Universitat Autònoma de Barcelona

The muonic hydrogen Lamb shift and the proton radius

arxiv: 1403.3408
arxiv: 1406.4524
work in collaboration with Clara Peset

PANIC14, Hamburg 2014

28th August 2014

Antonio Pineda

The proton radius puzzle

Value from μ-H: $r_p = 0.84087(39)$ fm *(Science '13)*

CODATA value (2012): $r_p = 0.8775(51)$ fm

$7.1\,\sigma$ *away*!

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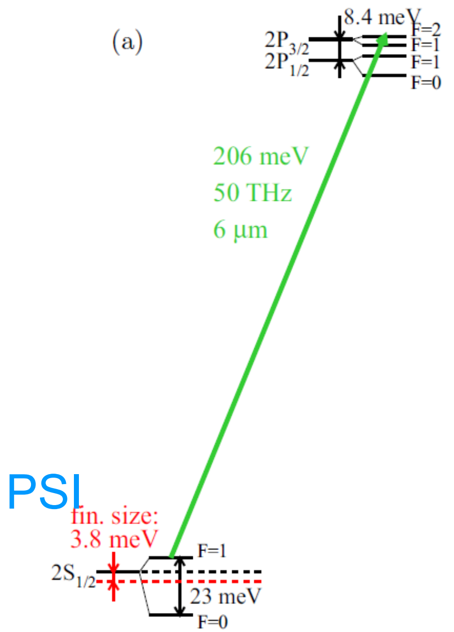

LAMB SHIFT: $2S_{1/2} \rightarrow 2P_{3/2}$

$$\delta E \sim m_\mu^3 r_p^2 \alpha^4$$

8 million times larger for μH

$m_\mu \approx 200\, m_e$

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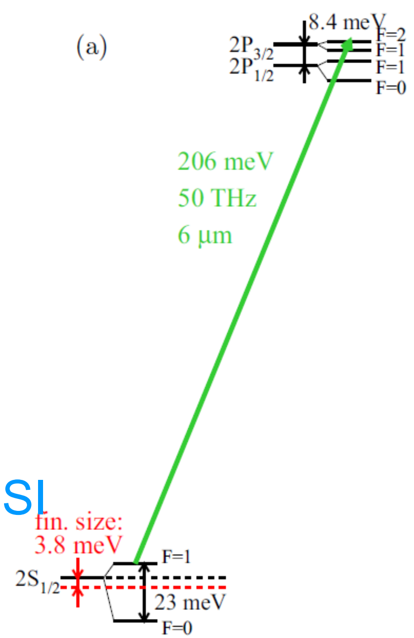
PSI



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(But also other hadronic effects are bigger ….)

Motivation

- Model independent prediction of the proton radius
- Effective field theory setup: HBET/NRQED/pNRQED

EFT for bound states (μp)

Scales in the bound state

- Hard scale: $m_r \longrightarrow m_r$
- Soft scale: $|\boldsymbol{p}| \longrightarrow m_r\alpha$
- Ultrasoft scale: $E \longrightarrow m_r\alpha^2$

⇒ Well separated scales

pNRQED

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- And: $m_p \sim m_\rho, \quad m_\mu \sim m_\pi \sim m_r, \quad m_r\alpha \sim m_e$

→ Small expansion parameters: $\frac{m_\mu}{m_p} \sim \frac{m_\pi}{m_p} \approx \frac{1}{9}, \quad \frac{m_e}{m_r} \sim \frac{m_r\alpha}{m_r} \sim \alpha \approx \frac{1}{137}$

Energy levels:

$$E(\mu p) = -\frac{\mathrm{m_r}\,\alpha^2}{2\mathrm{n}^2}(1 + \mathrm{c}_1\alpha^2 + \mathrm{c}_2\alpha^2 + \mathrm{c}_3\alpha^3 + \ \ldots)$$

$c_{1/2} = c_{1/2}\left(\frac{m_r\alpha}{m_e}\right)$ pure QED, and $c_{n>2} = c_{n>2}\left(\frac{m_r\alpha}{m_e}, \frac{m_\pi}{m_\rho}\right)$

pNRQED

pNRQED is a theory of ultrasoft photons

$$\text{HBET} \xrightarrow[m_\pi\,,\,\Delta]{} \text{NRQED} \xrightarrow[m_r\alpha]{} \text{pNRQED}$$

