Plasma effects on fast pair beams: Kinetic instability of parallel electrostatic waves

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Vlasov-Equation

$$\frac{\mathrm{d}f_a}{\mathrm{d}t} = \frac{\partial f_a}{\partial t} + \mathbf{v}\frac{\partial f_a}{\partial \mathbf{x}} + q_a\left[\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right]\frac{\partial f_a}{\partial \mathbf{p}} = S_{a,0}(\mathbf{x},\mathbf{p},t)$$

Maxwell-Equations

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = 0$$

$$\nabla\times\mathbf{B} - \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = \frac{4\pi}{c}\sum_a q_a\int \mathrm{d}^3\mathbf{p}\,\mathbf{v}f_a(\mathbf{x},\mathbf{p},t)$$

$$\nabla\times\mathbf{E} = 4\pi\sum_a q_a\int \mathrm{d}^3\mathbf{p}\,f_a(\mathbf{x},\mathbf{p},t)$$

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Collisions

$$g = \frac{\nu_{ee}}{\omega_{pe}} = 7.3 \cdot 10^{-4} \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{T_e}{\mathrm{K}}\right)^{-3/2}$$
$$= 2.3 \cdot 10^{-13} \left(\frac{n_e}{10^{-7}\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{T_e}{10^4\mathrm{K}}\right)^{-3/2}$$

Wavelength/-vector

$$k \leq k_{max} = 2\pi n_e^{1/3} = 0.03 \left(\frac{n_e}{10^{-7}\mathrm{cm}^{-3}}\right)^{1/3} \mathrm{cm}^{-1}$$

Test-Wave Approach

- initial state of particle distribution
- reaction to electromagnetic fields
- instabilities of particle distributions and properties of plasma-waves

Test-Particle Approach

- initial state of electromagnetic fields
- neglect impact of charged particles on fields
- transport of charged particles in magnetic fields, e.g. mean free path, diffusion-coefficients

Numerical Approach

- e.g. particle in cell (PIC)

Approaches

Test-Wave Approach

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- small pertubation in a field free and unmagnetized plasma

$$
\begin{aligned}
f_a &= f_{a,0} + \delta f_a \\
\mathbf{E} &= \delta \mathbf{E} \\
\mathbf{B} &= \delta \mathbf{B}
\end{aligned}
$$

$$
\frac{\partial f_{a,0}}{\partial t} + \mathbf{v} \frac{\partial \delta f_{a,0}}{\partial \mathbf{x}} - S_{a,0}(\mathbf{x}, \mathbf{p}, t) = 0
$$

$$
\Rightarrow \frac{\partial \delta f_a}{\partial t} + \mathbf{v} \frac{\partial \delta f_a}{\partial \mathbf{x}} + q_a \left[\delta \mathbf{E} + \frac{\mathbf{v} \times \delta \mathbf{B}}{c} \right] \frac{\partial f_{a,0}}{\partial \mathbf{p}} = 0 \qquad (1)
$$

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Maxwell-Equations

$$\mathbf{k} \cdot \mathbf{B}_1 = 0$$

$$\mathbf{B}_1 = \frac{c}{\omega} \mathbf{k} \times \mathbf{E}_1$$

$$\imath \mathbf{k} \times \mathbf{B}_1 = -\imath \frac{\omega}{c} \mathbf{E}_1 + \frac{4\pi}{c} \sum_a q_a \int \mathrm{d}^3 p \, \mathbf{v} f_{a,1}$$

$$\imath \mathbf{k} \cdot \mathbf{E}_1 = 4\pi \sum_a q_a \int \mathrm{d}^3 p \, f_{a,1}$$

Wave Equation

$$\frac{c}{\omega} \mathbf{k} \times (\mathbf{k} \times \mathbf{E}_1) + \frac{\omega}{c} \mathbf{E}_1 = \imath \frac{4\pi}{c} \sum_a q_a \int \mathrm{d}^3 p \, \mathbf{v} f_{a,1} \tag{2}$$

Maxwell-Operator

- $f_{a,1} = f_{response} + \delta N_a^0 = -\imath q_a \frac{\mathbf{E}_1 + \frac{\mathbf{v}\times(\mathbf{k}\times\mathbf{E}_1)}{\omega}}{(\omega - \mathbf{kv})} \frac{\partial f_{a,0}}{\partial \mathbf{p}} + \imath \frac{\delta f_a(\mathbf{k},\mathbf{p},t=0)}{(2\pi)^4(\omega - \mathbf{kv})}$
- $\mathbf{j}_1 = \overset{\leftrightarrow}{\sigma}\mathbf{E} = \sum_a q_a \int \mathrm{d}^3p\, \mathbf{v} f_{response}$
- $\Psi_{ij} = \delta_{ij} + \frac{4\pi\imath}{\omega}\sigma_{ij}$

$$\Lambda_{ij} E_{1j} = -\frac{4\pi\imath}{\omega}\delta_{ij} \sum_a q_a \int \mathrm{d}^3p\, v_j \delta N_a^0 \tag{3}$$

$$\Lambda_{ij} = \frac{k^2c^2}{\omega^2}\left[\frac{k_ik_j}{k^2} - \delta_{ij}\right] + \Psi_{ij} \tag{4}$$

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- $\mathbf{k} = (0, 0, k)^T = k\mathbf{e}_z$
- $\frac{(\mathbf{k}\times\mathbf{v})\times\mathbf{k}}{k^2} = (v_x, v_y, 0)^T$
- $\frac{\mathbf{k}}{k^2}(\mathbf{k}\cdot\mathbf{v}) = (0, 0, v_z)^T$

$$\Lambda_T \mathbf{E}_{1,\perp} = -\frac{4\pi\imath}{\omega k^2}\sum_a q_a \int \mathrm{d}^3p\, \delta N_a^0 (\mathbf{k}\times\mathbf{v})\times\mathbf{k} \tag{5}$$

$$\Lambda_e \mathbf{E}_{1,\parallel} = -\frac{4\pi\imath\mathbf{k}}{\omega k^2}\sum_a q_a \int \mathrm{d}^3p\, \delta N_a^0 (\mathbf{k}\cdot\mathbf{v}) \tag{6}$$

$$\Rightarrow \Lambda_{ij} = \begin{pmatrix} \Lambda_T & 0 & 0 \\ 0 & \Lambda_T & 0 \\ 0 & 0 & \Lambda_e \end{pmatrix}$$

