On parton number fluctuations

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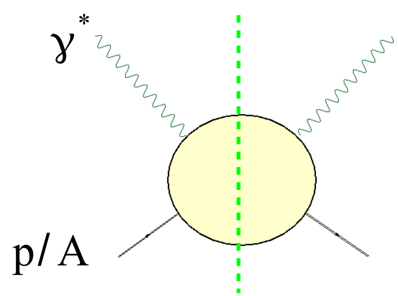
cnrs
dépasser les frontières

QCD at high density

- *Theoretically rich:* ***nonlinear physics, nontrivial fluctuations***
- *Phenomenologically: a lot of data to interpret*

"Goldmine for modern physics"
E. Scapparone, plenary talk

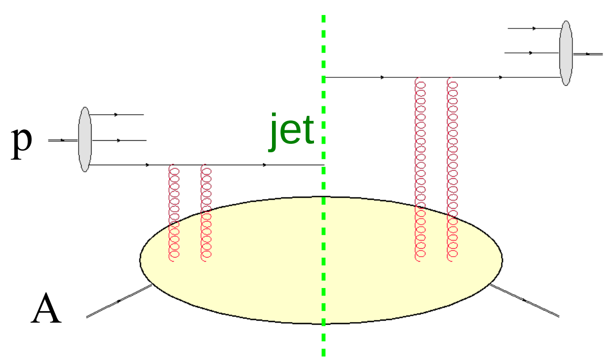
Deep-inelastic scattering



$\gamma^* p$ — *inclusive cross section*

$\gamma^* A$ — *diffractive cross section*

Hadron/nucleus scattering



pA — p_T *broadening*

dijet correlations

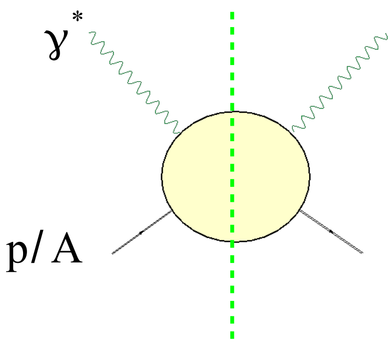
...

Universal object for this physics: ***dipole forward elastic amplitude***

Dipole amplitude

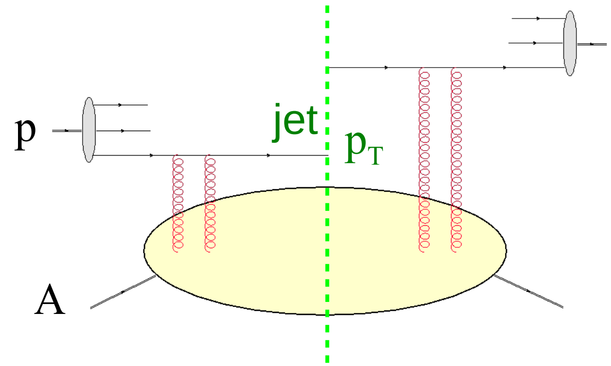
Deep-inelastic scattering

HERA+future ep,eA



Broadening

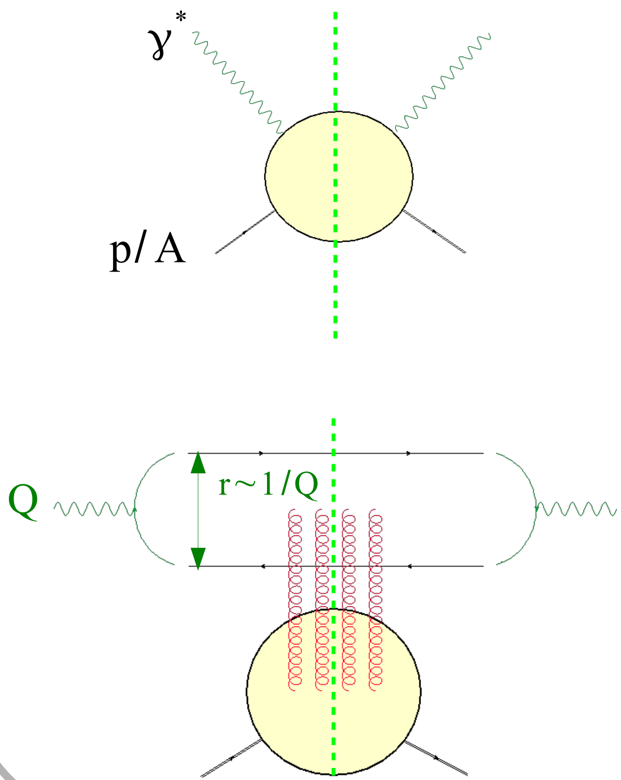
LHC



Dipole amplitude

Deep-inelastic scattering

HERA+future ep,eA



Broadening

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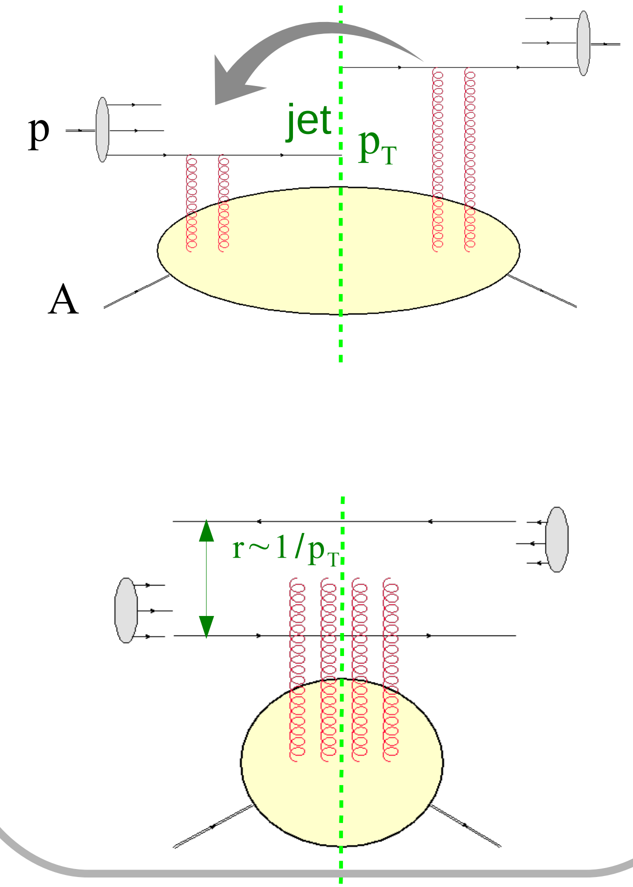
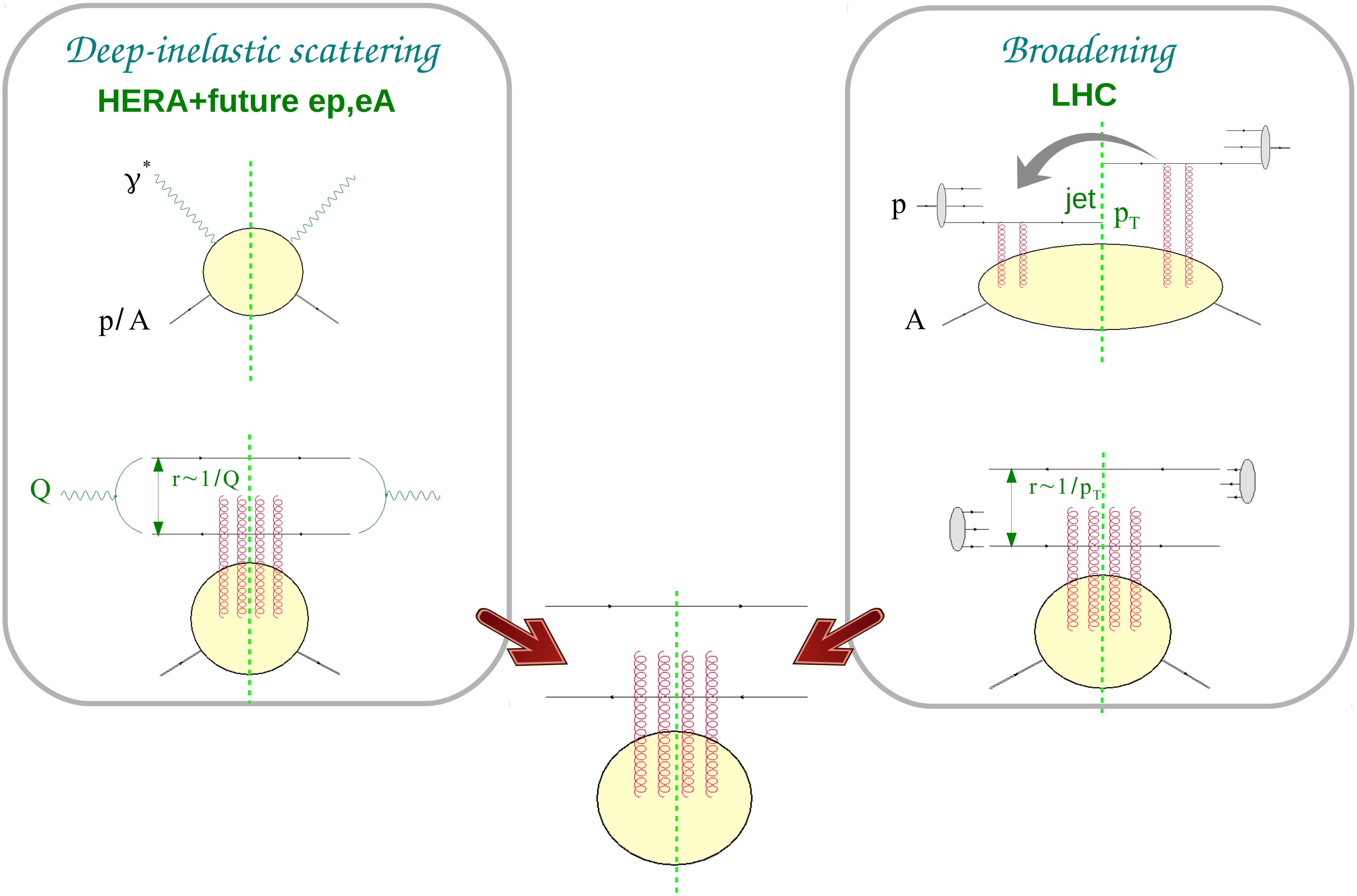


Dipole amplitude



Outline

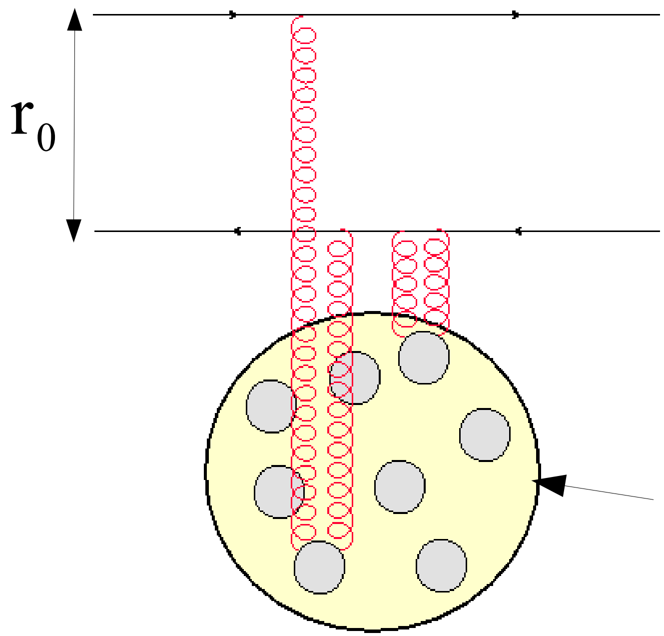
- Dipole-nucleus scattering

  The Balitsky-Kovchegov equation

  "How does it work?" Interpretation in terms of gluon number fluctuations

- Dipole-dipole scattering

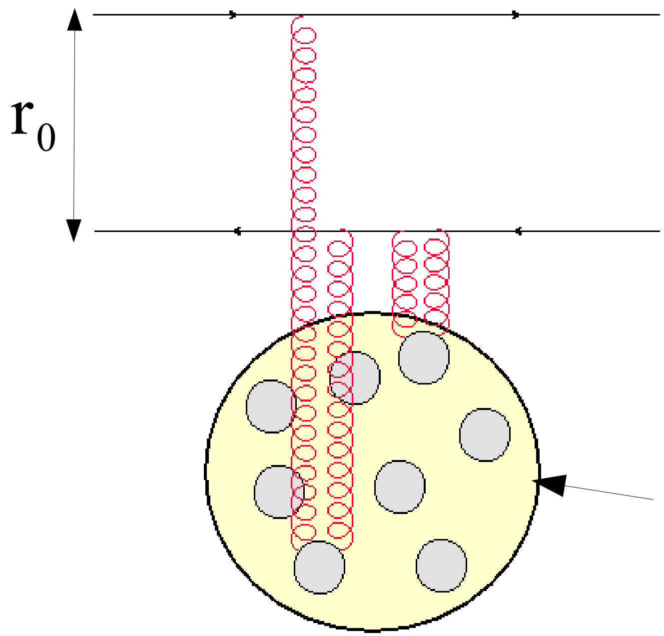
Dipole-nucleus scattering



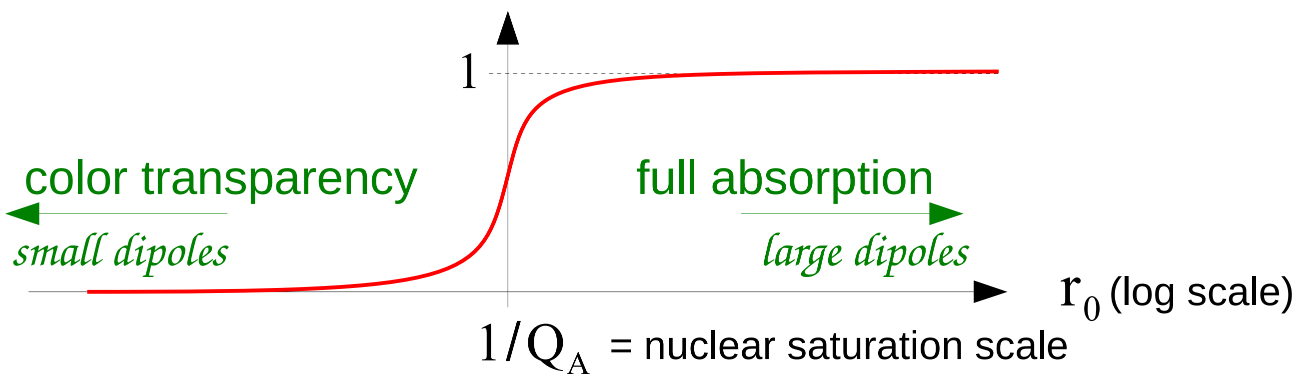
(low energy)

Nucleus=set of (many) independent nucleons

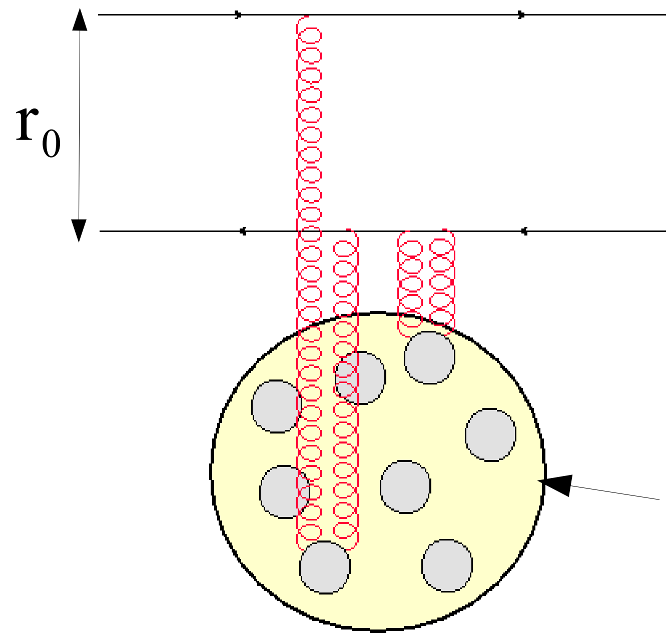
Dipole-nucleus scattering



Dipole amplitude: $$T(r_0) = 1 - e^{-\frac{r_0^2 Q_A^2}{4}}$$ *(McLerran-Venugopalan model)*



