Combining parton showers and fixed order calculations

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Outline

Introduction

- ◇ Scattering at hadron colliders.
- ◇ Jet evolution with parton showers.
- ◇ Jet production with matrix element calculations.

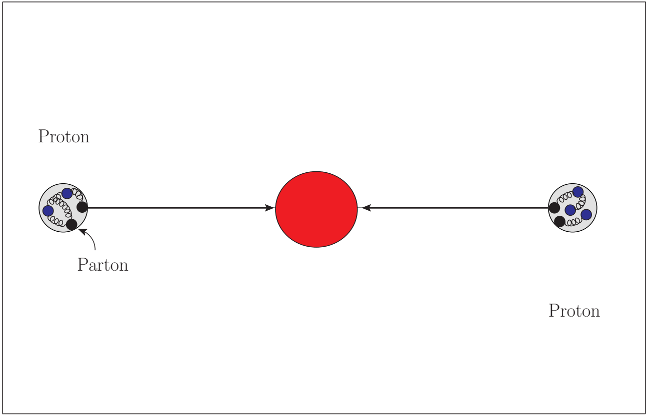
ME+PS merging: Combining the two approximations.

- ◇ Traditional approach for combining ME and PS.
- ◇ Unitarised merging.
- ◇ Next-to-leading order multi-jet merging.

Summary

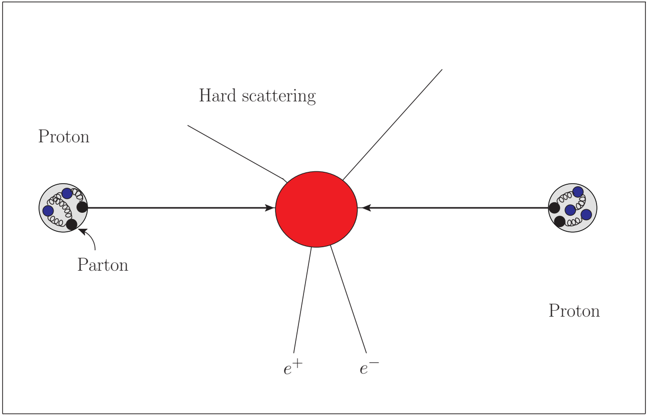
. . . let's see how far we get!

Scatterings in particle physics



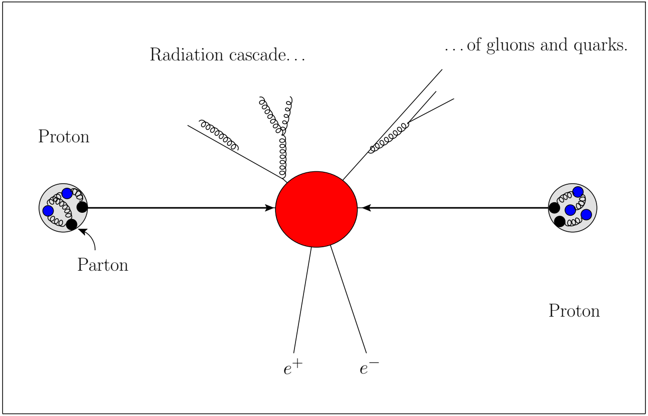
- ◇ We collide nucleons to investigate the interactions of their constituents.
- ◇ Nucleon constituents are called partons, and they are bound into a nucleon by QCD (Quantum Chromodynamics).

Scatterings in particle physics



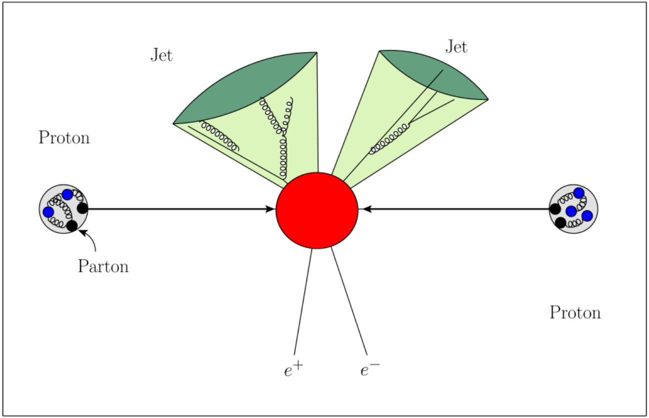
◇ At the very core of an interaction, only a few highly accelerated particles are produced by the interaction of two partons.

Scatterings in particle physics



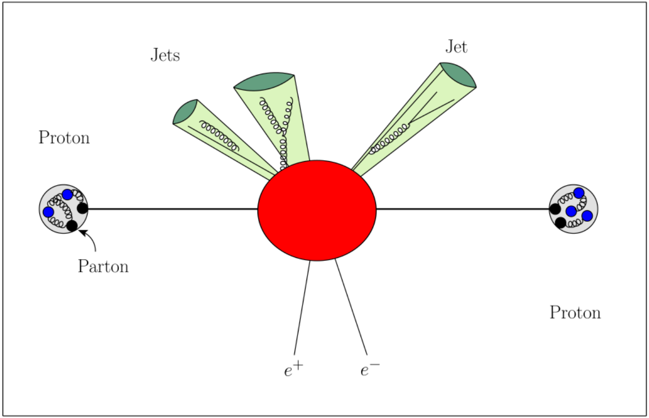
◇ The highly accelerated particles from the interaction decelerate by emitting radiation.

