The QCD critical end point driven by an external magnetic field in asymmetric quark matter

Pedro Costa

Márcio Ferreira, Constança Providência

Centro de Física Computacional

Departamento de Física, Universidade de Coimbra, Portugal

Hubert Hansen

Institut de Physique Nucléaire de Lyon, Université Claude Bernard de Lyon, CNRS/IN2P3, France

Débora P. Menezes

Depto de Física, CFM - Universidade Federal de Santa Catarina, Florianópolis, Brazil

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- Motivation;
- The PNJL model and formalism;
- PNJL vs. lattice calculations in the presence of an external magnetic field;
- Magnetic catalysis in the PNJL model;
- PNJL CEP in asymmetric quark matter in the presence of an external magnetic field;
- Inverse magnetic catalysis in the PNJL model;
- Summary and conclusions.

Motivation

- **Understanding the QCD phase structure is one of the most important challenges in the physics of strong interactions**

- **The very first QCD phase diagram taken from Cabibbo-Parisi (1975)**

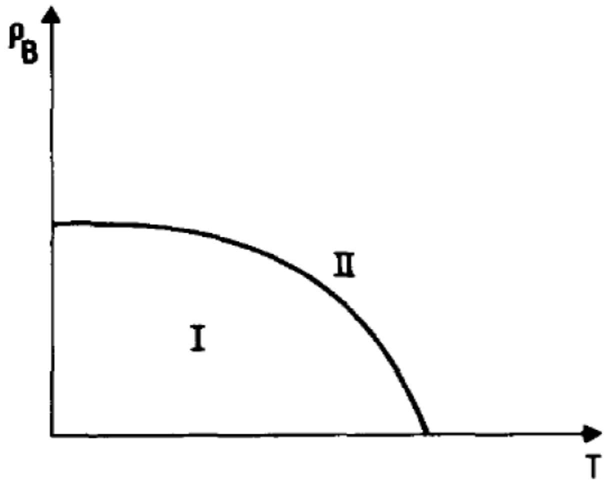
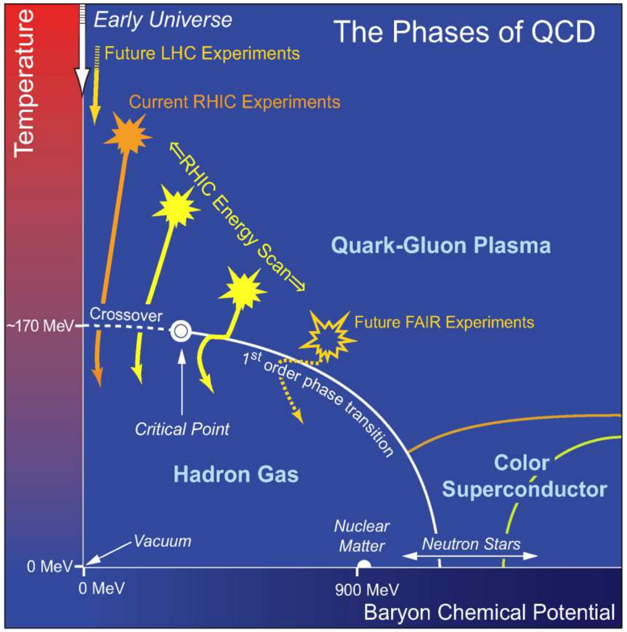


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

- **A schematic outline for the phase diagram of matter at ultrahigh density and temperature**



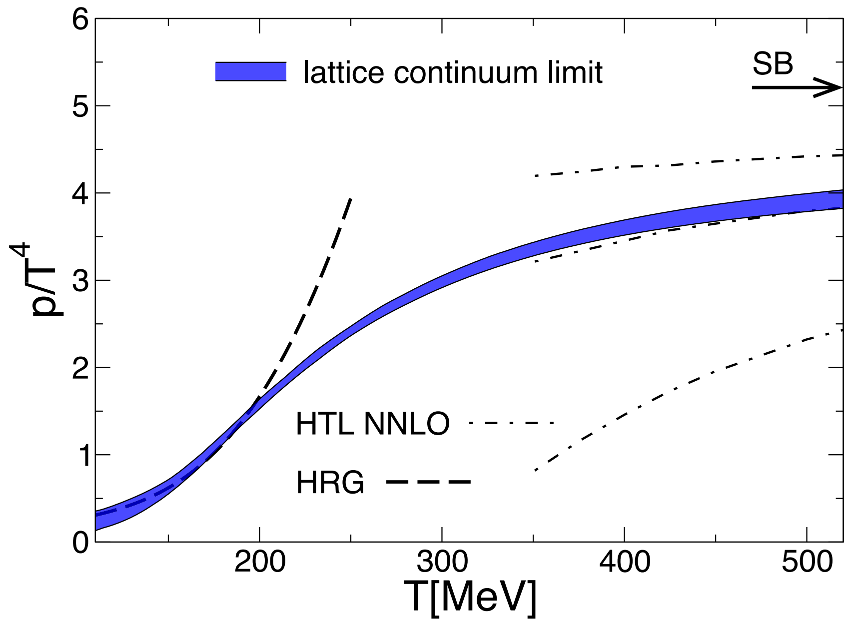
– N. Cabibbo, G. Parisi, PLB 59 (1975) 67

Motivation

- **Understanding the QCD phase structure is one of the most important challenges in the physics of strong interactions**

Theoretical approach:

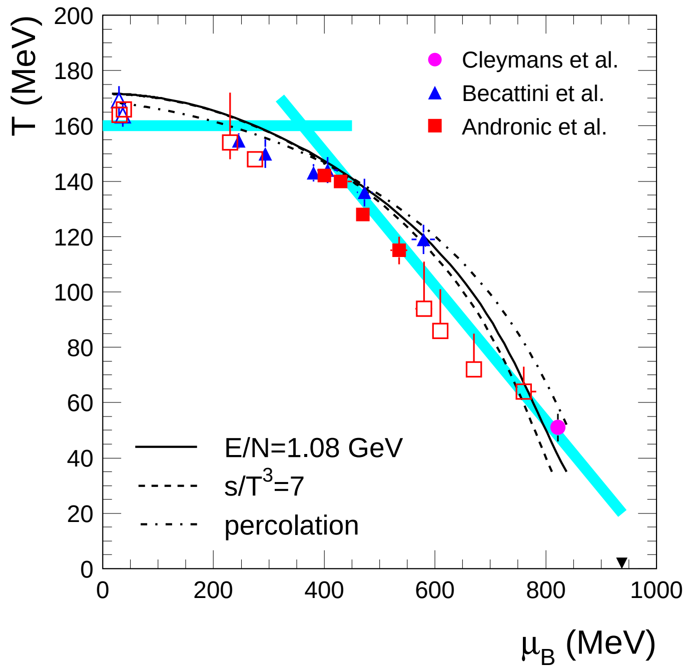
- Effective model calculations
- Lattice calculations



