Detailed study of the Ke4 decay mode properties with the NA48/2 experiment at CERN

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On behalf of the NA48/2 Collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

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OUTLINE

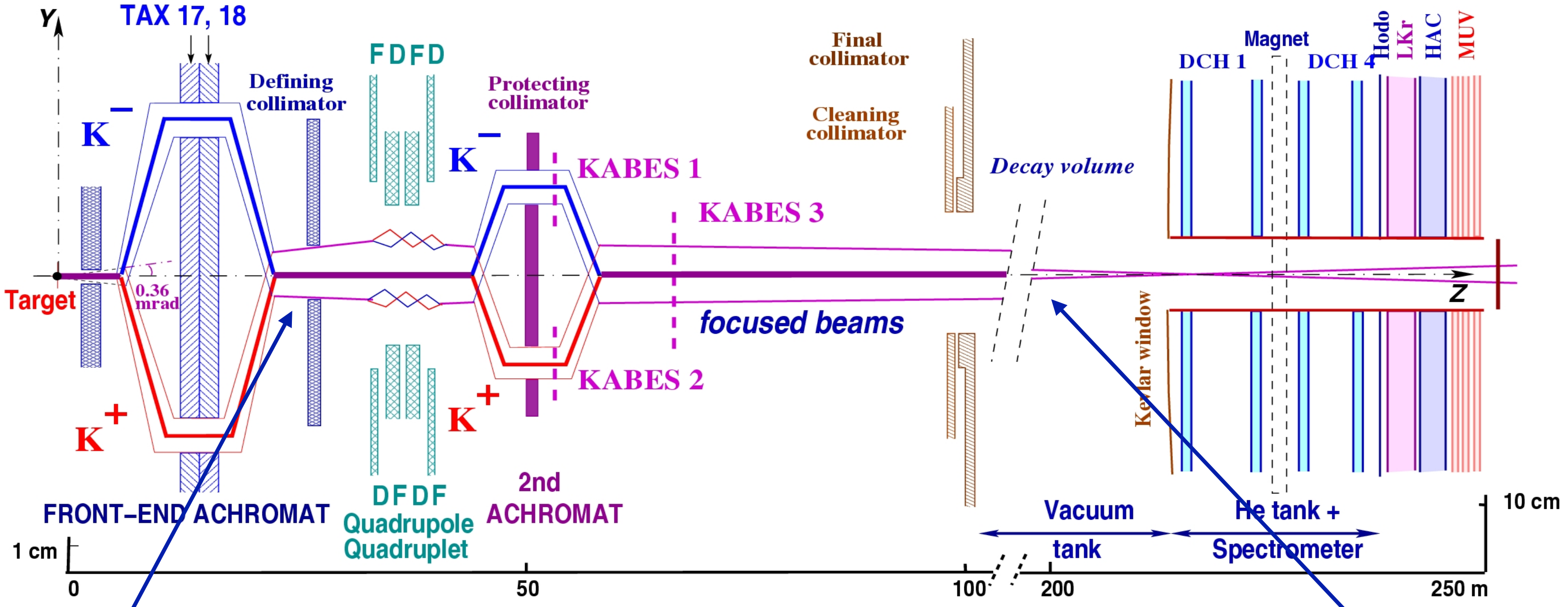
- The **NA48/2** experiment
- **Ke4 decay mode properties**: latest results
  - $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ **(Ke4(+−))** Branching Ratio and Form Factors
  - $K^{\pm} \rightarrow \pi^{0}\pi^{0}e^{\pm}\nu$ **(Ke4(00))** Branching Ratio and Form Factors NEW
- Results on $\pi\pi$ **scattering** from **Ke4** and **K3π** decays
- Conclusions

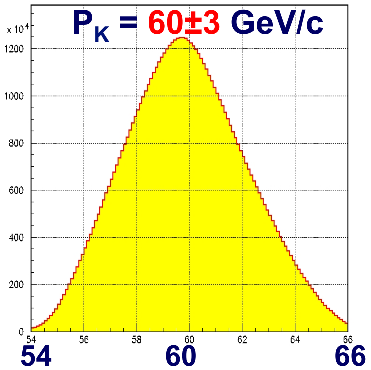
N.B. in this conference more results on Kaon Physics from NA48/2 and NA62 experiments at CERN by F. Costantini, G. Ruggiero and T. Spadaro

The NA48/2 Experiment at CERN

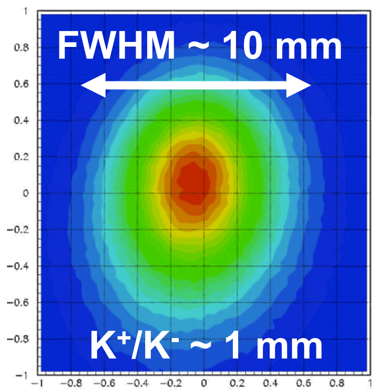
NA48/2: a fixed target experiment at the CERN SPS to search for direct CP violation in charged kaon 3π decays and to study rare decays

The NA48/2 Experimental Layout





- Primary beam:
  - SPS protons (~7×10^{11} p/spill, 400 GeV/c) on a Be target
- Secondary beam: simultaneous K^+ and K^- beams (~5-6% $K^\pm$)
  - focused at DCH1 with ~ 10 mm transverse size
  - superimposed beam axes within ~ 1 mm
  - flux ratio: K^+/K^- ~ 1.8
  - large charge symmetrization of experimental conditions
- Data taking: 2003 and 2004 (~ 4 months)



The NA48/2 detector

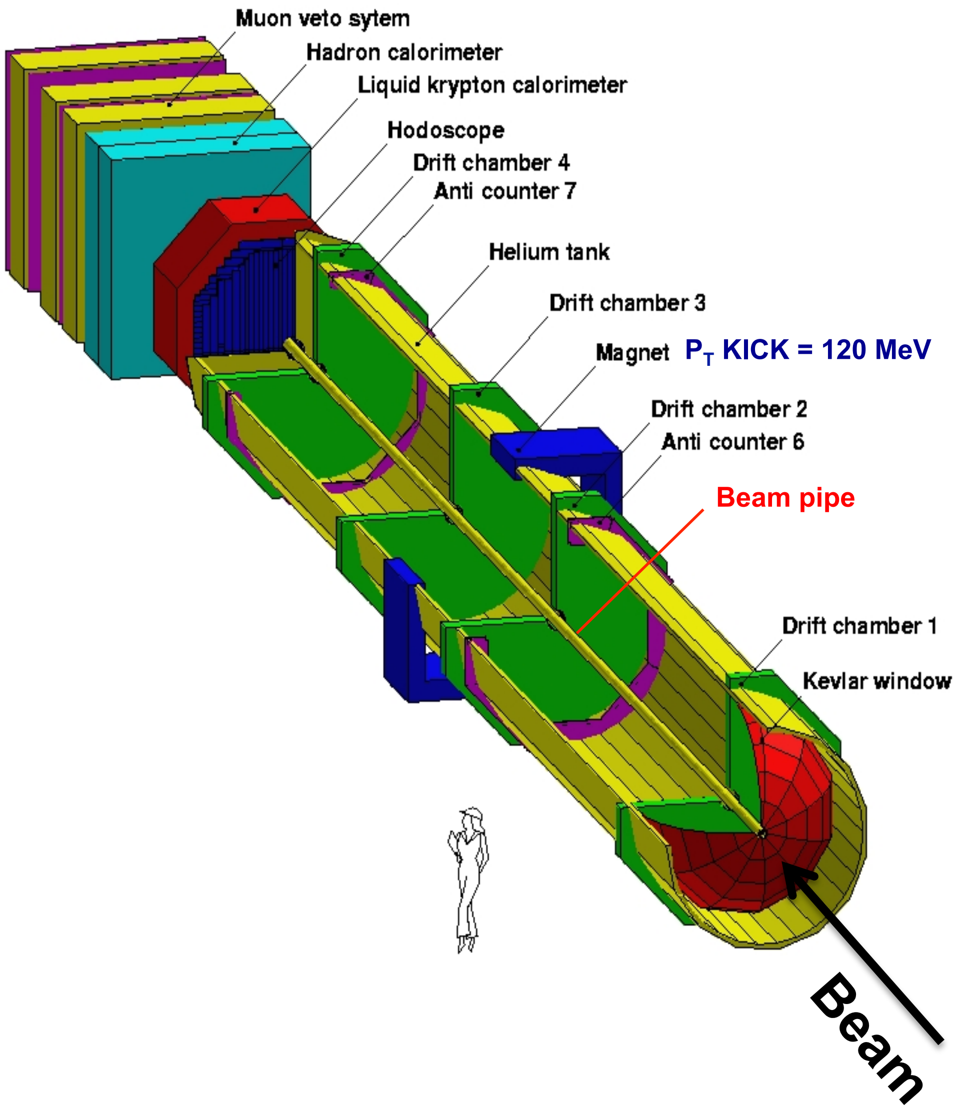
Magnetic spectrometer
- 4 DCHs , 4 views each + dipole magnet
    - redundancy, high efficiency
- $\Delta p/p = (1.02 + 0.044 \times p)\%$ (p in GeV/c)
- Mass resolution $\sigma(M3\pi^{\pm}) = 1.7$ MeV/c^2

Liquid Krypton EM calorimeter (LKr)
- High granularity, quasi-homogeneous
- $\Delta E/E = (3.2/\sqrt{E} + 9.0/E + 0.42)\%$ (E in GeV)
- $\sigma_x = \sigma_y \sim 1.5$ mm @ E = 10 GeV
- Mass resolution $\sigma(M\pi^0\pi^0) = 1.4$ MeV/c^2

- **E/p ratio used for e/π discrimination**

Charged Hodoscope
- Two orthogonal planes of scintillator
- Fast trigger $\sigma_t = 150$ ps

Trigger L1+L2 : 1 MHz → ~10 kHz



Decay region ~ 114 m, detector ~ 50 m

Similar acceptance between K^+ and K^- beams ensured reversing magnetic fields

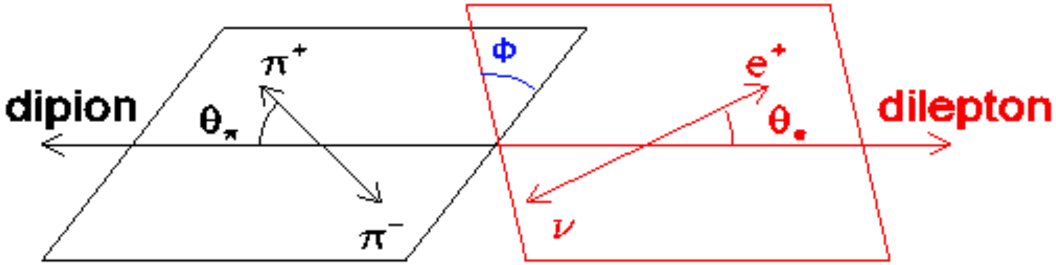
Ke4 decay mode properties

Ke4 Decays

Ke4 decays:

Ke4(+−): $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\,\nu$

Ke4(00): $K^{\pm} \rightarrow \pi^{0}\pi^{0}e^{\pm}\,\nu$



Two pion final state interaction in absence of any other hadron:

→ **Clean environment**

→ **Study the $\pi\pi$ system close to threshold (S- and P-wave states)**

$K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\,\nu$	$K^{\pm} \rightarrow \pi^{0}\pi^{0}e^{\pm}\,\nu$
Four-body decay described by **5 kinematic variables** (Cabibbo-Maksymowicz 1965): $S_\pi = M^2_{\pi\pi}$, $S_e = M^2_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$, ϕ	Four-body decay with two identical particles described by **3 kinematic variables:** $S_\pi = M^2_{\pi\pi}$, $S_e = M^2_{e\nu}$, $\cos\theta_e$

Ke4 Decay Amplitude

Ke4 Decay Amplitude: product of the weak current of the leptonic part and the (V-A) current of the hadronic part

$$\frac{G_F}{\sqrt{2}} V_{us}^* \bar{u_\nu} \gamma_\lambda (1-\gamma_5) v_e \langle \pi^+\pi^- | V^\lambda - A^\lambda | K^+ \rangle$$

where:

$$\langle \pi^+\pi^- | A^\lambda | K^+ \rangle = -\frac{i}{m_K} (F(\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})^\lambda + G(\mathbf{p}_{\pi^+} - \mathbf{p}_{\pi^-})^\lambda + R(\mathbf{p}_e + \mathbf{p}_\nu)^\lambda)$$

$$\langle \pi^+\pi^- | V^\lambda | K^+ \rangle = -\frac{H}{m_K^3} \epsilon^{\lambda\mu\rho\sigma} (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-} + \mathbf{p}_e + \mathbf{p}_\nu)_\mu) \times (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})_\rho)(\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})_\sigma)$$

- **p is the 4-momentum of each particle,**
- **F, G, R are 3 axial-vector complex Form Factors**
- **H is one vector complex Form Factor**

N.B. *in Ke4 decay rates R is multiplied by the squared lepton mass hence it is negligible*

➔ F, G, R and H Form Factors (FF) depend on the decay Lorentz invariants and are needed to describe data

Ke4 Formalism

Ke4 hadronic current described by the complex Hadronic FFs: F, G, H

Hadronic FFs: can be expressed as partial wave expansion of the decay amplitude into S- and P-waves *[Pais-Treiman PR168 (1968) 1858]* **with unique phases δs and δp** *[Watson theorem]*

2 Complex Axial-Vector FF: $F = F_s\, e^{i\delta s} + F_p\, e^{i\delta p} \cos\theta_\pi$, $G = G_p\, e^{i\delta p}$

1 Complex Vector FF: $H = H_p\, e^{i\delta p}$

$K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$

Fit the distribution of the **C-M** variables in the **five-dimensional space** with **4 real FFs and 1 phase δ** assuming identical phase for P-wave FFs

$$F_s, F_p, G_p\ H_p \text{ and } \delta = \delta_s - \delta_p$$

- ➔ 10 statistically independent fits (1 per each $S_\pi = M^2_{\pi\pi}$ bin) of the 5 variables
- ➔ 5-dimensional equi-populated boxes analyzed separately

$K^\pm \rightarrow \pi^0\pi^0 e^\pm \nu$

Dipion $\pi^0\pi^0$ system in S-wave state: only one complex axial FF simmetric in $\pi^0\pi^0$ exchange $F = F_s\, e^{i\delta s}$

Fit the distributions of the S_π, S_e variables in the **2-dimensional space** with **1 real FF F_s**

- ➔ use a grid of statistically independent boxes in the **(S_π, S_e) plane (Dalitz Plot)**
- ➔ Dalitz Plot density proportional to F_s^2

Final results: FFs and BR

Ke4(+−)

Eur. Phys. J. C70 (2010) 635
Phys. Lett. B715 (2012) 105

Ke4(00)

NEW

ArXiv 1406.4749v1, CERN-PH-EP-2014-145
(accepted for publication in JHEP)

Ke4(+−) Form Factors: fit result

- **Ke4(+−) relative FFs:** values normalized to the overall scale factor f_s (S-wave axial-vector form factor $F_s(q^2=0, S_e=0)$)
- **FFs energy dependence:** assuming isospin symmetry, FFs can expressed as Taylor series expansion of the dimensionless invariants:

$$q^2 = S_\pi/(4m_\pi^2) - 1 \quad \text{and} \quad S_e/(4m_\pi^2)$$

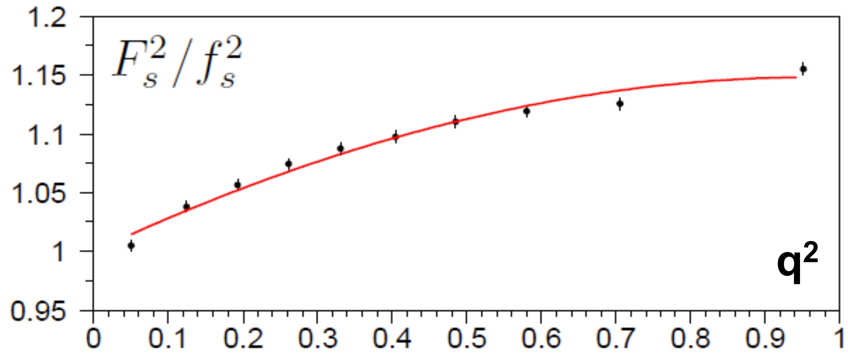
$$F_p = f_s\, f_p/f_s$$
$$G_p = f_s\,(g_p/f_s + g_p'/f_s\, q^2)$$
$$H_p = f_s\, h_p/f_s$$

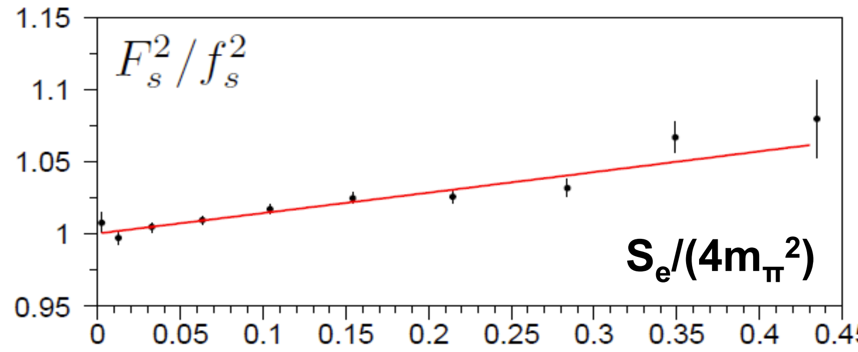
$$F_s^2 = f_s^2\,(1 + f_s'/f_s\, q^2 + f_s''/f_s\, q^4 + f_e'/f_s\, S_e/(4m_\pi^2))^2$$

FF	Value	Stat	Syst
f_s'/f_s	0.152	±0.007	±0.005
f_s''/f_s	-0.073	±0.007	±0.006
f_e'/f_s	0.068	±0.006	±0.007
f_p/f_s	-0.048	±0.003	±0.004
g_p/f_s	0.868	±0.010	±0.010
g_p'/f_s	0.089	±0.017	±0.013
h_p'/f_s	-0.398	±0.015	±0.008

First evidence!

[Eur.Phys. C70 (2010) 635]

