

## Space-time Thermodynamics without Hidden Degrees of Freedom

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Based on recent work in collaboration with H. Haggard, A. Riello, C. Rovelli

Jacobson 95, Jacobson Parentani 2003 Bianchi 2012, Bianchi Myers 2013 Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, Speziale

## OUTLINE

1. space-time "entanglement" thermodynamics and the Einstein's equation of state

2. Bekenstein-Hawking entropy from entanglement thermodynamics in perturbative quantum gravity

3. space-time thermodynamics fundamental ingredients from a non perturbative LQG perspective

## **BLACK HOLES THERMODYNAMICS**

BH solutions can be described as dynamical systems in terms of a small number of parameters: M, J, Qe (no hair theorem)

+

...

Israel 67, Christodoulou 71, Hawking 71, Bardeen, Carter 73

- gravity 
$$dM=rac{\kappa}{8\pi}dA$$

+ QFT

$$T_{H}=rac{\kappa\hbar}{2\pi}$$
 Hawking temperature



$$M + \delta E \rightarrow A + \delta A$$

$$S_{BH} = rac{A}{4G\hbar}$$
 Bekenstein-Hawking entropy

puzzling interplay between **GRAVITY / QFT / THERMODYNAMICS** 

#### NEAR HORIZON LIMIT => LOCAL CAUSAL HORIZON THERMODYNAMICS

Schwrzschild 
$$d au^2 = (1 - rac{2MG}{r})dt^2 - (1 - rac{2MG}{r})^{-1}dr^2 - r^2d\Omega^2$$

 $=g_{\mu
u}dx^{\mu}dx^{
u}.$ 



black hole thermodynamical relations holds for the event horizon of an asymptotic uniformly accelerated observer, simply derived from local killing field

$$dM = \frac{\kappa}{8\pi} dA$$

Jacobson Parentani 03, Perez 12

## FORGET BH => LOCAL CAUSAL HORIZON THERMODYNAMICS



- 4d Minkowski space

$$x^{\mu} = (T, Z, X, Y)$$

- two regions  $L = Z < |T|, \quad R = Z > |T|$
- QFT



- Minkowski vacuum is not separable: highly nonlocal space like correlations, high entanglement



- vacuum entanglement entropy

 $S_{ent}(|0
angle) = -Tr_R(
ho_0 \ln 
ho_0)$  UV divergent!

- Minkowski vacuum is not separable: highly nonlocal space like correlations, high entanglement



- restricted vacuum is thermal: Unruh effect

$$\rho_0 = Tr_L(|0\rangle\langle 0|) = \frac{e^{-2\pi K_R}}{Z}$$

Gibbs state

$$=\frac{e^{-\frac{2\pi}{\hbar a}a\,\hbar K_R}}{Z}\ =\frac{e^{-H/T}}{Z}$$

thermodynamics?

$$\delta S_{ent} = S_{ent}(|\Psi\rangle) - S_{ent}(|0\rangle)$$

- small perturbation of the vacuum

$$ho_1 = Tr_L(|\Psi\rangle\langle\Psi|)$$
  
 $\delta
ho = 
ho_1 - 
ho_0$ 

=> entanglement perturbation

$$\delta S_{ent} = -\delta Tr(\rho \ln \rho) = -Tr(\delta \rho \ln \rho_0) - Tr(\rho_0 \frac{1}{\rho_0} \delta \rho)$$
$$= 2\pi Tr(K_R \delta \rho) + \ln Z Tr(\delta \rho) = \frac{2\pi}{a\hbar} Tr(\hat{H}\delta \rho) = \frac{\delta E}{T}$$

 $H = \hbar K_R a$ Rindler Hamiltonian

L

T = Unruh temperature

R

$$|\Psi\rangle = | \qquad \rangle \rightarrow$$

Jacobson: can we invert the logic ? **QFT** + THERMODYNAMICS => **GRAVITY** 

hint:

entropy balance



## equation of state



## **EINSTEIN'S EQUATION OF STATE**

consider physical approach for the local vacuum perturbation:





$$S_{ent}(|0\rangle) = -Tr_R(
ho_0 \ln 
ho_0) = \alpha/l_p^2 A$$

- $\delta S_{ent} = \delta(\alpha/l_p^2 A)$
- assume universality  $\ \delta lpha = 0$ 
  - if  $\delta S_{ent} = \delta E/T$
  - $\Rightarrow \delta S_{ent} = \alpha / l_n^2 \delta A$  space-time cannot be inert !