– S. Borsányi, et al., PLB 730 (2014) 99

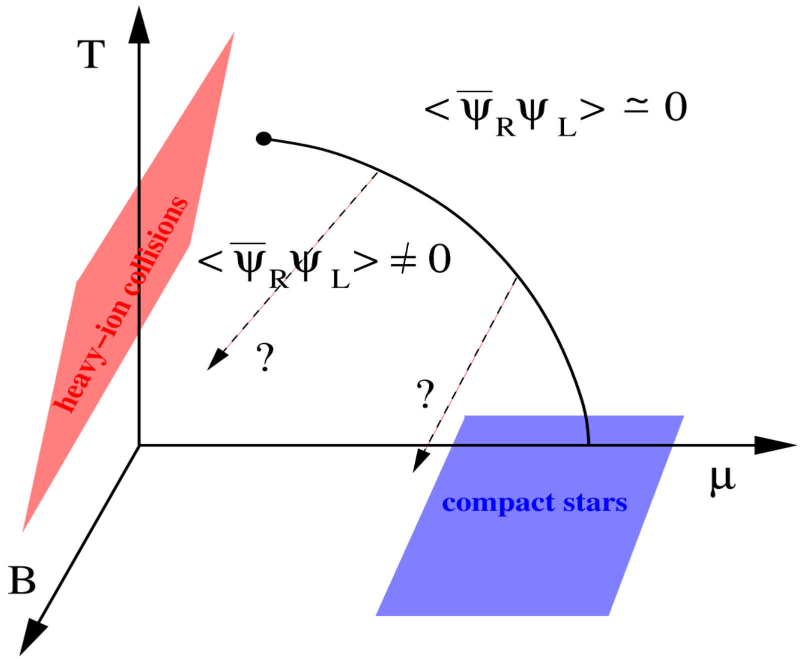
Experimental approach:

- Map the QCD phase boundary
- Localization of the CEP



– A. Andronic, et al., NPA 837 (2010) 65

Deconfinement and chiral restoration in an external magnetic field:



Important for:

- Physics of magnetars: ($B \sim 10^{18-20}$ G in the interior?)[1];
- Measurements in heavy ion collisions at very high energies;
  - RHIC energy scale: $eB_{max} \approx 5 \times 10^{18}$ G $(5 \times m_\pi^2)$[2]
  - LHC energy scale: $eB_{max} \approx 5 \times 10^{19}$ G $(15 \times m_\pi^2)$[2]
- Early stages of the universe.

1– E. J. Ferrer, et al., PRC 82 (2010) 065802

2– V. V. Skokov, et al., IJMPA 24 (2009) 5925

Motivation

- **QCD $\rightarrow$ two phase transitions**
  - restoration of chiral symmetry
    order parameters: **quark condensates**

$$\langle \bar{q}_i \, q_i \rangle \begin{cases} \neq 0 & \Leftrightarrow \quad \text{symmetry broken}, T < T_c \\ = 0 & \Leftrightarrow \quad \text{symmetry restored}, T > T_c \end{cases}$$

  - deconfinement
    order parameter: **Polyakov loop**

$$\Phi = \frac{1}{N_c} \mathrm{Tr}_c \left\langle \left\langle \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] \right\rangle \right\rangle$$

$$\Phi \begin{cases} = 0 & \Leftrightarrow \quad \text{confined phase}, T < T_c \\ \neq 0 & \Leftrightarrow \quad \text{deconfined phase}, T > T_c \end{cases}$$

- PURPOSE: Consider a model which describes both low and high temperature QCD behavior in a single picture $\rightarrow$ **PNJL model**

Open questions:

Restoration of chiral symmetry and deconfinement:

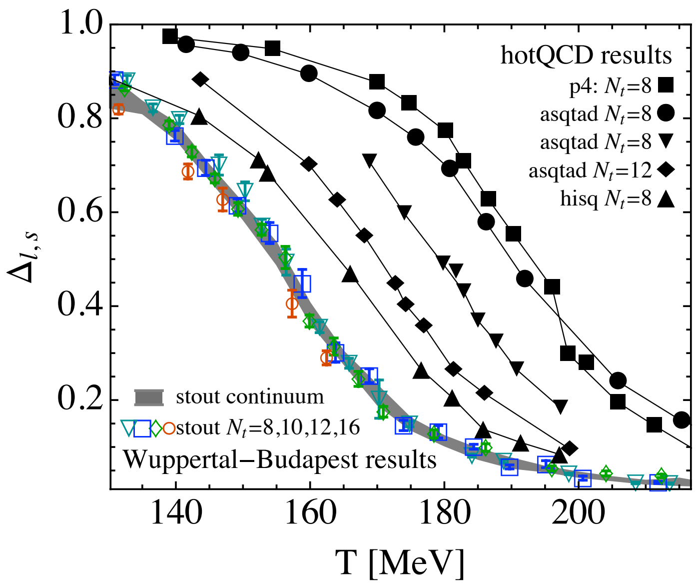
- Connection between chiral symmetry and confinement?
- Can both transitions occur simultaneously?
- $1^{st}-$ order chiral phase transition at high baryon density?
- Where is the CEP?
- Does an external magnetic field enhances the χ_S breaking? Magnetic catalysis (MC). (The magnetic field has a strong tendency to enhance ("catalyze") spin-zero fermion-antifermion condensates);
- Or, can the magnetic field suppress the quark condensate (inverse magnetic catalysis (IMC))?

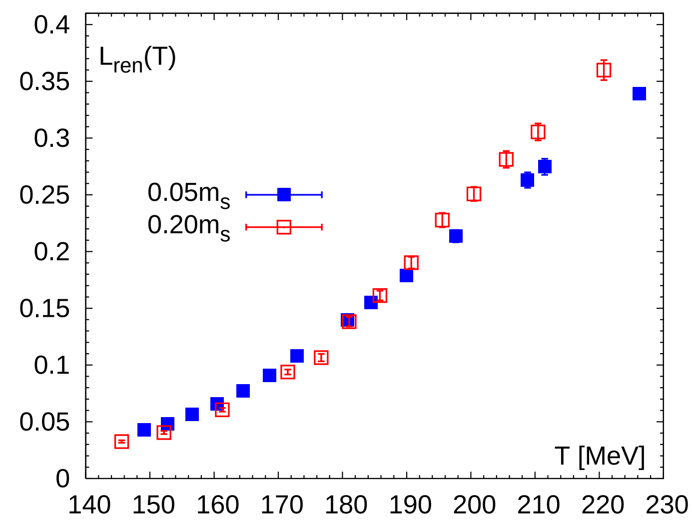
Lattice calculations at Finite Temperature:

Restoration of chiral symmetry and deconfinement: both transitions can occur simultaneously?

- **Phase transition:**

$N_f = 2+1$:





$T_c^\chi = 154 \pm 9$ MeV $\qquad$ $T_c^\Phi = 170(4)(3)$ MeV

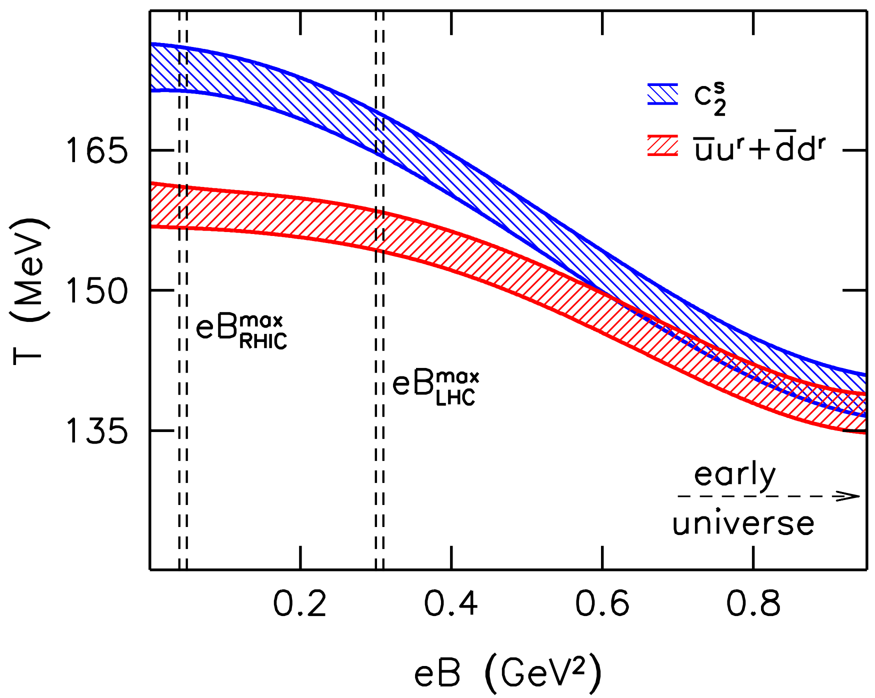
$\Delta_T \sim 16$ MeV

– A. Bazavov, et al., PRD 85 (2012) 054503 $\qquad$ – Y. Aoki, et al., JHEP 0906 (2009) 088