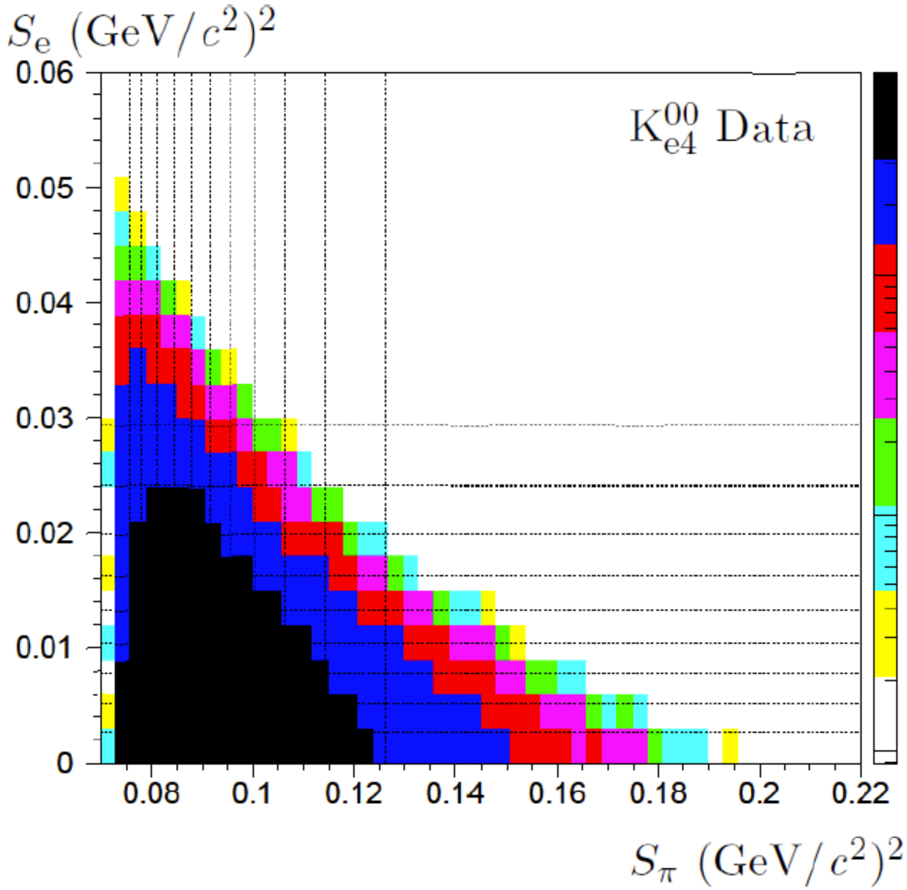


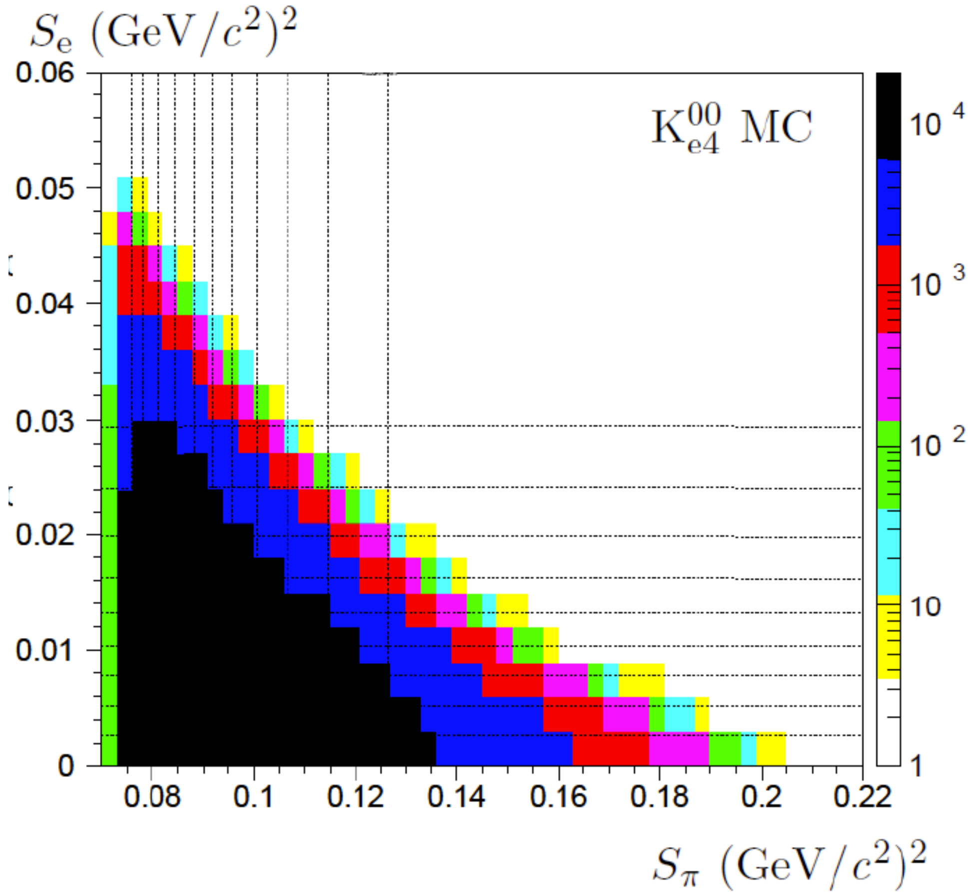


Ke4(00) FF: the (S_π , S_e) plane

Ke4(00) candidates
~65k events
(~1% background subtracted)

Ke4(00) MC data
(constant Fs)
10M simulated events



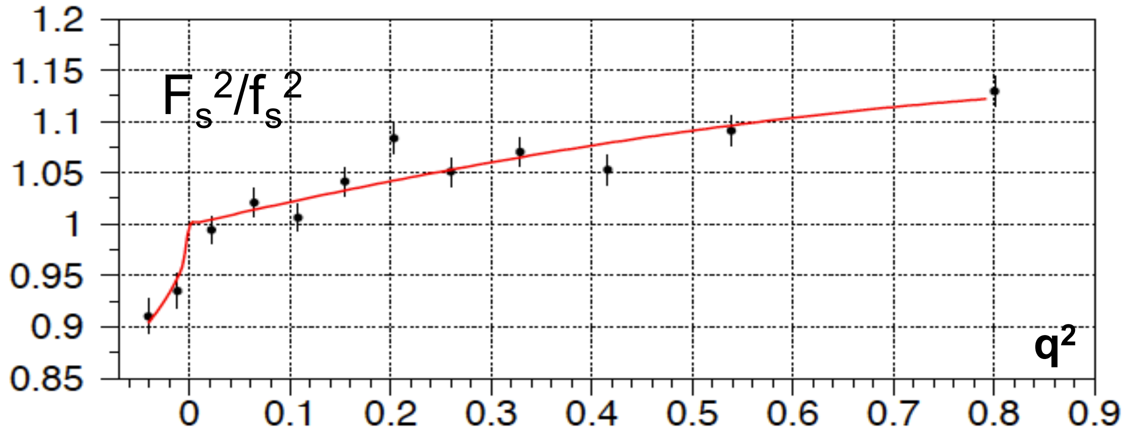


NEW: ArXiv 1406.4749v1, CERN-PH-EP-2014-145 (accepted in JHEP)

Ke4(00) Form Factor: fit result

$$F_s^{00} = f_s^{00}\,(1 + a q^2 + b q^4 + c y^2) \qquad q^2 \geq 0$$

$$F_s^{00} = f_s^{00}\,(1 + d\,\underbrace{\sqrt{|q^2| / (1 + q^2)}}_{\text{cusp-like function}} + c y^2) \qquad q^2 < 0$$



$F_s(q^2, S_e)$	Value	Stat	Syst
a	0.149	±0.033	±0.014
b	-0.070	±0.039	±0.013
c	0.113	±0.022	±0.007
d	-0.256	±0.049	±0.016
X^2/ndf	101.4/107 (Probability = 63%)		

Relative FF final result

FF expressed as series expansion of the dimensionless variables:

$q^2 = S_\pi/(4m_{\pi+}^2) - 1$

$y^2 = S_e/(4m_{\pi+}^2)$

q^2 > 0: series expansion in q^2 and y^2 as in the Ke4(+−) mode

q^2 < 0: cusp-like function and linear y^2 dependence

→first FF measurement

→deficit below $\pi^+\pi^-$ threshold compatible with final state $\pi\pi$ re-scattering: cusp-like behavior with a threshold at $4m_{\pi+}^2$

→S_e depencence of F_s

NEW Final Result

ArXiv 1406.4749v1

CERN-PH-EP-2014-145

(accepted for publication in JHEP)

Ke4(00) FF interpretation

Interpretation in progress:

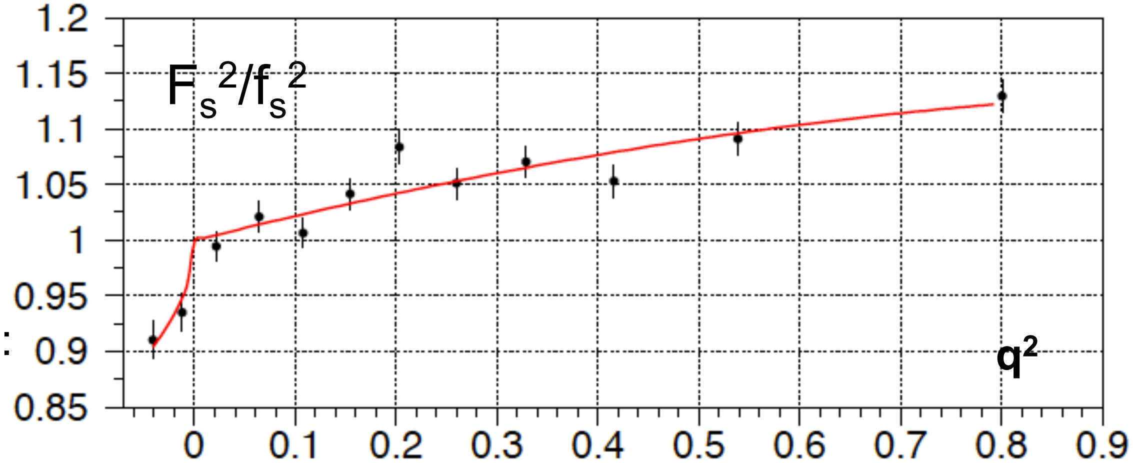
- naïve interpretation by analogy

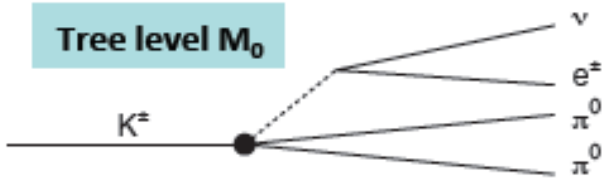
Hypothesis:

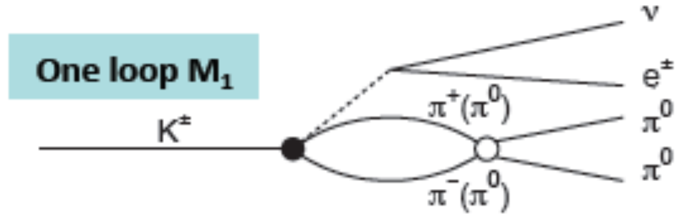
- charge exchange process $\pi^+\pi^-$ to $\pi^0\pi^0$ in the Ke4(+−) mode

  ➔ 1-loop calculation for K3π decays:

  N. Cabibbo, PRL 93(2004)121801







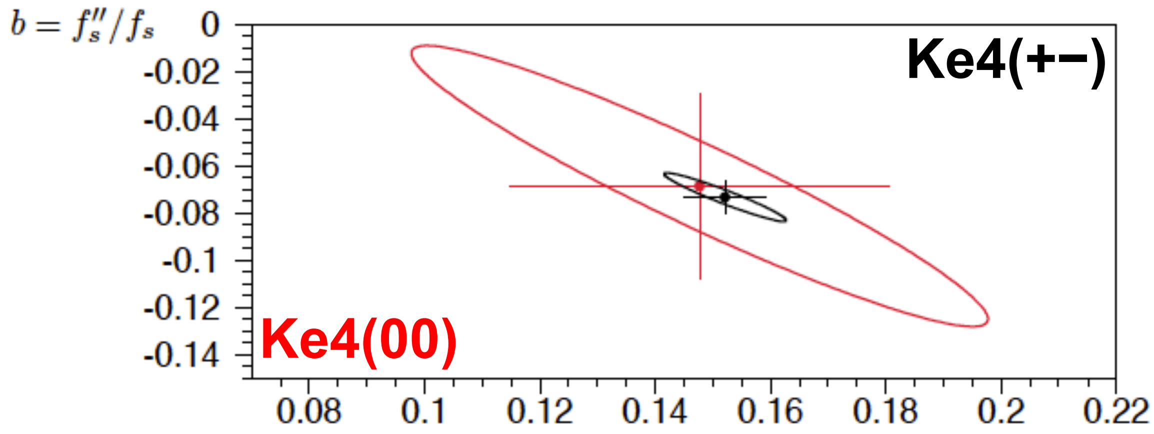
unperturbed amplitude	one loop contribution
$M_0 = F_s^{00}(q^2) = f_s(1+aq^2+b\,q^4+cS_e/4m_{\pi+}^2)$	$M_1 = -2/3\,(a^0_0-a^2_0)\,\sqrt{\lvert q^2/(1+q^2)\rvert}\,F_s^{+-}(q^2)$

$$|M|^2 = \begin{cases} |M_0 + i\,M_1|^2 = (M_0)^2 + (M_1)^2 & q^2>0,\text{ above } 2m_{\pi+} \text{ threshold} \\ |M_0+M_1|^2 = (M_0)^2+(M_1)^2 + 2\,M_0M_1 & q^2<0,\text{ below } 2m_{\pi+} \text{ threshold with destructive interference} \end{cases}$$

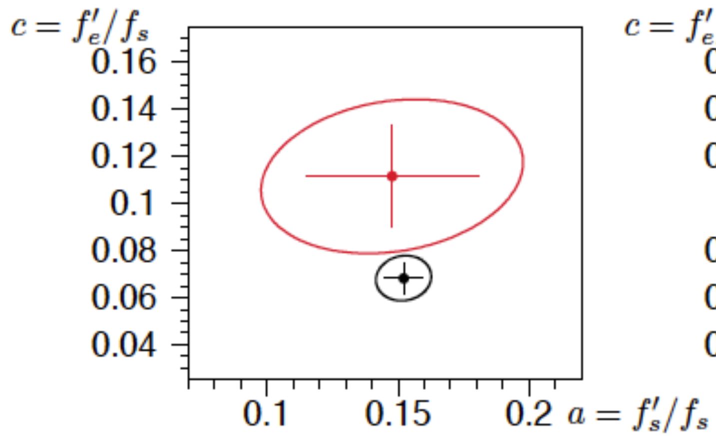
F_s form factors comparison ($q^2>0$)