Dipole-nucleus scattering

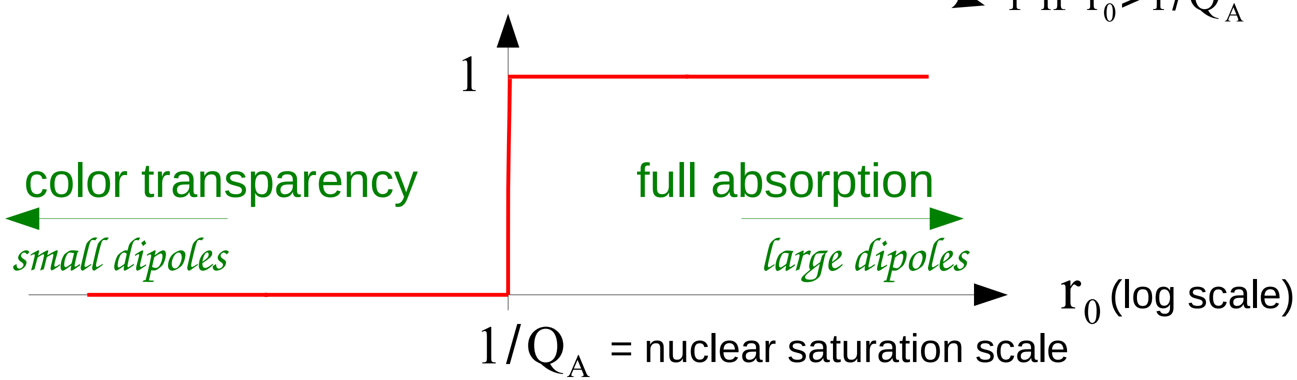


(low energy)

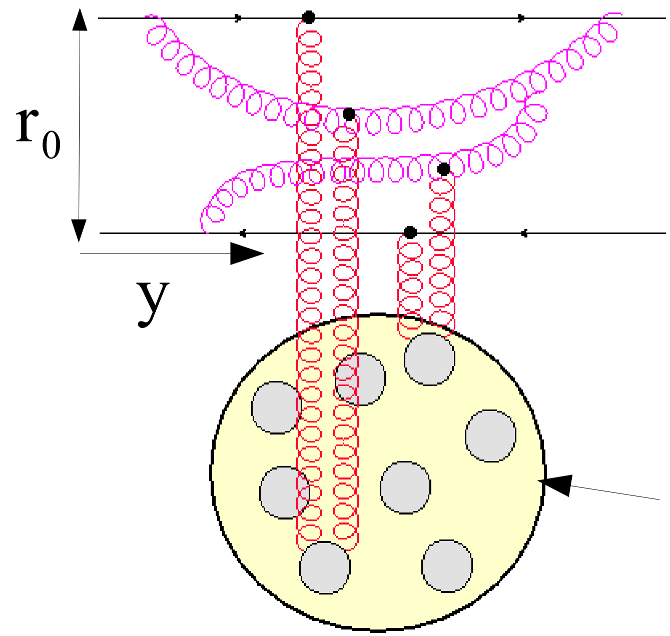
Nucleus=set of (many) independent nucleons

Dipole amplitude:

$$T(r_0) \simeq \Theta(\ln r_0^2 Q_A^2) = \begin{cases} 0 \text{ if } r_0 < 1/Q_A \\ 1 \text{ if } r_0 > 1/Q_A \end{cases}$$



Dipole-nucleus scattering

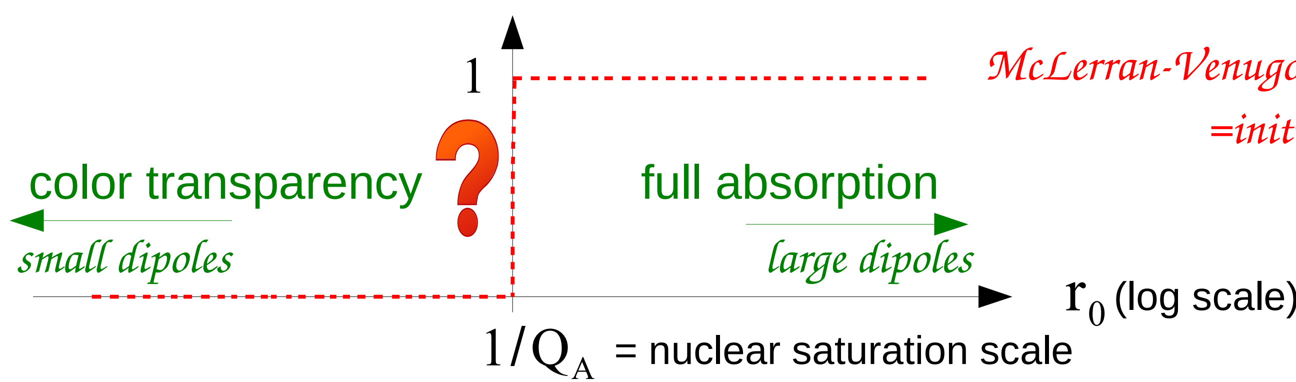


(higher energy: the dipole has ***rapidity y****)*

Nucleus=set of (many) independent nucleons

Dipole amplitude:

$T(r_0, y)$ solves the Balitsky-Kovchegov equation



McLerran-Venugopalan model =initial condition

Reminder: the BK equation and its solution

$$\partial_y T(r_0, y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \left[T(r_1, y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y) T(r_0 - r_1, y) \right]$$

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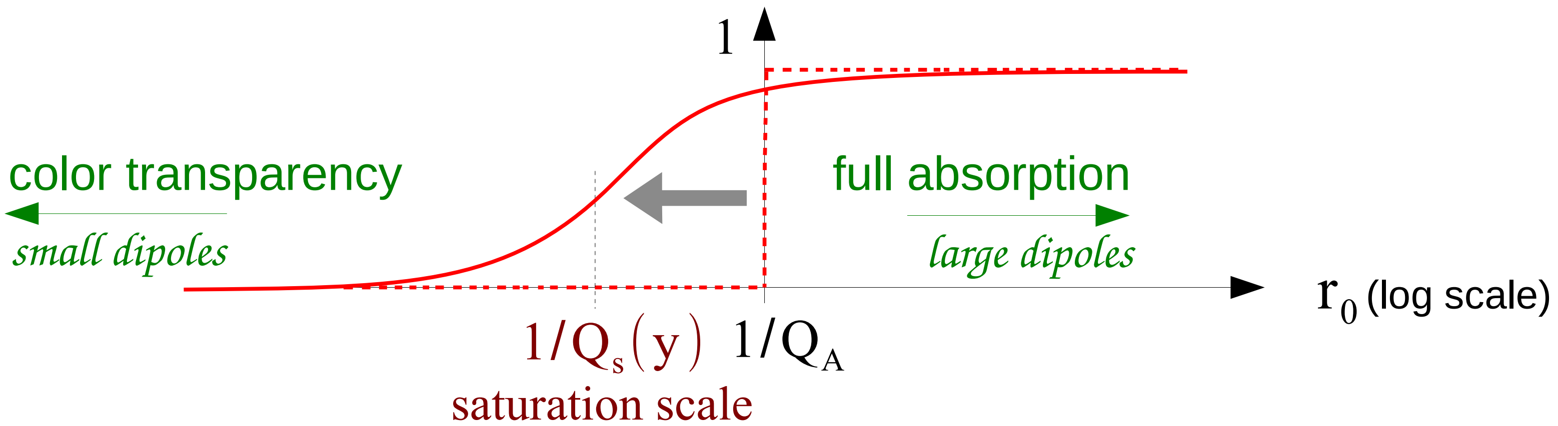

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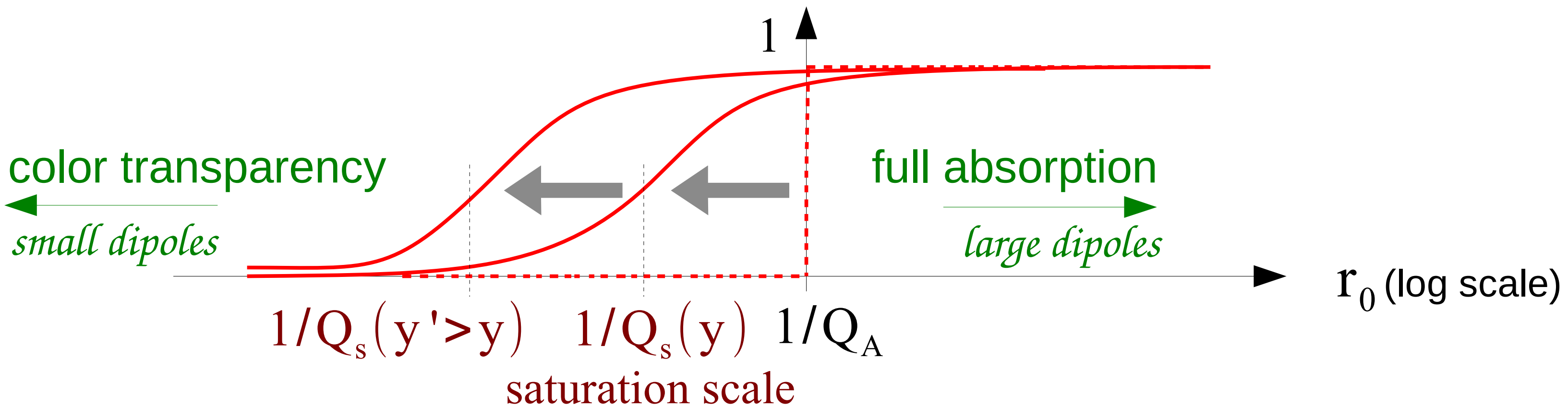

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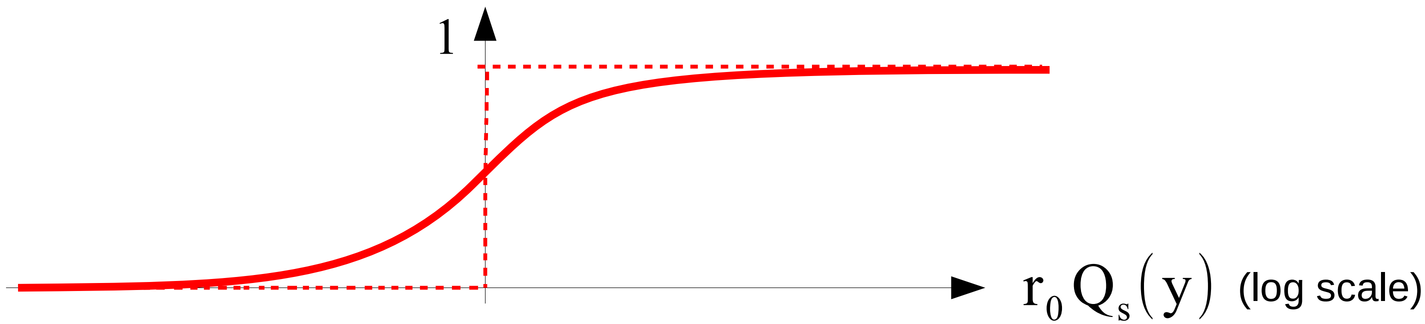

$$T(r_0,y)\underset{y\gg1}{\simeq}\text{function of }(r_0 Q_s(y))$$

Traveling wave property = geometric scaling

Reminder: the BK equation and its solution

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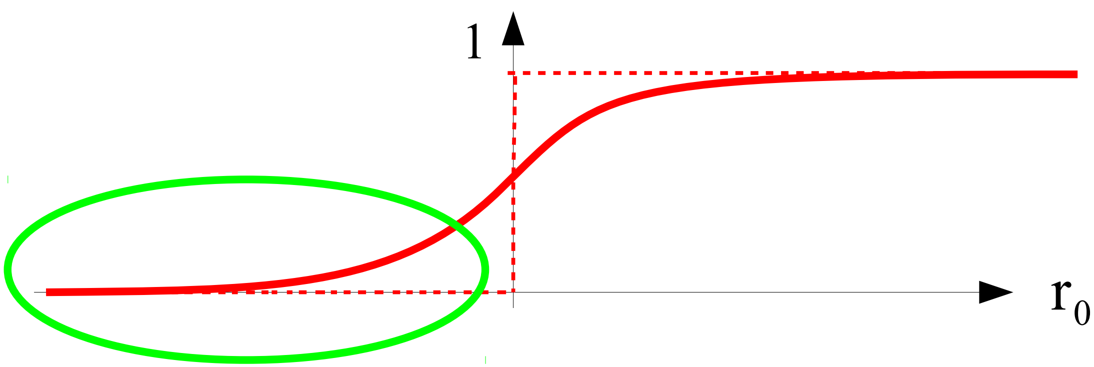
Linear part: BFKL equation

eigenfunctions $T(r_0)=e^{\gamma \ln r_0^2}$

eigenvalues $\frac{\alpha_s N_c}{\pi}\chi(\gamma)$, $\chi(\gamma)\equiv 2\psi(1)-\psi(\gamma)-\psi(1-\gamma)$



$$T(r_0,y)\underset{y\gg 1}{\simeq}\text{function of }(r_0 Q_s(y))$$

Traveling wave property = geometric scaling

$$T(r_0,y)\underset{r_0 Q_s(y)\ll 1}{\sim}\ln\frac{1}{r_0^2 Q_s^2(y)}\,e^{\gamma_0 \ln[r_0^2 Q_s^2(y)]}$$

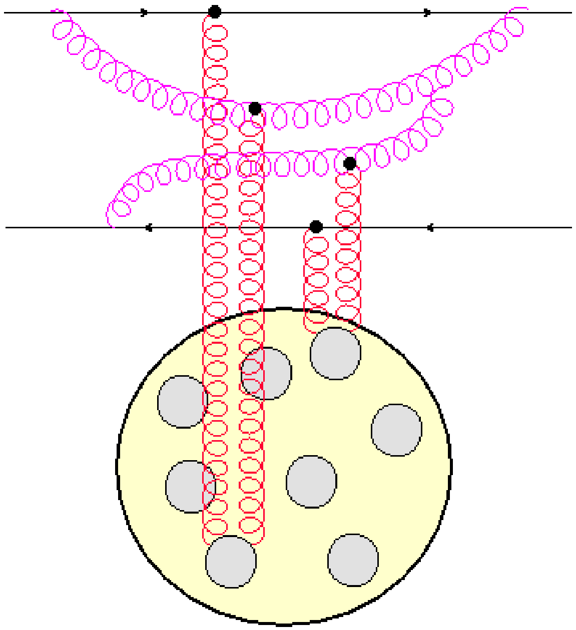
$$Q_s^2(y)\simeq Q_A^2\, e^{\frac{\alpha_s N_c}{\pi}\chi'(\gamma_0) y}$$

γ_0 solves $\gamma_0\chi'(\gamma_0)=\chi(\gamma_0)$

The BK equation: how does it work?

$$\partial_y T(r_0,y)=\frac{\alpha_s N_c}{\pi}\int \frac{d^2 r_1}{2\pi}\frac{r_0^2}{r_1^2(r_0-r_1)^2}\left[T(r_1,y)+T(r_0-r_1,y)-T(r_0,y)-T(r_1,y)T(r_0-r_1,y)\right]$$

Quantum evolution of the dipole by gluon radiation



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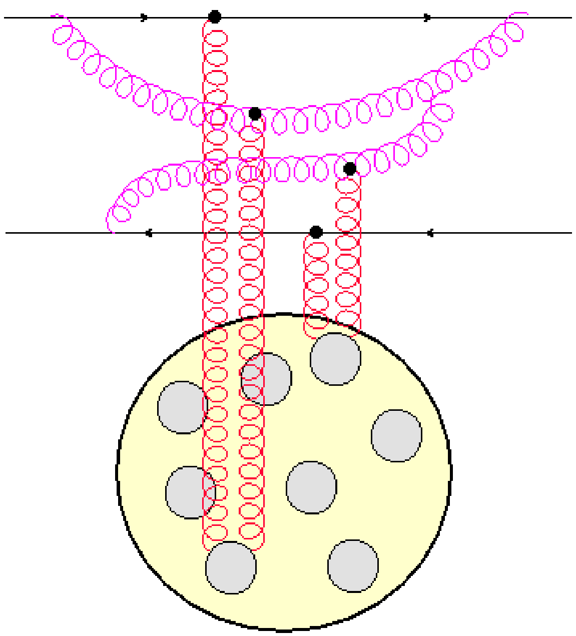
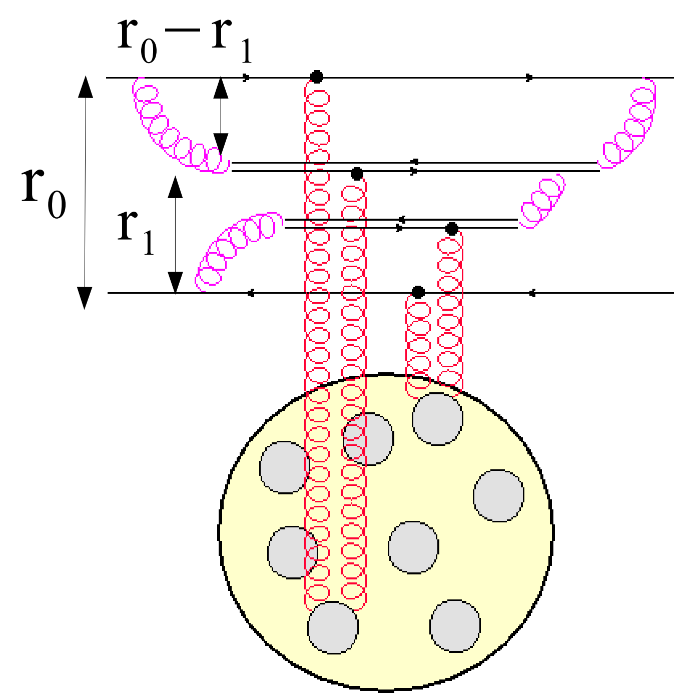


large number-of-color limit

Successive splittings of dipoles



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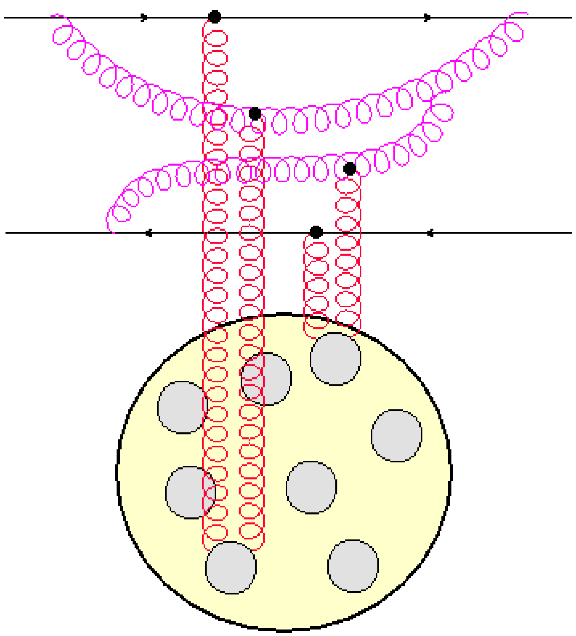

