bar{D}^0 \pi^0$ decay widths.

Outline

Conclusions II

- We studied the $X(3872) \to D^0 \bar{D}^0 \pi^0$ decay using an EFT based on the hadronic molecule assumption for the $X(3872)$.
- This decay is unique in the sense that it is sensitive to the long-distance structure of the $X(3872)$ as well as the strength of the S-wave interaction between the D and $\bar{D}$.
- If there was a near threshold pole in the $D\bar{D}$ system, the partial decay width can be very different from the result neglecting the FSI effects. This decay may be used to measure the so far unknown counter-term C_{0A}.
- A future measurement of the $\frac{\partial \Gamma}{\partial |\vec{p}_D|}$ distribution might provide valuable information on the $X(3872)$ wave function at the fixed momentum $\Psi(\vec{p}_D)$.

THANK YOU FOR YOUR ATTENTION

Wave functions for meson-antimeson systems.

Meson-antimeson wave functions depend on the spin of the heavy (light) quark pair $\mathcal{S} = 0, 1$ ($\mathcal{L} = 0, 1$), the total spin $J = \mathcal{L} \oplus \mathcal{S}$, the isospin $I = 0, 1$, the strangeness... An adequate basis is of the form: $|\mathcal{S}, \mathcal{L}, J, \alpha\rangle$, as shown by Voloshin in *PRD.84.031502*. Thus, for example:

$$\begin{cases} \left|B\bar{B}(0^{++}), I\right\rangle = \frac{1}{2}\left|\mathcal{S} = 0, \mathcal{L} = 0, J = 0, I\right\rangle + \frac{\sqrt{3}}{2}\left|S = 1, \mathcal{L} = 1, J = 0, I\right\rangle \\ \left|B^*\bar{B}^*(2^{++}), I\right\rangle = \left|\mathcal{S} = 1, \mathcal{L} = 1, J = 2, I\right\rangle \end{cases}$$

In the HQSS, $\mathrm{H_{QCD}}$ is diagonal and taking into account that the dynamics only depend on the isospin and spin of the light degrees of freedom, we find:

$$\left.\begin{array}{l} \left\langle B\bar{B}(0^{++}), I\right| V \left|B\bar{B}(0^{++}), I\right\rangle = \frac{1}{4}V_{\mathcal{L}=0} + \frac{3}{4}V_{\mathcal{L}=1} = C_{IA} \\ \left\langle B^*\bar{B}^*(2^{++}), I\right| V \left|B^*\bar{B}^*(2^{++}), I\right\rangle = V_{\mathcal{L}=1} = C_{IA} + C_{IB} \end{array}\right\}$$

$$\Rightarrow \begin{cases} V_{\mathcal{L}=0} = C_{IA} - 3C_{IB} \\ V_{\mathcal{L}=1} = C_{IA} + C_{IB} \end{cases}$$

Wave functions for triply heavy meson-baryon molecules.

Analogously to the meson-antimeson system, meson-baryon wave functions depend on the spin of the heavy (light) quark pair $\mathcal{S}$ ($\mathcal{L} = 0, 1$), the total spin $J = \mathcal{L} \oplus \mathcal{S}$, the isospin $I = 0, 1$, the strangeness... The only difference is that, now, $S = \frac{1}{2}, \frac{3}{2}$.

If the two heavy quarks in the baryons are the same, they can only couple into $\mathcal{S}_{baryon} = 1$ (Pauli's exclusion principle), and, therefore, only seven different channels are possible (being $P, P^* = D, D^*$ or $\bar{B}, \bar{B}^*$):

$$
\begin{cases}
\left|P\Xi_{QQ}\left(\frac{1}{2}^-\right); I\right\rangle = \frac{1}{2}\left|\mathcal{S} = \frac{1}{2}, \mathcal{L} = 0, J = \frac{1}{2}, I\right\rangle + \frac{1}{2\sqrt{3}}\left|\mathcal{S} = \frac{1}{2}\mathcal{L}L = 1, J = \frac{1}{2}, I\right\rangle \\
\qquad + \sqrt{\frac{2}{3}}\left|\mathcal{S} = \frac{3}{2}, \mathcal{L} = 1, J = \frac{1}{2}, I\right\rangle \\
\left|P^*\Xi_{QQ}\left(\frac{1}{2}^-\right); I\right\rangle = \frac{1}{2\sqrt{3}}\left|\mathcal{S} = \frac{1}{2}, L = 0, J = \frac{1}{2}, I\right\rangle + \frac{5}{6}\left|\mathcal{S} = \frac{1}{2}, \mathcal{L} = 1, J = \frac{1}{2}, I\right\rangle \\
\qquad - \frac{\sqrt{2}}{3}\left|\mathcal{S} = \frac{3}{2}, \mathcal{L} = 1, J = \frac{1}{2}, I\right\rangle \\
\left|P^*\Xi^*_{QQ}\left(\frac{1}{2}^-\right); I\right\rangle = \sqrt{\frac{2}{3}}\left|\mathcal{S} = \frac{1}{2}, \mathcal{L} = 0, J = \frac{1}{2}, I\right\rangle - \frac{\sqrt{2}}{3}\left|\mathcal{S} = \frac{1}{2}, \mathcal{L} = 1, J = \frac{1}{2}, I\right\rangle \\
\qquad - \frac{1}{3}\left|\mathcal{S} = \frac{3}{2}, \mathcal{L} = 1, J = \frac{1}{2}, I\right\rangle
\end{cases}
$$