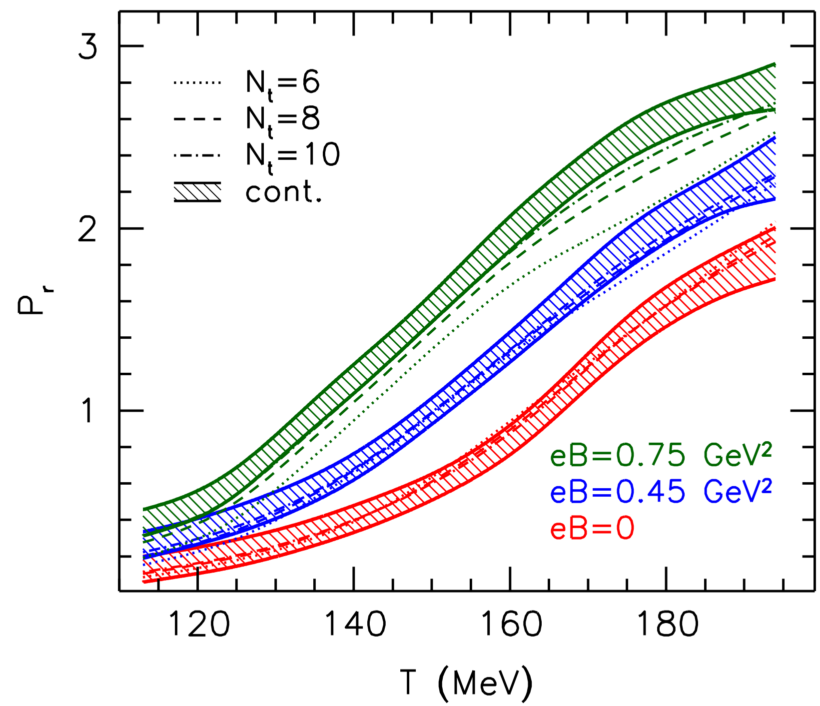
Lattice calculations at Finite Temperature:

Effect of an external magnetic field on the finite temperature transition of QCD

- **Phase transition deduced from peaks in susceptibilities**



- **Polyakov Loop**



– G. S. Bali, et al., JHEP 1202 (2012) 044 – F. Bruckmann, et al., JHEP 1304 (2013) 112

Model and formalism

PNJL model in the presence of an external magnetic field

$$\mathcal{L}_{PNJL} = \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}\right)q + \frac{g_S}{2}\sum_{a=0}^{8}\left[(\bar{q}\lambda^a q)^2 + (\bar{q}(i\gamma_5)\lambda^a q)^2\right]$$

$$+ \; g_D\left[\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right] + \mathcal{U}\left(\Phi,\bar{\Phi};T\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix

- $D^\mu = \partial^\mu - iq_f A^\mu_{EM} - igA^\mu; \quad A^\mu = \delta^\mu_0 A^0$ (Polyakov gauge)
- $F_{\mu\nu} = \partial_\mu A^{EM}_\nu - \partial_\nu A^{EM}_\mu$
  - $A^{EM}_\mu = \delta_{\mu 2} x_1 B$ static and constant magnetic field in the z direction

Coupling between Polyakov loop and quarks uniquely determined by covariant derivative D^μ

$$\Phi(\vec{x}) = \frac{1}{N_c}\mathrm{Tr}_c\left\langle\left\langle \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right]\right\rangle\right\rangle$$

Effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$

- Effective potential for the (complex) Φ field: is conveniently chosen to reproduce results obtained in lattice calculations

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}; T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2]$$

with

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

a_0	a_1	a_2	b_3
3.51	-2.47	15.2	-1.75

and $T_0 = 270$ MeV

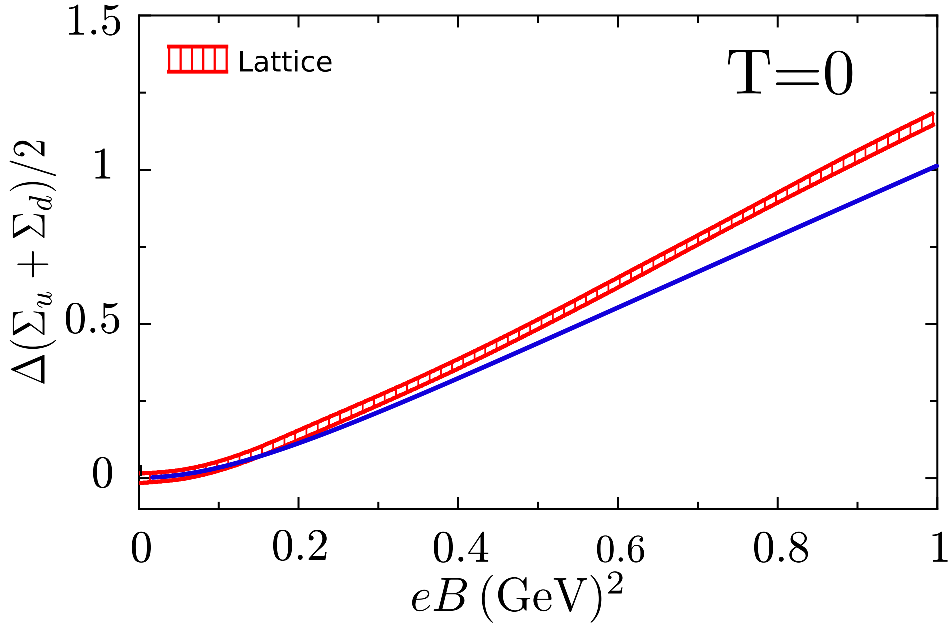
– S. Roessner, C. Ratti, W. Weise, PRD 75 (2007) 034007

Polyakov loop extended NJL model

- The model includes features of both **chiral** and $\mathbb{Z}_3$ symmetry breaking;
- The coupling is fundamental for reproducing lattice results concerning QCD thermodynamics: it originates a **suppression** of the **unconfined quarks** in the hadronic phase [1] (low temperature);
- A non-zero **Polyakov loop** reflects the **spontaneously broken** $\mathbb{Z}_3$ symmetry characteristic of deconfinement (high temperature);
  - $\mathbb{Z}_3$ is **broken** in the **deconfined phase** ($\Phi \to 1$);
  - $\mathbb{Z}_3$ is **restored** in the **confined one** ($\Phi \to 0$);
- At $T = 0$: $\Phi = \bar{\Phi} = 0 \longmapsto$ both sectors decouple.

1– C. Ratti, et al., PRD 73 (2006) 014019

PNJL vs. lattice calculations



Change of the light condensate due to eB:

$$\Delta\Sigma_f(eB,T)=\Sigma_f(eB,T)-\Sigma_f(0,T),$$

with

$$\Sigma_f(eB,T)=\frac{2m_f}{m_\pi^2 f_\pi^2}\left[\langle\bar{q}_f q_f\rangle(eB,T)-\langle\bar{q}_f q_f\rangle(0,0)\right]+1$$

- For $T=0$ NJL and PNJL models coincide;
- Results quantitatively agree with lattice[1] and even at $eB=1$ GeV^2 there is a discrepancy of the order of $\sim 15\,\%$.
- At $T=\mu_B=0$ chiral symmetry is always broken with eB.