Collective Modes

$$\det \Lambda_{ij} = \Lambda_T^2 \Lambda_e = 0 \tag{7}$$

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Electromagnetic Mode

$$\Lambda_T = 1 - \frac{k^2 c^2}{\omega^2} + \pi \sum_a \frac{\omega_{p,a}^2}{\omega^2 n_a} \int_{-\infty}^{\infty} \mathrm{d}p_\parallel \int_0^\infty \mathrm{d}p_\perp \frac{p_\perp^2}{\Gamma_a} \times \left[\frac{\partial f_a}{\partial p_\perp} + \frac{k v_\perp}{\omega - k v_\parallel} \frac{\partial f_a}{\partial p_\parallel} \right] \tag{8}$$

Electrostatic Mode

$$\Lambda_e = 1 + 2\pi \sum_a \frac{\omega_{p,a}^2}{\omega n_a} \int_{-\infty}^{\infty} \mathrm{d}p_\parallel p_\parallel \int_0^\infty \mathrm{d}p_\perp \frac{p_\perp}{\Gamma_a(\omega - k v_\parallel)} \frac{\partial f_a}{\partial p_\parallel} \tag{9}$$

Electromagnetic Fluctuation Spectrum

$$\begin{pmatrix} < \delta E_{\parallel}^2 >_{k,\omega} \\ < \delta E_{\perp}^2 >_{k,\omega} \\ < \delta B_{\perp}^2 >_{k,\omega} \end{pmatrix} = \sum_a \frac{\omega_{p,a}^2 m_a}{4\pi^3 k^2} \begin{pmatrix} \frac{|K_{\parallel}(k,\omega)|}{|\omega\Lambda_e(\mathbf{k},\omega)|^2} \\ \frac{|K_{\perp}(k,\omega)|}{|\omega\Lambda_T(\mathbf{k},\omega)|^2} \\ \frac{c^2k^2|K_{\perp}(k,\omega)|}{|\omega^2\Lambda_T(\mathbf{k},\omega)|^2} \end{pmatrix} \tag{10}$$

Form Factors for gyrotropic distribution functions $f_a(p_\parallel, p_\perp)$

$$\begin{pmatrix} K_\parallel \\ K_\perp \end{pmatrix} = k^2 \Re \left[\imath \int_{-\infty}^{\infty} \mathrm{d}p_\parallel \int_0^\infty \mathrm{d}p_\perp \frac{p_\perp f_a(p_\parallel, p_\perp)}{\omega - kv_\parallel} \begin{pmatrix} v_\parallel^2 \\ v_\perp^2 \end{pmatrix} \right] \tag{11}$$

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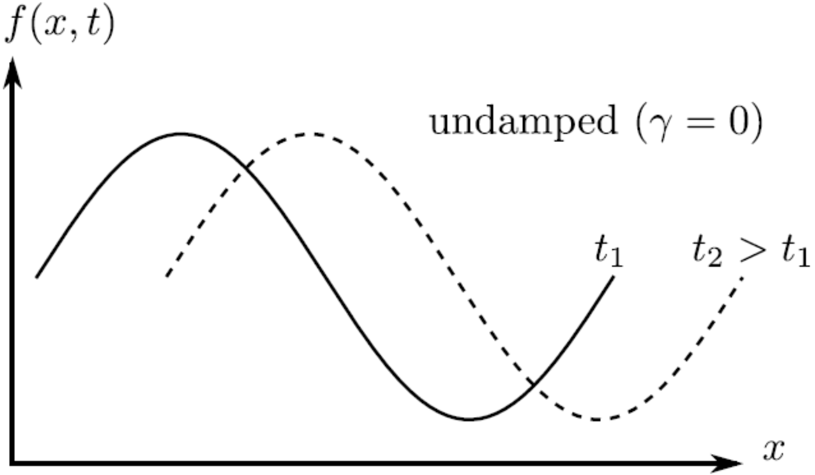
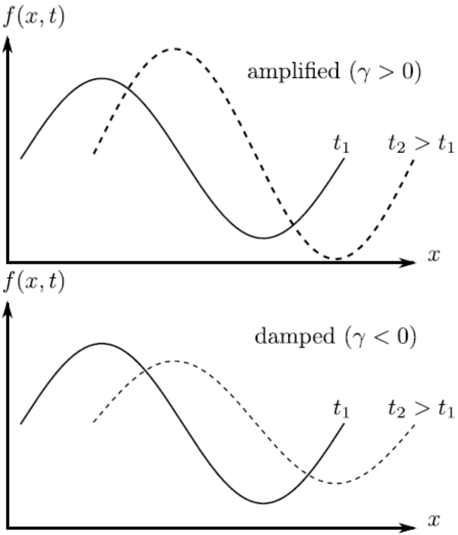
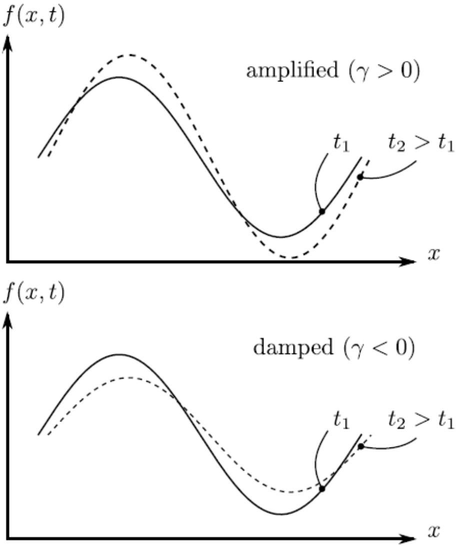
- complex frequency $\omega = \omega_r + \imath\gamma$
- $|\gamma| \ll |\omega_r| \rightarrow$ propagating plasma waves
  - $\gamma > 0$ weakly amplified
  - $\gamma < 0$ weakly damped
- $|\gamma| \gg |\omega_r| \rightarrow$ weakly propagating plasma waves
  - $\gamma > 0$ amplified
  - $\gamma < 0$ dampled
- an electrostatic instability changes the distribution function to the stable parallel plateau distribution with $\partial f / \partial p_{||} = 0$, providing a unique differential energy spectrum $N(E) \propto E^2$
- an electromagnetic instability isotropize the distribution function