Wave functions and potential for meson-baryon systems.

$$\begin{cases} \left|P\Xi^*_{QQ}\left(\tfrac{3}{2}^-\right);I\right\rangle = \frac{1}{\sqrt{3}}\left|\mathcal{S}=\tfrac{1}{2},\mathcal{L}=1,J=\tfrac{3}{2},I\right\rangle - \frac{1}{2}\left|\mathcal{S}=\tfrac{3}{2},\mathcal{L}=0,J=\tfrac{3}{2},I\right\rangle \\ \qquad +\sqrt{\frac{5}{12}}\left|\mathcal{S}=\tfrac{3}{2},\mathcal{L}=1,J=\tfrac{3}{2},I\right\rangle \\ \left|P^*\Xi_{QQ}\left(\tfrac{3}{2}^-\right);I\right\rangle = \frac{-1}{3}\left|\mathcal{S}=\tfrac{1}{2},\mathcal{L}=1,J=\tfrac{3}{2},I\right\rangle + \frac{1}{\sqrt{3}}\left|\mathcal{S}=\tfrac{3}{2},\mathcal{L}=0,J=\tfrac{3}{2},I\right\rangle \\ \qquad +\frac{\sqrt{5}}{3}\left|\mathcal{S}=\tfrac{3}{2},\mathcal{L}=1,J=\tfrac{3}{2},I\right\rangle \\ \left|P^*\Xi^*_{QQ}\left(\tfrac{3}{2}^-\right);I\right\rangle = \frac{\sqrt{5}}{3}\left|\mathcal{S}=\tfrac{1}{2},\mathcal{L}=1,J=\tfrac{3}{2},I\right\rangle + \sqrt{\frac{5}{2}}\left|\mathcal{S}=\tfrac{3}{2},\mathcal{L}=0,J=\tfrac{3}{2},I\right\rangle \\ \qquad -\frac{1}{6}\left|\mathcal{S}=\tfrac{3}{2},\mathcal{L}=1,J=\tfrac{3}{2},I\right\rangle \\ \left|P^*\Xi^*_{QQ}\left(\tfrac{5}{2}^-\right);I\right\rangle = \left|\mathcal{S}=\tfrac{3}{2},\mathcal{L}=1,J=\tfrac{5}{2},I\right\rangle \end{cases}$$

Wave functions and potential for meson-baryon systems.

Since our dynamics only depends on the light degrees of freedom, the potentials can be taken from the previous relations in meson-antimeson system, that is:

$$\begin{cases} V_{\mathcal{L}=0} = C_{IA} - 3C_{IB} \\ V_{\mathcal{L}=1} = C_{IA} + C_{IB} \end{cases}$$

$$\Rightarrow \begin{cases} \left\langle P\Xi_{QQ}\left(\frac{1}{2}^-\right);I\right| V \left|P\Xi_{QQ}\left(\frac{1}{2}^-\right);I\right\rangle = C_{IA} \\ \left\langle P^*\Xi_{QQ}\left(\frac{1}{2}^-\right);I\right| V \left|P^*\Xi_{QQ}\left(\frac{1}{2}^-\right);I\right\rangle = C_{IA} + \frac{2}{3}C_{IB} \\ \left\langle P\Xi^*_{QQ}\left(\frac{1}{2}^-\right);I\right| V \left|P\Xi^*_{QQ}\left(\frac{1}{2}^-\right);I\right\rangle = C_{IA} - \frac{5}{3}C_{IB} \\ \left\langle P\Xi^*_{QQ}\left(\frac{3}{2}^-\right);I\right| V \left|P\Xi^*_{QQ}\left(\frac{3}{2}^-\right);I\right\rangle = C_{IA} \\ \left\langle P^*\Xi_{QQ}\left(\frac{3}{2}^-\right);I\right| V \left|P^*\Xi_{QQ}\left(\frac{3}{2}^-\right);I\right\rangle = C_{IA} - \frac{1}{3}C_{IB} \\ \left\langle P^*\Xi^*_{QQ}\left(\frac{3}{2}^-\right);I\right| V \left|P^*\Xi^*_{QQ}\left(\frac{3}{2}^-\right);I\right\rangle = C_{IA} - \frac{2}{3}C_{IB} \\ \left\langle P^*\Xi^*_{QQ}\left(\frac{5}{2}^-\right);I\right| V \left|P^*\Xi^*_{QQ}\left(\frac{5}{2}^-\right);I\right\rangle = C_{IA} + C_{IB} \end{cases}$$

Wave functions and potential for meson-baryon systems.