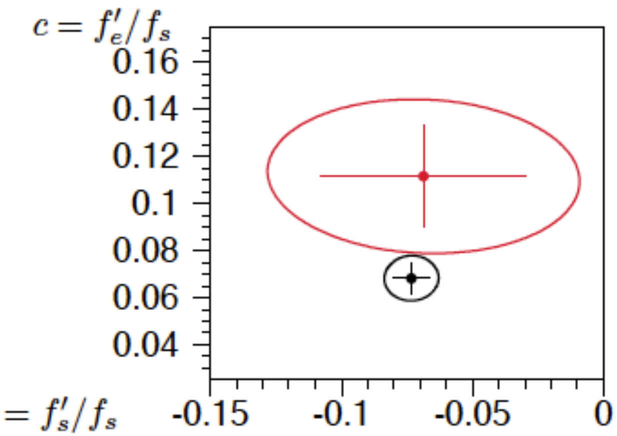
Ke4(+−): $F_s^{+-} = f_s^{+-} + f_s' q^2 + f_s'' q^4 + f_e' y^2)$

Ke4(00): $F_s^{00} = f_s^{00} (1+aq^2 + bq^4 + cy^2)$



$a = f_s'/f_s$





$b = f_s''/f_s$

- Similar $q^2 = S_\pi/(4m_{\pi+}^2)-1$ and $y^2 = S_e/(4m_{\pi+}^2)$ dependence
- Contours: 68% CL
- Statistical errors only
- Same correlations between fitted parameters
- Results consistent within statistical errors

[ArXiv 1406.4749v1, CERN-PH-EP-2014-145]

Ke4 Branching Ratio

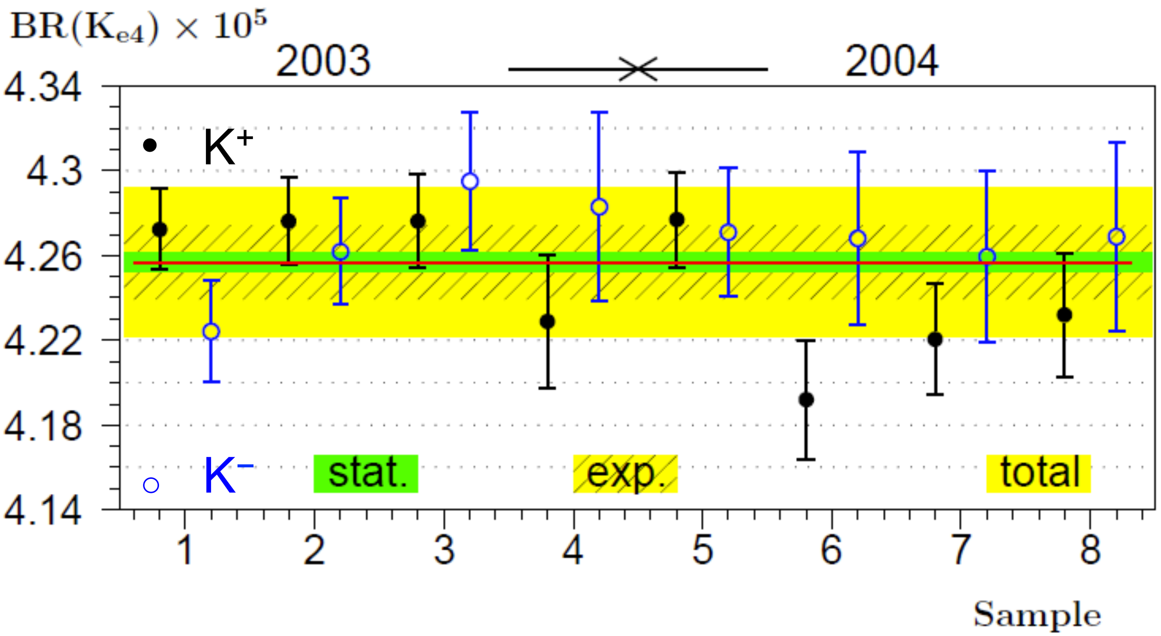
$$BR(K_{e4}) = [(N_s - N_b)/N_n] \cdot [A_n \varepsilon_n / A_s \varepsilon_s] \cdot BR(n)$$

BR(K_{e4}(+−))	Input quantities	BR(K_{e4}(00))
1.1 × 10^6	N_s = K_{e4} candidates	65210
0.95% × N_s	N_b = background to K_{e4}	1.00% × N_s
1.9 × 10^9 ($\pi^+\pi^+\pi^\pm$)	N_n = $K_{3\pi}$ candidates	($\pi^0\pi^0\pi^\pm$) 93.5 × 10^6
18.22%	A_s = Acceptance for K_{e4}	1.93%
24.18%	A_n = Acceptance for $K_{3\pi}$	4.05%
98.3%	ε_s = trigger efficiency for K_{e4}	96.1%
97.5%	ε_n = trigger efficiency for $K_{3\pi}$	97.4%
(5.59 ± 0.04)%	$BR(K_{3\pi})$ = normalization BR	(1.761 ± 0.022)%

Ke4(+−) Branching Ratio

Final measurement: 1.1 ×10^6 signal events, ~0.6% background
use $K^\pm \to \pi^+\pi^-\pi^\pm$ channel for normalization

BR(Ke4(+−) = $(4.257 \pm 0.004_{stat} \pm 0.016_{syst} \pm 0.031_{ext}) \times 10^{-5}$



Systematic uncertainties (%)	δBR/BR	$\delta f_s/f_s$
Acceptance	0.18	0.23
Muon vetoing	0.16	0.08
Accidental	0.21	0.10
Trigger efficiency	0.11	0.06
Particle-ID	0.09	0.05
Background	0.07	0.03
Radiative effects	0.08	0.06
External sources	0.72	0.54

PDG 2012 BR = $(4.09 \pm 0.10) \times 10^{-5}$ (2.4% precision) [PL B715 (2012) 105]

➔ world average precision improved by a factor 3

➔ 0.8% relative uncertainty, dominated by external errors (0.7%)

Absolute Form Factor can be obtained from the BR measurement

Ke4(+−) Absolute Form Factors

The Ke4 BR measurement allows to translate relative Form Factors (obtained by fitting) into absolute Form Factors

$$BR(Ke4\pm) = \tau_{K\pm} \, |V_{US}|^2 f_s^2 \int d\Gamma_5 / (|V_{US}| f_s)^2$$

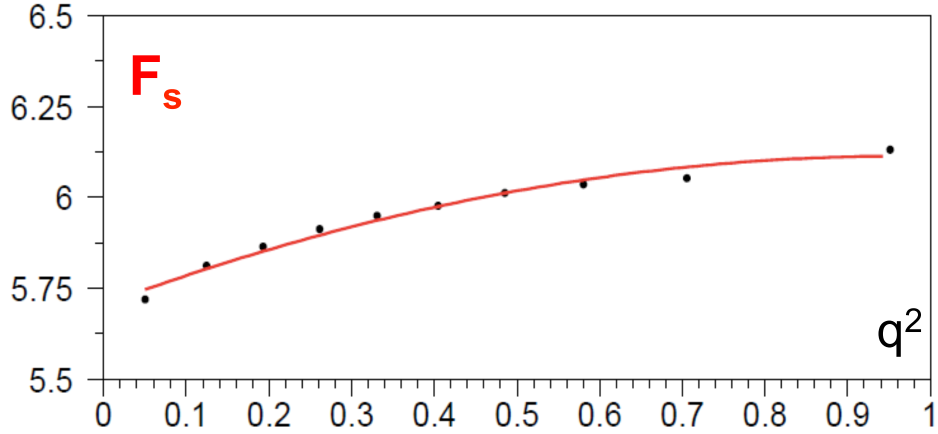
$|V_{US}| = 0.2252 \pm 0.0009$ (PDG 2012)
$\tau_{K\pm} = (1.2380 \pm 0.0021) \times 10^{-8}$ s

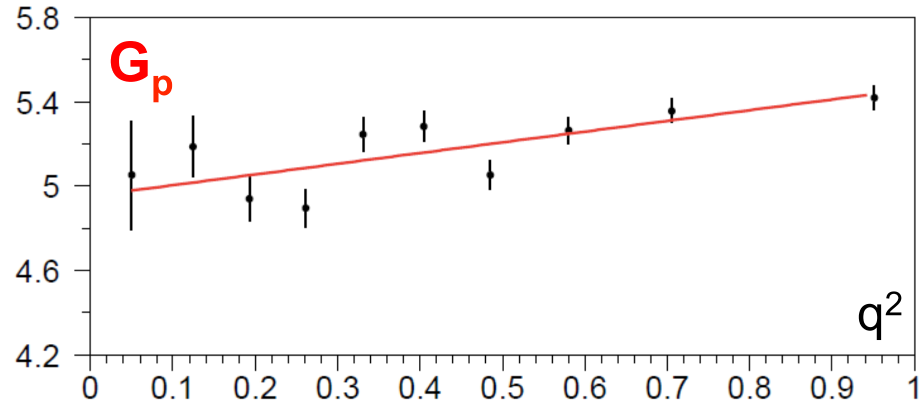
$$|V_{US}| \times f_s = 1.285 \pm 0.004_{exp} \pm 0.005_{ext}$$
$$f_s = 5.705 \pm 0.017_{exp} \pm 0.031_{ext}$$

$f_s = 5.705 \pm 0.003_{stat} \pm 0.017_{syst} \pm 0.031_{external}$

$= 5.705 \pm 0.035$ ➔ **0.6% relative precision**

f_s'	=	$0.867 \pm 0.040_{stat} \pm 0.029_{syst} \pm 0.005_{norm}$
f_s''	=	$-0.416 \pm 0.040_{stat} \pm 0.034_{syst} \pm 0.003_{norm}$
f_e'	=	$0.388 \pm 0.034_{stat} \pm 0.040_{syst} \pm 0.002_{norm}$
f_p	=	$-0.274 \pm 0.017_{stat} \pm 0.023_{syst} \pm 0.002_{norm}$
g_p	=	$4.952 \pm 0.057_{stat} \pm 0.057_{syst} \pm 0.031_{norm}$
g_p'	=	$0.508 \pm 0.097_{stat} \pm 0.074_{syst} \pm 0.003_{norm}$
h_p	=	$-2.271 \pm 0.086_{stat} \pm 0.046_{syst} \pm 0.014_{norm}$