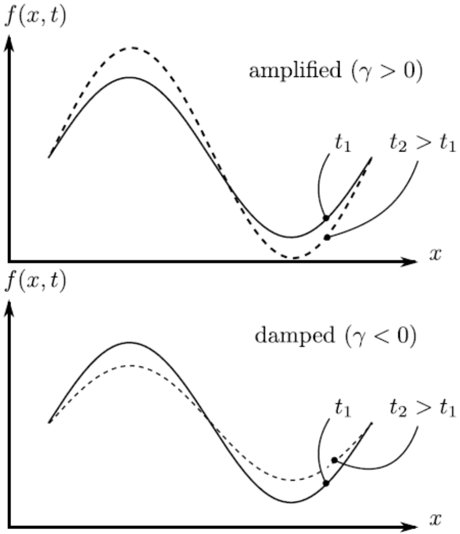
Undamped Plasma Waves




$f(x,t)$
amplified ($\gamma > 0$)
t_1
$t_2 > t_1$
x
$f(x,t)$
damped ($\gamma < 0$)
t_1
$t_2 > t_1$
x


$f(x,t)$
amplified ($\gamma > 0$)
t_1
$t_2 > t_1$
x
$f(x,t)$
damped ($\gamma < 0$)
t_1
$t_2 > t_1$
x

Aperiodic Plasma Waves



- two photons can create an electron positron pair, if

$$E_\gamma \simeq 250 \left(\frac{\epsilon}{1\text{eV}}\right)^{-1} \text{GeV} \tag{12}$$

- mean free path for photon photon annihilation in the EBL

$$\lambda_{\gamma\gamma} \simeq \frac{1}{\sigma_{\gamma\gamma} n_{EBL}} \tag{13}$$

- Schickeiser et al 2012 analytically calculated the pair production spectrum ($x = p_{||}/m_e c$)

$$n(x) = A_1 e^{-\frac{x_c}{x}} \frac{x^{\frac{1}{2}-p}}{[1 + (\frac{x}{x_b})^{3/2}]} H(x) \tag{14}$$

Photon-Photon Annihilation in IGM

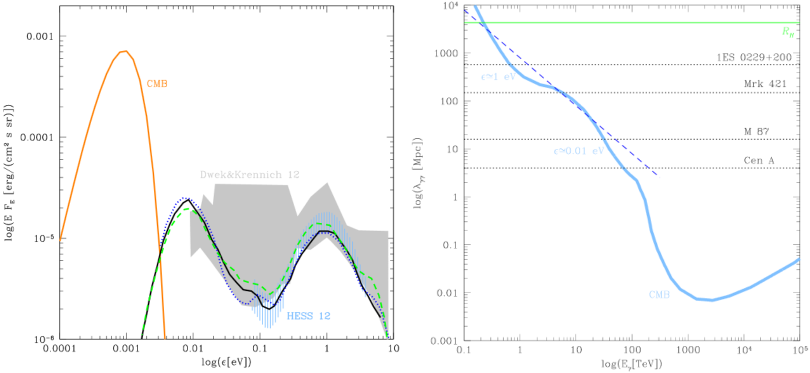


Figure 1: Left panel: Spectral energy distribution of the CMB and EBL (measurements, limits and model calculations). Right panel: Mean free path for photon photon annihilation. From R. Durrer and A. Neronov (2013)

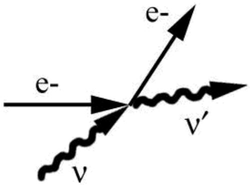


Figure 2: Inverse Compton scattering ($\nu < \nu'$).

- electron positron pairs lose their energy due to inverse Compton (IC) scattering with the CMB photons with the mean distance

$$D_{IC} \simeq 0.3 \left(\frac{E_e}{1\,\mathrm{TeV}} \right)^{-1} \mathrm{Mpc} \tag{15}$$

- mean energy of the inverse Compton photons

$$E_{IC} \simeq 3 \left(\frac{E_e}{1\,\mathrm{TeV}} \right)^{2} \mathrm{GeV} \tag{16}$$

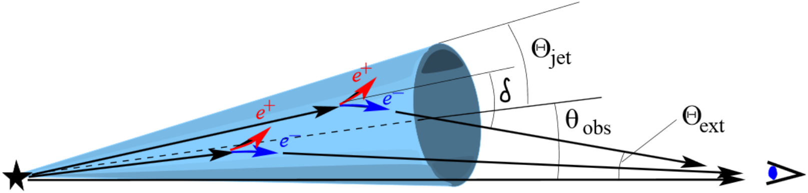


Figure 3: Geometry of secondary blazer emission from electromagnetic cascades. From Neronov et al (2010).