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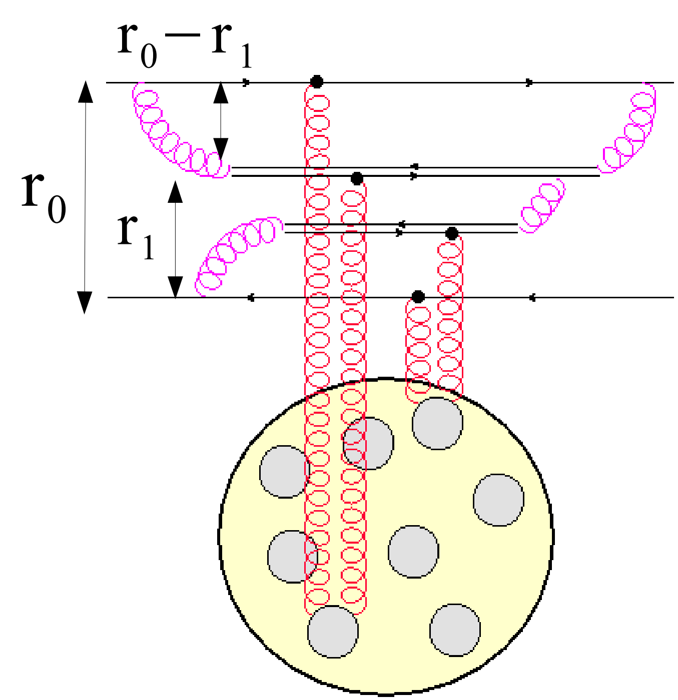

QCD dipole model: *dipole splitting rate =*

$$\frac{dp}{dy}=\frac{\alpha_s N_c}{\pi}\frac{r_0^2}{r_1^2(r_0-r_1)^2}\frac{d^2 r_1}{2\pi}$$

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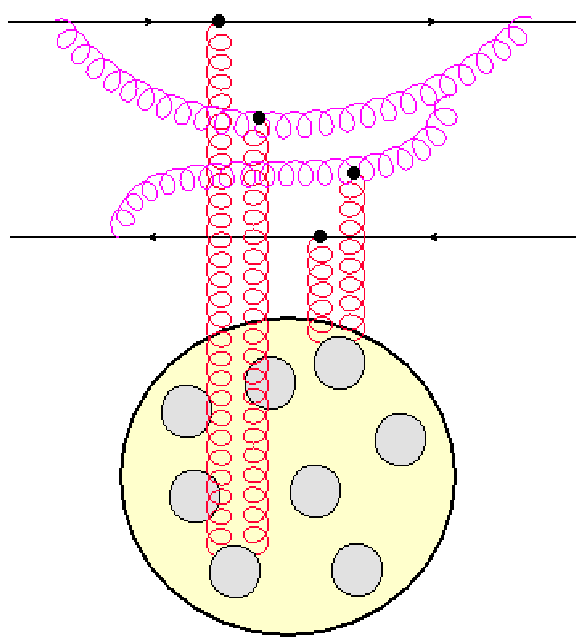

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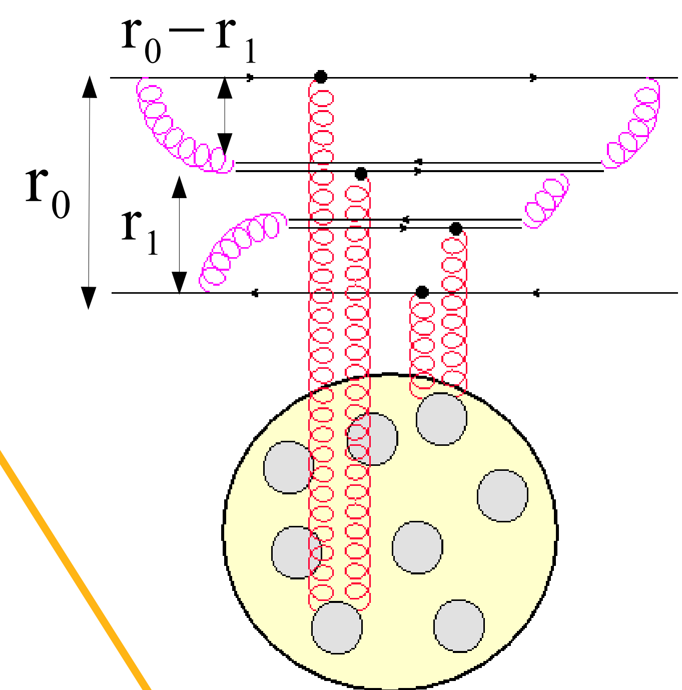

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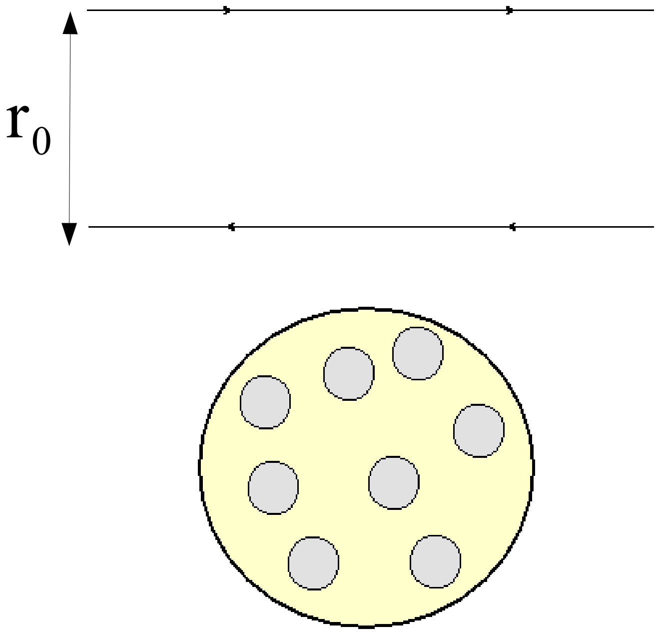

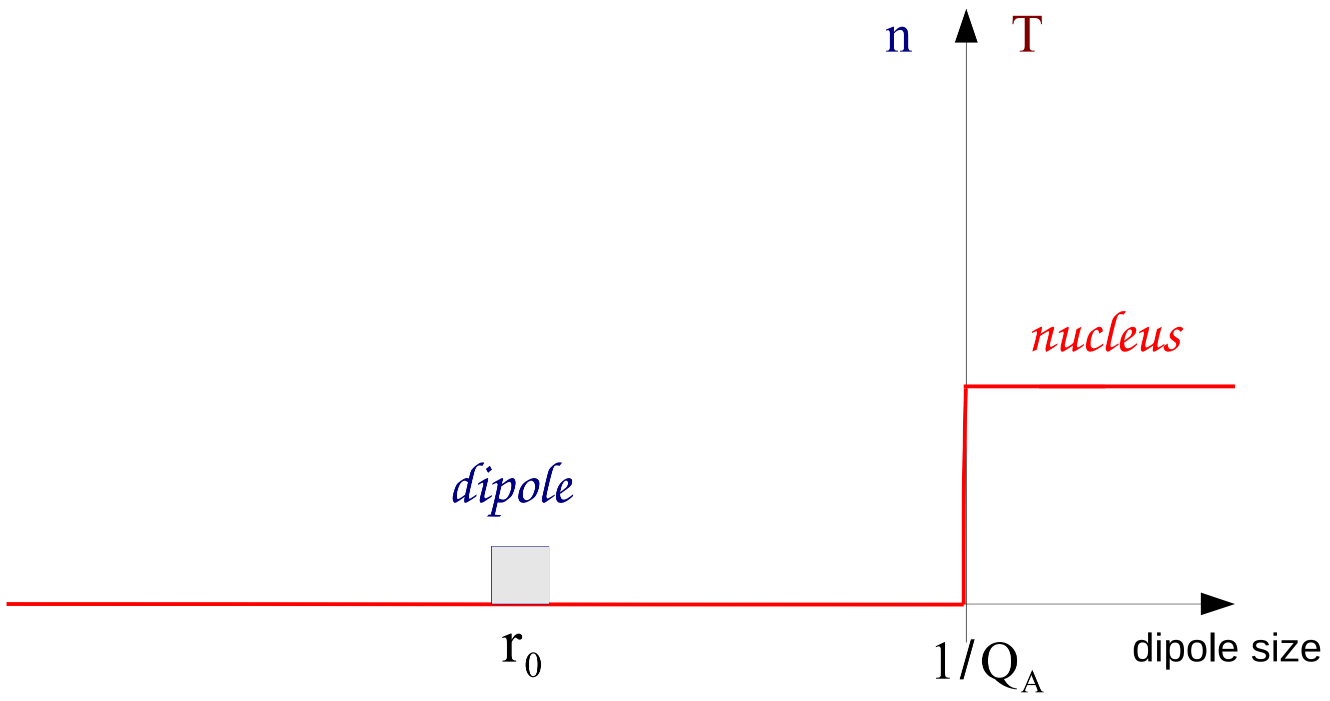

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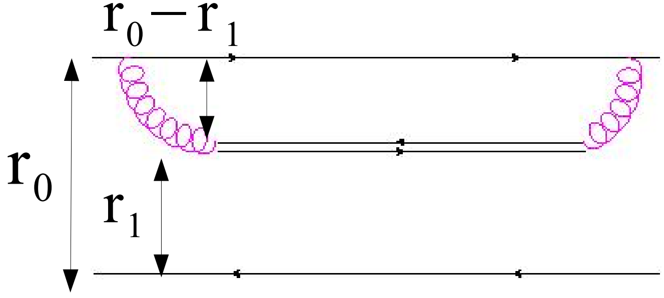

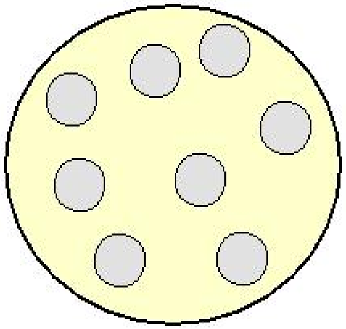

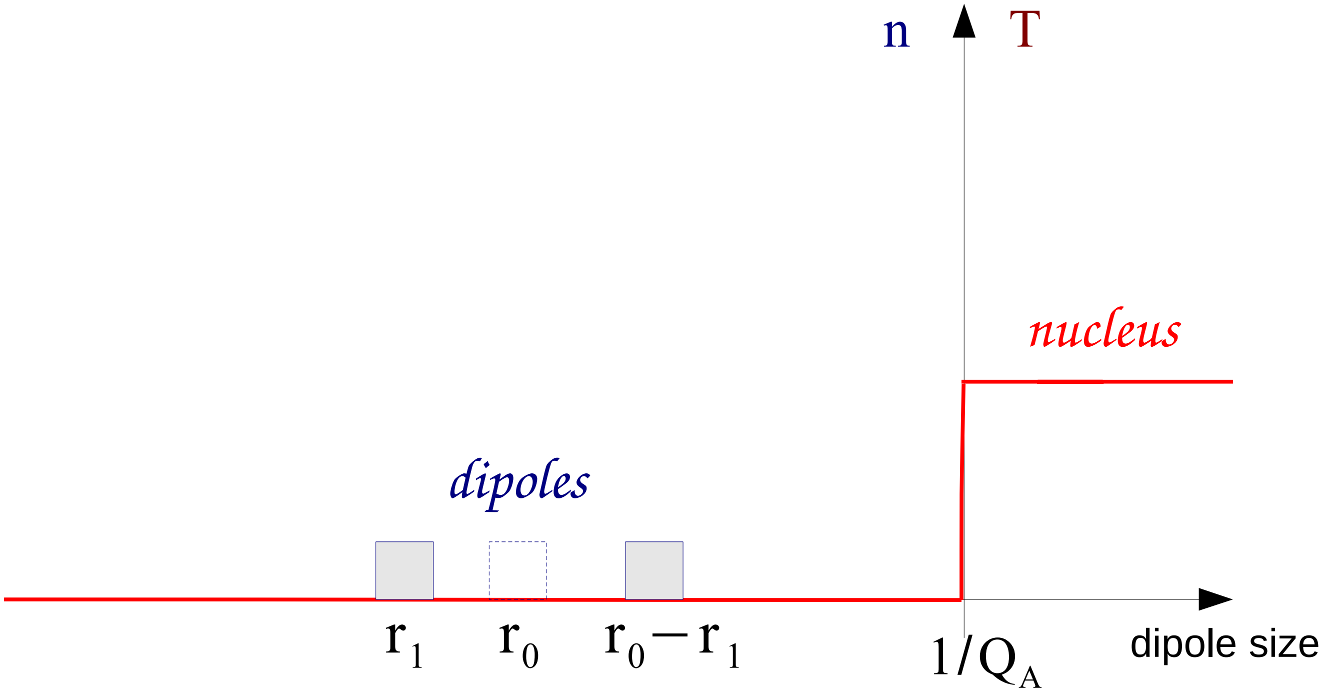

dipole splitting rate =

$$\frac{dp}{dy} = \frac{\alpha_s N_c}{\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \frac{d^2 r_1}{2\pi}$$

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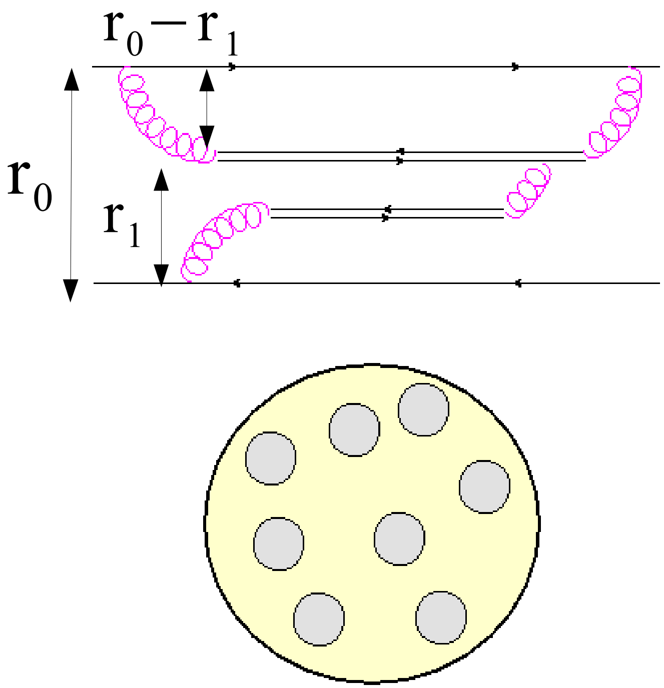