1– G. S. Bali, et al., PRD 86 (2012) 071502

PNJL *Pseudocritical* temperatures

Pseudocritical temperatures:

⇓

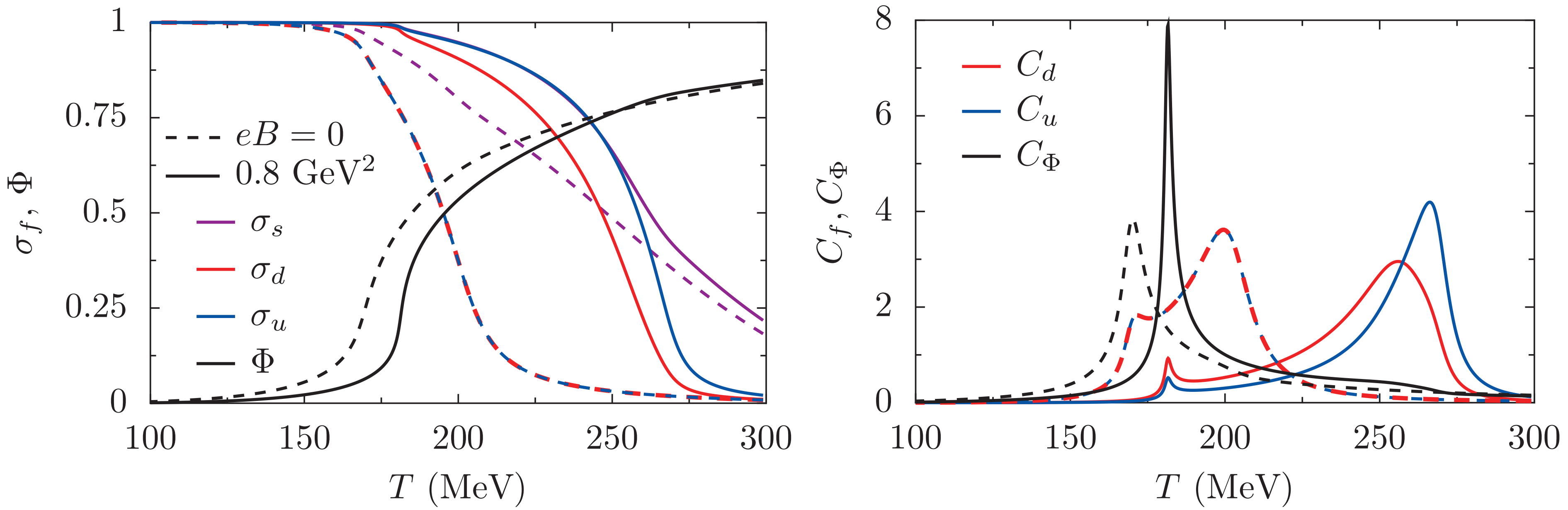
Separate the different phases in PNJL model

Criteria to identify the ***partial*** restoration of chiral symmetry and the transition to the deconfinement:

⇓

$$\frac{\partial^2 \langle \bar{u}u \rangle}{\partial T^2} = \frac{\partial^2 \langle \bar{d}d \rangle}{\partial T^2} = \frac{\partial^2 \Phi}{\partial T^2} = 0$$

PNJL *Pseudocritical* temperatures



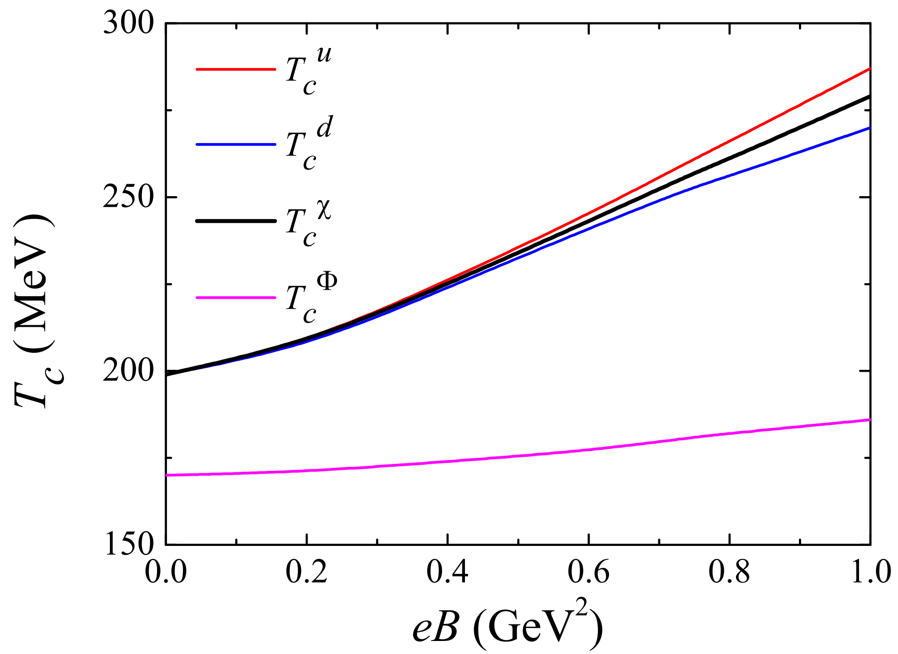
$C_f = -m_\pi \partial\sigma_f/\partial T$ (where $\sigma_f =< \bar{q}_f q_f > (eB,T)/ < \bar{q}_f q_f > (B,0)$); $C_\Phi = m_\pi \partial\Phi/\partial T$

- Smooth **crossover** from the chirally broken to the chirally symmetric phase: ***partial* restoration of** χ_S
  - T^χ for u and d quark transitions become different as eB increases;
  - $q_u = 2e/3$; $q_d = -e/3 \longmapsto M_u$ becomes larger and the restoration of χ_S in the u sector is delayed:
    - T_u^χ is higher than T_d^χ.
- C_Φ becomes narrower as eB increases $\longmapsto$ eventually for sufficient strong eB a $1^{st}-$ order phase transition takes place.

PNJL *Pseudocritical* temperatures

Pseudocritical temperatures for the chiral transition $\left(T_c^\chi = (T_u^\chi + T_d^\chi)/2\right)$ and for the deconfinement (T_c^Φ) with $\mathbf{T_0 = 210}$ MeV.

	PNJL			
eB [GeV^2]	T_u^χ [MeV]	T_d^χ [MeV]	T_c^χ [MeV]	T_c^Φ [MeV]
0	199	199	199	**170**
0.2	208	207	208	171
0.4	226	224	225	174
0.6	245	241	243	177
0.75	261	253	257	181
0.8	266	256	261	182
1	287	270	279	186

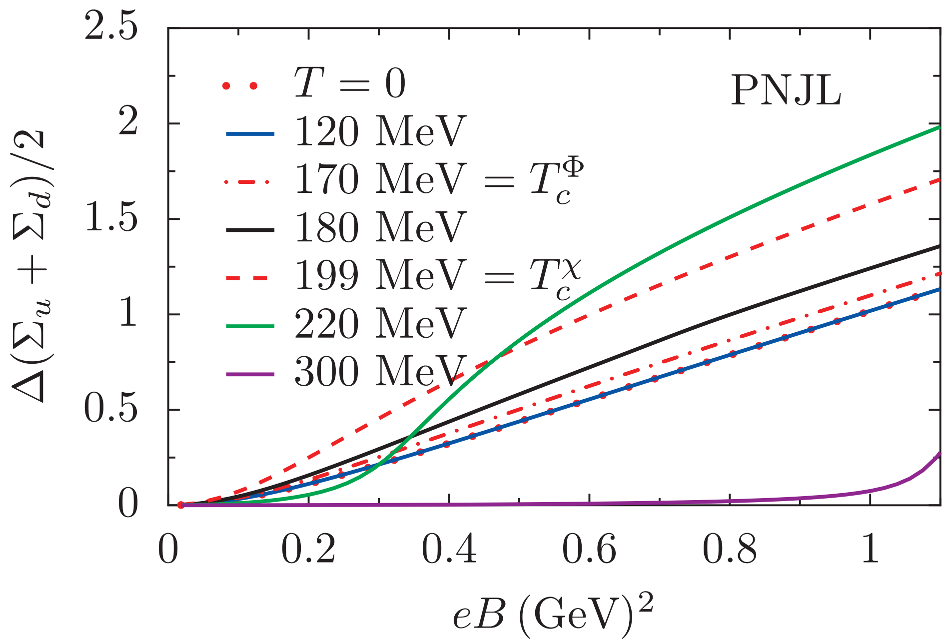


- As eB becomes stronger, the separation between T_c^χ and T_c^Φ increases;
- eB has a smaller impact in the location of the deconfinement crossover: T_c^Φ has just a weaker increase.