- blazar jets are almost aligned with the line of side $\Theta_{obs} \leq \Theta_{Jet} \sim 0.1^{\circ}$
- secondary cascade emission comes from the locations within the cone, rather than directly from the primary point source
- γ-ray emission in the range of 1-100 GeV is, in principle, detectable by FERMI (Atwood et al (2009)) and above by HESS,MAGIC and VERITAS (Aharonian et al (2008))

- but: no detection of the IC-component!
- first explanation by Neronov et al 2010: Existence of an intergalactic magnetic field (IGMF), which lead to an additional misalignment of the primary γ-ray direction.

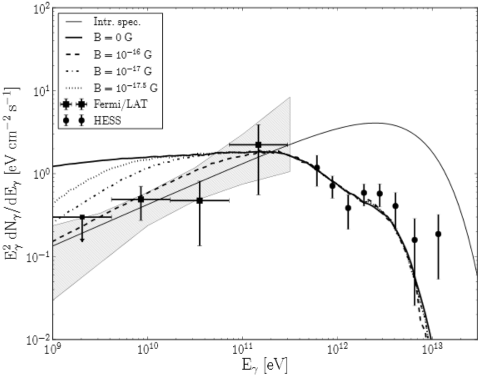


Figure 4: The γ-ray spectrum of the blazar 1ES 0229+200. From Vovk et al (2012).

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- is the non detection of the IC-component only explainable with an IGMF, or is their any mechanism which could also explain the deficit?

$\Rightarrow$ Yes, electron positron beams lose energy due to plasma effects.

- is the non detection of the IC-component only explainable with an IGMF, or is their any mechanism which could also explain the deficit?

⇒ Yes, electron positron beams lose energy due to plasma effects.

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- the electron positron beam cannot propagate stable through the intergalactic medium (Schlickeiser et al 2012)
  - → the beam lose energy due to the electrostatic relaxation to a stable distribution

Reactive beam distribution function in initial γ-ray direction

$$f(p_\perp^\star, p_\parallel^\star) = \frac{\delta(p_\perp^\star)}{\pi p_\perp}\Big(n_{IGM}\delta(p_\parallel^\star) + n_b\delta(p_\parallel^\star - \Gamma m_b v_b)\Big) \tag{17}$$

Reactive beam distribution function in wavevector direction

$$\begin{aligned} f(p_\perp, p_\parallel) = \frac{1}{\pi p_\perp}&(n_{IGM}\delta(p_\parallel)\delta(p_\perp) \\ &+ n_b\delta(p_\parallel - \Gamma m_b v_b \cos\theta)\delta(p_\perp - \Gamma m_b v_b \sin\theta)) \end{aligned} \tag{18}$$

Electrostatic collective mode (Eq. (9))

$$\Lambda_e = 1 + 2\pi \sum_a \frac{\omega_{p,a}^2}{\omega n_a} \int_{-\infty}^{\infty} \mathrm{d}p_{\|} p_{\|} \int_0^{\infty} \mathrm{d}p_{\perp} \frac{p_{\perp}}{\Gamma_a(\omega - kv_{\|})} \frac{\partial f_a}{\partial p_{\|}} = 0$$

- the normalized momentum variables $p_{\|} = m_b c x$ and $p_{\perp} = m_b c y$, $t_a = \Gamma\beta\cos\theta$ and $q_a = \Gamma\beta\sin\theta$ and the inverse index of refraction $z = \omega/kc$
- the plasma distribution function is of the form

$$f_a(p_{\|}, p_{\perp}) = \frac{n_a}{2\pi y}\delta(x - t_a)\delta(y - q_a) \tag{19}$$

- it follows for the electrostatic dispersion function

$$0 = \Lambda_e = 1 + \sum_a \frac{\omega_{p,a}}{zk^2c^2} J_e(z, t_a, q_a) \;, \tag{20}$$

with

$$J_e(z,t_a,q_a) = \int_{-\infty}^{\infty} dx\, U_e(x) \frac{d}{dx}\delta(x-t_a) = -\left[\frac{dU_e}{dx}\right]_{x=t_a} \qquad (21)$$

$$U_e(x) = \frac{x}{z\sqrt{1+q_a^2+x^2}-x}$$

$$\frac{dU_e}{dx} = \frac{z(1+q_a^2)}{(1+q_a^2+x^2)^{3/2}\left[z-\frac{x}{(1+q_a^2+x^2)^{1/2}}\right]^2}$$

- with $\Gamma_a = \sqrt{1+q_a^2+t_a^2}$ one yields

$$J_e(z,t_a,q_a) = -\frac{z(1+q_a^2)}{\Gamma_a^3\left[z-\frac{t_a}{\Gamma_a}\right]^2}$$

$$\Rightarrow 0 = \Lambda_e = 1 - \sum_a \frac{\omega_{p,a}^2}{k^2c^2}\frac{1+(\Gamma_a^2-1)\sin^2\theta}{\Gamma_a^3[z-\beta_a\cos\theta]^2}$$

Electrostatic dispersion function

$$\Lambda_e = 1 - \frac{\omega_{p,e}^2}{\omega^2} - \frac{2\omega_{p,e}^2 \chi (1 - \beta_{\|}^2)}{\Gamma(\omega - k v_{\|})^2} = 1 - R(\omega) = 0 \tag{22}$$

with

$$\chi = \frac{n_b}{n_e} \simeq 10^{-15}, \quad \omega_{p,b}^2 = 2\omega_{p,e}^2 \chi, \quad v_{\|} = \beta_{\|} c = v \cos\theta$$

- $R(\omega)$ has a minimum at

$$\omega_0 = \frac{k v_{\|}}{1 + (2\chi(1 - \beta_{\|}^2)/\Gamma)^{2/3}} = \frac{k v_{\|}}{1 + \xi^{2/3}}$$

- for an instability $R(\omega_0) > 1$
- maximum growth at

$$\frac{\partial}{\partial k}(\Im \omega) = 0 \tag{23}$$

- differentiating Eq. (22) with respect to k yields

$$\frac{\partial \omega}{\partial k} = \frac{v\xi^2}{\xi^2 + \left(1 - \frac{kv_{\|}}{\omega}\right)^3} \tag{24}$$