The pNRQED lagrangian:

$$\mathcal{L}_{pNRQED} = \int d^3\mathbf{x} d^3\mathbf{X} S^\dagger(\mathbf{x},\mathbf{X},t) \left\{ i\partial_0 - \frac{\mathbf{p}^2}{m_r} + \frac{\mathbf{p}^4}{m_\mu^3} + \frac{\mathbf{p}^4}{m_p^3} - \frac{\mathbf{P}^2}{2M} - V(\mathbf{x},\mathbf{p},\sigma_1,\sigma_2) + e\mathbf{x}\cdot\mathbf{E}(\mathbf{X},t) \right\} S(\mathbf{x},\mathbf{X},t) - \int d^3\mathbf{x} \frac{d_1}{4} F_{\mu\nu}F^{\mu\nu}$$

The potential:

$$V = V^{(0)} + V^{(1)} + V^{(2)} + V^{(3)} + \ldots$$

$V^{(i)} \propto \left(\frac{1}{m}\right)^i$ + expansions in the small parameters: α

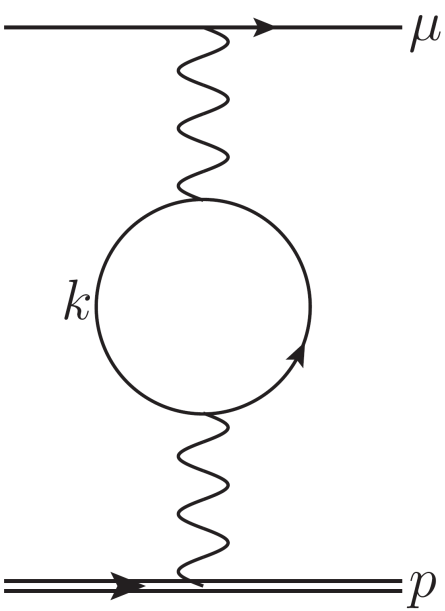
"Pure" QED corrections

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6)\text{meV}$$

Peset, AP

μH QED leading contribution: ELECTRON VACUUM POLARIZATION



$$\langle V_{VP} \rangle \propto \mathcal{O}(m_r \alpha^3)$$

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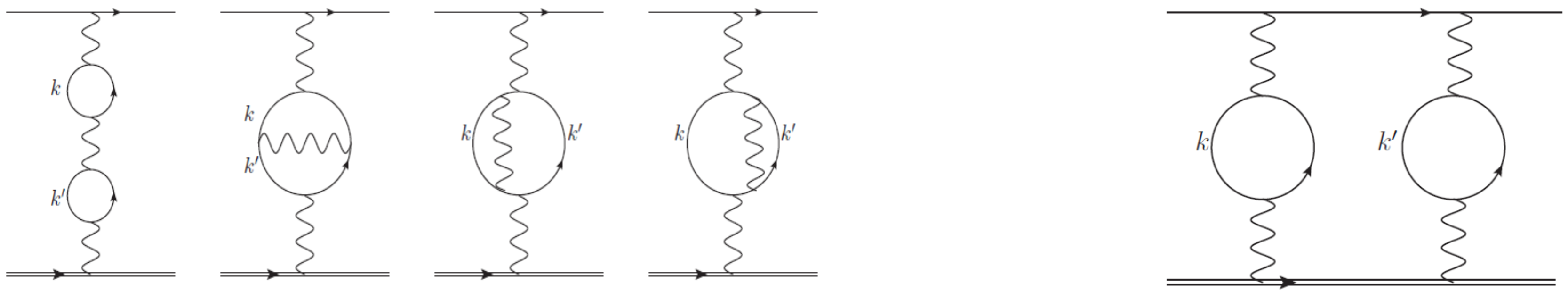
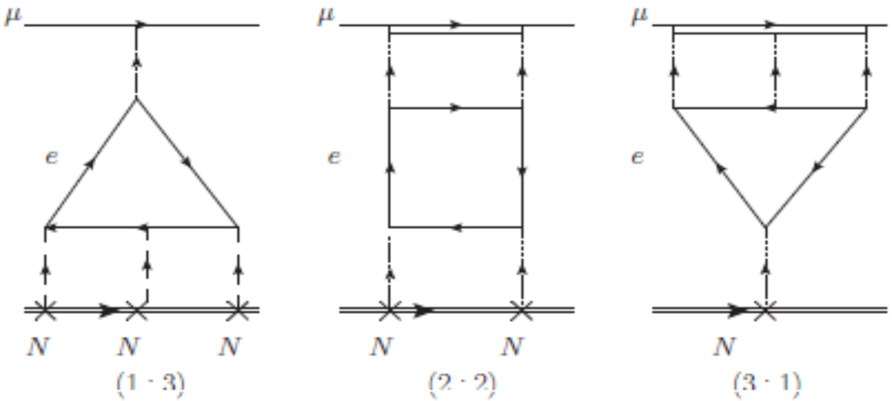
Peset, AP