Scatterings in particle physics



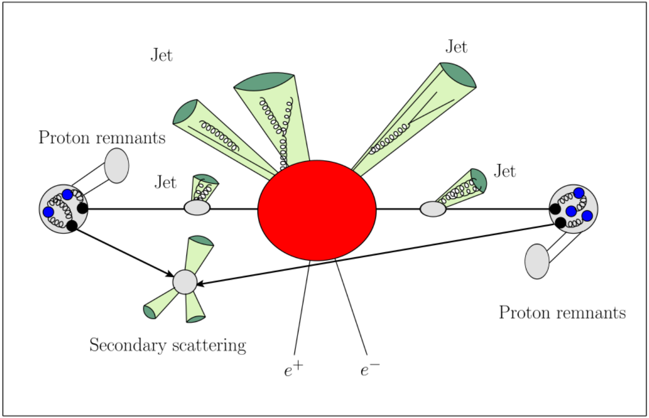
- The highly accelerated particles from the interaction decelerate by emitting radiation.
- The products are bound into hadrons $(p, \pi, \Lambda, \ldots)$, which may decay further. This produces **collimated sprays** of particles called **jets**.

Scatterings in particle physics



- The number of resolvable jets is unknown.
- **Jet evolution** has to be approximated.
- Scattering cross sections for few "seed" partons, i.e. **jet production**, can be calculated "exactly".

Scatterings in particle physics

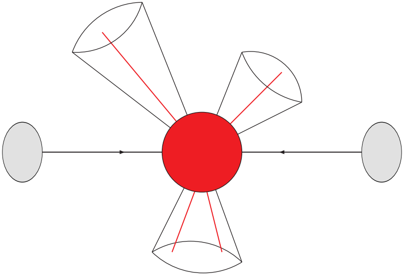


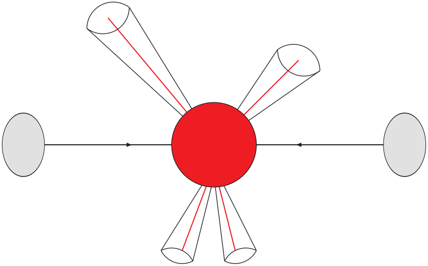
if that's already complicated, there is further

- . . . initial state radiation.
- . . . multiple scatterings of proton constituents.
- . . . the remnants of the smashed proton.

Jet evolution vs. jet production

So how do we interpret an event?



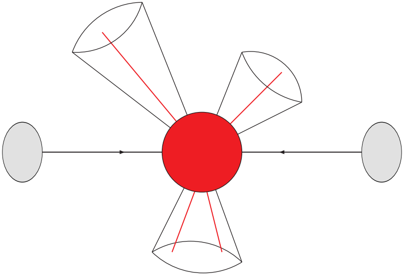


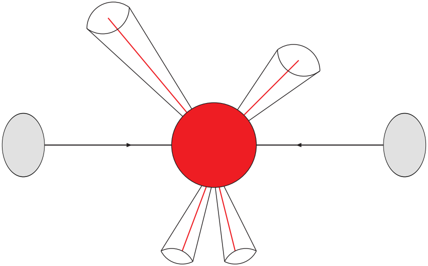
This is a problem of resolution:

Does the radiation only shift the properties of a jet (= jet evolution)?
Do we resolve the radiation as additional jet (= jet production)?

Jet evolution vs. jet production

So how do we interpret an event?



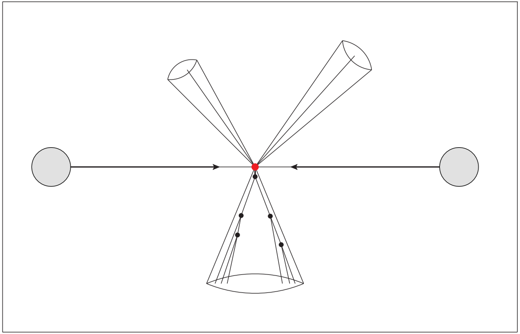


This is a problem of calculability:

Jet evolution described by infinite number of (quasi-classical) partons.
Jet production described by fixed number of QM-interfering partons.

Relying exclusively on either will fail to describe LHC data.

Jet evolution: Parton showers



- Based on collinear approximation. Any number of emissions is possible.
- Re-sums leading terms of the perturbative series to all orders into multiplicative no-emission probabilities ($\approx$ Sudakovs).
- Interface to hadronisation and decay of composite particles.