- with positive α and integer n one gets

$$\omega = \frac{kv_{\|}}{1 + \alpha \exp(-\imath n\pi/3)} \tag{25}$$

- $n = 1$ yields

$$\begin{aligned} \frac{1+2\alpha}{\alpha^3(2+\alpha)} &= \xi^{-2} \\ \frac{k^2 v_{\|}^2}{\omega_{p,e}^2 \chi^2} &= 1 + \frac{1+2\alpha+3\alpha^2}{\alpha^3(2+\alpha)} \end{aligned} \tag{26}$$

- values of the real part and growth rate at maximum growth

$$(\omega)_{max} = \frac{kv_{\parallel}}{1+\alpha+\alpha^2}\left[1+\frac{\alpha}{2}+\imath\frac{\sqrt{3}\alpha}{2}\right] \tag{27}$$

- solution of Eq. (26)

$$\alpha = \left(\frac{\xi}{2}\right)^{1/3} \simeq 10^{-7}\left(\frac{\chi_{-15}}{\Gamma_6}\right)^{1/3}(1-\beta_{\parallel}^2) \ll 1$$

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Real part and growth rate at maximum growth

$$
\begin{aligned}
(\Re\omega)_{max} &\simeq \omega_{p,e}\left(1+\frac{\alpha}{2}\right) \approx \omega_{p,e} \\
(\Im\omega)_{max} &= \gamma_e \simeq \frac{\sqrt{3}}{2}\omega_{p,e}\alpha \\
&= 8.7\cdot 10^{-8}\omega_{p,e}\left(\frac{\chi_{-15}}{\Gamma_6}\right)^{1/3}(1-\beta_{\parallel}^2)^{1/3} \\
&= 1.6\cdot 10^{-6}\left(\frac{\chi_{-15}}{\Gamma_6}\right)^{1/3}(1-\beta_{\parallel}^2)^{1/3}\mathrm{Hz}
\end{aligned}
\tag{28}
$$

- numerical simulations (Grognard 1975, Pavan et al 2011) estimate the relaxation time

$$
\tau_r = 100\gamma_e^{-1}\zeta^{-1} \; , \tag{29}
$$

where $\zeta \leq 1$ accounts nonlinear plasma effects like NLD

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Electromagnetic collective mode (Eq. (8))

$$\Lambda_T = 1 - \frac{k^2c^2}{\omega^2} + \pi \sum_a \frac{\omega_{p,a}^2}{\omega^2 n_a} \int_{-\infty}^{\infty} \mathrm{d}p_\parallel \int_0^\infty \mathrm{d}p_\perp \frac{p_\perp^2}{\Gamma_a} \times \left[\frac{\partial f_a}{\partial p_\perp} + \frac{k v_\perp}{\omega - k v_\parallel} \frac{\partial f_a}{\partial p_\parallel} \right] = 0$$

$$\Lambda_T = 1 - \frac{k^2c^2 + \omega_{p,e}^2 (1 + 2\chi/\Gamma)}{\omega^2} - \frac{\omega_{p,e}^2}{\chi\omega^2} \frac{(k^2c^2 - \omega^2)\beta_\perp^2}{\Gamma\omega^2(\omega - k v_\parallel)^2} = 0$$

- purely aperiodic fluctuations ($\Re\omega = 0$) with the solution

Electromagnetic instability

$$\gamma_{em} = 8 \cdot 10^{-10} \beta_1 \sqrt{\frac{n_{-22}}{\Gamma_6}} \mathrm{Hz} = \frac{\gamma_e}{1375} \left(\frac{\chi_{-15}}{\Gamma_6} \right)^{1/6} \qquad (30)$$

Contents

- the IGM is described by the Maxwell-Juettner distribution, where the finite temperature ($T_e = 10^4 T_4 \mathrm{K}$) is taking into account
- with $p = \sqrt{p_\parallel^2 + p_\perp^2}$ and $\mu_a = m_a c^2/(k_B T_a) = 2/\beta_a^2$, where $\beta_a = \sqrt{2 k_b T_a/(m_a c^2)}$ is the thermal IGM velocity in units of the speed of light, one yields

IGM distribution function

$$F_a(p) = \frac{N_a \mu_a}{4\pi (m_a c)^3 K_2(\mu_a)} e^{-\mu_a \sqrt{1+\frac{p^2}{m_a^2 c^2}}} \tag{31}$$

- for further calculation we simplify Eq. (14) to ($s = p - 1/2$)

$$n(x) = A_0 g(x), \quad g(x) = x^{-s} e^{-\frac{x_c}{x}} H(x) \ ,$$

with

$$A_0 = \frac{x_c^{s-1}}{\Gamma(s-1)} \quad \text{and} \quad x_c = \frac{M_c}{\ln \tau_0} \approx 10^6$$

- adopting a perpendicular spread of

$$G(p_\perp, b) = \frac{H[bm_e c - p_\perp]}{bm_e c}$$

Distribution function of the pair beam

$$f_b(p_\perp, x) = \frac{n_b}{2\pi p_\perp m_e c} A_0 g(x) G(p_\perp, b) \tag{32}$$

Plasma Distribution Function

- from the maximum angular spread (Miniati & Elyiv 2013)

$$\Delta\phi = 10^{-5} \ ,$$

we can assume

$$b \simeq x_c \Delta\phi = \frac{7.2 \cdot 10^{-2}}{1 + \ln \tau_3/(3 \ln 10)} \tag{33}$$