Other contributions from the static potential up to $\mathcal{O}(m_r\alpha^5)$





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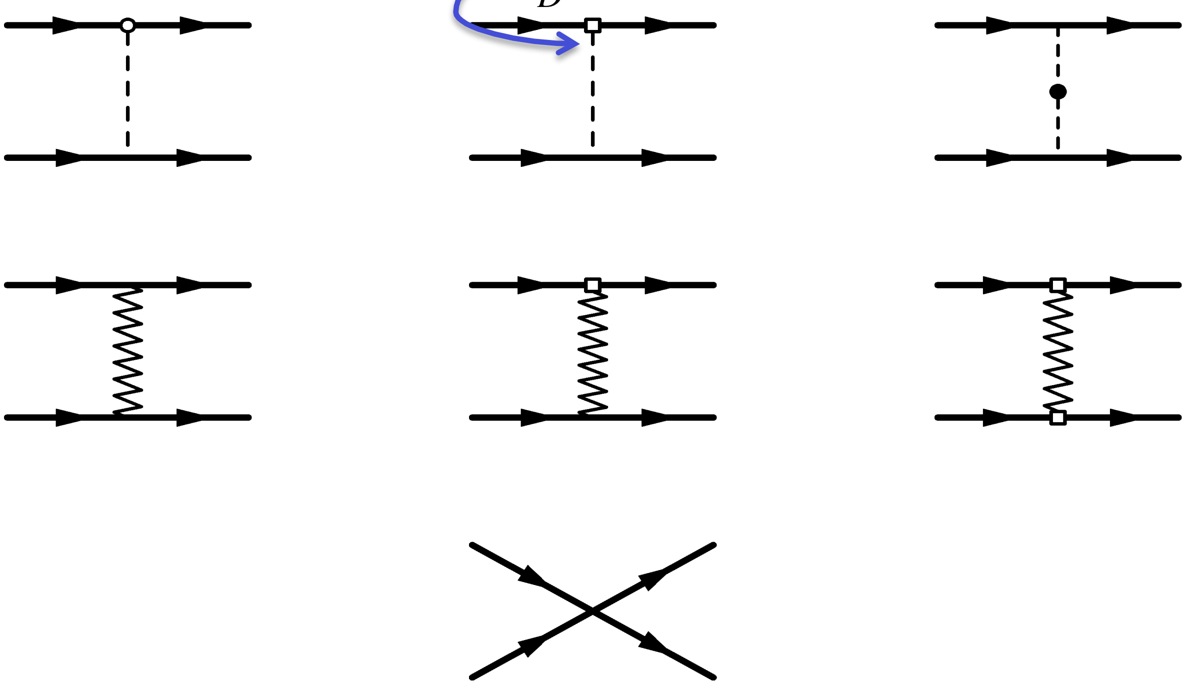
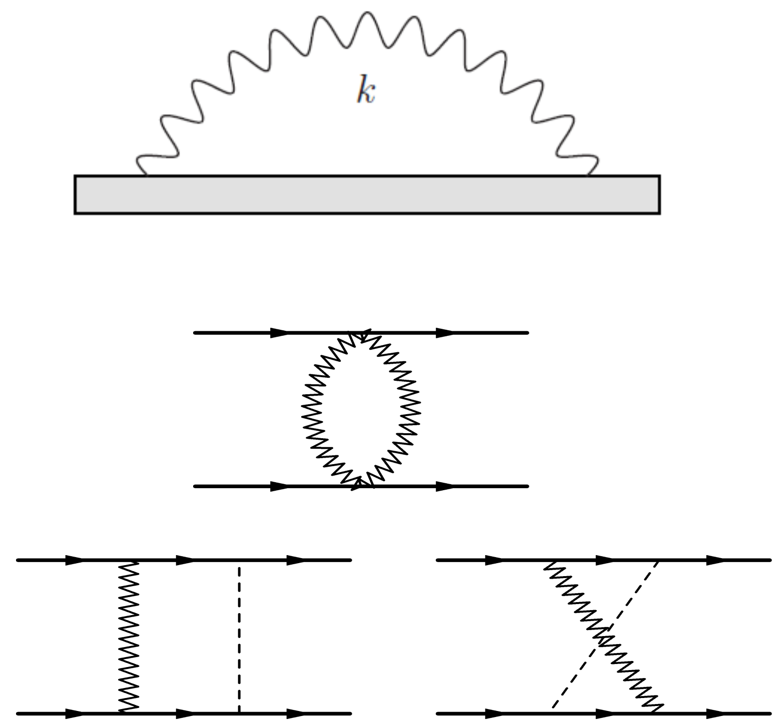
Peset, AP