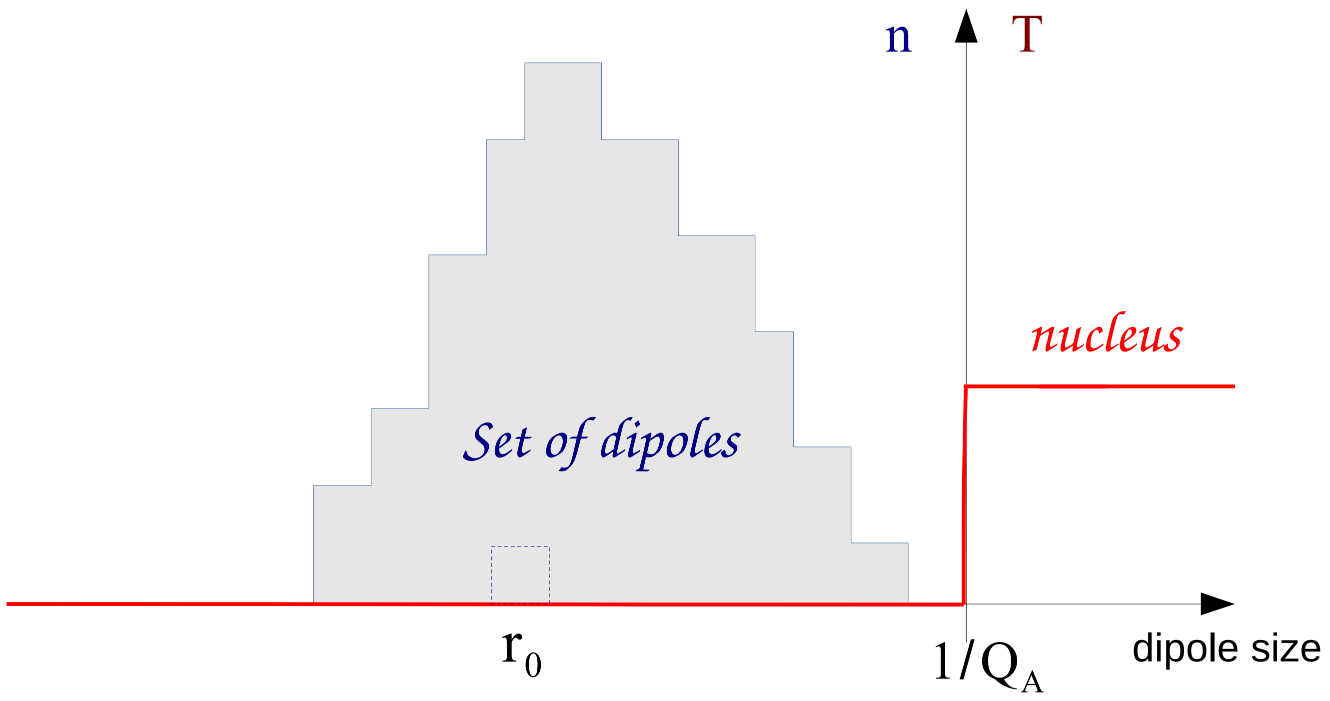

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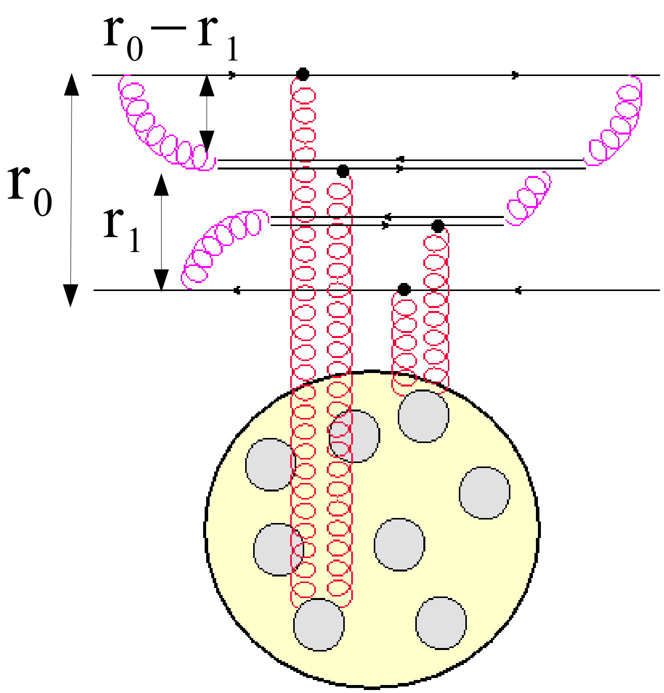

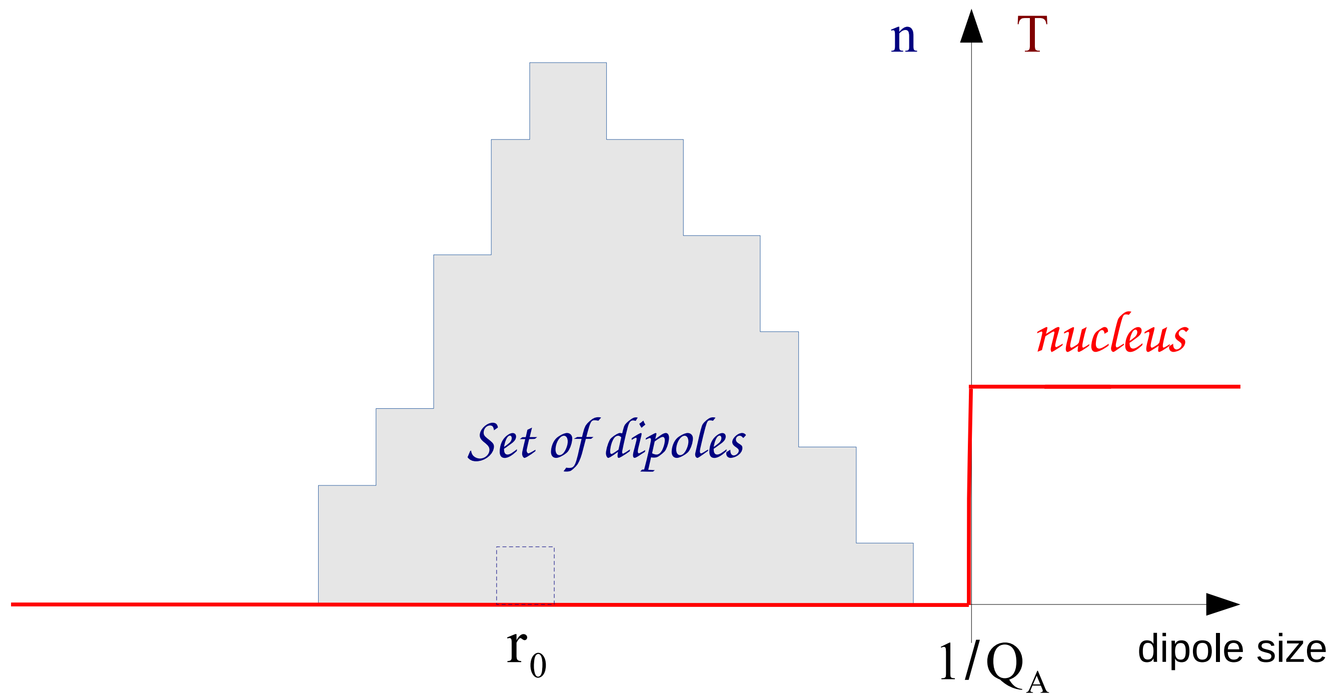

$$T_{\text{1-event}}(r_0, y) = \begin{cases} 0 \text{ if all dipoles have size } r < 1/Q_A \\ 1 \text{ if at least one dipole has size } r > 1/Q_A \end{cases}$$

dipole splitting rate =

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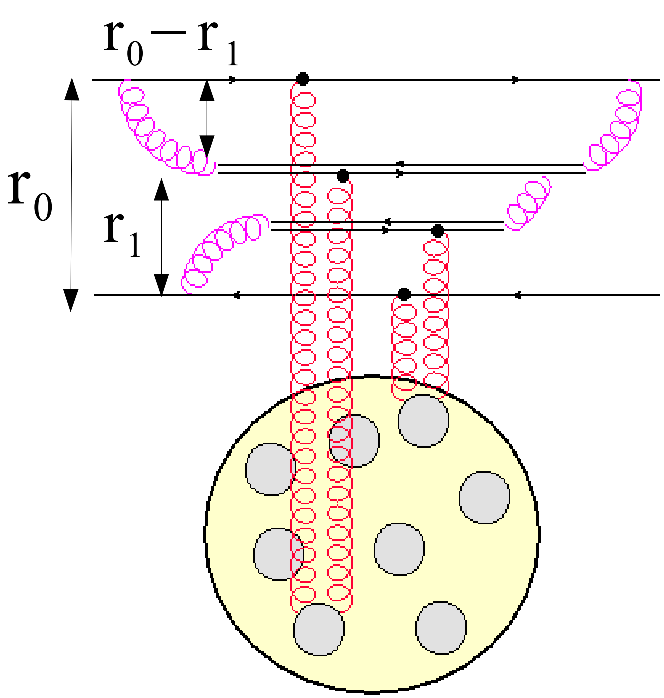

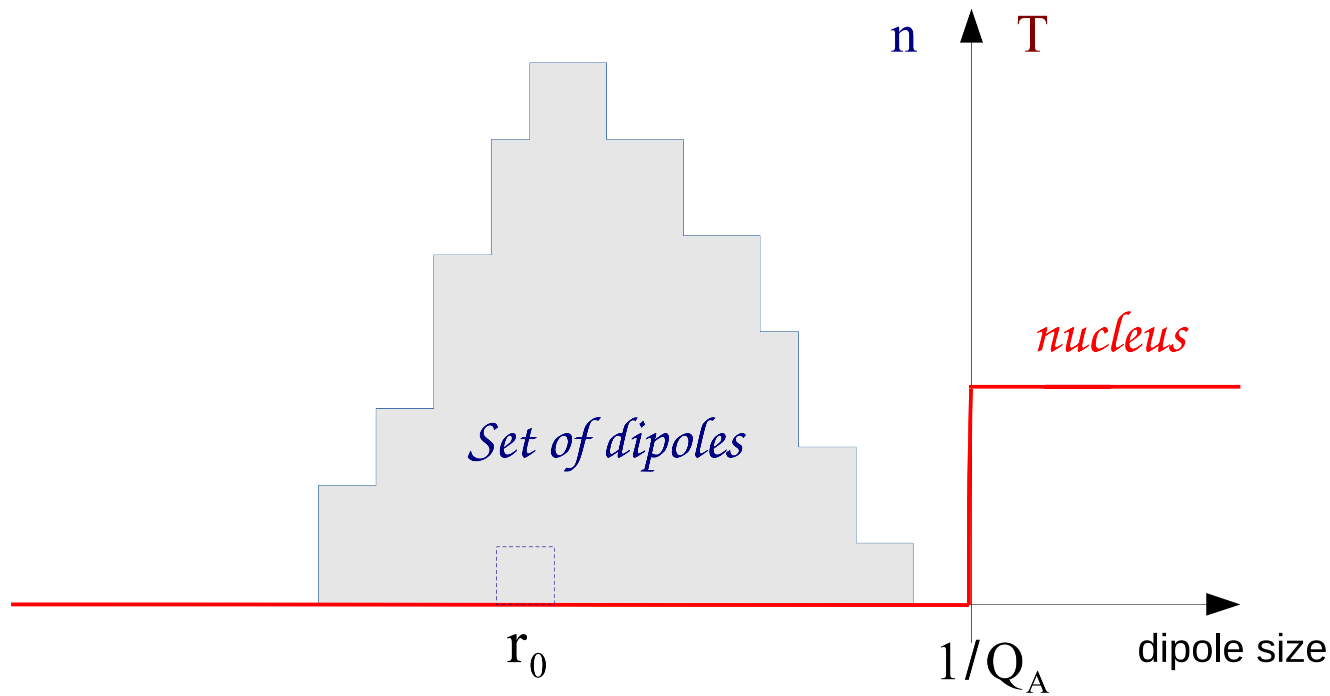

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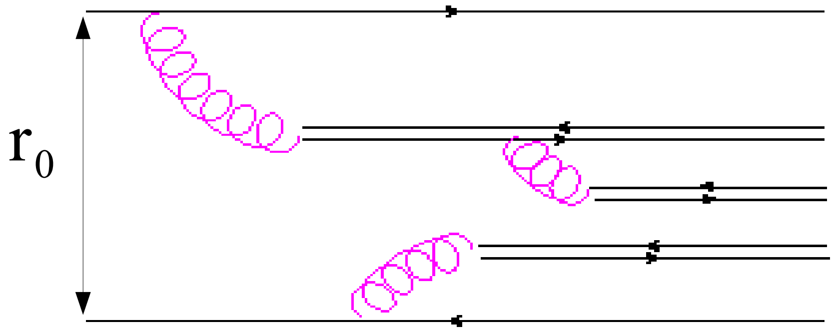
$T_{\text{1-event}}(r_0, y) =$
- 0 if all dipoles have size $r < 1/Q_A$
- 1 if at least one dipole has size $r > 1/Q_A$

$T(r_0, y) = \langle T_{\text{1-event}}(r_0, y) \rangle$ solves the BK equation

Interpretation: *probability that the largest dipole has a size larger than the inverse nuclear saturation momentum*

More on dipole evolution



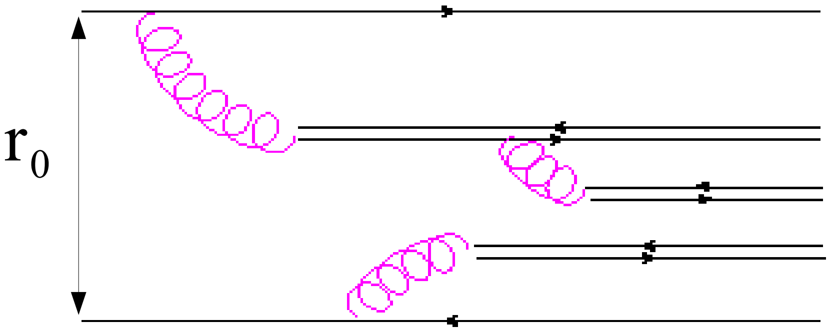
Dipole evolution

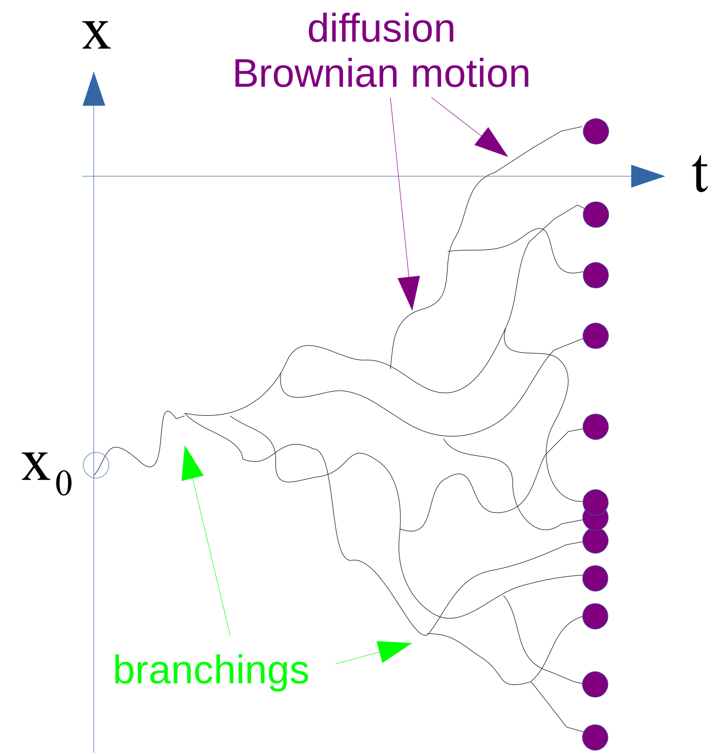
*Scattering amplitude with a nucleus: solves the **BK equation***

$$\partial_y T(r_0,y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2}$$

$$\times \left[T(r_1,y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y) T(r_0 - r_1, y) \right]$$

More on dipole evolution: a useful analogy





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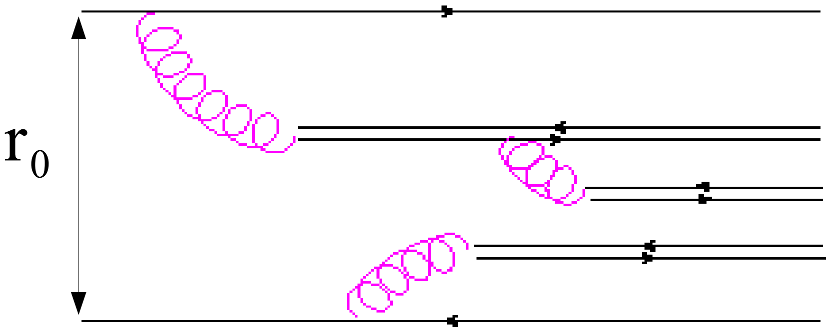
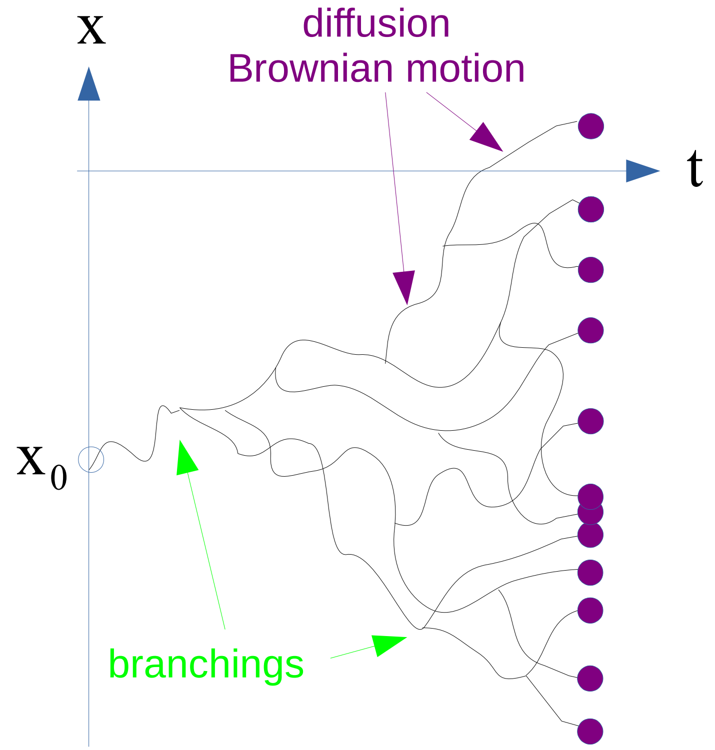