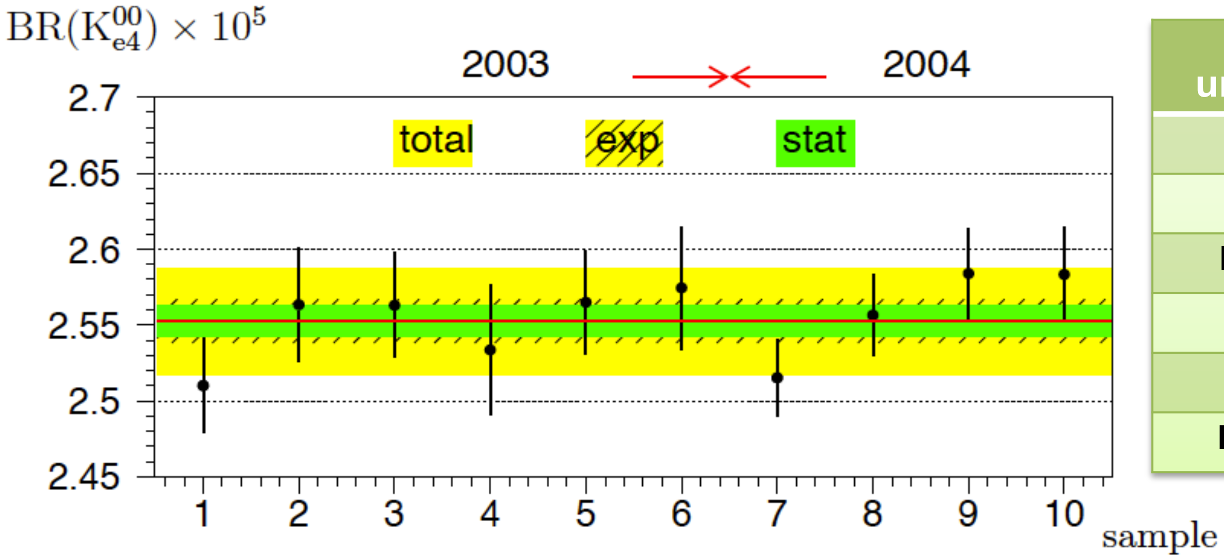


[PL B715 (2012) 105]

Ke4(00) Branching Ratio

Final measurement: 65210 signal events, (1.00±0.02)% background
use $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ channel for normalization

BR(Ke4(00) = $(2.552 \pm 0.010_{stat} \pm 0.010_{syst} \pm 0.032_{ext}) \times 10^{-5}$



Systematic uncertainties (%)	δBR/BR	$\delta f_s/f_s$
Acceptance	0.23	0.42
Trigger	0.05	0.03
Radiative effects	0.19	0.14
Particle-ID	0.25	0.13
Background	0.02	0.01
External sources	1.25	0.75

PDG 2012 BR = $(2.2 \pm 0.4) \times 10^{-5}$ (18% precision)

➔ **world average precision improved by more than 1 order of magnitude**

➔ **1.4% relative uncertainty, dominated by external errors (1.25%)**

[ArXiv 1406.4749v1, CERN-PH-EP-2014-145]

Absolute FF value ($|V_{us}|$ from PDG 2012; δ_{EM}= long distance elm corrections to total rate):
$(1+\delta_{EM}) \times |V_{us}| \times f_s = 1.369 \pm 0.007_{exp} \pm 0.009_{ext}$, $(1+\delta_{EM}) \times f_s = 6.079 \pm 0.030_{exp} \pm 0.046_{ext}$

N.B. Absolute FF value in Ke4(00) shows some tension with f_s in Ke4(+−) mode: detailed Ke4 radiative corrections not available yet, could need more refined modeling of isospin breaking effects

$\pi\pi$ **scattering** from **K3π** and **Ke4** decays

$\pi\pi$ scattering lengths from K decays

$K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$
- large BR (1.7%) → 60 Millions of events
- 3 pions: $\pi^0\pi^0$ system (S-wave) + nearby hadron
→ final state interactions
- accessible $M_{\pi\pi}$ range from $\pi^0\pi^0$ threshold to $(M_K - M_\pi)$

$K^{\pm} \rightarrow \pi^+\pi^- e^{\pm} \nu$
- small BR (~ 4×10^{-5}) → 1.1 Millions of events
- 2 pions: $\pi^+\pi^-$ (measure difference between S- and P-wave)
→ clean hadronic environment
- accessible $M_{\pi\pi}$ range from $\pi^+\pi^-$ threshold to $(M_K - M_e) \cong M_K$

Two different but complementary approaches to $\pi\pi$ scattering near threshold used to extract S-wave scattering lengths (a0, a2) for Isospin I=0 and I=2

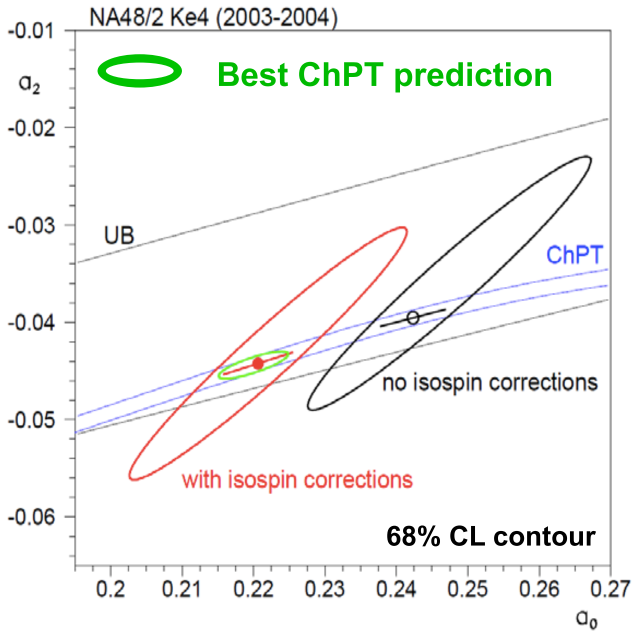
Combination of these results:

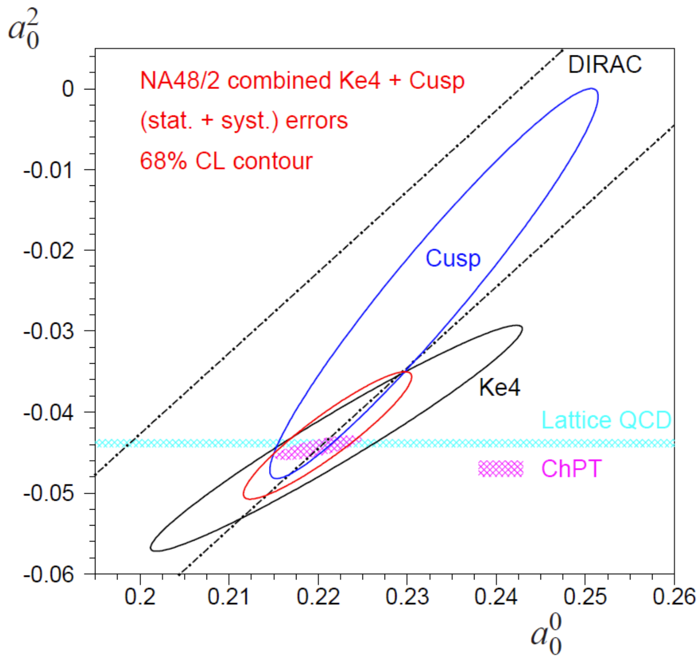
- **improved determination of a0 and first precise measurement of a2**
- **new inputs to low energy QCD calculations**
- **stringent tests of existing predictions from ChPT and lattice QCD calculations**

Ke4: from phase shift to scattering lengths

Ke4: $\pi\pi$ scattering lengths can be obtained from the phase shift $\delta = \delta_s - \delta_p$ measurement (Roy equations)

Isospin corrections related to mass effects are not negligible compared to the present statistical accuracy

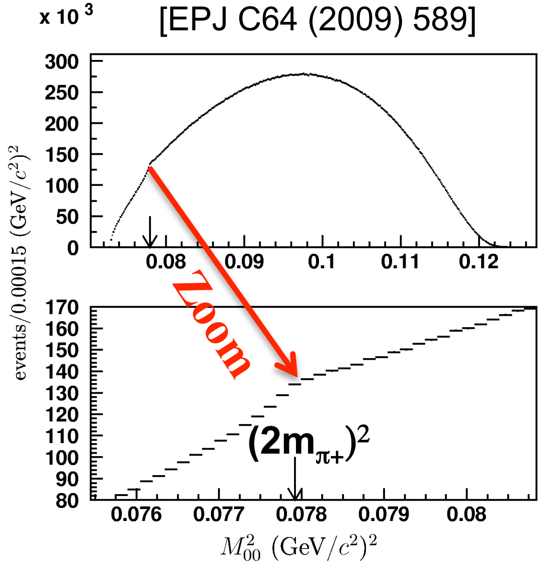




[EPJ C70 (2010) 635]

Cusp in $K^\pm \to \pi^\pm\pi^0\pi^0$

[EPJ C64 (2009) 589]



First evidence

- Cusp in $(M_{\pi 0 \pi 0})^2$ spectrum
- Rescattering at 1 and 2 loop + radiative corrections
- Fit to extract a_2 and a_0-a_2

NA48/2 combined Ke4 + Cusp results:

$a_0 = 0.2210 \pm 0.0047_{stat} \pm 0.0040_{syst}$

$a_2 = -0.0429 \pm 0.0044_{stat} \pm 0.0028_{syst}$

$a_0 - a_2 = 0.2639 \pm 0.0020_{stat} \pm 0.0015_{syst}$

➔ **Data in very good agreement with ChPT and Lattice QCD calculations**

DIRAC Pionium scattering lengths: Phys.Lett. B704 (2011)
ChPT prediction Colangelo,Gasser,Leutwyler: Phys.Lett. B488 (2000)
Lattice QCD: ETM Phys.Lett. B684 (2010); NPLQCD Phys.Rev. D77 (2008)

Conclusions

- **NA48/2** performed high precision measurements with a very large statistics of **Ke4 and K3π** events collected in 2003 and 2004, bringing new remarkable input to low energy QCD calculations:
  - ➔ **1.1 x 10^6** reconstructed **Ke4(+−)** candidates with ~**0.6% background** level
  - ➔ **65 x 10^3** reconstructed **Ke4(00)** candidates with ~**1% background** level

- **Improved measurements of branching ratios**
  - ➔ **Ke4(+−) most precise BR** (0.8% relative precision), 3 times better than PDG
  - ➔ **Ke4(00) most precise BR** (1.4% relative precision), 13 times better than PDG

- **Form Factors**
  - ➔ **Ke4(+−) absolute Form Factor determination**
    - *evidence for small negative F_p contribution and F_s dependence on S_e*
  - ➔ **Ke4(00) first Form Factor determination**
    - *significant dependence on q^2 and S_e*
    - *evidence for re-scattering effects in the final state*