Relativistic contributions up to $\mathcal{O}(m_r\alpha^5)$

$$c_D^{(\mu)} = 1 + \#\,\alpha + \#\,\alpha^2 + \ldots$$





Summary and error estimates

Theoretical equation for the Lamb Shift:

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Peset, AP

$\mathcal{O}(m_r\alpha^3)$	$V_{\rm VP}^{(0)}$	205.00745
$\mathcal{O}(m_r\alpha^4)$	$V_{\rm VP}^{(0)}$	1.50795
$\mathcal{O}(m_r\alpha^4)$	$V_{\rm VP}^{(0)}$	0.15090
$\mathcal{O}(m_r\alpha^5)$	$V_{\rm VP}^{(0)}$	0.00752
$\mathcal{O}(m_r\alpha^5)$	$V_{LbL}^{(0)}$	-0.00089
$\mathcal{O}(m_r\alpha^4 \times \frac{m_\mu^2}{m_p^2})$	$V^{(2)} + V^{(3)}$	0.05747
$\mathcal{O}(m_r\alpha^5)$	$V_{\rm soft}^{(2)}$/ultrasoft	-0.71903
$\mathcal{O}(m_r\alpha^5)$	$V_{\rm VP}^{(2)}$	0.01876
$\mathcal{O}(m_\mu\alpha^6 \times \ln(\frac{m_\mu}{m_e}))$	$V^{(2)}; c_D^{(\mu)}$	-0.00127
$\mathcal{O}(m_\mu\alpha^6 \times \ln\alpha)$	$V_{\rm VP}^{(2)}; c_D^{(\mu)}$	-0.00454

Hadronic corrections

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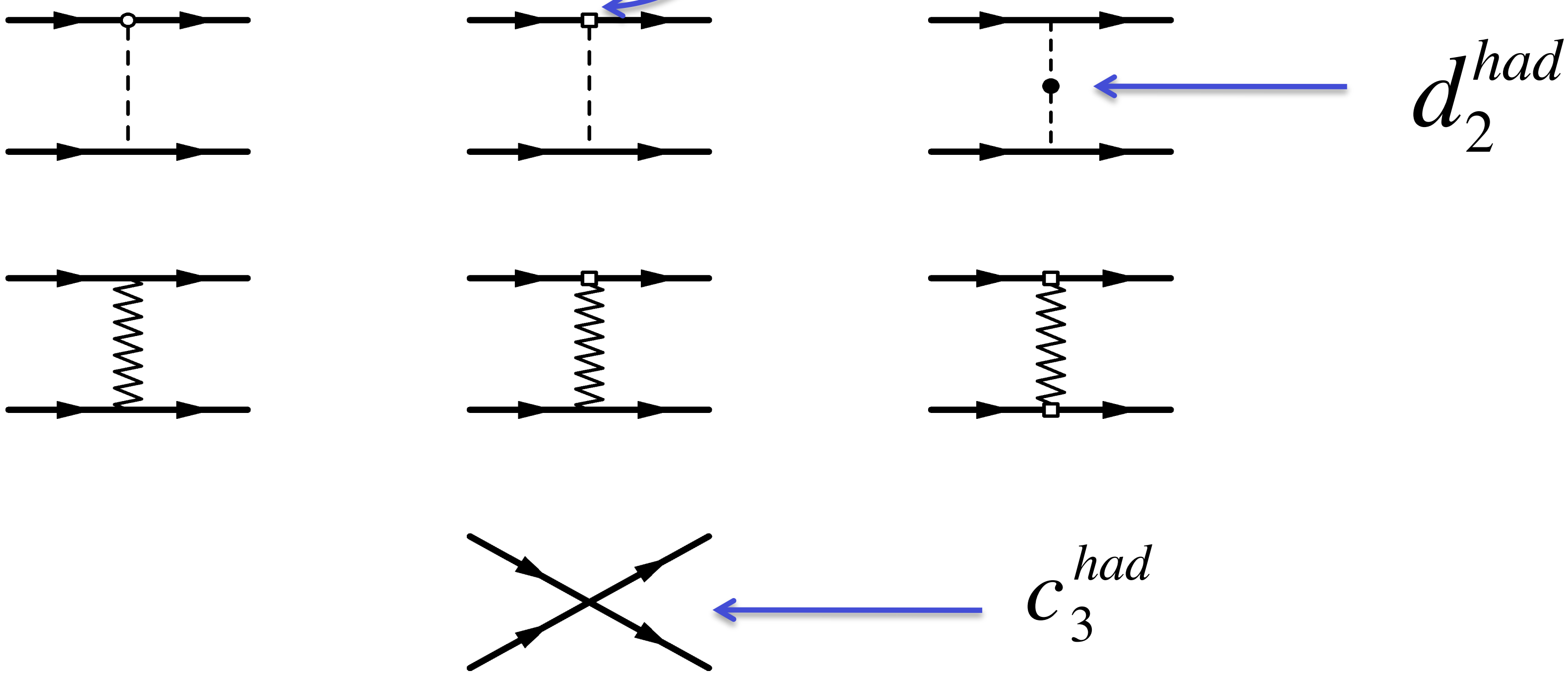
Peset, AP