Dipole evolution

Scattering amplitude with a nucleus: solves the ***BK equation***

$$\partial_y T(r_0, y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \times \left[T(r_1, y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y) T(r_0 - r_1, y) \right]$$

Branching random walk

Probability P that the upmost particle has a position > 0: solves the ***FKPP equation***

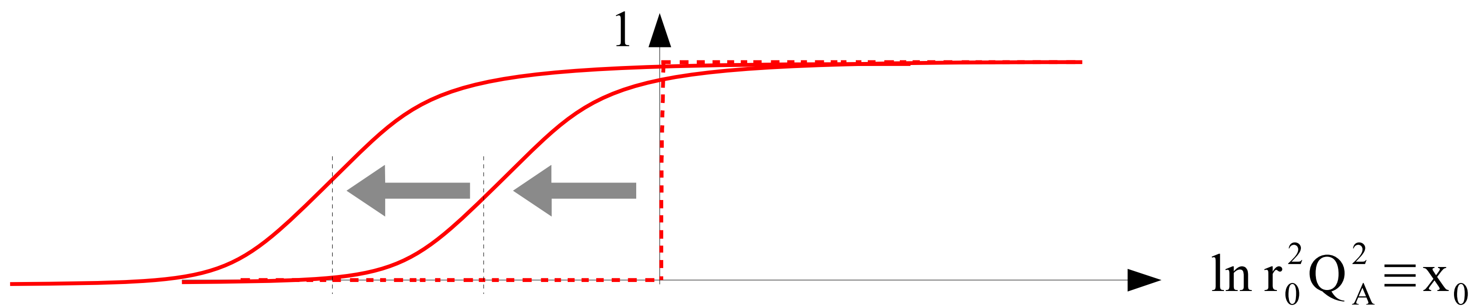
(Fisher-Kolmogorov-Petrovsky-Piscounov)

$$\partial_t P = \partial_{x_0}^2 P + P - P^2$$

(diffusion constant=1, splitting rate=1)

Rapidity evolution of dipole-nucleus amplitude $\simeq$ *Time evolution of the boundary of a branching random walk*

$T(\ln r_0^2 Q_A^2, y)$ $\quad\quad$ $P(x_0, t)$



$$T(\ln r_0^2 Q_A^2, y) = \text{function of } (\ln r_0^2 Q_A^2 - \ln Q_A^2/Q_s^2(y))$$

$$P(x_0, t) = \text{function of } (x_0 - X(t))$$

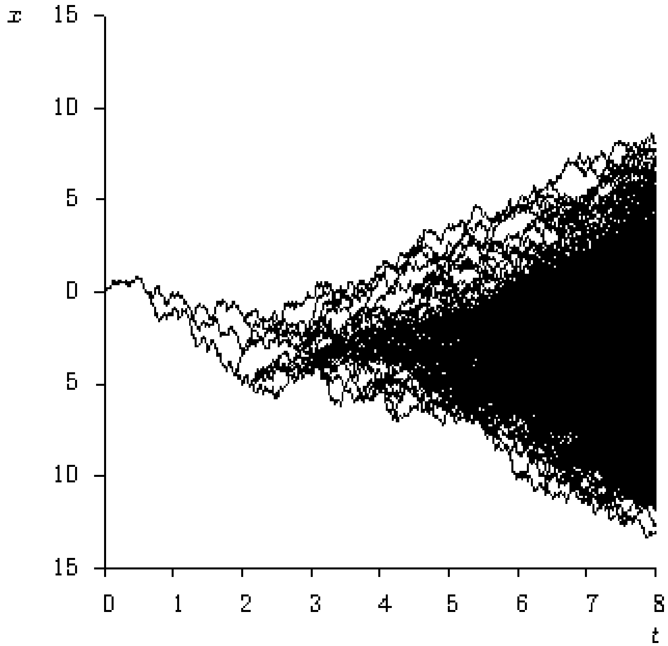
$$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} e^{\gamma_0 \ln[r_0^2 Q_s^2(y)]}$$

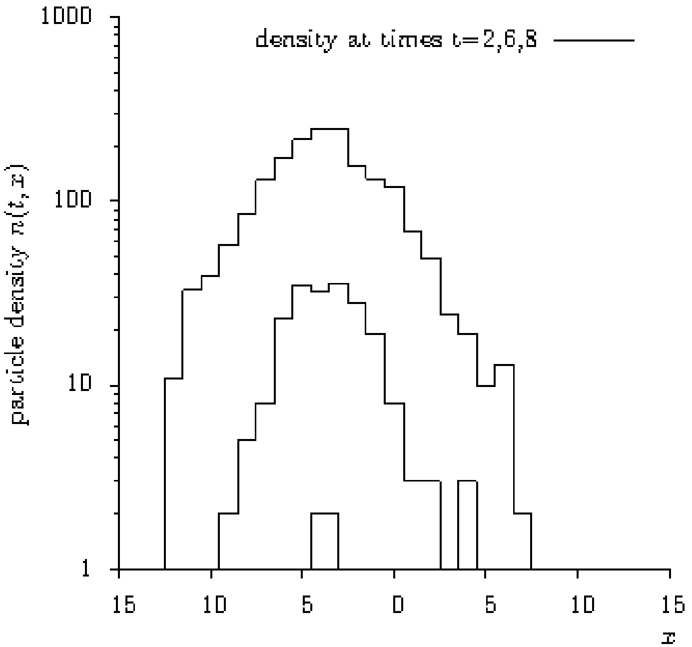
$$\ln \frac{Q_A^2}{Q_s^2(y)} \sim -\frac{\alpha_s N_c}{\pi} \chi'(\gamma_0) y$$