- area change

area change 
$$\delta A = \int \delta heta \, dv dy_1 dy_2$$
 $\delta heta = heta_p + rac{\partial heta}{\partial v}|_p \, \delta v$ 

- Raychaudhuri equation

$$rac{\partial heta}{\partial v} = -R_{\mu
u}k^{\mu}k^{
u}$$





EINSTEIN'S EQUATION OF STATE

- so 
$$\delta S_{ent} = \delta E/T$$
 =>  $\delta E_H = \alpha \frac{\hbar \kappa}{2\pi l_p^2} \, \delta A_H$ 

- for all null  $\,k^a\,$ 

$$\begin{array}{lll} \mbox{local constitutive} & \frac{2\pi l_p^2}{\hbar \alpha} T_{\mu\nu} = R_{\mu\nu} + \Phi \, g_{\mu\nu} \\ \mbox{-define} & \Phi = -\frac{1}{2}R - \Lambda & \mbox{by} & \mbox{local energy} & \nabla^{\nu}T_{\mu\nu} = 0 \\ \mbox{-define} & \Phi = -\frac{1}{2}R - \Lambda & \mbox{by} & \mbox{local energy} & \nabla^{\nu}R_{\mu\nu} = 0 \\ \mbox{-Bianchi} & \nabla^{\nu}R_{\mu\nu} = \frac{1}{2}\nabla_{\mu}R \\ \mbox{=>} & 8\pi G \, T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} \, g_{\mu\nu} - \Lambda \, g_{\mu\nu} & \mbox{then extended to the whole} \\ \mbox{if only} & \alpha = 1/4 \\ \mbox{Bekenstein-Hawking} & \mbox{=>} & S_{ent} = S_{BH} = A/4l_p^2 \end{array}$$

just an amazing SEMICLASSICAL RESULT



finiteness & universality <=> QUANTUM GRAVITY UV cut-off problems Solodukhin Liv. Rev. Rel. 2011

a straightforward way to see it...

- space-time explicitly dynamical but at quantum level:add the graviton to the previous picture

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G} h_{\mu\nu}$$

the graviton field is defined as the perturbation of the space-time metric about the Minkowski flat metric

- at leading order in a perturbative expansion in Newton's constant G:

$$I = I_{grav}[h_{\mu\nu}] + I_{matt}[\phi] + \sqrt{8\pi G} \int d^4x h_{\mu\nu} T^{\mu\nu}$$

**Fierz-Pauli action** 

universal coupling

- equations of motion

$$\Box h_{\mu\nu} = -\sqrt{8\pi G} (T^{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^{\sigma}_{\sigma})$$
 harmonic gauge

- look at small perturbations of the interacting Minkowski vacuum of matter fields and gravitons

- boost energy flux

$$K = \int_{H} T_{\mu
u} \chi^{\mu} dH^{
u}$$

- horizon cross section area at leading order in a perturbative expansion in Newton's constant G

$$A_H = A_0 + \sqrt{8\pi G} \int (-\Box h_{\mu\nu} k^\mu k^\nu) v dv dy_1 dy_2$$

- entropy universality recovered dynamically  $\delta_{\cdot}$ 

$$E=2\pi Tr(\int_{H}T_{\mu
u}vk^{\mu}dH^{
u}\delta
ho)$$

$$\delta S_{ent} = \frac{2\pi}{\sqrt{8\pi G}} Tr(\int_H (-\Box h_{\mu\nu} k^\mu k^\nu) v \, dv dy_1 dy_2 \,\delta\rho)$$

$$\Rightarrow \quad \delta S_{ent} = 2\pi \frac{1}{\sqrt{8\pi G}} Tr(A_H \delta \rho) = \frac{\delta A}{4G}$$

**Bekenstein-Hawking** 

## **QUANTUM GRAVITY: PERTURBATIVE APPROACH**

#### what do we learn?

$$\delta E = \frac{a}{8\pi G} \,\delta A \quad \checkmark$$

- 1. is not only a strict consequence of Einstein's equations, but is also a sufficient condition for the Einstein's equation to hold
- 2. a universal entanglement entropy area relation a la Bekenstein-Hawking requires gravity playing an active role at quantum level

Einstein's equations are encoded in the relation of proportionality between classical averages of quantum fluctuations of wedge fields energy and horizon area, as measured by a uniformly accelerating observer

... 3. the role of the area operator suggest to look at a non-perturbative quantum gravity scenario