Scheme dependent $c_D^{(p)} \equiv 1 + \frac{4}{3} r_p^2 m_p^2 + \#\,\alpha + \ldots$

$$\delta\mathcal{L} = -e\frac{c_D^{(p)}}{m_p^2} N_p^\dagger \nabla \cdot \mathbf{E} N_p$$



Hadronic Effects

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Peset, AP

$\mathcal{O}(m_r\alpha^4 \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-5.1975\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r\alpha^5 \times m_r^2 r_p^2)$	$V_{\text{VP}}^{(2)}; c_D^{(p)}; r_p^2$	$-0.0283\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r\alpha^6 \ln\alpha \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-0.0014\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r\alpha^5 \times \frac{m_r^2}{m_\rho^2})$	$V_{\text{VP}_{\text{had}}}^{(2)}; d_2^{\text{had}}$	$0.0111(2)$
$\mathcal{O}(m_r\alpha^5 \times \frac{m_r^2}{m_\rho^2}\frac{m_\mu}{m_\pi})$	$V^{(2)}; c_3^{\text{had}}$	$0.0344(125)$

$$\delta V^{had} = \frac{D_d^{had}}{m_p^2}\delta^{(3)}(\mathbf{r})$$

$$D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2}c_D^{had}$$

Definition of the proton radius

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F. Jegerlehner

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Hadronic Vacuum polarization:
Obtained from Dispersion Relations (DR)

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$$c_3^{\mathrm{had}} = c_3^{\mathrm{Born}} + c_3^{\mathrm{pol}} \sim \alpha^2 \frac{m_\mu}{m_\pi}\left[1 + \# \frac{m_\pi}{\Delta} + \ldots\right] + O\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right)$$

Large-Nc limit: we expect a large contribution from Δ(1232)

$c_3^{\rm had}$: Born/Zemach contribution

$$c_{3,\rm Born}^{pl_i} = 4(4\pi\alpha)^2 M_p^2 m_{l_i} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{1}{\mathbf{q}^6} G_E^{(0)} G_E^{(2)}(-\mathbf{q}^2) = \frac{\pi}{3}\alpha^2 M_p m_\mu \langle r^3 \rangle_{(2)}$$

Zemach third momentum

Zemach momenta:

		$\langle r^3 \rangle$	$\langle r^4 \rangle$	$\langle r^5 \rangle$	$\langle r^6 \rangle$	$\langle r^7 \rangle$	$\langle r^3 \rangle_{(2)}$
EFT: Peset, AP	π	0.4980	0.6877	1.619	5.203	20.92	0.9960
	$\pi\&\Delta$	0.4071	0.6228	1.522	4.978	20.22	0.8142
FITS:	Dipole	0.7706	1.083	1.775	3.325	7.006	2.023
	Kelly	0.9838	1.621	3.209	7.440	19.69	2.526
	Distler et al	1.16(4)	2.59(19)(04)	8.0(1.2)(1.0)	29.8(7.6)(12.6)	$---$	2.85(8)

large dependence on the fitted function & large difference with EFT

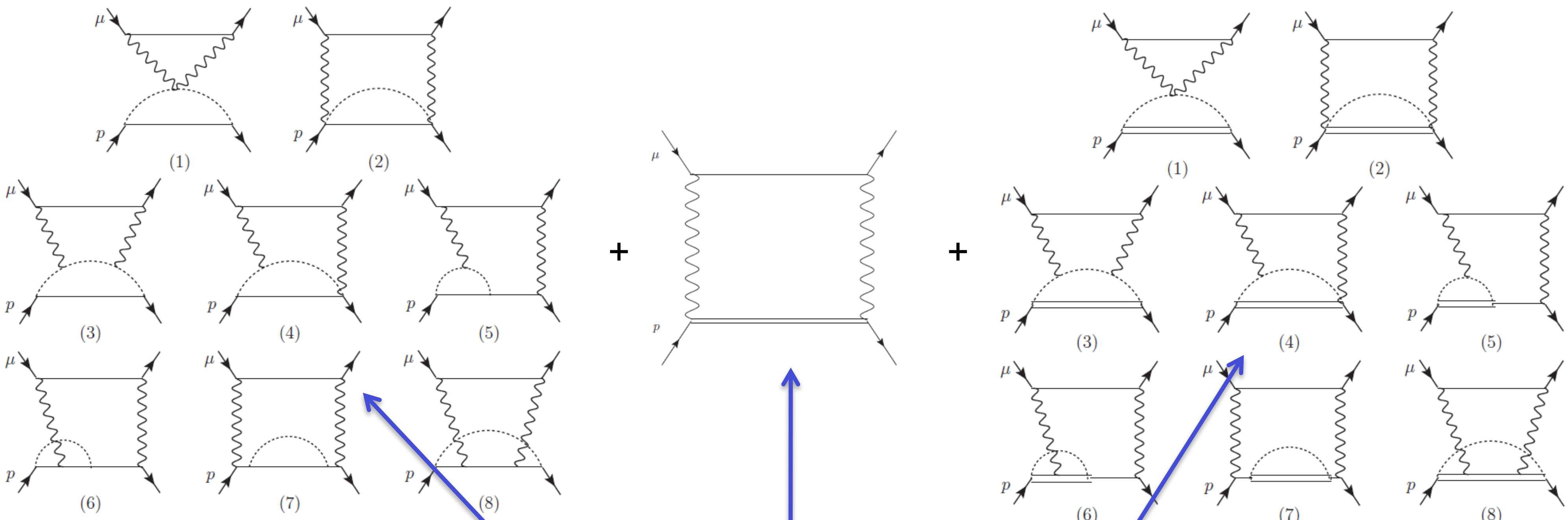
Born energy shift:

μeV	DR	Pachucki	Carlson et al	HBET	AP (π)	Peset, AP $(\pi\&\Delta)$
$\Delta E_{\rm Born}$		23.2(1.0)	24.7(1.6)		10.1(5.1)	8.3(4.3)

we would expect less difference with the DR analysis

$c_3^{\rm had}$: Polarizability

$$T_S^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) S_1(\rho, q^2) + \frac{1}{M_p^2}\left(p^\mu - \frac{M_p\rho}{q^2}q^\mu\right)\left(p^\nu - \frac{M_p\rho}{q^2}q^\nu\right) S_2(\rho, q^2)$$



$$\Delta E_{\rm pol} = \frac{c_{3,\rm pol}^{pl_\mu}}{M_p^2}\frac{1}{\pi}\left(\frac{m_r\alpha}{2}\right)^3 = 18.51(\pi{-}{\rm loop}) - 1.58(\Delta{-}{\rm tree}) + 9.25(\pi\Delta{-}{\rm loop}) = 26.2(10.0)\mu{\rm eV}$$

Peset, AP

$c_3^{\rm had}$: Polarizability

Different results for the polarizability contribution:

(μeV)	DR + Model				BχPT (π)	HBET (π)	(π&Δ)
$\Delta E_{\rm pol}$	12(2)	11.5	7.4(2.4)	15.3(5.6)	$8.2(^{+1.2}_{-2.5})$	18.5(9.3)	26.2(10.0)
	Pachucki	Martynenko	Carlson et al	Gorchtein et al	Alarcon et al	Nevado, AP	Peset, AP

corrections to HBET are suppressed by $\sim \frac{m_\pi}{m_\rho}, \frac{1}{N_c}$

The polarizability contribution from EFT is larger than the one computed using combinations of DR and different models

Total TPE energy shift:

$$\Delta E_{\rm TPE} = \Delta E_{\rm Born} + \Delta E_{\rm pol} = 28.59(\pi) + 5.86(\pi\&\Delta) = 34.4(12.5)\mu{\rm eV}$$

(LO) (NLO)

Peset, AP

The total contribution of TPE agrees with DR results better than when arbitrarily split into Born and polarizability pieces

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-**The proton radius puzzle survives the EFT analysis.** The hadronic contributions are the main source of uncertainty, although it is not enough to account for the discrepancy with ep.

- The main radius-independent hadronic contribution is the Two-Photon-Exchange term

  - The EFT approach gives Born & polarizability contributions which are quite different from the ones obtained by DR (+ models)

  - Both the agreement and the disagreement should be further understood

Zemach momenta

Even momenta:

$$G_E(-\mathbf{k}^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\mathbf{k}^{2n} \int_0^{\infty} dr (4\pi) r^{2n} \rho_e(r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\mathbf{k}^{2n} \langle r^{2n} \rangle$$

Odd momenta:

$$\langle r^{2k+1} \rangle = \frac{\pi^{3/2}\Gamma[2+k]}{\Gamma[-1/2-k]} 2^{4+2k} \int \frac{d^3q}{(2\pi)^3} \frac{1}{\mathbf{q}^{2(2+k)}} \left[G_E(-\mathbf{q}^2) - \sum_{n=0}^{k} \frac{\mathbf{q}^{2n}}{n!} \left(\frac{d}{d\mathbf{q}^2}\right)^n G_E(-\mathbf{q}^2)\Big|_{\mathbf{q}^2=0} \right]$$

Polarizability energy shift

$$c_3^{pl_i} = -e^4 M_p m_{l_i} \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^4} \frac{1}{k_E^4 + 4m_{l_i}^2 k_{0,E}^2} \left\{(3k_{0,E}^2 + \mathbf{k}^2) S_1(ik_{0,E}, -k_E^2) - \mathbf{k}^2 S_2(ik_{0,E}, -k_E^2)\right\} + \mathcal{O}(\alpha^3)\,. \quad (4.1)$$

Matching coefficients

$\sim \frac{m_\mu}{\Lambda_{QCD}}$

Hadronic contributions: d_2^{had} c_D^{had} c_3^{had}

EXAMPLE:

HBET:

$$\delta\mathcal{L}_{(N,\Delta)l_\mu} = \frac{1}{m_p^2} c_{3,R}^{pl_\mu} \bar{N}_p \gamma^0 N_p \bar{l}_\mu \gamma_0 l_\mu$$

↓

NRQED:

$$\mathcal{L}_{Nl_\mu} = \frac{1}{m_p^2} c_{3,NR}^{pl_\mu} N_p^\dagger N_p l_\mu^\dagger l_\mu$$

↓

pNRQED:

$$V^{(2)}(r) = c_{3,NR}^{pl_\mu} \frac{\delta^{(3)}(\mathbf{r})}{m_p m_\mu}$$

$$c_{3,NR}^{pl_\mu} = c_{3,R}^{pl_\mu} + c_{3,point-like}^{pl_\mu} + \boxed{c_3^{had}}$$

Definition of the proton radius

$$r_p^2 = 6\frac{dG_{p,E}(q^2)}{dq^2}\Big|_{q^2=0}$$

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NRQED Lagrangian:

$$\delta\mathcal{L} = -e\frac{c_D^{(p)}}{m_p^2}N_p^\dagger \nabla\cdot\mathbf{E}N_p$$

Caswell, Lepage '86

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2)$$

$$c_D = 1 + 2F_2 + 8F_1'$$

$$r^2_{p,\overline{MS}}(\nu) = \frac{3}{4m_p^2}\left(c_D^{\overline{MS}}(\nu) - 1\right)$$

General expression:

$$c_D^{(p)}(\nu) - c^{(p)}_{D,point-like}(\nu) \equiv \frac{4}{3}m_p^2 r_p^2$$