$$P(x_0, y) \sim (X(t) - x_0) e^{x_0 - X(t)}$$

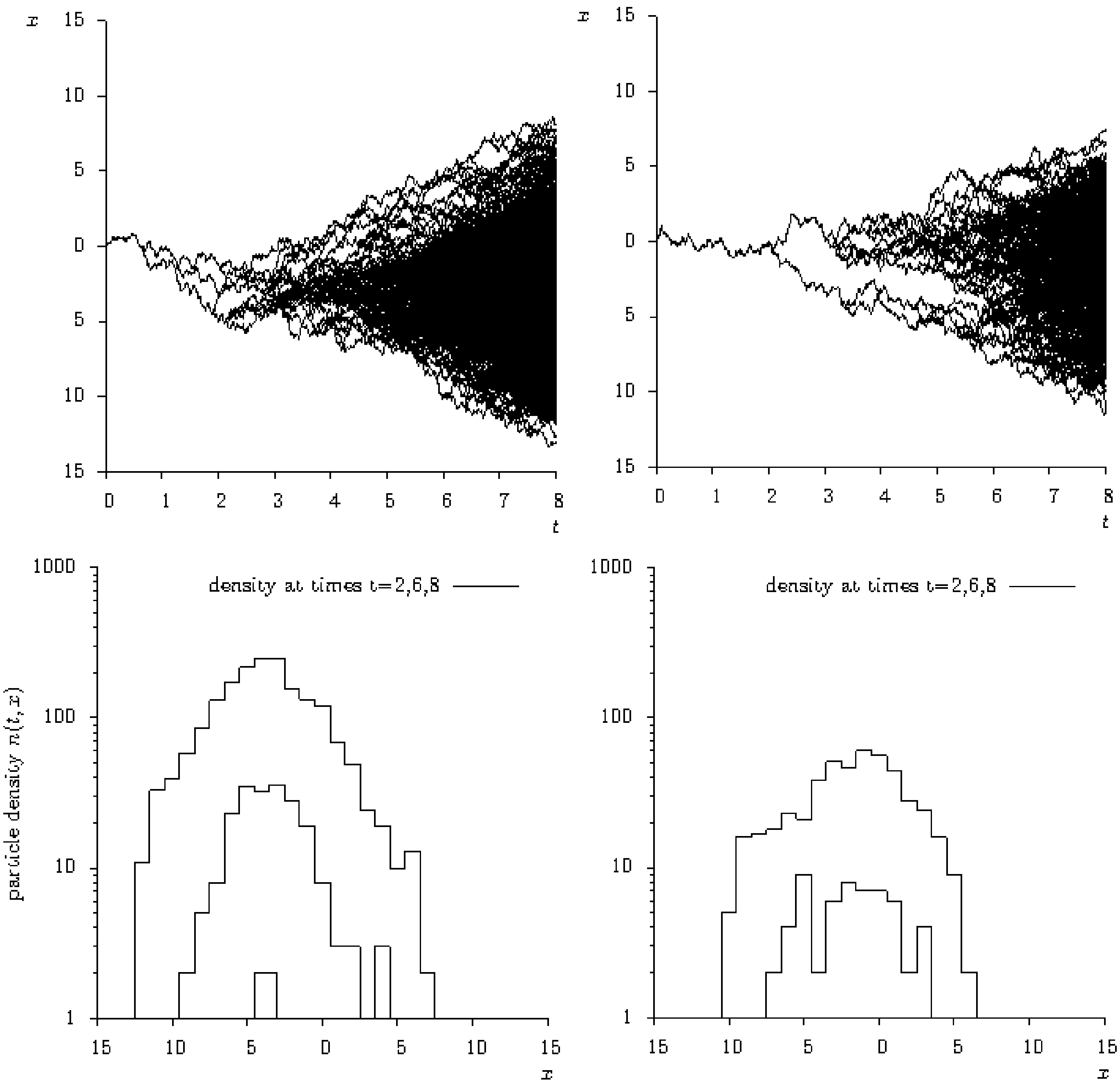
$$X(t) \sim -2t$$

Branching random walk

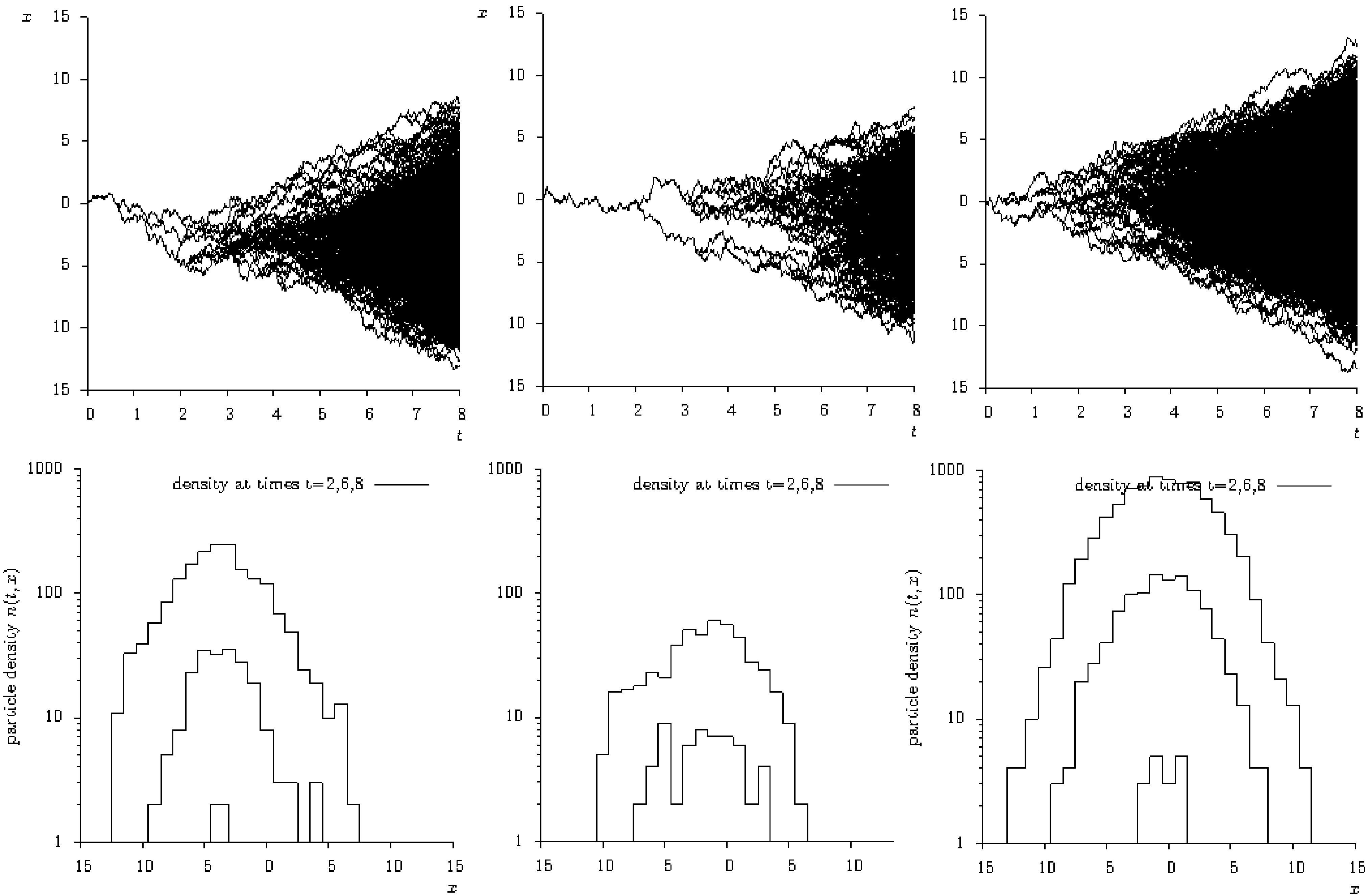




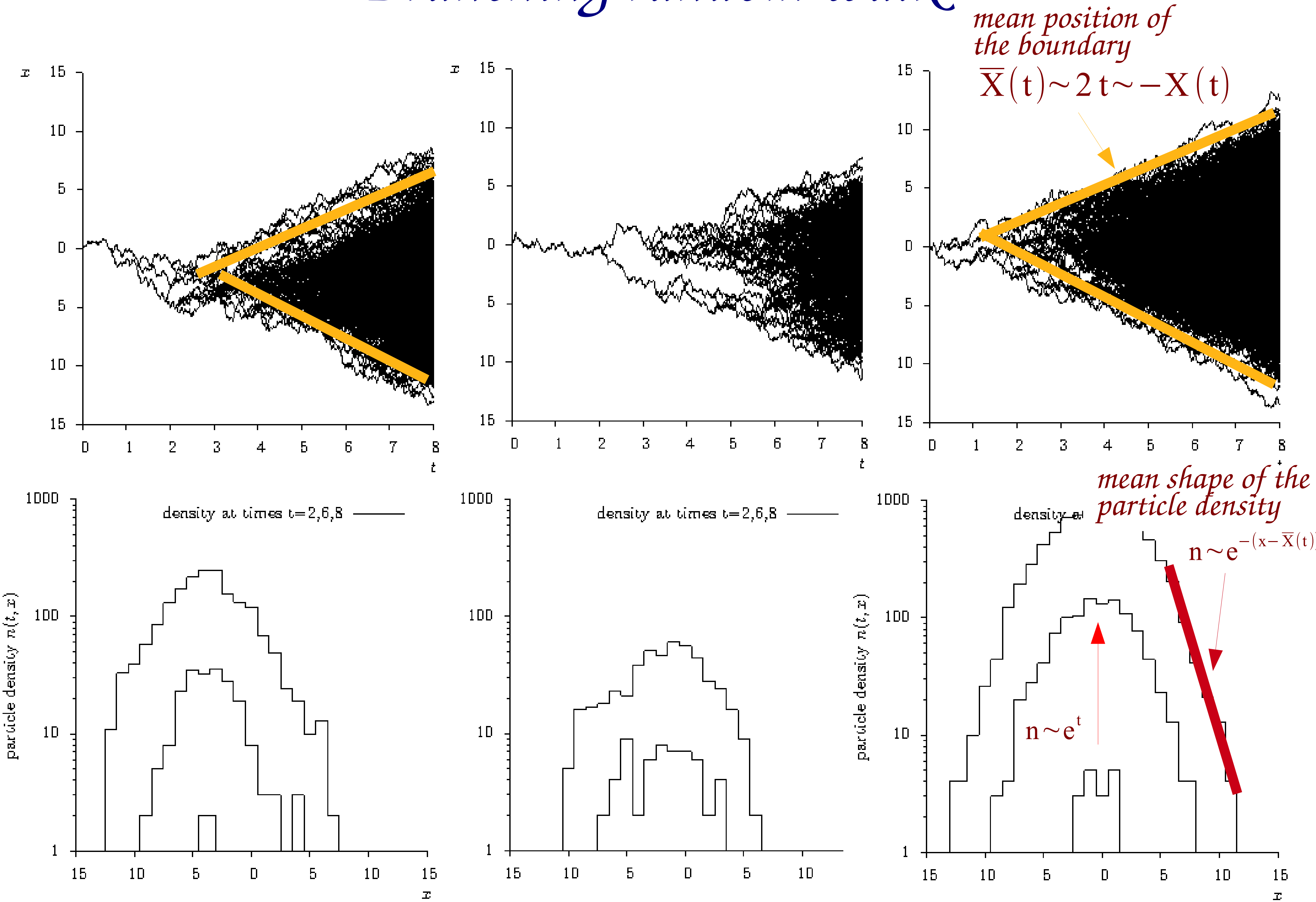
Branching random walk



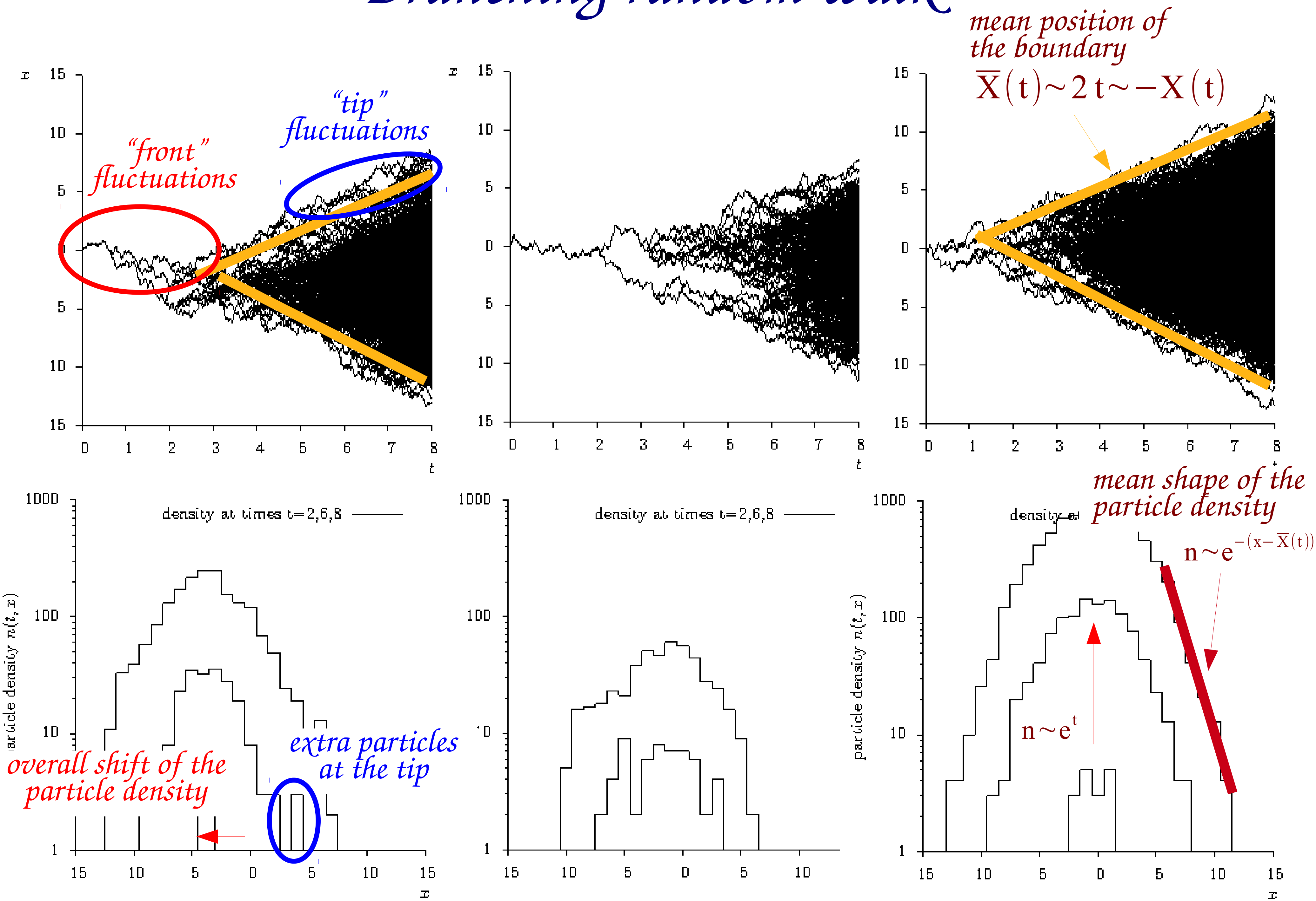
Branching random walk



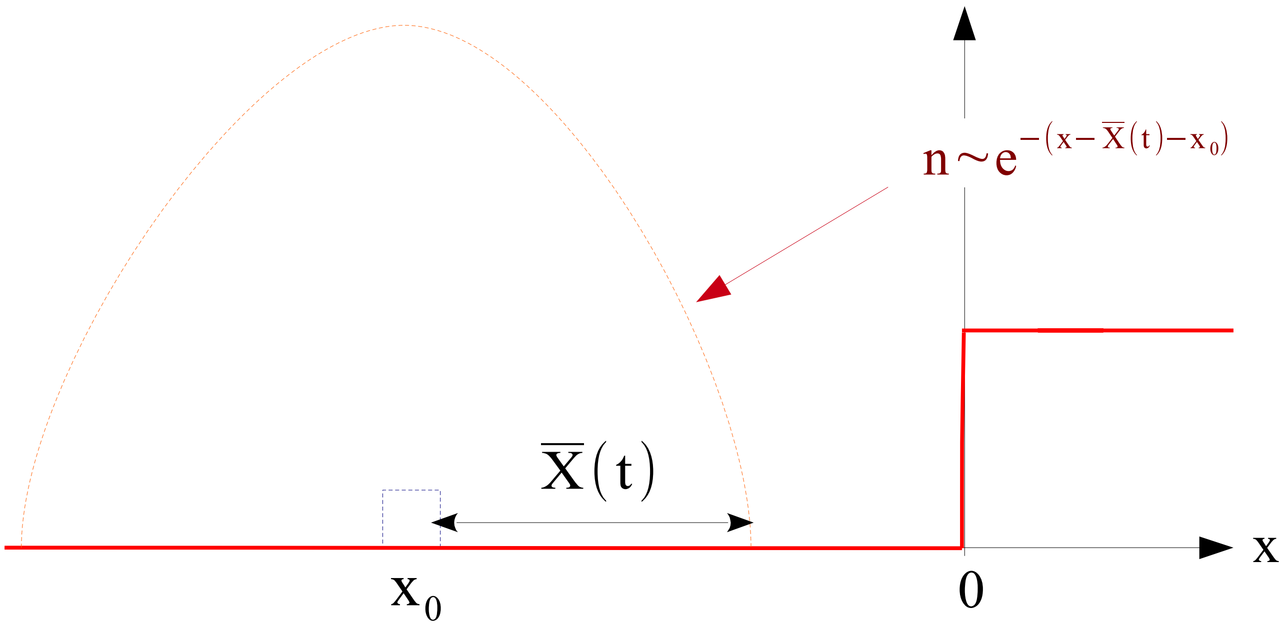
Branching random walk



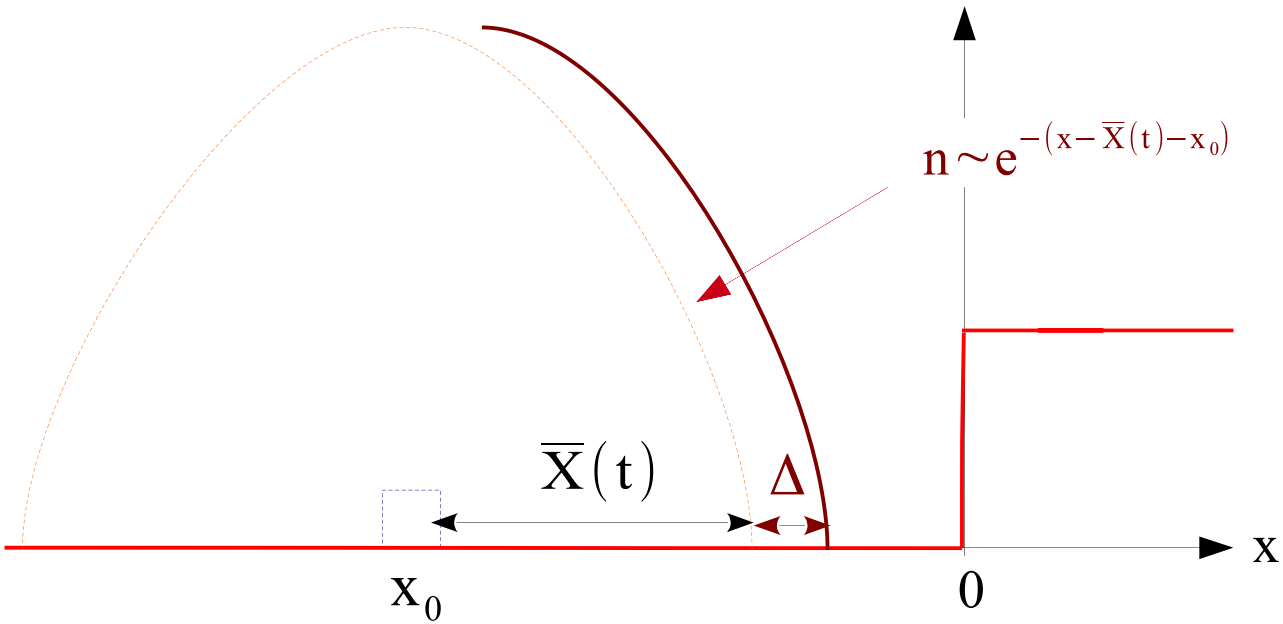
Branching random walk



Branching random walk

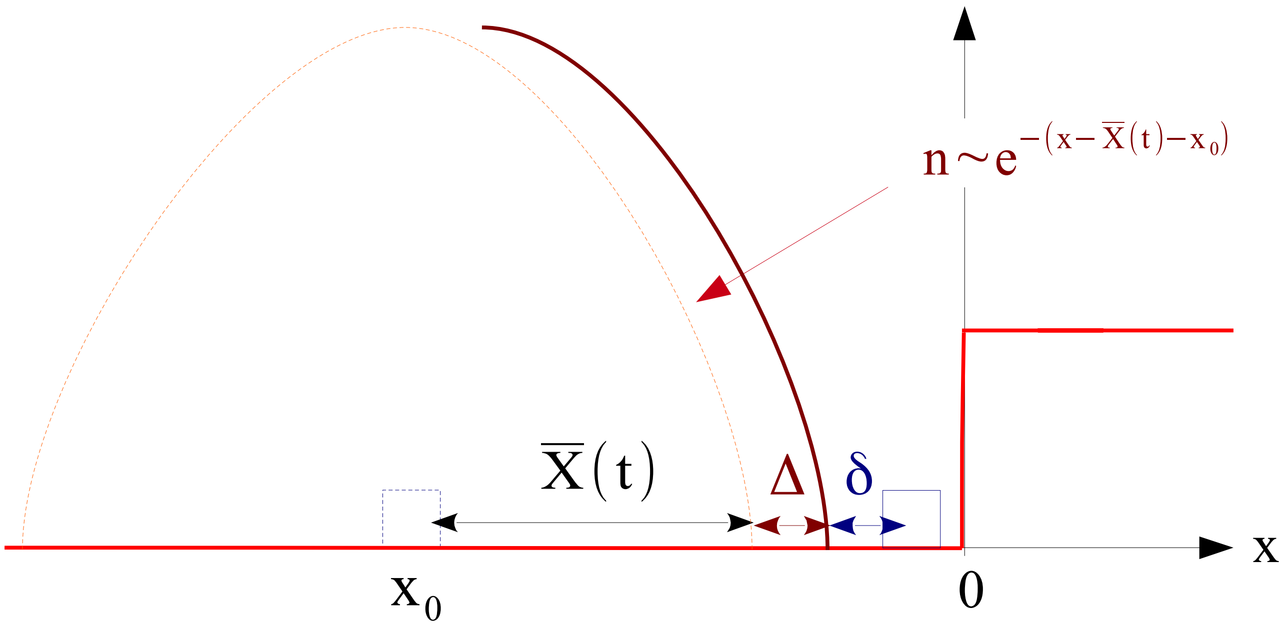


Branching random walk



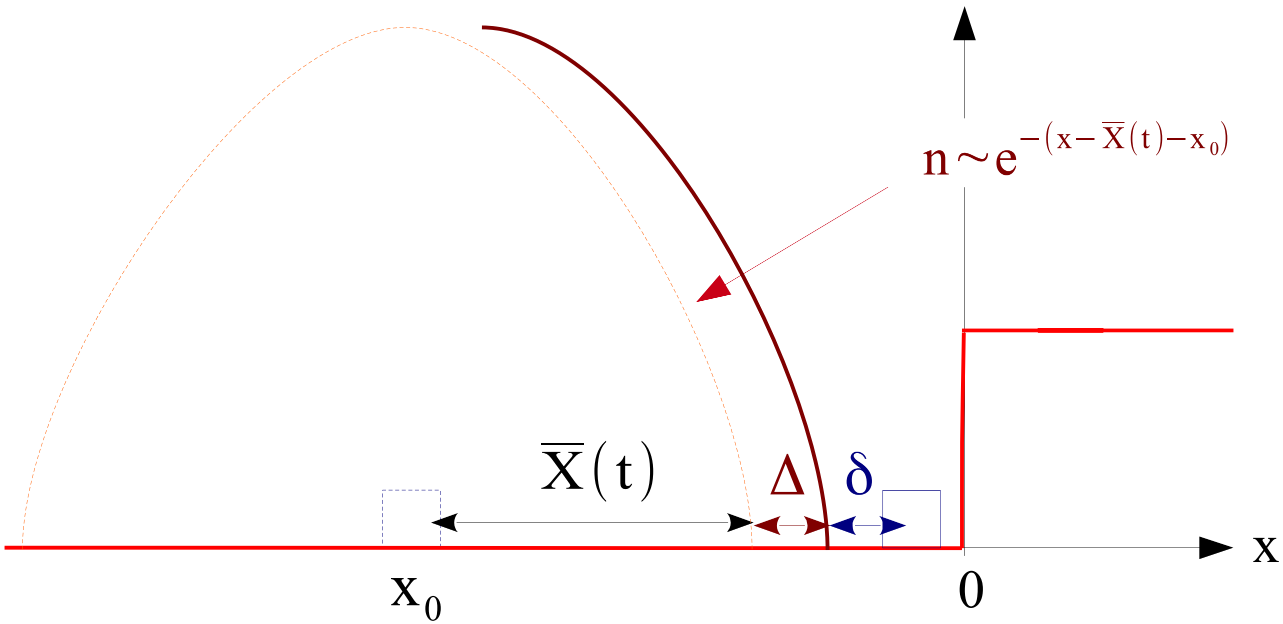
Conjecture: $p(\Delta) \sim e^{-\Delta}$

Branching random walk



Conjecture: $p(\Delta) \sim e^{-\Delta}$, $p(\delta) \sim e^{-\delta}$

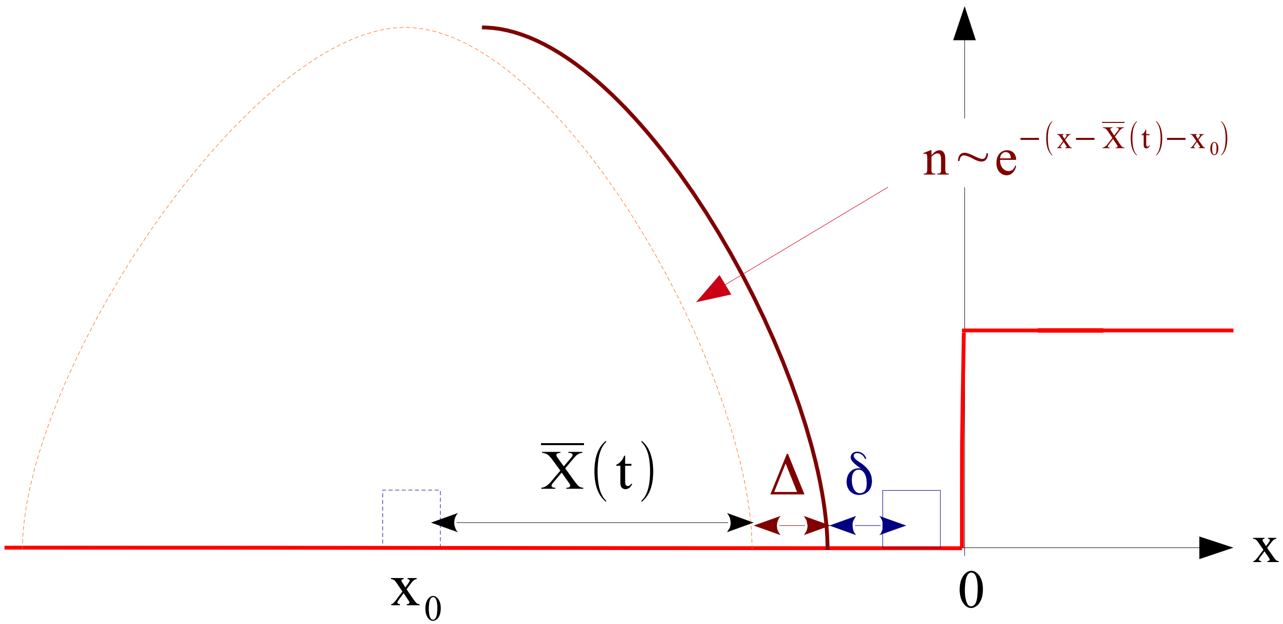
Branching random walk



$P =$ fraction of realizations in which $x_0 + \overline{X}(t) + \Delta + \delta > 0 = \left\langle \Theta\left(x_0 + \overline{X}(t) + \Delta + \delta\right) \right\rangle$