#### our variables

- Einstein formulation

$$\mathcal{S}[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R - 2\Lambda \right)$$

tetrad formulation

$$S[e] = \int e^{I} \wedge e^{J} \wedge F^{KL} \varepsilon_{IJKL}$$

- Palatini: first order formulation

$$\mathcal{S}[e,\omega] = \int e \wedge e \wedge F[\omega]^*$$

**Holst action** 

$$g_{\mu\nu}(x) = e^I_\mu(x)e^J_\nu(x)\eta_{IJ}$$

it admits a purely geometric formulation in terms of forms

Lorentz connection is an object in the Lorentz algebra

$$\mathcal{S}[e,\omega] = \int e \wedge e \wedge F^{\star} + \frac{1}{\gamma} \int e \wedge e \wedge F$$

the coupling constant  $\gamma$  is called the Barbero-Immirzi coupling constant. This is the action with which we do 4-dimensional quantum gravity, as it is the most generic one: it is polynomial and has all the relevant symmetries. The Holst term has no effect on classical physics, but plays a substantial role in the quantum theory.

## A NON - PERTURBATIVE PERSPECTIVE

Rovelli

#### in 3D

- discretize the connection, as in Yang Mills theory, by associating an SU(2) group element Ue to each edge e of the two complex. We discretize the triad by associating a vector  $e^{i}_{b}$  of R<sup>3</sup> to each bone b of the original triangulation



 $\begin{array}{ccc} \omega \longrightarrow U_e \\ e \longrightarrow L_f \end{array}$ 

- the formal relation between the continuous and the discrete variables can be taken to be the following
- the boundary variables  $(L_l, \mathcal{U}_l) \in su(2) \times SU(2)$
- the algebra is also the cotangent space of the group, we can write

 $su(2) \times SU(2) \sim T_*SU(2) \implies T_*SU(2)^L$ 

- a cotangent space carries a natural symplectic structure

holonomy of the connection along the edge, namely the matrix of the parallel transport generated by the connection along the edge

$$U_e = P \ e^{\int_e \omega} \in SL(2, C)$$
$$B_f = \int_{t_f} B \ \in sl(2, C)$$

=>

boundary phase space

quantization

analog to angular momentum

- Loop gravity quantum geometry



- abstract graph  $\Gamma$
- SU(2) spin jj at every link
- SU(2) invariant tensor ι<sub>n</sub> at every node (intertwiner)

the group SU(2) encodes the symmetries of the quantized space

GR on a lattice:

- Hilbert space  ${\cal H}$
- Feynmann rules defining the transition amplitudes  $\mathcal{A}_v$
- BUT gravitational field forms the space-time metric: quanta of gravity are also quanta of space



SL(2,C) spin foams: quantum gravitational processes between spin network states

- interaction vertices among quanta of space

$$Z = \sum_{j_f, j_e} \prod_f (2j_f + 1) \prod_V \mathcal{A}_v(j_e, j_v)$$

semiclassical limit of a sum over 4-geometries

$$\Rightarrow \qquad Z \sim \int Dg \, e^{\frac{i}{\hbar} \int d^4 x \sqrt{g} \, R}$$

#### - characterize the Hilbert space

consider here for simplicity a single link of the spin network. The corresponding quantum state if a function  $\psi(U)$  on SU(2): a basis

 $\langle U|j,m,n\rangle = D^j_{mn}(U)$ 

the state space H associated to the link decomposes as

 $\mathcal{H} = \oplus_j (\mathcal{H}_j^* \otimes \mathcal{H}_j)$ 

the dynamics of the theory is obtained mapping these states to a particular subspace of unitary representations of SL(2,C)

Wigner matrices

$$Y_{\gamma} : \mathcal{H}^{j} \to \mathcal{H}_{(\gamma j), j)}$$
$$|j, m\rangle \to |(\gamma j, j); j, m\rangle$$

 $\boldsymbol{\gamma}$  is the Immirzi parameter



• on the image of this map the boost generator and the rotation generator satisfy

$$\vec{K} = \gamma \vec{L}$$

the spin j component of the state space live in a subspace of

 $\mathcal{H}^{j}_{\gamma} = (\mathcal{H}^{j}_{\gamma j, j})_{\mathcal{S}} \otimes (\mathcal{H}^{j}_{\gamma j, j})_{\mathcal{T}} = \mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{T}}$ 



U has the classical interpretation of the path integral of the gravitational Ashtekar connection along the link

## A NON - PERTURBATIVE PERSPECTIVE

- describe a quantum surface  $|s\rangle$  embedded in 4d

SU(2) spin-network states represent the geometry of a spatial section of space-time with normal  $t^{\mu}$ 

if we measure the geometry of a space-like surface S with a certain precision, we can describe the result of our measurements by tessellating with N facets f



each facet is going to be described by a spin network link, which we can visualise as puncturing the surface: the state describing a quantum surface of a given area and with normal in direction  $\vec{n}$  has the form

$$|s
angle = \otimes_f |s_f
angle$$

a single puncture state  $|sf \ \rangle$  is an eigenstate of the area operator of the corresponding facet

$$|A_f|=8\pi G\hbar\gamma |L_f|$$
 =>  $A_f=8\pi G\hbar\gamma j$  quantised area facet

- For a single puncture, the relation  $|K_f| = \gamma |L_f|$  then gives

This relation is the basis of the second part of Jacobson argument. From this relation we can obtain the Einstein equations. Notice that this is a relation between between the area of a space like surface (say in the x-y plane) and the boost (in the dual t – z plane).

not only ...