- we restrict our calculation for parallel wave vector orientations ($\theta = 0$)
- for Eq. (26) and Eq. (28) follow

$$\alpha(\theta = 0) = 10^{-11} \left(\frac{\chi_{-15}}{\Gamma_6}\right)^{1/3}$$

and

$$(\Im\omega)_{max} = 1.5 \cdot 10^{-10} \left(\frac{\chi_{-15}}{\Gamma_6}\right)^{1/3} \text{Hz}$$

PDF of the IGM and pair beam

$$f(x, p_\perp) = \frac{N_e \mu_e}{4\pi (m_e c)^3} \left[\frac{\exp\left(-\mu_e \sqrt{1 + \frac{p^2}{m_e^2 c^2}}\right)}{K_2(\mu_e)} \right.$$
$$\left. + \frac{\exp\left(-\frac{\mu_e}{\eta\epsilon} \sqrt{1 + \frac{\epsilon^2 p^2}{m_e^2 c^2}}\right)}{\eta \epsilon^4 K_2\left(\frac{\mu_e}{\eta\epsilon}\right)} \right]$$
$$+ \frac{n_b}{2\pi p_\perp m_e c} A_0 g(x) G(p_\perp, b) \tag{34}$$

with $\eta = T_p / T_e$ and $\epsilon = m_e / m_p$.

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- after some calculations one gets (for detailed discussion see Schlickeiser et al 2013)

Electrostatic dispersion function

$$0 = \Lambda_e(R, I) \simeq 1 - \frac{1}{\kappa^2 \beta_e^2}\left[Z^{'}\left(\frac{R + \imath I}{\beta_e}\right) + \frac{1}{\eta} Z^{'}\left(\frac{R + \imath I}{\sqrt{\eta\epsilon}\beta_e}\right)\right] - \imath \frac{2\pi n_b}{N_e} H[1 - |R|] \frac{x_c^{s-1}}{\kappa^2 \Gamma(s-1)(1-R^2)^{3/2}} J(b) \quad (35)$$

where we introduce $\kappa = kc/\omega_{p,e}$ and the real and imaginary part of the space velocity $R = \omega_r/kc$, $I = \gamma/kc$

- the plasma function is defined as

$$Z(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \frac{e^{-x^2}}{x - t} \quad (36)$$

and

$$Z'(t) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \frac{xe^{-x^2}}{x-t} \tag{37}$$

- the function $J(b)$ takes the spread into account

$$\begin{aligned} J(b) &= \frac{1}{b} \int_0^b dq \sqrt{1+q^2} \left[\frac{dg(x)}{dx} \right]_{x_0(R,q)} \\ &\simeq J(0)B(X) \end{aligned} \tag{38}$$

with

$$X = \sqrt{\frac{x_c\sqrt{1-R^2}}{2R} b} = \sqrt{\frac{A}{2} b} \, ,$$

$$B(X) = \frac{\exp(X^2)}{X} \left[F(X) + \frac{(s-1)A - s(s-2)(F(X)-X)}{2A(A-s)} \right] \tag{39}$$

- since $(\Re\omega)_{max} \gg (\Im\omega)_{max}$, the wave is weakly amplified
- the imaginary phase speed part of the fluctuations are given by (Schlickeiser 2002)

$$I = \frac{\gamma}{kc} = -\frac{\Im\Lambda(R, I=0)}{\frac{\partial\Re\Lambda(R,I=0)}{\partial R}} \tag{40}$$

$$\Re\Lambda(R, I=0) = 0$$

- the plasma function can be simplified to

$$Z^{'}(t) \simeq -2\imath\pi^{1/2} t e^{-t^2} H[1-|R|] - 2(1-2t^2), \quad \text{for } |t| \ll 1,$$

$$Z^{'}(t) \simeq -2\imath\pi^{1/2} t e^{-t^2} H[1-|R|] + \frac{1}{t^2}[1+\frac{3}{2t^2}], \quad \text{for } |t| \gg 1$$

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- separating into real and imaginary parts $\Lambda_e = \Re\Lambda + \imath\Im\Lambda$

Real and imaginary part of Λ_e

$$\Re\Lambda_e(R,I) = 1 - \frac{1}{\kappa^2\beta_e^2}\left[\Re Z'\left(\frac{R+\imath I}{\beta_e}\right) + \frac{1}{\eta}\Re Z'\left(\frac{R+\imath I}{\sqrt{\eta\epsilon}\beta_e}\right)\right]$$

$$\Im\Lambda(R,I) = -\frac{1}{\kappa^2\beta_e^2}\left[\Im Z'\left(\frac{R+\imath I}{\beta_e}\right) + \frac{1}{\eta}\Im Z'\left(\frac{R+\imath I}{\sqrt{\eta\epsilon}\beta_e}\right)\right] - \frac{2\pi n_b}{N_e}H[1-|R|]\frac{x_c^{s-1}}{\kappa^2\Gamma(s-1)(1-R^2)^{3/2}}J(b) \tag{41}$$

- Eq. (40) yields

Growing/damping rate

$$\gamma(\kappa) = -\omega_{p,e}\kappa\frac{\Im\Lambda(R,I=0)}{\frac{\partial\Re\Lambda(R,I=0)}{\partial R}} = \gamma_b(\kappa) - \gamma_L(\kappa) \tag{42}$$

Growth/Damping Rate

- growth rate from the anisotropic relativistic pair distribution

Growth rate

$$\gamma_p(\kappa, b) = \frac{2\pi\omega_{p,e} n_b}{\frac{\partial \Re\Lambda(R, l=0)}{\partial R} N_e} \frac{H[1-R] x_c^{s-1}}{\Gamma(s-1)\kappa(1-R^2)^{3/2}} J(0) B(X(b)) \tag{43}$$