Conjecture: $p(\Delta) \sim e^{-\Delta}$, $p(\delta) \sim e^{-\delta}$

Branching random walk



$$P = \text{fraction of realizations in which } x_0 + \overline{X}(t) + \Delta + \delta > 0 = \langle \Theta(x_0 + \overline{X}(t) + \Delta + \delta) \rangle$$

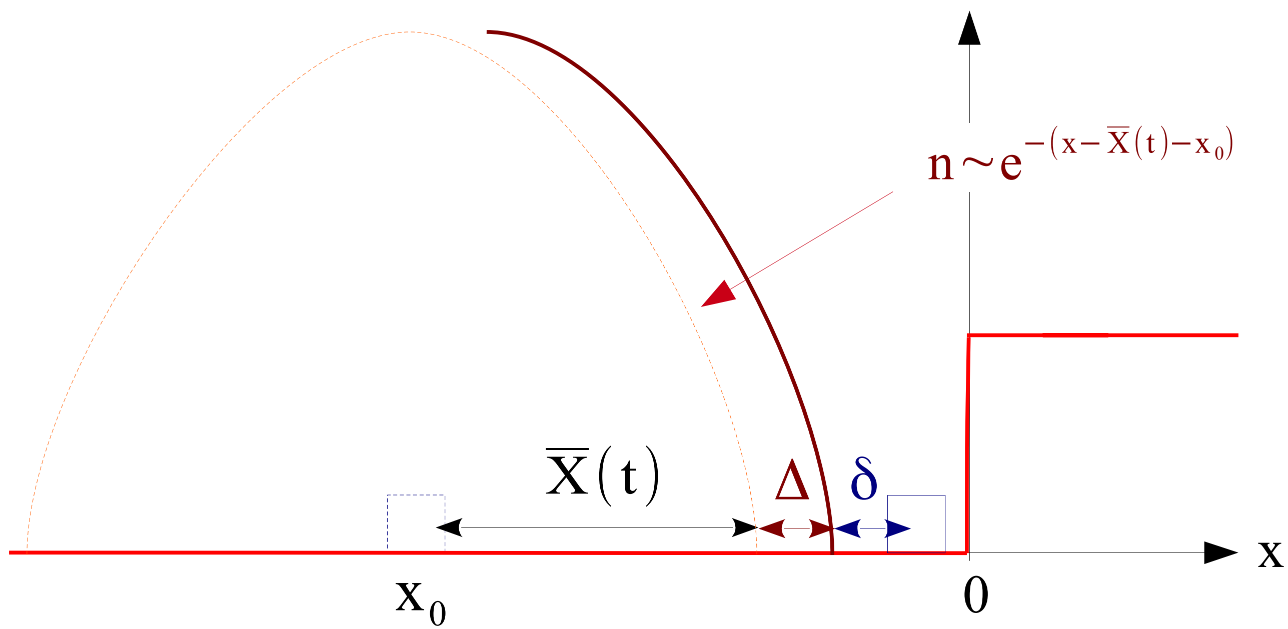
Conjecture: $p(\Delta) \sim e^{-\Delta}$, $p(\delta) \sim e^{-\delta}$

$$P(x_0, t) = \int d\Delta\, p(\Delta) \int d\delta\, p(\delta)\, \Theta(x_0 + \overline{X}(t) + \Delta + \delta)$$

$$\simeq (-\overline{X}(t) - x_0)\, e^{x_0 + \overline{X}(t)}$$

$$P(x_0, t) \simeq (X(t) - x_0)\, e^{x_0 - X(t)}$$

Branching random walk



$P =$ fraction of realizations in which $x_0+\overline{X}(t)+\Delta+\delta>0=\langle\Theta(x_0+\overline{X}(t)+\Delta+\delta)\rangle$

Conjecture: $p(\Delta)\sim e^{-\Delta}$, $p(\delta)\sim e^{-\delta}$

$$P(x_0,t)=\int d\Delta\, p(\Delta)\int d\delta\, p(\delta)\,\Theta(x_0+\overline{X}(t)+\Delta+\delta)$$

$$\simeq(-\overline{X}(t)-x_0)\,e^{x_0+\overline{X}(t)}$$

$$P(x_0,t)\simeq(X(t)-x_0)\,e^{x_0-X(t)}$$

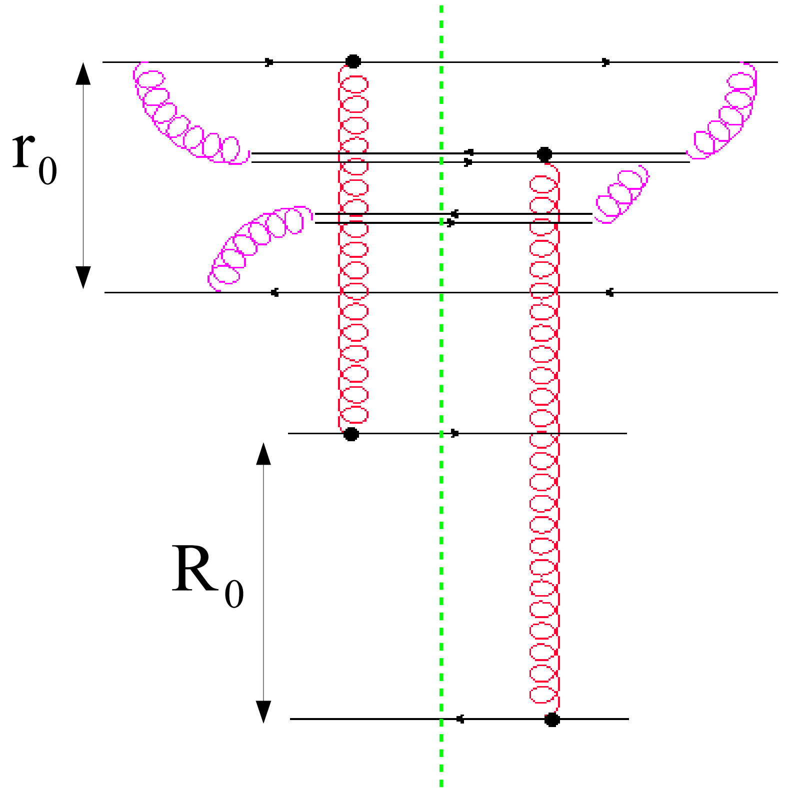
Consistent with solution to BK!

$$T(r_0,y)\simeq\ln\frac{1}{r_0^2 Q_s^2(y)}\,e^{\gamma_0\ln[r_0^2 Q_s^2(y)]}$$

The ***shape of the dipole-nucleus scattering amplitude*** *as a function of the dipole size is related to the* ***fluctuations of the number of gluons*** *in the QCD evolution*

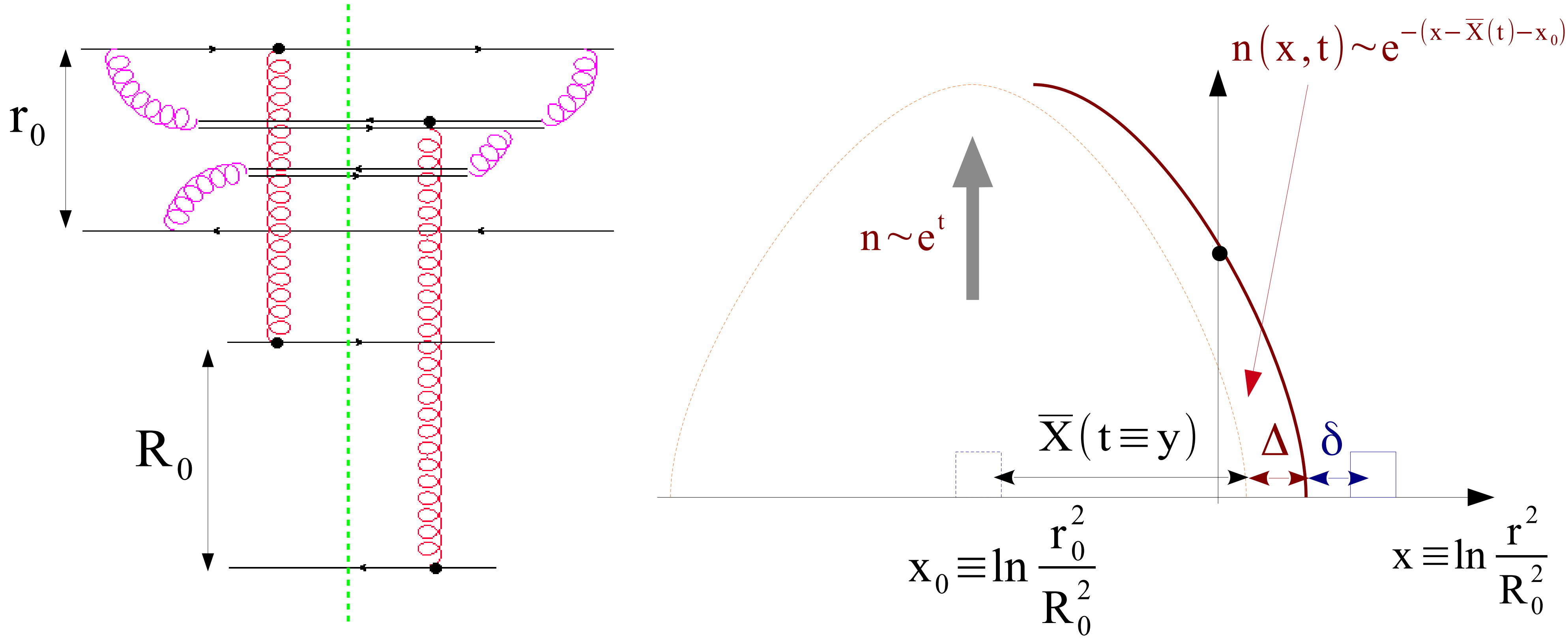
$$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} \, e^{\gamma_0 \ln\left[r_0^2 Q_s^2(y)\right]}$$

Dipole-dipole scattering



$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution}$$

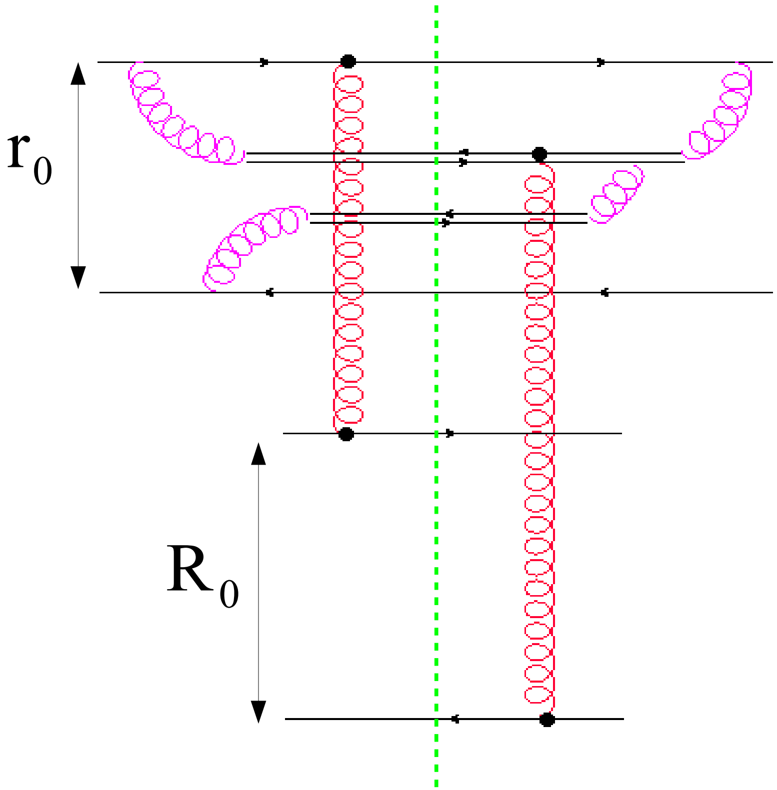
Dipole-dipole scattering

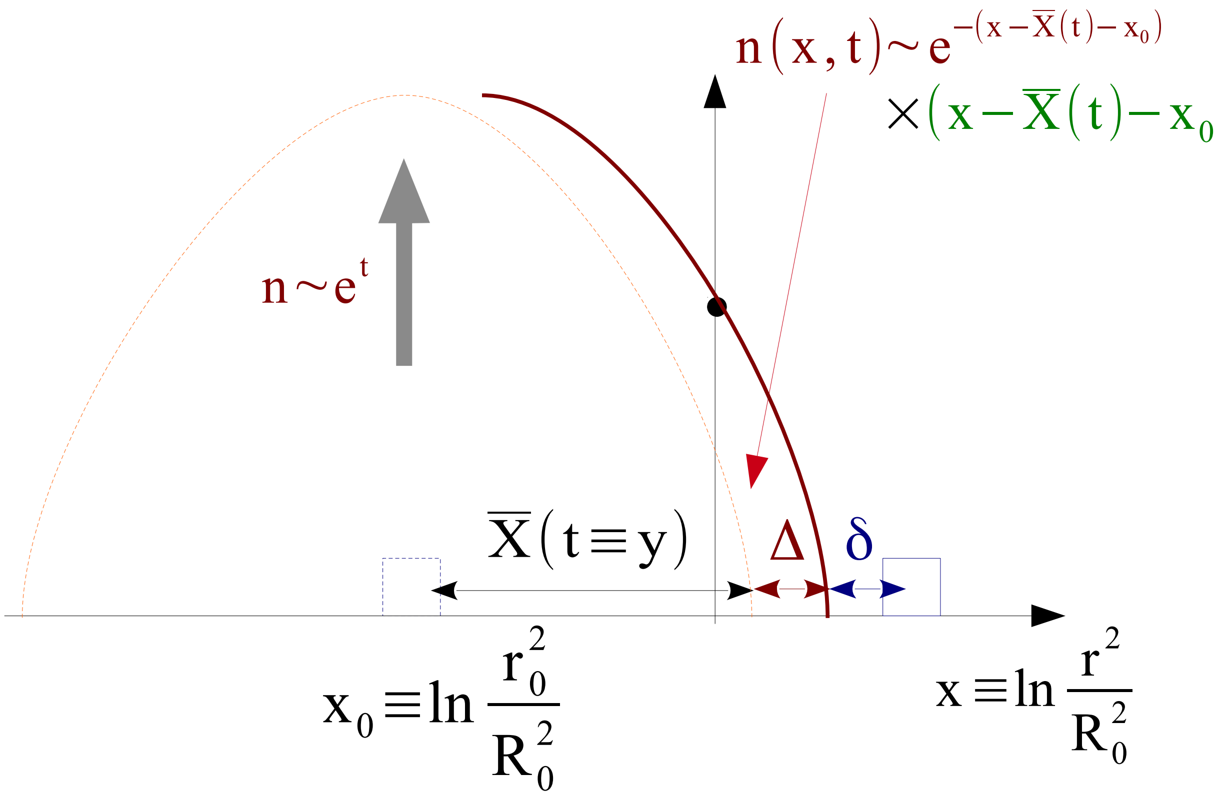


$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

Probes the shape of the density of dipoles!