- the tensor product structure

$$\mathcal{H}^j_\gamma = \mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{T}$$

is related to a local "partition of space" in two chunks, associated to the source and target nodes at the two ends of the link

 the short-scale correlations of the theory are then captured by the correlations between these two Hilbert states





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$$K_f^z = rac{A_f}{8\pi G\hbar}$$
  $K_f^z = ec{K} \cdot ec{n}$  no Immirzi

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again the localization scheme...

- the short-scale correlations of the theory are then captured by the correlations between these two Hilbert states





$$K_f^z = \vec{K} \cdot \vec{n}$$

single link entanglement thermodynamics!

- If  $|\Phi\rangle \in H_{Y}^{j}$  is a pure state of the link, then its restriction to one half-link is the density matrix

$$\rho_{\mathcal{T}} = Tr_{\mathcal{H}_{\mathcal{S}}}[|\Phi\rangle\langle\Phi|]$$

measurements performed on one side only of the facet

- consider a particular family of states

$$\rho_{\mathcal{T}} = e^{-2\pi K}$$

- these states have interesting properties:

$$S = -Tr[\rho_f \ln \rho_f] = 2\pi Tr[\rho_f K_f^z] = 2\pi \langle K \rangle$$

measures the entanglement across the facet

where  $\langle K \rangle = \langle jf | K_f^z | jf \rangle = \gamma jf$  is the expectation value of  $K_f^z$  on  $| sf \rangle$ 

- a variation  $\delta \rho$  relates a change in the entropy to a change in the expectation value of K

$$\delta S = 2\pi\,\delta\langle K
angle \qquad ext{given}\,\langle ext{K}
angle = \gamma\langle ext{L}
angle \qquad ext{=>} \qquad \delta S = rac{\delta A_f}{4G\hbar}$$



- area/energy is given by an algebraic relation at the root of quantum gravitational dynamics

$$K_f^z = \frac{A_f}{8\pi G\hbar}$$

- finiteness and universality of the entanglement entropy density per unit horizon area rely on discreteness of geometry at UV scale and on finiteness of spectrum of the area operator  $\delta S = \frac{\delta A_f}{4G\hbar}$ 

same relations we find at classical level?

basic structure of quantum theory : relations should refer to results of a measurement performed by an external system. here the quantum system is a portion of quantum space-time the external observer is an accelerated observer which can measure local aspects of the geometry

 the states |sf > describe a portion of a quantum surface, a small facet with given area Af and normal in direction. The evolution of the Lorentz invariant facet state in space-time as seeing by an accelerated observer is generated by the unitary operator representing a Lorentz boost



 a classical measurement of the facet state dynamics is associated to the covariant dynamics of the trajectory of the accelerated observer

$$\rho_f = N e^{-\frac{2\pi}{\hbar a}H} \quad \Longrightarrow \quad E = \frac{A_f a}{8\pi G} \quad T = \frac{\delta E}{\delta S} = \frac{a\hbar}{2\pi}$$

ensemble of single facet states

- 1. The derivation of the Einstein's equations by Jacobson admits an interpretation different from the common one indicating the existence of microscopic states for which the Einstein equations would be an equation of state.
  - QFT entanglement thermodynamics (modular localization +KMS)
  - finiteness & universality of Bekenstein-Hawking area law from gravity
  - relevant microscopic degrees of freedom are the quanta of the gravitational field
- 2. the derivation requires quantum gravity: the area law indicates that there should be a physical (real) quantum mechanical cutoff at the Planck scale. This is precisely the conclusion that one obtains from a full quantum gravitational treatment of the problem in the context of loop quantum gravity, where the discreteness of the geometry at the Planck scale appears as a conventional quantization effect
- 3. we can identify quantum states of the semiclassical field that have a semiclassical behaviour and appears as continuous geometries when probed at large scale: in terms of these state we show that the fundamental input for the thermodynamical derivation of EE can be recovered starting from the elementary quantum dynamics of the theory
  - relation between the boost generator and the area: it contains G and gives the dynamics

# thank you.