- Landau damping rate from the thermal IGM plasma

Damping rate

$$\begin{aligned}\gamma_L(\kappa) &= \frac{2\pi^{1/2}\omega_{p,e} R H[1-R]}{\frac{\partial \Re\Lambda(R,\kappa)}{\partial R}\kappa\beta_e^3}\left[e^{-\frac{R^2}{\beta_e^2}} + \frac{1}{\xi^{1/2}\chi^{3/2}} e^{-\frac{R^2}{\xi\chi\beta_e^2}}\right] \\ &\simeq \frac{2\pi^{1/2}\omega_{p,e} R H[1-R]}{\frac{\partial \Re\Lambda(R,\kappa)}{\partial R}\kappa\beta_e^3} e^{-\frac{R^2}{\beta_e^2}} \end{aligned} \tag{44}$$

Growth/Damping Rate without spread

- for no spread, Eq. (43) reduces to

Maximum growth rate

$$\gamma_p^{\max}(b=0) = 2.8 \cdot 10^{-9} \chi_{-15} x_{c,6} \ \text{Hz} \approx 18 \cdot (\Im\omega)_{max} \tag{45}$$

- at the same value of R_0 and κ_0 one gets

Landau damping

$$\begin{aligned} \gamma_L(R_0) &= \pi^{1/2} \omega_{p,e} \kappa_0 R_0 \left(\frac{R_0}{\beta_e}\right)^3 e^{-\frac{R_0^2}{\beta_e^2}} \\ &\simeq \frac{\pi^{1/2}\omega_{p,e}}{\beta_e^3} e^{-\frac{1}{\beta_e^2}} < 10^{-10^5} \end{aligned} \tag{46}$$

Growth Rate with spread

Table 1: Values of the zeros $A_N(s) = s$, location of maxima $A_0(s)$ and minimum correction function $B(A_0(s), b = 0.1, s) - 1$ for different values of s and $b = 0.1$.

s	A_0	$B(A_0(s), b = 0.1, s) - 1$
1.5	2.32	$-1.35 \cdot 10^{-5}$
2.0	3.00	$-2.25 \cdot 10^{-5}$
2.5	3.66	$-3.35 \cdot 10^{-5}$
3.0	4.30	$-4.62 \cdot 10^{-5}$
4.0	5.56	$-7.62 \cdot 10^{-5}$

$\Rightarrow$ Contrary to the claims of Miniati and Elyiv (2013) we find that neither the longitudinal nor the perpendicular spread in the relativistic pair distribution function do significantly affect the electrostatic growth rates.

Table 1: Values of the zeros $A_N(s) = s$, location of maxima $A_0(s)$ and minimum correction function $B(A_0(s), b = 0.1, s) - 1$ for different values of s and $b = 0.1$.

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$\Rightarrow$ Contrary to the claims of Miniati and Elyiv (2013) we find that neither the longitudinal nor the perpendicular spread in the relativistic pair distribution function do significantly affect the electrostatic growth rates.

Contents

- we take nonlinear Landau damping for low and the modulation instability for high energy densities (W_e) into account
- if the beam is unaffected by nonlinear processes, half of the initial energy density is transfered to electrostatic fluctuation

$$W_e = n_b(m_e c^2)(\Gamma - 1) = 8.2 \cdot 10^{-23} n_{-22} \Gamma_6 \text{ erg cm}^{-3} \quad (47)$$

- thermal energy density of the IGM

$$W_{\mathrm{th}} = N_e k_B T = 1.4 \cdot 10^{-19} N_{-7} T_4 \text{ erg cm}^{-3} \quad (48)$$

- for the ratio

$$\psi = \frac{W_e}{W_{\mathrm{th}}} = 5.9 \cdot 10^{-4} \frac{n_{-22}\Gamma_6}{N_{-7} T_4} \quad (49)$$

- NLD is a process resulting from weak turbulence theory, where the wave energy is small enough to identify individual wave modes
- interaction between thermal ions and the beat of two Langmuir waves (LW) (Sagdeev & Galeev 1969)
- if $v^{\phi}_{beat} = v^{th}_{ion}$, energy is transfered from the waves to the ions
- since $\omega_r \simeq \omega_{p,e}$, the wave number is reduced and the waves are "scattered" to higher phase velocities

→ NLD removes the phase velocity from the resonant spectral range to nonresonant spectral regions (superluminal phase velocity)

→ the number of scattering processes between relativistic electrons and waves is drastically reduced.

- the modulation instability starts, if the turbulence gets stronger
- LW have a pressure associated with them in the same manner as radiation
- the pressure expel electrons from a local plasma region, where the density of plasma waves exceed the average value
- due to ambipolar diffusion, the ions follows the electrons and a depleted density region is formed
- if the energy density is bigger than $W_e = k^2 \lambda_d^2 n_e k_B T_e$, the diffusive properties of the plasma are no longer strong enough and the waves produce density modulations
- since the local phase velocity of the LW is smaller, waves are trapped into the region (see figure 5) and more waves get refracted into this region

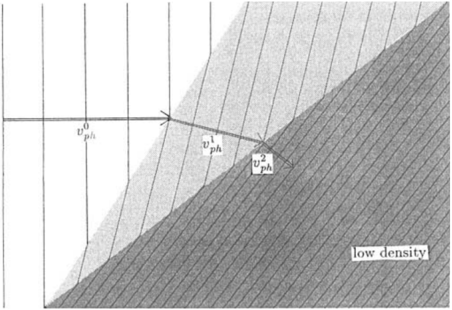

Figure 5: Wave fronts of are bent toward the region of low phase velocity. Since the phase velocity of LW depends strongly on the density, the waves are concentrated into low density regions. From Benz 2002.