Dipole-dipole scattering





$$T_{1\text{-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

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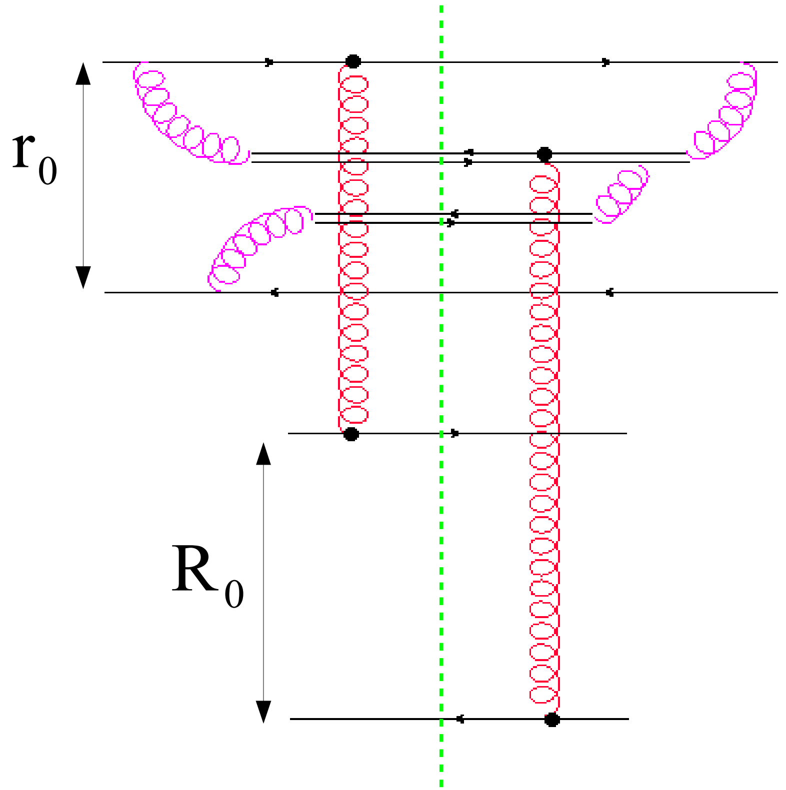
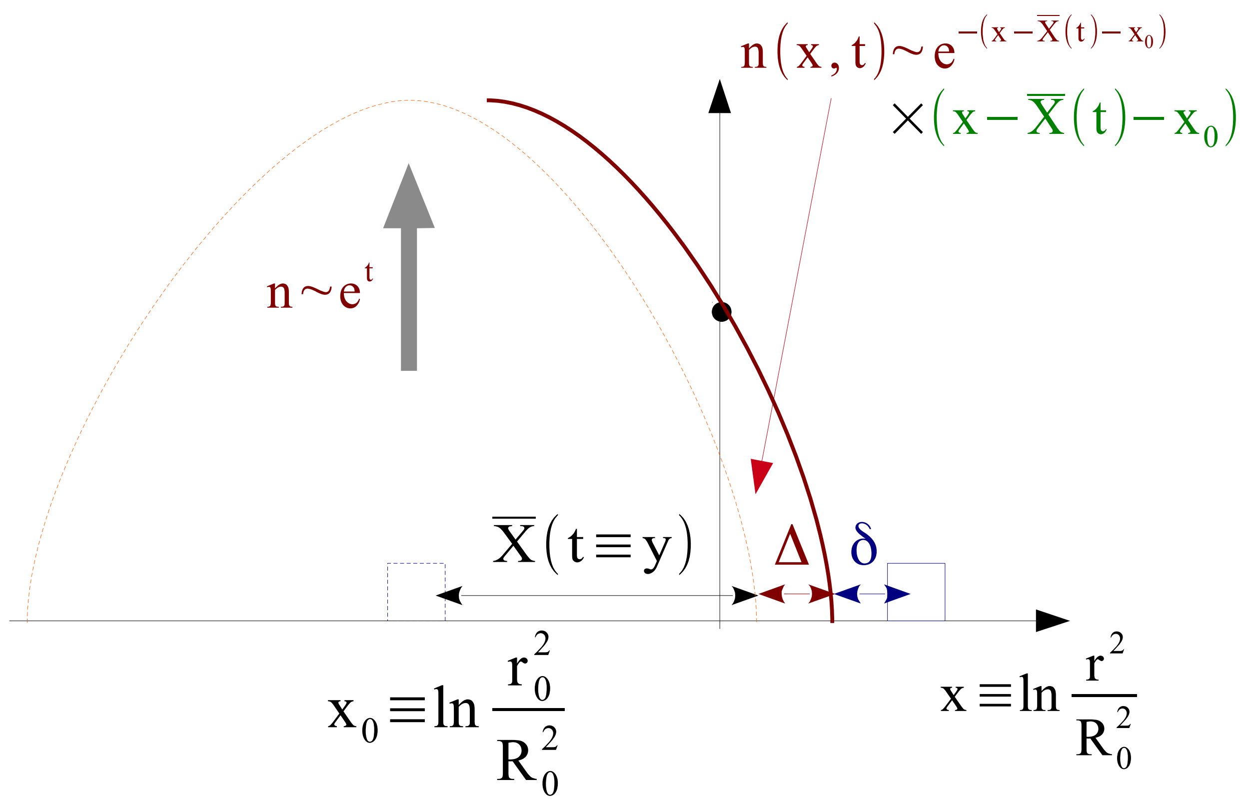
Dipole-dipole scattering





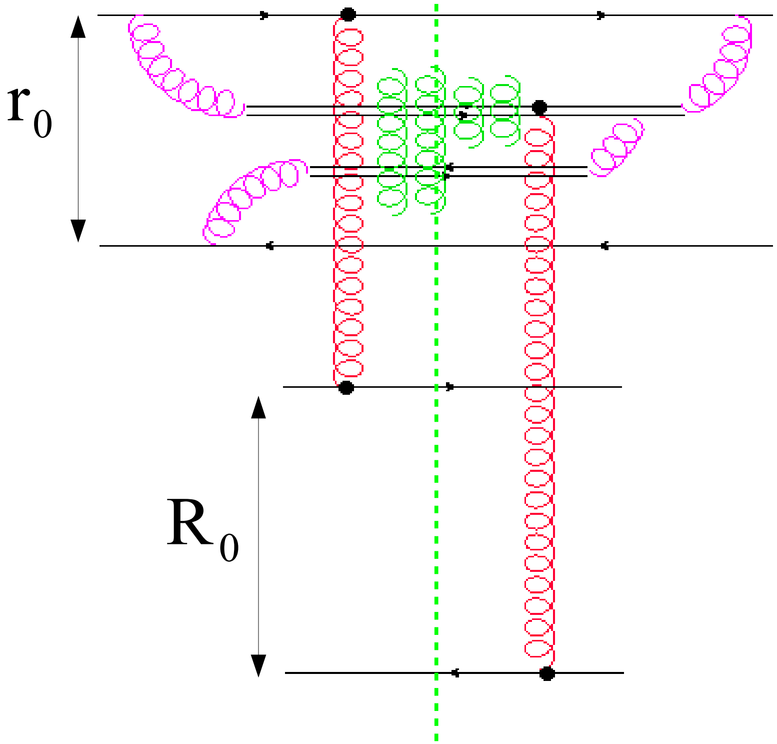
$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

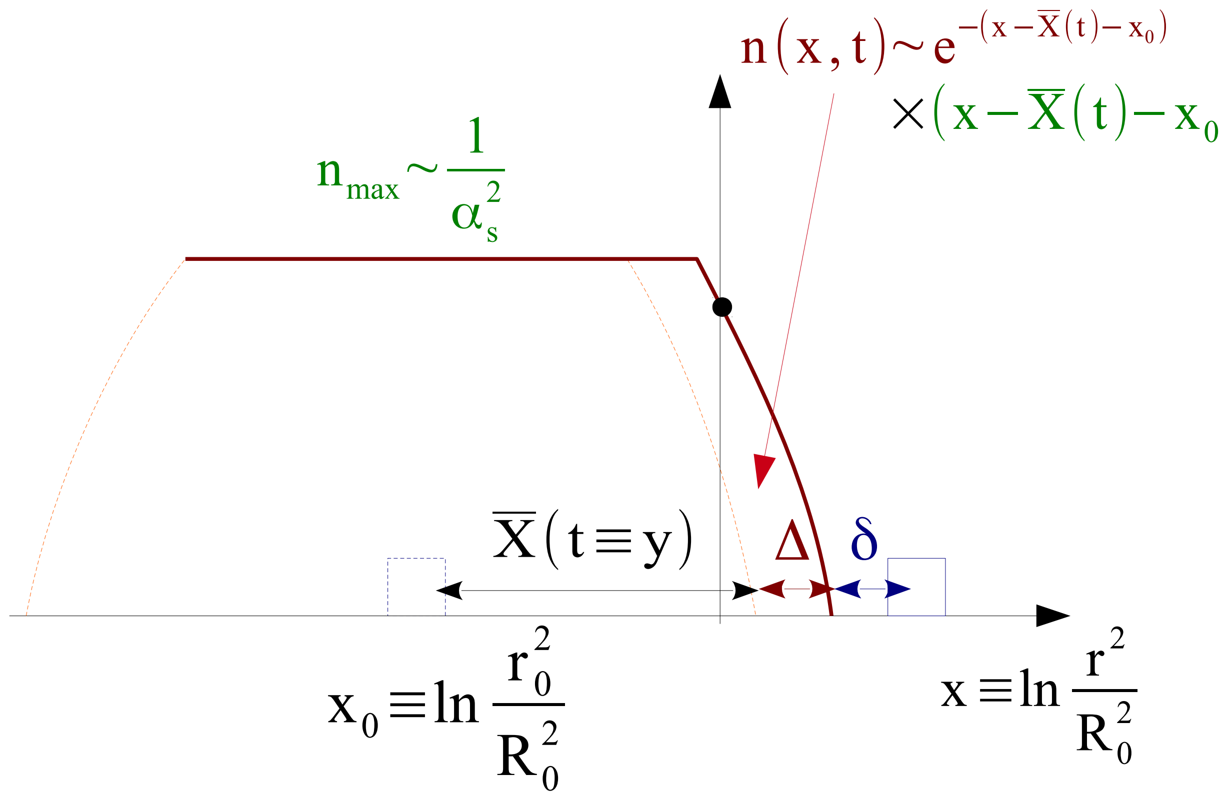
Probes the shape of the density of dipoles!

Since n grows exponentially, at some rapidity unitarity would be violated!

Dipole-dipole scattering

Need nonlinear effects which are not described by the BK equation





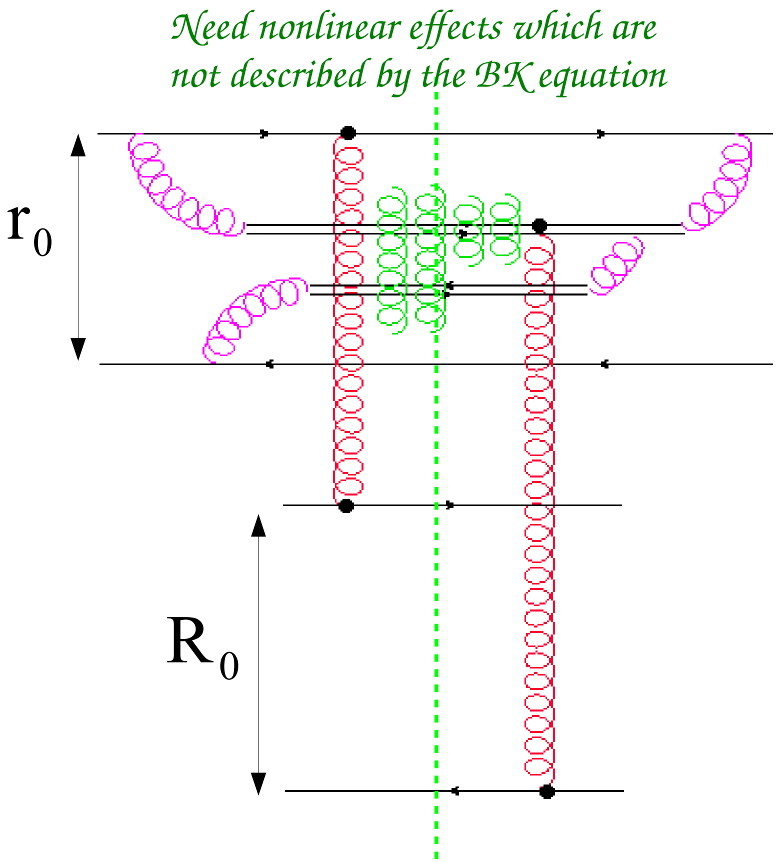
$$T_{1\text{-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

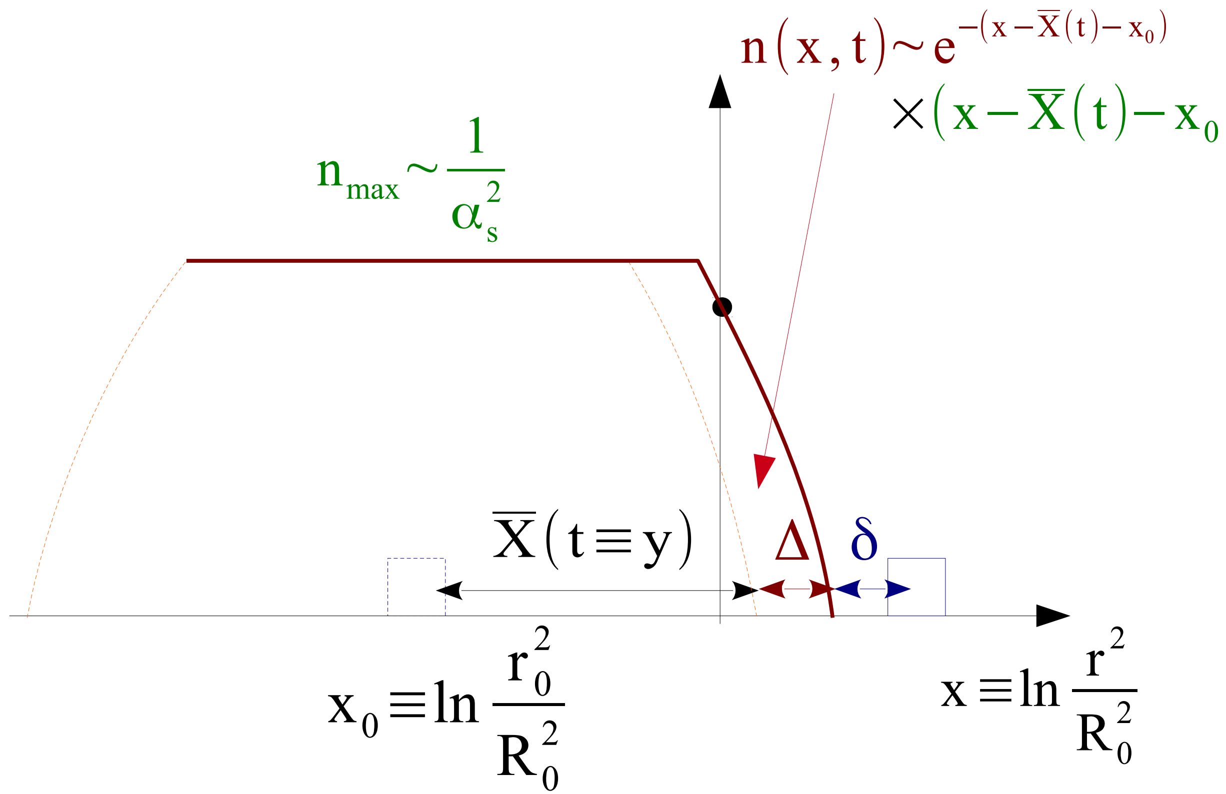
Probes the shape of the density of dipoles!

Since n grows exponentially, at some rapidity unitarity would be violated!

One needs saturation of the gluons

Dipole-dipole scattering





$$T_{1\text{-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

$$T(r_0, y) = \langle T_{1\text{-event}}(r_0, y) \rangle \sim \left(\ln \frac{1}{r_0^2 Q_s^2(y)} \right)^2 e^{\gamma_0 \ln\left[r_0^2 Q_s^2(y)\right]}$$

The ***shape of the dipole-dipole scattering amplitude*** *as a function of the dipole size is related to the* ***shape of the gluon number density*** *and to its* ***fluctuations*** *in the QCD evolution*

$$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln^2 \frac{1}{r_0^2 Q_s^2(y)} \, e^{\gamma_0 \ln[r_0^2 Q_s^2(y)]}$$

Summary

The shape of dipole amplitudes is intimately related to the probability distribution of the fluctuations of the gluon number density, whose rapidity evolution is a branching random walk.

There is a qualitative difference between the dipole-nucleus and the dipole-dipole cases:

Dipole-nucleus: $$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} e^{\gamma_0 \ln[r_0^2 Q_s^2(y)]}$$

Dipole-dipole: $$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln^2 \frac{1}{r_0^2 Q_s^2(y)} e^{\gamma_0 \ln[r_0^2 Q_s^2(y)]}$$

**valid in an intermediate rapidity regime:* $y \ll \frac{1}{\alpha_s N_c} \ln^3 \frac{1}{\alpha_s^2}$. *Beyond, everything becomes universal!*

Much more on fluctuations in parton evolution/branching random walks in

A.H. Mueller, S. Munier, arXiv:1404.5500, arXiv:1405.3131

(In particular: parametric expressions for the saturation scale, picture of the fluctuations in different frames etc...)