– M. Ferreira, et al., PRD 89 (2014) 036006

PNJL magnetic catalysis

Change of the renormalized condensates as a function of eB for several temperatures:



$T < T_c^\chi(eB = 0)$:

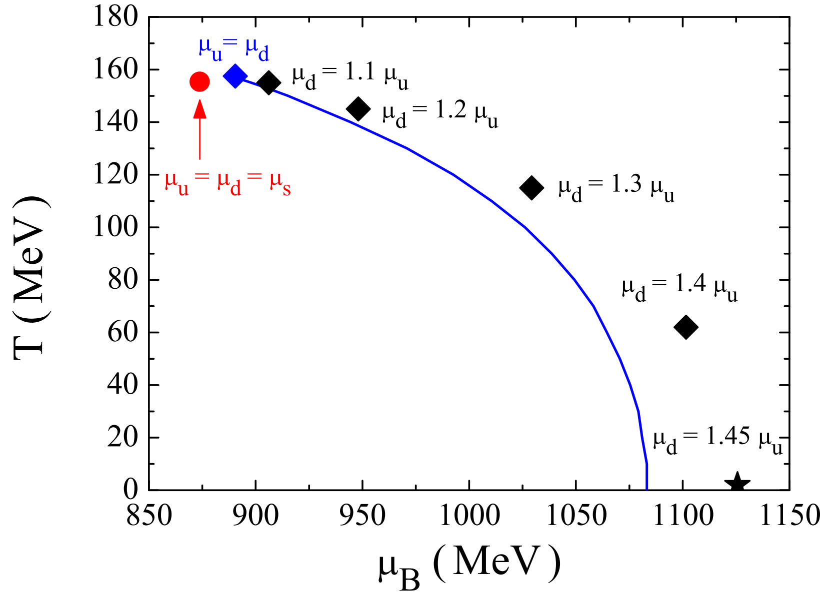
- condensates average increase with eB

  its value is greater the higher the temperature
  (due to the magnetic catalysis effect)

- $T > T_c^\chi(eB = 0)$:
  - Two competitive effects: partial restoration of χ_S and magnetic catalysis.
    - Partial restoration of χ_S prevails at lower values of eB:
      the change of the renormalized condensates is approximately zero;
    - The magnetic catalysis becomes dominant as eB increases:
      the change of the renormalized condensates condensate becomes nonzero.

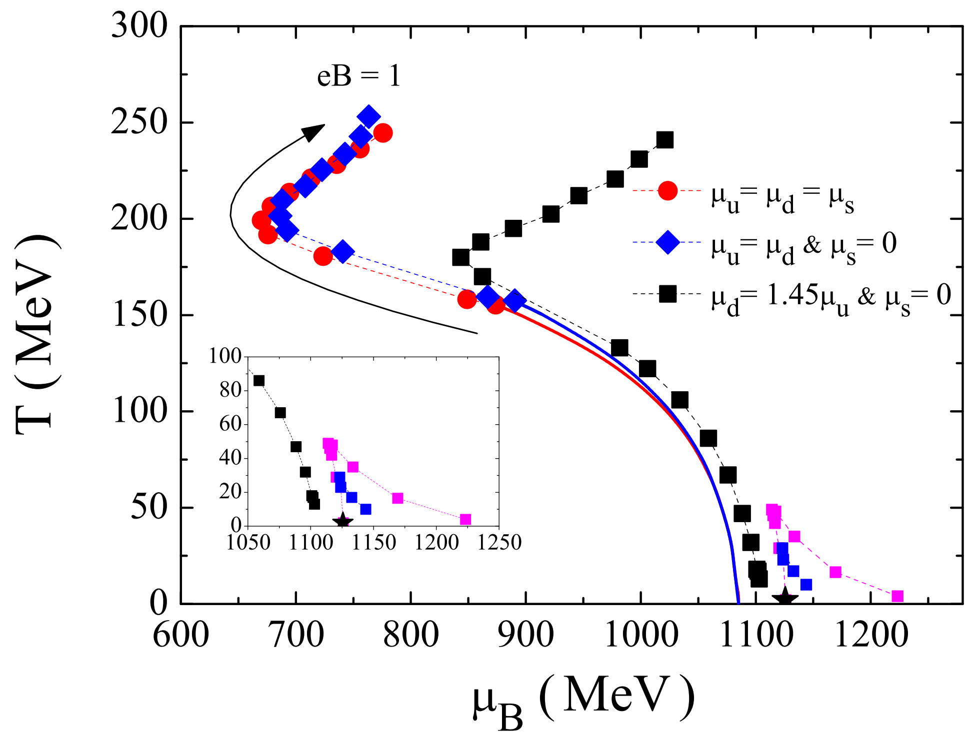
Asymmetric quark matter: location of the CEP depends on the isospin

- *d*-quark rich matter as it occurs in:
  - HIC – asymmetry presently attained in HIC: $\mu_u < \mu_d < 1.1\mu_u$;
  - neutron stars – neutron matter has $\mu_d \sim 1.2\mu_u$;



- CEPs calculated at $\mu_s = 0$:
  - Increasing the isospin asymmetry moves the CEP to smaller T and larger μ_B;
  - Matter being less symmetric is less bound: the transition to a chirally symmetric phase occurs at a smaller temperature and density than the symmetric case;
  - $\mu_d \sim 1.45\mu_u$: asymmetry large enough $\rightarrow$ the CEP disappears.