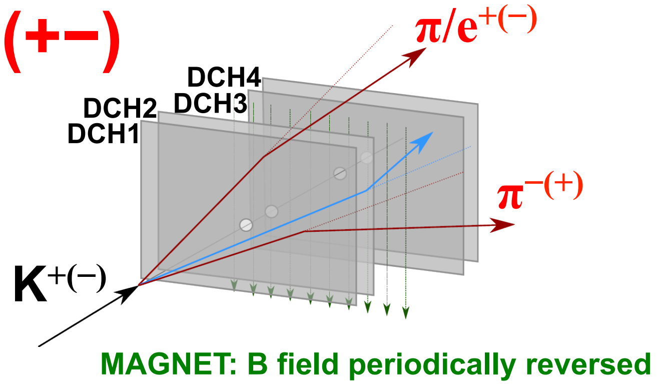
- **Converging results on ππ scattering lenght a0 and a2**
  - ➔ **Ke4(00): cusp-like structure in S_π** (work in progress)
  - ➔ Very good **agreement with ChPT and Lattice QCD** calculations

- **NA62 will start taking data at fall 2014**
  - ➔ We may be able to collect ~**10 times the present Ke4(00) statistics** in 2015-17

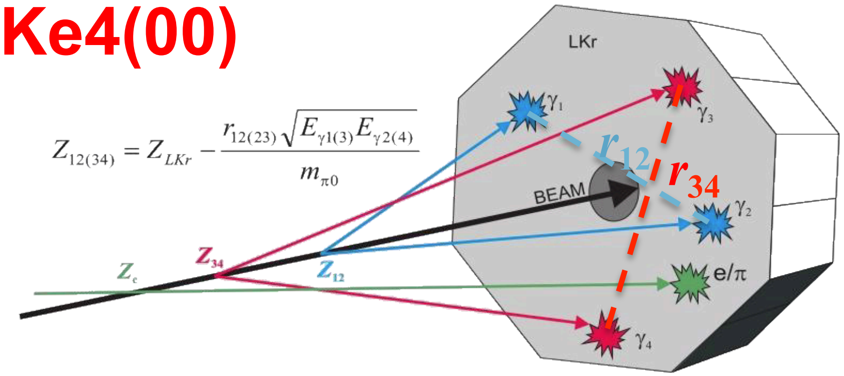
SPARES

Ke4 Event Selection

Ke4(+−)



Ke4(00)



Decay mode for normalization: $K^{\pm}\rightarrow\pi^{\pm}\pi^{+}\pi^{-}$

Event Reconstruction:

- 3 tracks in the magnetic spectrometer
- vertex within the decay volume
- opposite charge sign $\pi\pi$
- 1 electron (E/p $\sim$ 1)
- No MUV hit associated with tracks.

Background:

- $K^{\pm}\rightarrow\pi^{\pm}\pi^{+}\pi^{-}$ with fake-electron or $\pi^{\pm}\rightarrow e^{\pm}\nu$ decay
- Accidental track
- $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}(\pi^{0})$ followed by $\pi^{0}\rightarrow e^{+}e^{-}\gamma$ decay with fake-π and undetected γ

Data statistics (2003-2004):

- 1.1×10^{6} candidates: 0.7×10^{6} K^{+}, 0.4×10^{6} K^{-}
- $\sim$0.6% background level

Decay mode for normalization: $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$

Event Reconstruction:

- 2 LKr cluster pairs (γ1, γ2) and (γ3, γ4), in time (±2.5 ns), with Energy > 3 GeV, satisfying π^{0} mass constraint
- decay positions:
  - $|Z_{12} - Z_{34}| < 500$ cm,
  - $Z_n = (Z_{12} + Z_{34})/2$ within decay volume
- combined with charged track:
  - Z_c at CDA of charged track to the beam line within $|Z_c - Z_n| < 800$ cm

Background:

- $K_{3\pi}$ with fake-electron or $\pi^{\pm}\rightarrow e^{\pm}\nu$ decay
- Accidental track/photon

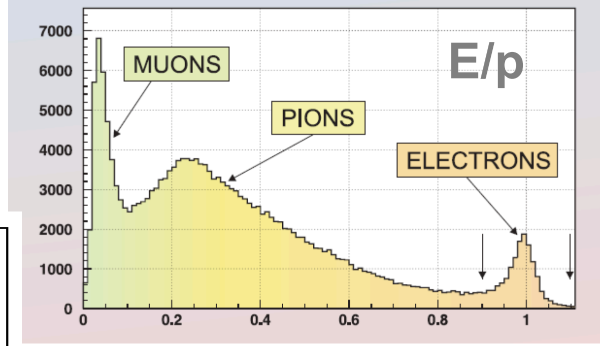
Data statistics (2003-2004)

- 65210 Ke4(00) candidates
- (1.00±0.02)% background level

Ke4 Event Selection - II

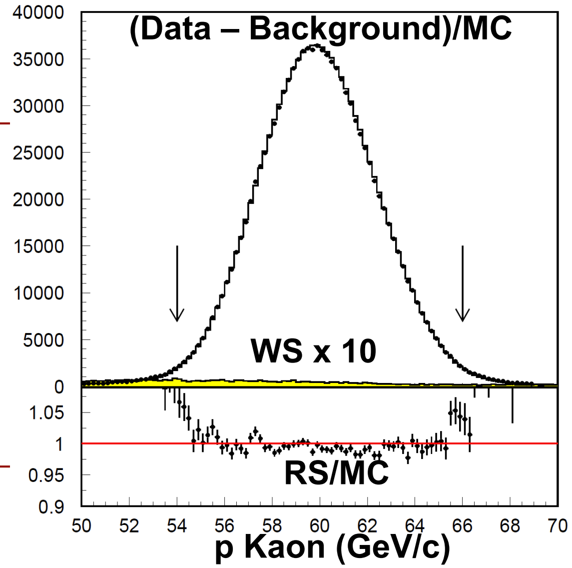
Particle Identification

- ***electron-ID:*** *0.9 < E/p < 1.1 + shower-shape properties*
- ***charged pion-ID:*** *(Ke4(+−) mode) E/p < 0.8*



Kaon momentum reconstruction:

- *energy-momentum conservation hypothesis*
- *assume K mass and missing neutrino*
- *get the two solutions for K momentum*
- *pick the closest to the nominal beam momentum (60 GeV/c)*



Main Background $K^{\pm}\rightarrow\pi^{\pm}\pi^{+}\pi^{-}$

Right Sign events

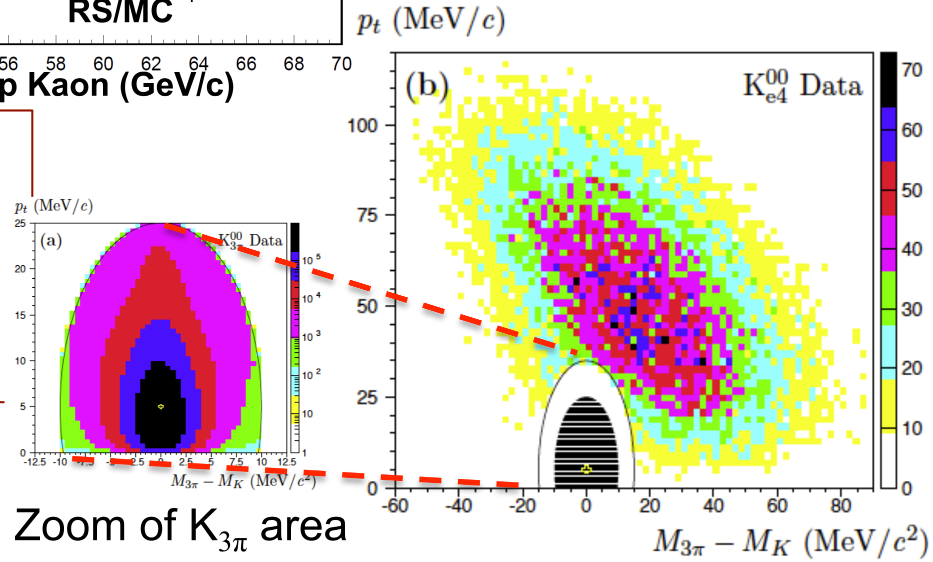
RS: $e^{+}\pi^{+}\pi^{-}$

Wrong Sign events

WS: $e^{-}\pi^{+}\pi^{+}$

Signal/normalization kinematic separation:

- *Compute the invariant mass and p_t (wrt the beam line) in pion hypothesis.*
- *An elliptic cut separates 1.9 (0.1) billions $K_{3\pi}$ candidates from 1.1 (0.07) millions Ke4(+−) (Ke4(00)) candidates.*



Zoom of $K_{3\pi}$ area

Ke4(+−): BR and FFs

K⁻ first measurement

$BR(K_{e4}(+)) = (4.255 \pm 0.008) \times 10^{-5}$ $BR(K_{e4}(-)) = (4.261 \pm 0.011) \times 10^{-5}$

BR (Ke4) = (N4 −Nbkg)/ N3 × A3/A4 × ε(K3π) /ε(Ke4) × BR (K3π)

(5.59 ± 0.04) %

N4 Signal events	1.1×10^6
Nbkg background events	2 × 5276 (<1% relative)
N3 normalization events K3π ($\pi^\pm \pi^+ \pi^-$)	1.9×10^9
A3 normalization Acceptance	23.97%
A4 signal Acceptance	18.19%
ε (K3π) normalization trigger eff	97.65 %
ε (Ke4) signal trigger eff	98.52%

Relative systematic uncertainty	%
Acceptance, beam geom.	0.18
Muon vetoing	0.16
Accidental activity	0.21
Particle ID	0.09
background	0.07
Radiative effects	0.08
Trigger efficiency	0.11
Simulation statistics	0.05
Total systematics	0.37
External error [Br(K3π)]	0.72

Correlation:

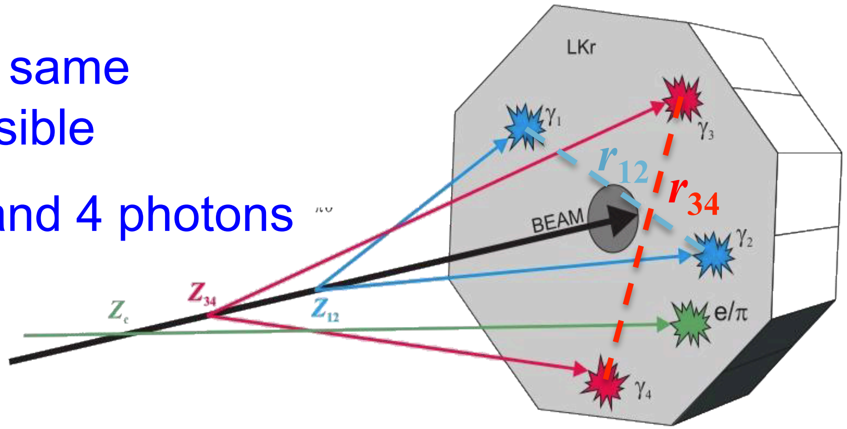
	f''_s/f_s	f'_e/f_s		g_p/f_s
f'_s/f_s	-0.954	0.080	g'_p/f_s	-0.914
f''_e/f_s		0.019		