- dispersion relation of the LW (with $\Delta n = W_{tot}/k_B T$)

$$\omega_r^2 = \omega_{p,e}^2 \left(1 + 3k^2\lambda_D^2 - \frac{\Delta n}{n_0}\right) \tag{50}$$

- one can define a critical ratio (Galeev et al 1975), where the modulation instability becomes relevant

$$\psi_c = (k\lambda_d)^2 = \frac{5v_{\mathrm{th}}^2}{3c^2} = 2.8 \cdot 10^{-6}\, T_4 \qquad (51)$$

- by comparing Eq. (49) and Eq. (51) one gets a critical density and IGM temperature as the onset condition for the modulation instability

$$n_{-22} > n_c(T) = 4.8 \cdot 10^{-3} \frac{N_{-7}}{\Gamma_6} T_4^2 \qquad (52)$$

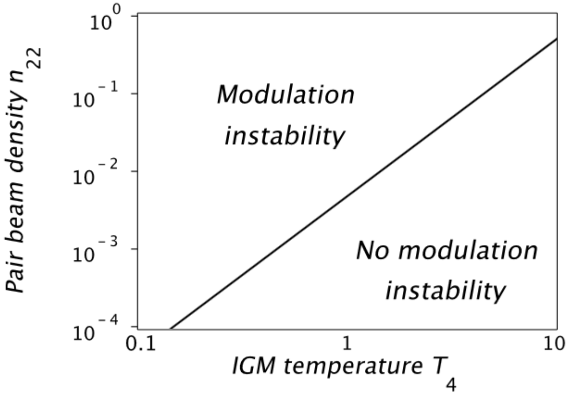

Figure 6: Values of the dimensionless IGM temperatures T_4 and the dimensionless pair densities n_{-22} for the occurrence and non-occurrence of the modulation instability calculated for $(N_{-7}/\Gamma_6) = 1$. From Schlickeiser et al 2012.

NLD for Weak Blazers

- NLD competes as alternative dissipation process to quasi-linear relaxation with the characteristic damping rate (Breizman & Ryutov 1974)

$$\gamma_{NLD} = \frac{m_e}{m_p} W_e \frac{\omega_{p,e}}{(k\lambda_d)^2 N_e k_B T_e} = \frac{m_e \omega_{p,e} \psi}{m_p \psi_c} = 2.1 \frac{n_{-22} \Gamma_6}{N_{-7}^{1/2} T_4^2} \text{ Hz} \qquad (53)$$

- the relaxation of the beam is strongly affected by the NLD
- the relaxation time increases drastically (Eq. (29))

$$\zeta = \frac{\gamma_{e,max}}{\gamma_{NLD}} = 5.2 \cdot 10^{-7} \frac{N_{-7}^{2/3} T_4^2}{n_{-22}^{2/3} \Gamma_6^{4/3}} \qquad (54)$$

- with densities below the critical density Eq. (52) one yields:

$$\Rightarrow \tau_r \leq 8.9 \cdot 10^5 \frac{\Gamma_6^{4/3}}{N_{-7} T_4^{4/3}} \text{ yr} \qquad (55)$$

- the inverse Compton cooling time is

$$2.4 \cdot 10^6 \Gamma_6^{-1} (1+z)^{-4} \text{ yr} , \tag{56}$$

which is a factor 2.7 higher than the relaxation time of the electrostatic instability for near blazars ($z \approx 0$)

- recently, Saveliev et al (2013) simulate the impact of the electrostatic instability on the IC-radiation, see figure 7

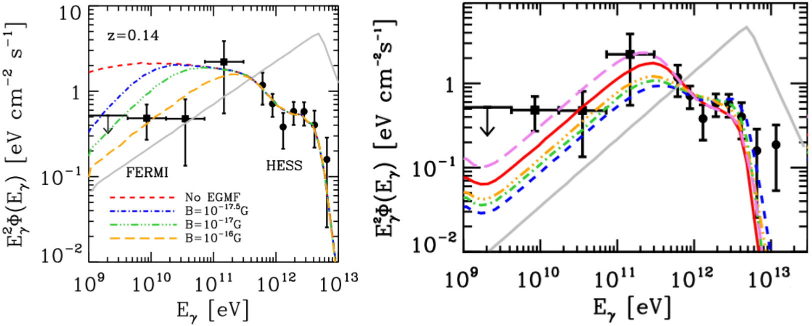

Figure 7: Left panel: Effect of the IGMF on the photon spectrum of the BL Lac 1ES 0229+200 at $z = 0.14$ neglecting the plasma effects of the IGM. Right panel: Gamma-ray spectrum for different temperatures ($T = 10^3$ K, $T = 5 \cdot 10^3$ K, $T = 10^4$ K, $T = 5 \cdot 10^4$ K, $T = 10^5$ K, from top to bottom). From Saveliev et al (2013)

Modulation Instability for strong Blazers

- the growth rate of the modulation instability is (Rose et al 1984)

$$\gamma_{MI} = \frac{\omega_{p,e}}{4}\psi \,, \tag{57}$$

for $(k\lambda_D)^2 < \psi < m_e/m_i$ and

$$\gamma_{MI} = \omega_{p,e}\sqrt{\frac{\psi}{3}\frac{m_e}{m_p}} \tag{58}$$

for $\psi \geq m_e/m_p$.

- for $\psi = 5.9 \cdot 10^{-4}$

$$\zeta = \frac{\gamma_{e,max}}{\gamma_{MI}} \approx 4.2 \cdot 10^{-4} \tag{59}$$

- the relaxation time is

$$\tau_r = 2.1 \cdot 10^{11}\ \mathrm{s} = 6.3 \cdot 10^3\ \mathrm{yr} \tag{60}$$

Contents

- the instability of electron positron beams, which are created due to the annihilation of γ-rays with the EBL-photons, is investigated
- we show, that the electrostatic instability change the propagation of the beam drastically and the beam lose a significant energy fraction and the IC-radiation is suppressed
- we find that neither the longitudinal nor the perpendicular spread in the relativistic pair distribution function do significantly affect the electrostatic growth rates
- as a consequence, an intergalactic magnetic field is not necessary to explain the non-detection of the IC-component
- in future, we will investigate nonlinear effects like the modulation instability in strong blazers more detailed