If, however, the two heavy quarks within the baryon are different, there are three extra combinations since now $S_{baryon} = 0$ is allowed:

$$\begin{cases} \left|P\Xi'_{QQ}\left(\frac{1}{2}^-\right);I\right\rangle = -\frac{1}{2}\left|\mathcal{S}=\frac{1}{2},\mathcal{L}=0,J=\frac{1}{2},I\right\rangle + \frac{\sqrt{3}}{2}\left|\mathcal{S}=\frac{1}{2},\mathcal{L}=1,J=\frac{1}{2},I\right\rangle \\ \left|P^*\Xi'_{QQ}\left(\frac{1}{2}^-\right);I\right\rangle = \frac{\sqrt{3}}{2}\left|\mathcal{S}=\frac{1}{2},\mathcal{L}=0,J=\frac{1}{2},I\right\rangle + \frac{1}{2}\left|\mathcal{S}=\frac{1}{2},\mathcal{L}=1,J=\frac{1}{2},I\right\rangle \\ \left|P^*\Xi'_{QQ}\left(\frac{3}{2}^-\right);I\right\rangle = \left|\mathcal{S}=\frac{1}{2},\mathcal{L}=1,J=\frac{1}{2},I\right\rangle \end{cases}$$

And their corresponding potentials:

$$\begin{cases} V_{\left|P\Xi'_{QQ}\left(\frac{1}{2}^-\right);I\right\rangle} = C_{IA} \\ V_{\left|P^*\Xi'_{QQ}\left(\frac{1}{2}^-\right);I\right\rangle} = C_{IA} - 2C_{IB} \\ V_{\left|P^*\Xi'_{QQ}\left(\frac{3}{2}^-\right);I\right\rangle} = C_{IA} + C_{IB} \end{cases}$$

“Isospin violation” in X(3872) decays

A more detailed analysis of X(3872) decays (Hanhart et al. 2012) gives the ratio between the amplitudes of the decays taking into account the different widths of intermediate vector bosons ρ and ω:

$$R_X = \frac{\mathcal{M}(X \to J/\psi\ \rho)}{\mathcal{M}(X \to J/\psi\ \omega)} = 0.26^{+0.08}_{-0.05}$$

"Isospin violation" in X(3872) decays

And, in our model (depicted in the figure), the same ratio is given by:

$$R_X = \frac{\mathcal{M}(X \to J/\psi\ \rho)}{\mathcal{M}(X \to J/\psi\ \omega)} = \frac{g_\rho}{g_\omega}\left(\frac{\hat{\psi}_1 - \hat{\psi}_2}{\hat{\psi}_1 + \hat{\psi}_2}\right)$$

being

$$g_\omega = \mathcal{M}_\omega(D\bar{D}^*(I=0) \to J/\psi\ \omega) \qquad g_\rho = \mathcal{M}_\rho(D\bar{D}^*(I=1) \to J/\psi\ \rho)$$

and $\hat{\psi}_1$ y $\hat{\psi}_2$ an average of the neutral and charged $D^*\bar{D}$ wave functions in the vicinities of the origin.

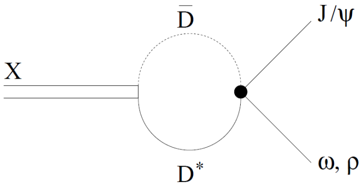

Figure: X(3872) decay mechanism in this model.

"Isospin violation" in X(3872) decays

Because of the light flavour SU(3) symmetry of QCD Hamiltonian, it is satisfied:

$$\Rightarrow g_\omega - g_\rho = \sqrt{2} g_\phi$$

And, using OZI rule ($s\bar{s}$ pair creation is suppressed) then: $g_\phi \to 0$

$$\Rightarrow g_\omega = g_\rho$$

$$\Rightarrow R_X = \frac{\mathcal{M}(X \to J/\psi\ \rho)}{\mathcal{M}(X \to J/\psi\ \omega)} = \left(\frac{\hat{\psi}_1 - \hat{\psi}_2}{\hat{\psi}_1 + \hat{\psi}_2} \right) = 0.26^{+0.08}_{-0.05}$$

"Isospin violation" in X(3872) decays

If we now assume a vanishing $D\bar{D}^*$ interaction in the isospin $I = 1$ sector, it is found that the ratio R_X only depends on the masses of the resonance and the thresholds of the different channels (Gammermann et al.,2010) and takes the value:

$$R_X \simeq 0.13$$

We improve on this, and consider a non-vanishing $I = 1$ interaction that we fit to the value for R_X reported by Hanhart et al. Hence, we'll work with two coupled channels (neutral and charged channels) contact potential:

$$V_{coupled} = \frac{1}{2} \begin{pmatrix} V_0 + V_1 & V_0 - V_1 \\ V_0 - V_1 & V_0 + V_1 \end{pmatrix}$$

being V_0 and V_1, the well defined isospin potentials $I = 0$ and $I = 1$, respectively.

Therefore, the experimental ratio of the ρ and ω decay widths of the X(3872) provides further constraints to the counter-terms.