Ke4(00) event selection

$K^\pm \to \pi^0\pi^0\pi^\pm$ (K3π): normalization channel
➔ Ke4 and K3π samples collected concurrently: same trigger logic and common selection as far as possible

Final states reconstructed from 1 charged track and 4 photons forming two π^0 pointing to the same decay vertex

Reconstruct the $\pi^\pm\pi^0\pi^0$ invariant mass assuming the charged track to be a pion

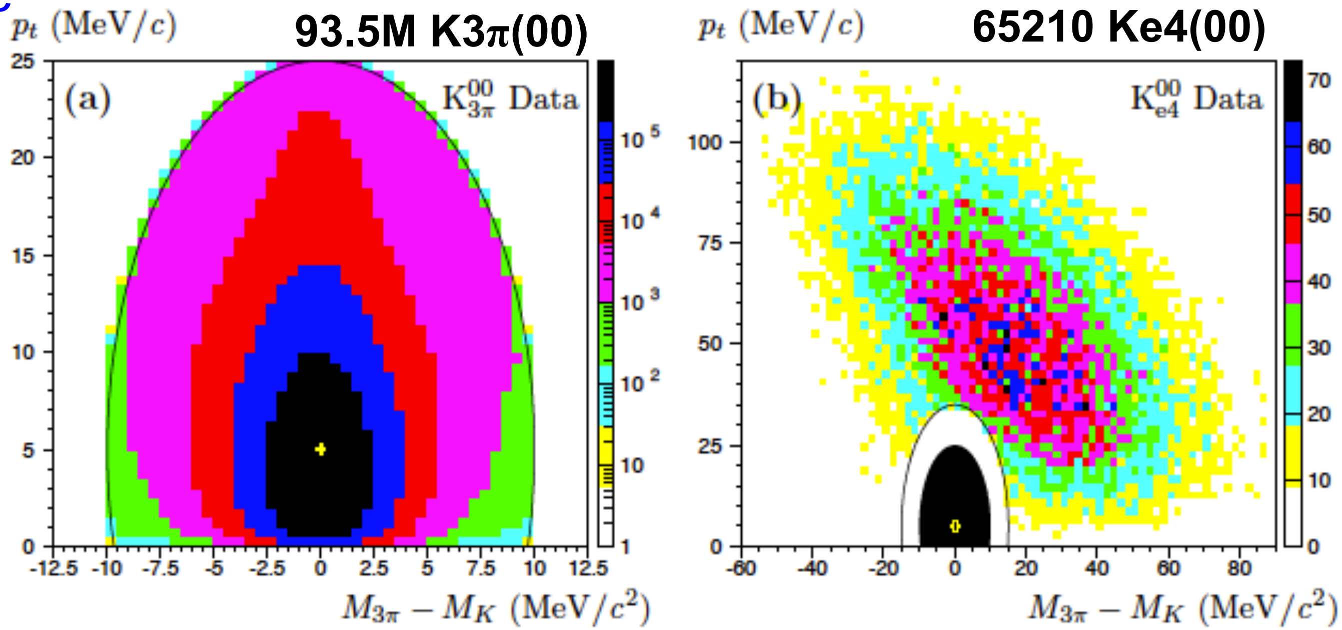


In the plane **pt vs (M3π – MK)** the Ke4(00) candidates are well separated from K3π events (small pt and Kaon PDG mass)

Pt(GeV/c) vs (M3π — MK) (GeV/c2)

93.5M K3π(00)

65210 Ke4(00)



Ke4(00) background:

- K3π with fake-electron or $\pi^\pm \to e^\pm\nu$ decay
- Accidental track/photon
- (1.00±0.02)% bkgd level

Ke4(00) FF and BR: measurement principle

- Ke4(00) rate measured wrt to $K^\pm \to \pi^0\pi^0\pi^\pm$ (K3π(00)) normalization channel as:

$$\Gamma(\text{Ke4(00)}/\Gamma(\text{K3}\pi(00)) = \text{BR(Ke4(00)/BR(K3}\pi(00)) = \frac{N_s - N_B(s)}{N_n - N_B(n)} \cdot \frac{A_n \times \varepsilon_n}{A_s \times \varepsilon_s}$$

N_s, N_n: numbers of signal and normalization candidates
$N_B(s)$, $N_B(n)$: numbers of background events in signal and normalization samples
A_s, ε_s, A_n, ε_n: geometric acceptances and trigger efficency for signal and normalization samples

- $BR^\pm$ (K3π(00)) from PDG
- Samples collected concurrently: same trigger logic and common selection as far as possible
  - ➔ partial cancellation of systematic effects
- Geometrical acceptances non uniform over the kinematical space, depend on the dynamics characterizing each decay is needed
  - ➔ detailed study of Ke4(00) FF in the kinematical space accessible thanks to the available statistics
- Geometrical acceptances and trigger efficiency non uniform over the whole dta sample due to different data taking conditions
  - ➔ 10 independent sub-samples recorded with stable conditions are analyzed separately and statistically combined to obtain BR

Ke4(00): BR and FF

$BR(Ke4) = (N4 - Nbkg)/ N3 \times A3/A4 \times \varepsilon(K3\pi) / \varepsilon(Ke4) \times BR(K3\pi)$

(1.761 ± 0.022) %

(1% relative)

N4 Signal events	65210
Nbkg background events	650
N3 normalization events $K3\pi$ ($\pi^{\pm}\pi^{+}\pi^{-}$)	93.5×10^6
A3 normalization Acceptance	4.05%
A4 signal Acceptance	1.92%
$\varepsilon(K3\pi)$ normalization trigger eff	97.4%
$\varepsilon(Ke4)$ signal trigger eff	96.1%

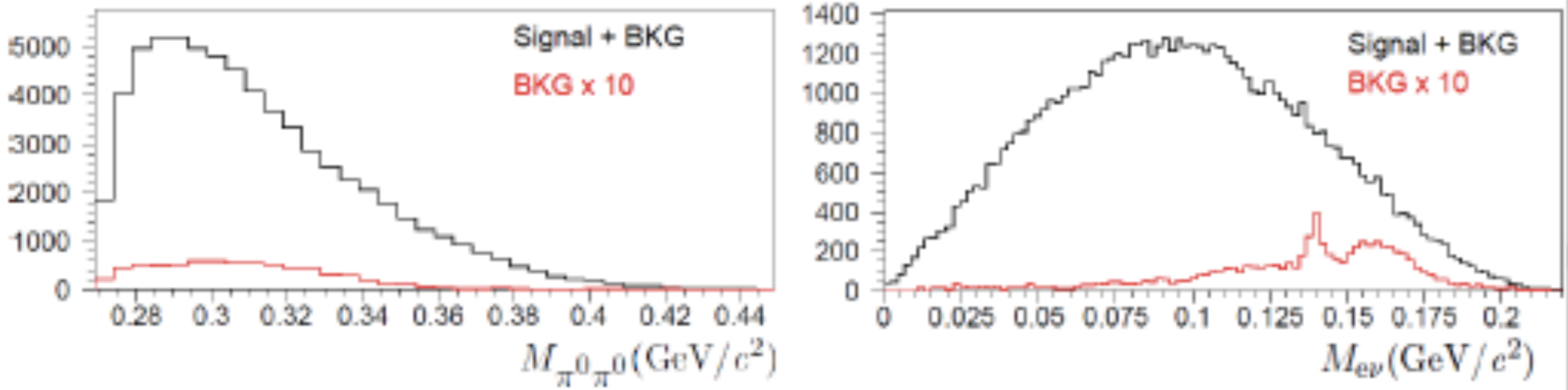
Source	δa	δb	δc	δd
Background control	0.0140	0.0122	0.0062	0.0164
Radiative events modeling	0.0037	0.0035	0.0033	0.0013
Fit procedure	–	–	–	–
Reconstruction/resolution	–	–	–	–
Trigger simulation	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Acceptance control	–	–	–	–
Total systematics	0.014	0.013	0.007	0.016
Parameter	a	b	c	d
Value	0.149	-0.070	0.113	-0.256
Statistical error	0.033	0.039	0.022	0.049

Systematic Uncertainty (% to Br value)	
Acceptance	0.15
Form Factor	0.17
Background	0.25
Trigger cut	0.04
Radiative effects	0.20
Simulation statistics	0.09
Trigger efficiency	0.03
Total	0.40

External error ~1.25%
Statistical error ~ 0.39%

Ke4(00): FF measurement

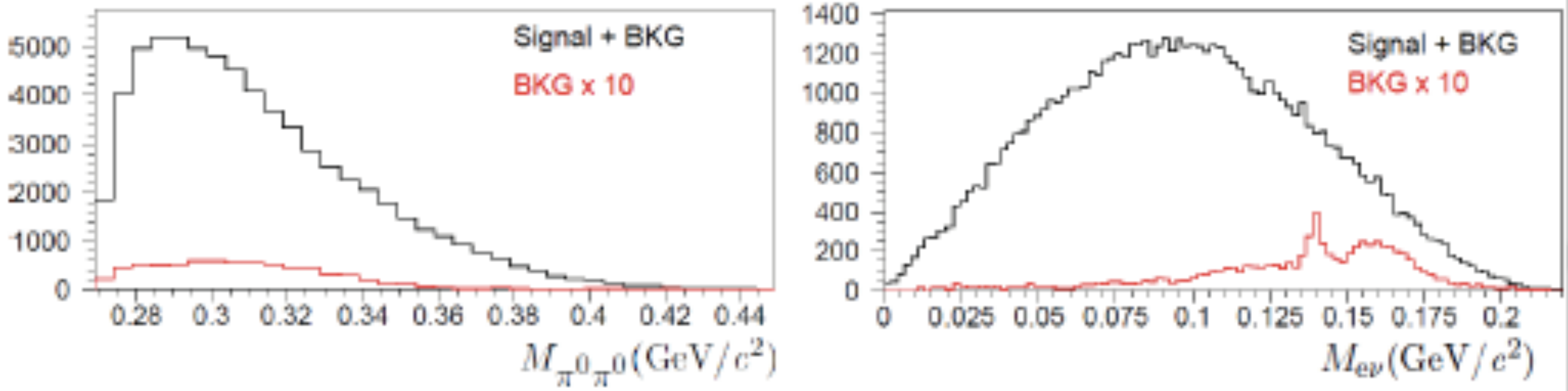
- Differential rate in the ($S\pi$,Se) plane is proportional to $|Fs|^2$
- Subtract background in the 2d-plane



- Compare to the same distribution obtained from simulation including acceptance, resolution, trigger efficiency, radiative corrections (real photon emission at decay vertex) and kinematic factors but using a constant form factor
- switch to dimensionless variables: $q2 = S\pi/4m^2_{\pi+} - 1$, $Se/4m^2_{\pi+}$
- Define a grid of 10 equal population bins in q2 above q2=0 ($2m_{\pi+}$ threshold) and two equal population bins below (10 bins with 6000 events each, 2 bins with 3000 events each), 10 bins in Se (300 or 600 events in 2d-bins)