PNJL phase diagram

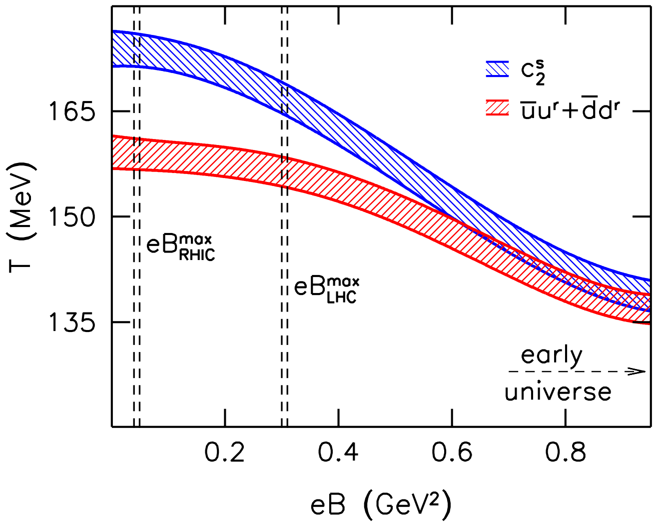


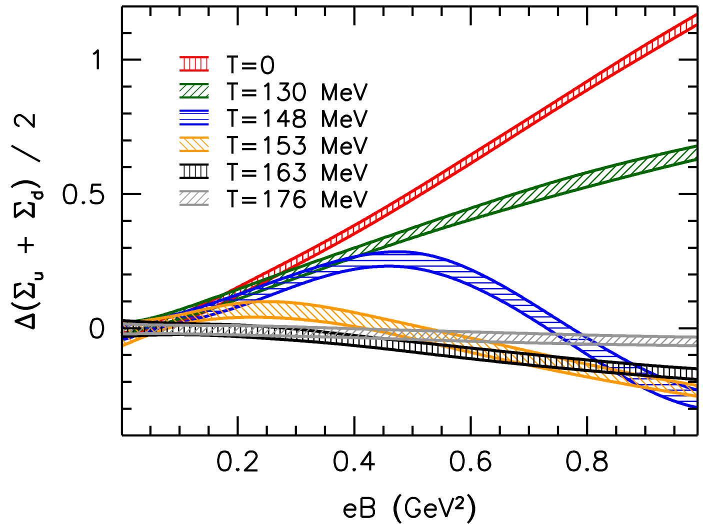
- Phase transition driven by the magnetic field will occur (for $\mu_d > 1.45\mu_u$):
  - possible appearance of multiple CEPs for for sufficiently small values of eB and T.
- The trend is very similar for different scenarios:
  - as the intensity of the magnetic field increases, T^{CEP} increases and μ_B^{CEP} decreases until $eB \sim 0.3$ GeV^2;
  - for stronger magnetic fields both T^{CEP} and μ_B^{CEP} increase.

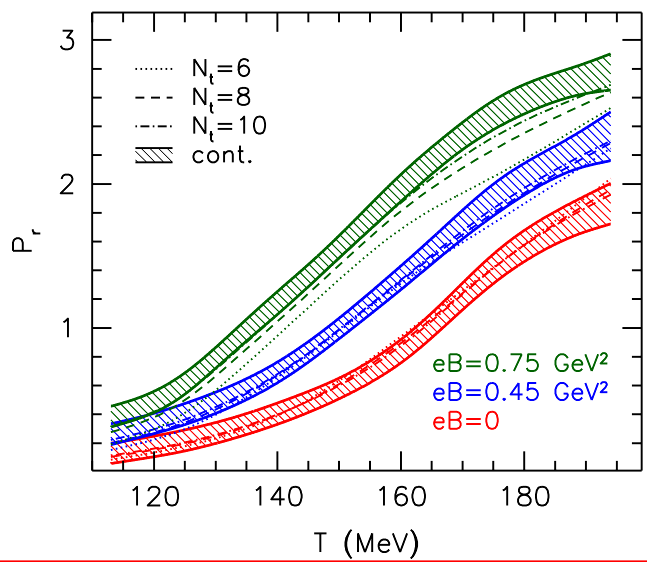
– P. Costa, et al., PRD 89 (2014) 056013

Lattice calculations

Inverse Magnetic Catalysis (IMC):







– G. S. Bali, et al., JHEP 1202 (2012) 044 – F. Bruckmann, et al., JHEP 1304 (2013) 112

Strong coupling: the influence of eB

- In the lower p region, relevant for the chiral symmetry breaking dynamics: effect of screening of the gluon interactions in a magnetic field;
- The strong coupling α_s decreases with eB[1]:

$$\alpha_s(eB) = \frac{1}{(11N_c - 2N_f)/6\pi \ln(|eB|/\Lambda^2_{QCD})};$$

- Presence of a magnetic field: weakening of the interaction between quarks;
- In the NJL model: $G_s \propto \alpha_s \longmapsto G_s(eB)$;
- In the presence of a magnetic field $\longmapsto G_s$ decreases with eB.

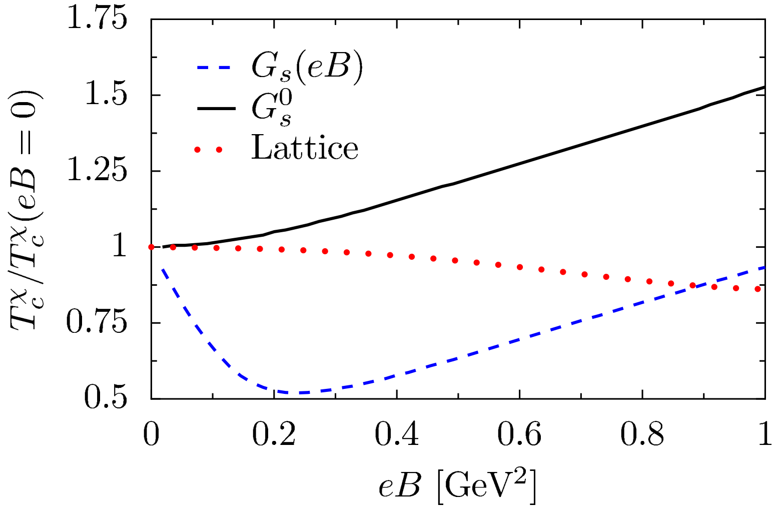
• *"The Importance of Asymptotic Freedom for the Pseudocritical Temperature in Magnetized Quark Matter"*, R. L. S. Farias, et al., arXiv:1404.3931 [hep-ph]

• *"Inverse magnetic catalysis in the (2+1)-flavor Nambu–Jona-Lasinio and Polyakov–Nambu–Jona-Lasinio models"*, M. Ferreira, P. Costa, O. Lourenço, T. Frederico, C. Providência, PRD 89 (2014) 116011

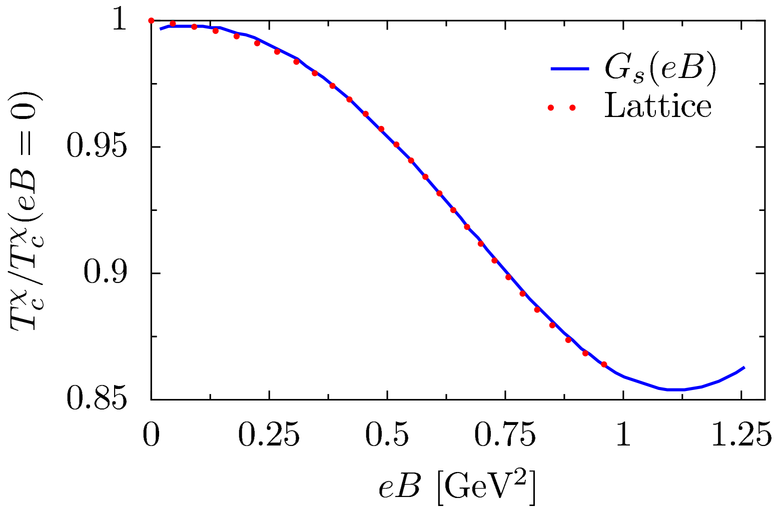
[1] V. A. Miransky et al., PRD 66 (2002) 045006

NJL with $G_s(eB)$ - IMC

- Pseudocritical transition temperatures:



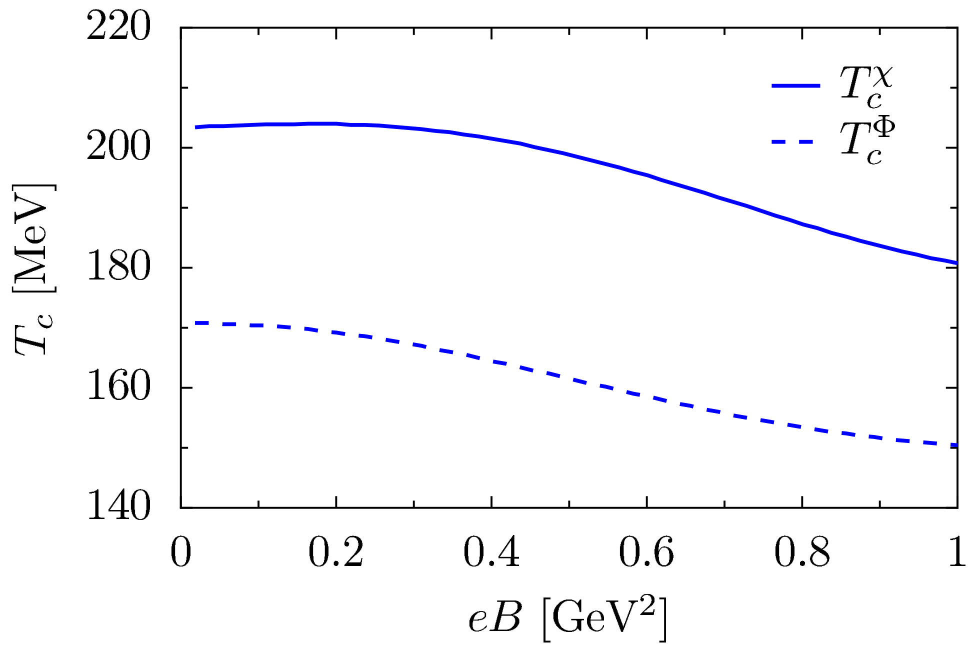
- $G_s(eB)$ is fitted in order to reproduce $T_c^\chi(eB)$ obtained in LQCD:



– M. Ferreira, et al., PRD 89 (2014) 116011

PNJL with $G_s(eB)$ - IMC

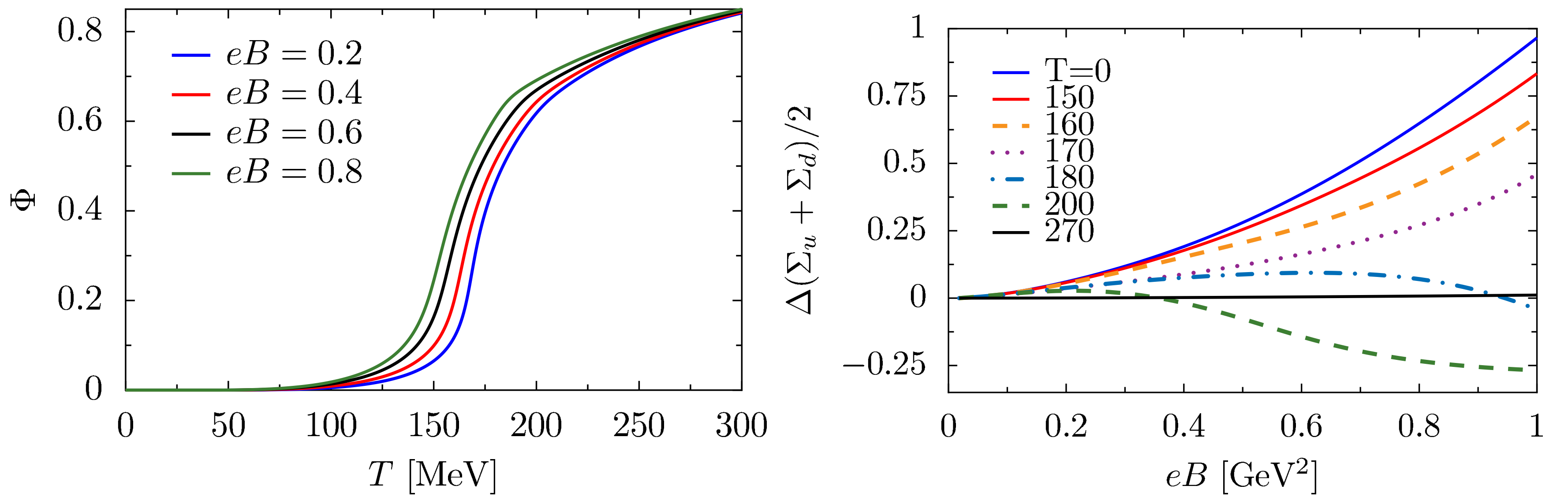
Inserting $G_s(eB)$ in the PNJL model:



- Increasing $eB \longmapsto$ both transitions occur at lower temperatures.

– M. Ferreira, et al., PRD 89 (2014) 116011

PNJL with $G_s(eB)$ - IMC

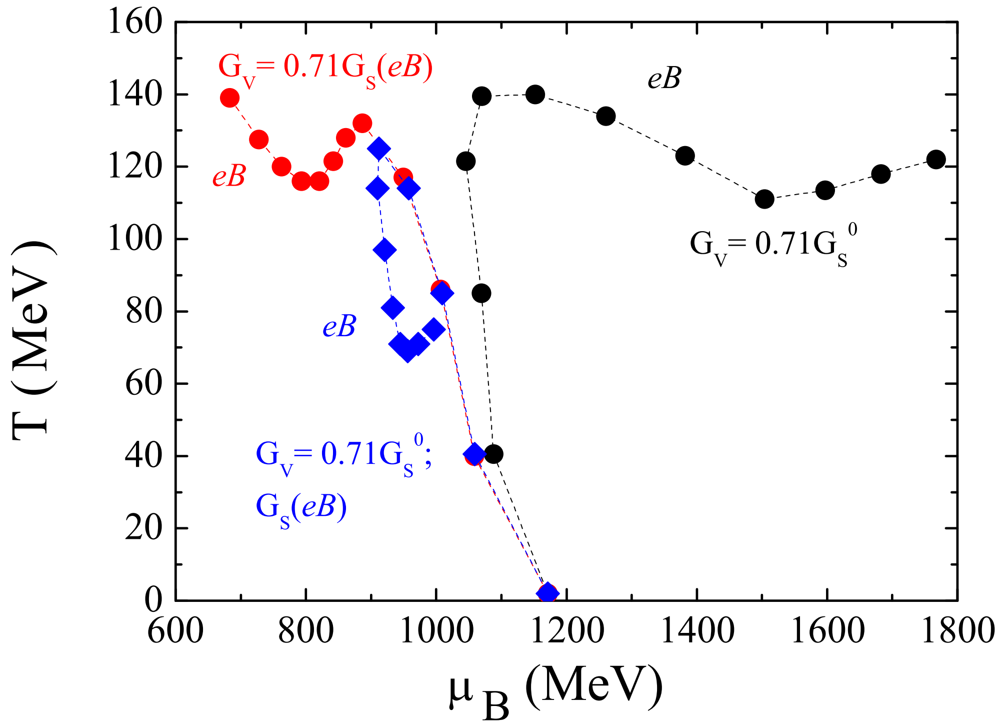


- The Polyakov loop and chiral condensate behaviors as functions of eB reproduce qualitatively LQCD results:
  - At a fixed T the Polyakov loop increases with eB;
  - The chiral condensate has a nonmonotonic behavior.

PNJL with $G_s(eB)$ - IMC

Introducing a repulsive vector contribution:

$$\mathcal{L}_{vec} = -G_V \sum_{a=0}^{8} \left[(\bar{\psi}\gamma^{\mu}\lambda_a\psi)^2 + (\bar{\psi}\gamma^{\mu}\gamma_5\lambda_a\psi)^2 \right]$$



Summary and Conclusions

- In the presence of an external magnetic field at $T = 0$, the quantitative behavior of SU(3) PNJL is closer to the lattice results;

- Chiral and deconfinement transition temperatures increase (magnetic catalysis) in the presence of an external magnetic field, although the deconfinement transition temperature suffers a much weaker effect;

- In the presence of a large enough isospin asymmetry the CEP does not exist for a zero external magnetic field. A sufficiently high external magnetic field can drive the system into a first order phase transition again;

- A running coupling $G_s(eB)$, motivated by asymptotic freedom and that reproduces the qualitative behavior of chiral pseudocritical temperature given by LQCD, leads to the inverse magnetic catalysis in the PNJL model.