Ke4(00): FF measurement

- Differential rate in the ($S\pi$,Se) plane is proportional to $|Fs|^2$
- Subtract background in the 2d-plane



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Ke4(00): FF fit procedure

2d fit function:

$G = N\,(1 + a\,X + b\,X^2 + c\,Y)^2$ X > 0, above treshold

$G = N\,(1 + d\,(|X/(1+X)|)^{1/2} + c\,Y)^2$ X < 0, below treshold

Dimensionless variables:

$X = q^2 = S_\pi/(4m^2_{\pi+}) - 1$

$Y = S_e/(4m^2_{\pi+})$

To minimize:

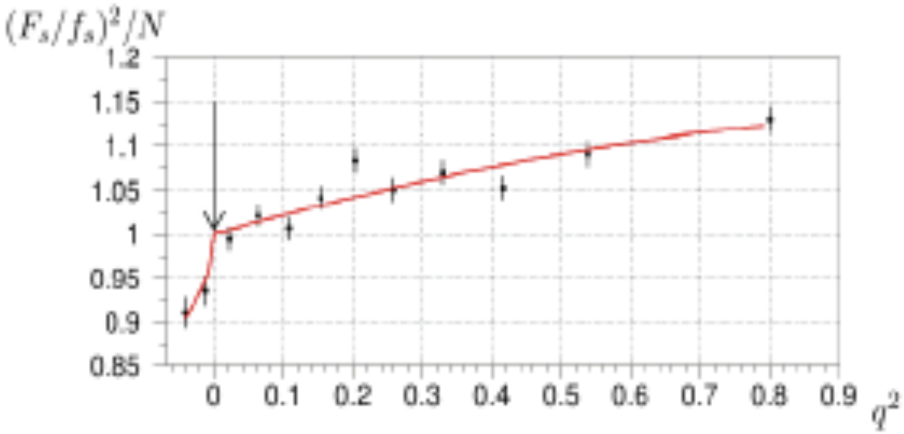
$$\chi^2 = \sum_{i=1}^{12}\sum_{j=1}^{10}((n_{ij}/m_{ij} - G(X_i, Y_j, \hat{p}))/\sigma_{ij})^2$$

n_{ij} = Data - BGR

m_{ij} = MC with F_s=1

X_i, Y_j are the barycenters of the bin ij.

fit parameters = a,b,c,d



We observe the cusp-like behavior of Form Factor S_π dependence with a threshold at $4m^2_{\pi+}$.

Ke4(00) BR and FF: experiments and theory

Previous experiments had very low statistics (PDG 2012)

37 events from 3 experiments: BR = $(2.2 \pm 0.4)10^{-5}$ (18% rel. error)
214 events from KEK E470 (not considered): BR = $(2.29 \pm 0.34)10^{-5}$ (large syst.)

No form factor determination so far, just a relation between partial rate and a constant form factor value :

$$\Gamma = 0.8|V_{us} \cdot F|^2 10^3 \mathrm{s}^{-1}$$

Using the kaon mean life time $(1.2380 \pm 0.00021)\ 10^{-8}$ s, it translates to
$|Vus| . F = 1.49 \pm 0.13$ or $F = 6.61 \pm 0.58$ for $|Vus| = 0.2252 \pm 0.0009$

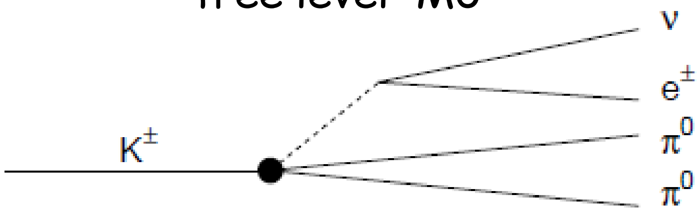
Theoretical predictions :

Isospin symmetry (mu=md= 0, αQED = 0) predicts a relation between rates
Γ(Kl4 +-) = $\frac{1}{2}\Gamma$ (Kl4 0±) + 2 Γ (Kl4 00) (valid for lepton = e,μ)
$K^\pm$ (2.4% now 0.8%) K^0 (2.1%) $K^\pm$ (18%)
ChPT calculations O(p2,p4,p6) from Bijnens Colangelo Gasser (*NPB 427 (1994) 427*)
using available 1977 Ke4(+-) form factors predict :
BR(Ke400) = $(2.01 \pm 0.11)\ 10^{-5}$ (~5% precision)

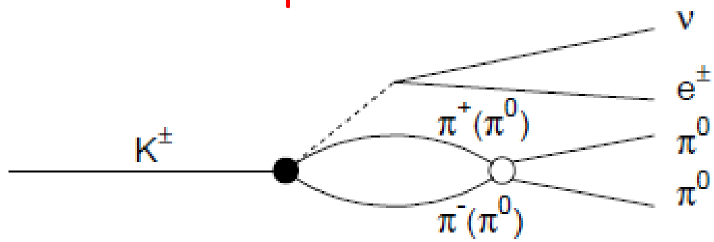
Ke4(00) FF below $q^2 = 0$

The 10% drop (cusp-like) for q2<0 can be interpreted as final state charge exchange scattering in the Ke4(+-) mode :

tree level M0



1-loop M1



Follow papers by Cabibbo (PRL 93 (2004)) and Cabibbo-Isidori (JHEP 03 (2005)) to write the amplitudes :

$$\boxed{M_0 = f_s(1 + a\, q^2 + b\, q^4 + c\, S_e/4m^2_{\pi^+}),} \qquad \boxed{M_1 = -2/3\ (a_0^0 - a_0^2)\ f_s\ \sigma_\pi,}$$

$$q^2 = S_\pi/4m^2_{\pi^+} - 1 \text{ and } \sigma_\pi = \sqrt{1 - 4m^2_{\pi^+}/S_\pi} = \sqrt{|q^2/(1+q^2)|}$$

above threshold ($q^2 > 0$): $|M|^2 = |M0 + iM1|^2 = M0^2 + M1^2$

below threshold ($q^2 < 0$): $|M|^2 = |M0 + M1|^2 = M0^2 + M1^2 + 2M0\,M1$

M is reduced as M1 < 0

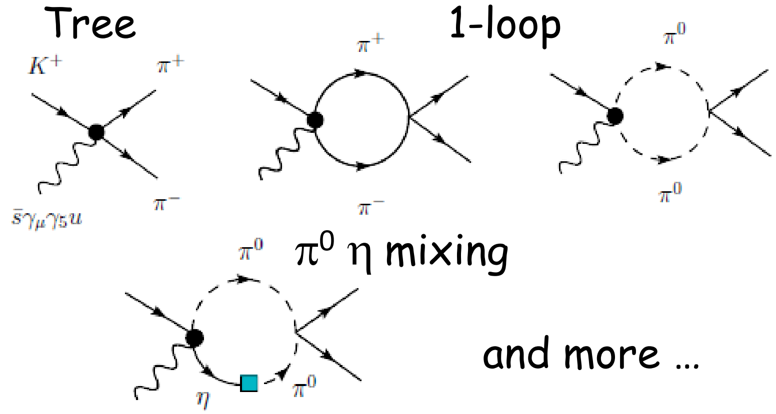
Ke4: Isospin corrections (mass effects) to phase shift

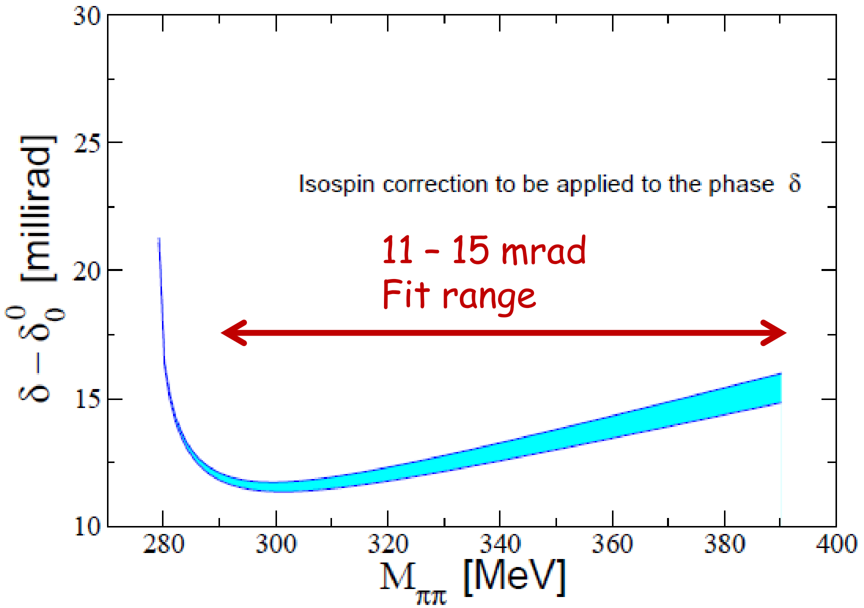
Colangelo Gasser Rusetsky EPJ C59,777(2009)
ChPT approach

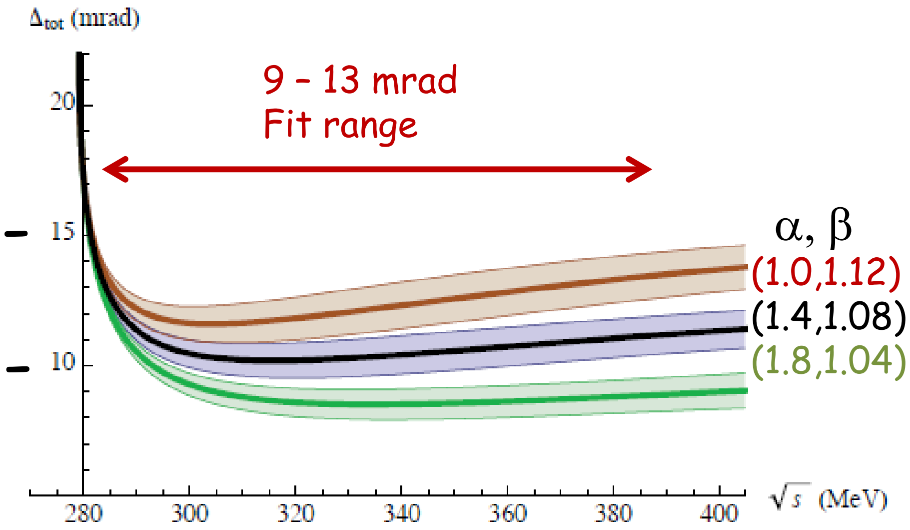
Descotes Knecht EPJ C72 (2012) 1962
Dispersive approach

$$\delta = \frac{1}{32\pi F^2}\left\{(4\Delta_\pi + s)\sigma + (s - M_{\pi^0}^2)\left(1+\frac{3}{2R}\right)\sigma_0\right\} - \delta_1^1 + O(p^4)$$

where $\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2$, $\sigma = \sqrt{1 - \frac{4M_\pi^2}{s}}$, $R = \frac{m_s - \hat{m}}{m_d - m_u}$







Very similar impact on our final result !