Acknowledgments

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Backup slides

Appendix: Model and formalism

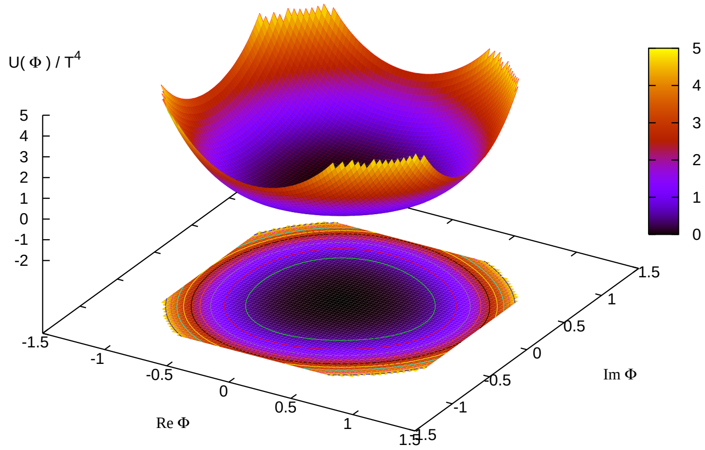
Parameters and results:

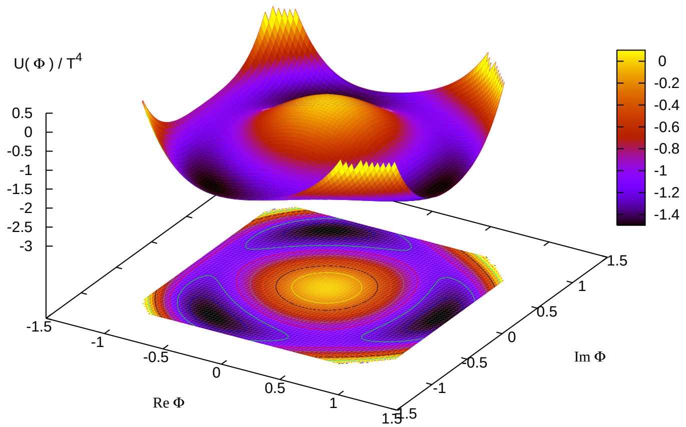
Physical quantities	Parameter set and constituent quark masses
$f_\pi = 92.4$ MeV	$m_u = m_d = 5.5$ MeV
$M_\pi = 135.0$ MeV	$m_s = 140.7$ MeV
$M_K = 497.7$ MeV	$\Lambda = 602.3$ MeV
$M_{\eta'} = 960.8$ MeV	$g_S \Lambda^2 = 3.67$
$M_\eta = 514.8$ MeV*	$g_D \Lambda^5 = -12.36$
$f_K = 97.7$ MeV*	$M_u = M_d = 367.7$ MeV*
$M_\sigma = 728.8$ MeV*	$M_s = 549.5$ MeV*
$M_{a_0} = 873.3$ MeV*	
$M_\kappa = 1045.4$ MeV*	
$M_{f_0} = 1194.3$ MeV*	
$\theta_P = -5.8^{o*}$; $\theta_S = 16^{o*}$	

– S. P. Klevansky, et al., PRC 53 (1996) 410 – P. Costa, et al. PRD 71 (2005) 116002

Effective potential $U(\Phi, \bar{\Phi}, T)$:

$\Phi \to 0$: confined phase | **$\Phi \to 1$: deconfined phase**





- **Low temperature**: $\mathbb{Z}_3$ symmetric, confined phase ($\mathbb{Z}_3$ center of $SU_c(3)$ symmetry);
- **High temperature**: deconfined phase characterized by the spontaneous breaking of the $\mathbb{Z}_3$ symmetry.

– H. Hansen, et al., PRD 75 (2007) 065004

Appendix: Model and formalism

Model and formalism:

The calculations in NJL model can be generalized to the PNJL one by introducing the modified Fermi–Dirac distribution functions (with $\beta = 1/T$):

$$f(E_i - \mu) = \frac{1}{1 + e^{\beta(E_i - \mu)}} \mapsto f_\Phi^+(E_i) = \frac{\left(\bar{\Phi} + 2\Phi e^{-\beta(E_i-\mu)}\right) e^{-\beta(E_i-\mu)} + e^{-3\beta(E_i-\mu)}}{1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_i-\mu)}\right) e^{-\beta(E_i-\mu)} + e^{-3\beta(E_i-\mu)}}$$

$$f(E_i + \mu) = \frac{1}{1 + e^{\beta(E_i + \mu)}} \mapsto f_\Phi^-(E_i) = \frac{\left(\Phi + 2\bar{\Phi} e^{-\beta(E_i+\mu)}\right) e^{-\beta(E_i+\mu)} + e^{-3\beta(E_i+\mu)}}{1 + 3\left(\Phi + \bar{\Phi} e^{-\beta(E_i+\mu)}\right) e^{-\beta(E_i+\mu)} + e^{-3\beta(E_i+\mu)}}$$

For example the quark condensates are calculated according to:

$$\langle\langle \bar{q}_i q_i \rangle\rangle = -2N_c \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{M_i}{E_i} [\theta(\Lambda^2 - \mathbf{p}^2) - f_\Phi^+(E_i) - f_\Phi^-(E_i)]$$

– H. Hansen, et al., PRD 75 (2007) 065004

Appendix: Model and formalism

- In the presence of an external magnetic field $B = B\hat{z}$:

$$E_i \to \sqrt{p_z^2 + 2|q_i B| n + M_i^2}$$

  - $n = 0, 1, 2, ...$ is the Landau level.
- Dimensional reduction: $D \to D - 2 \longmapsto k_x, k_y, k_z \to k_z$.
- The model is modified in the following:

$$2\int \frac{d^3p}{(2\pi)^3} f(E_i) \to \frac{|qB|}{2\pi} \sum_{n=0}^{\infty} \alpha_n \int \frac{dp_z}{2\pi} f\left(\sqrt{p_z^2 + 2|qB|n + M_i^2}\right)$$

with $\alpha_0 = 1$ and $\alpha_{n\neq 0} = 2$

Appendix: Model and formalism

PNJL model at finite T, μ in an external magnetic field eB

The thermodynamic potential is:

$$
\begin{aligned}
\Omega(T,\mu_i) \;=\;& \mathcal{U}(\Phi,\bar{\Phi},T) - N_c \sum_{i=u,d,s} \frac{|q_i|eB}{2\pi} \sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \Bigg(E_i \\
&+ \frac{T}{3} \ln\left\{1 + 3\bar{\Phi}e^{-(E_i-\mu_i)/T} + 3\Phi e^{-2(E_i-\mu_i)/T} + e^{-3(E_i-\mu_i)/T}\right\} \\
&+ \frac{T}{3} \ln\left\{1 + 3\Phi e^{-(E_i+\mu_i)/T} + 3\bar{\Phi} e^{-2(E_i+\mu_i)/T} + e^{-3(E_i+\mu_i)/T}\right\} \Bigg) \\
&+ g_S \sum_{\{i=u,d,s\}} \langle \bar{q}_i q_i \rangle^2 - 4 g_D \langle \bar{q}_u q_u \rangle \langle \bar{q}_d q_d \rangle \langle \bar{q}_s q_s \rangle
\end{aligned}
$$

with $E_i = \sqrt{2n|q_i|eB + p_z^2 + M_i^2}$

Appendix: Model and formalism

Methodology:

Minimization of $\Omega(T, \mu_f)$ with respect to M_f $(f = u, d, s)$

⬇

"Gap" equations:

$$M_f = m_f - 2 g_S \langle \bar{q}_f q_f \rangle + 2 g_D \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle$$

Effective action for the scalar and pseudoscalar mesons

⬇

Meson propagators, $g_{M\bar{q}q}$, $f_{M\bar{q}q}, \ldots$