$\Longrightarrow$ Describes jet evolution.

The PS is an all-order object

$\mathbf{PS}\left[\sigma_{+0}^{\mathsf{ME}}\right]$

Parton shower concepts

The PS is an all-order object

$$\mathbf{PS}\left[\sigma_{+0}^{\mathrm{ME}}\right] = \sigma_{+0}^{\mathrm{PS}} +$$

$$= \sigma_{+0}^{\mathrm{ME}} \Pi_{S_{+0}}(\rho_0, \rho_{\mathrm{min}}) \quad \longleftarrow \text{0 emissions in } [\rho_0, \rho_{\mathrm{min}}]$$

$$+$$

Parton shower concepts

The PS is an all-order object

$$\mathbf{PS}\left[\sigma^{\mathsf{ME}}_{+0}\right] = \sigma^{\mathsf{PS}}_{+0} + \sigma^{\mathsf{PS}}_{+1} +$$

$$= \sigma^{\mathsf{ME}}_{+0}\Pi_{S_{+0}}(\rho_0, \rho_{\mathsf{min}}) \longleftarrow \text{0 emissions in } [\rho_0, \rho_{\mathsf{min}}]$$

$$+ \sigma^{\mathsf{ME}}_{+0}\Pi_{S_{+0}}(\rho_0, \rho_1)\,\alpha_{\mathrm{s}} P_0 \Pi_{S_{+1}}(\rho_1, \rho_{\mathsf{min}}) \longleftarrow \text{1 emission in } [\rho_0, \rho_{\mathsf{min}}]$$

$$+$$

Parton shower concepts

The PS is an all-order object

$$
\begin{aligned}
\mathbf{PS}\left[\sigma_{+0}^{\mathrm{ME}}\right] &= \sigma_{+0}^{\mathrm{PS}} + \sigma_{+1}^{\mathrm{PS}} + \sigma_{+\geq 2} \\
&= \sigma_{+0}^{\mathrm{ME}} \Pi_{S_{+0}}(\rho_0, \rho_{\mathrm{min}}) \qquad \longleftarrow \text{0 emissions in } [\rho_0, \rho_{\mathrm{min}}] \\
&+ \sigma_{+0}^{\mathrm{ME}} \Pi_{S_{+0}}(\rho_0, \rho_1)\, \alpha_s P_0 \Pi_{S_{+1}}(\rho_1, \rho_{\mathrm{min}}) \qquad \longleftarrow \text{1 emission in } [\rho_0, \rho_{\mathrm{min}}] \\
&+ \sigma_{+0}^{\mathrm{ME}} \Pi_{S_{+0}}(\rho_0, \rho_1)\, \alpha_s P_0 \Pi_{S_{+1}}(\rho_1, \rho_2)\, \alpha_s P_1 \left[\Pi_{S_{+2}}(\rho_2, \rho_{\mathrm{min}}) + \ldots\right]
\end{aligned}
$$

↑ 2 or more emissions in $[\rho_0, \rho_{\mathrm{min}}]$

with the no-emission probability

$$
\Pi_{S_{+i}}(\rho_1, \rho_2) = \exp\left\{-\int_{\rho_2}^{\rho_1} d\rho\, \alpha_s P_i\right\}
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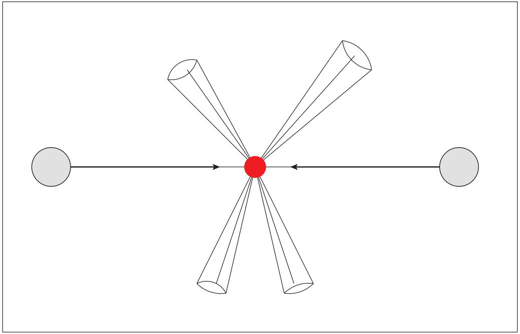
PS cross sections for a *fixed* number of emissions are *exclusive*

$$\mathbf{PS}\left[\sigma^{\mathrm{ME}}_{+0}\right] = \underbrace{\sigma^{\mathrm{PS}}_{+0}}_{\text{exclusive: 0 and only 0 emissions}} + \underbrace{\sigma^{\mathrm{PS}}_{+1}}_{\text{exclusive: 1 and only 1 emission}} + \underbrace{\sigma^{\mathrm{PS}}_{+\geq 2}}_{\text{inclusive: 2 or more emissions}}$$

and obey PS unitarity $\mathbf{PS}\left[\sigma^{\mathrm{ME}}_{+0}\right] = \sigma^{\mathrm{PS}}_{+0} + \sigma^{\mathrm{PS}}_{+1} + \sigma^{\mathrm{PS}}_{+\geq 2} = \sigma^{\mathrm{ME}}_{+0}$

Jet production: Matrix element calculations



- Calculate the QM transition probability for a (small) number of **well-separated** partons.
- Such configurations are very likely in high energy collisions.
- Calculation "complete" up to a fixed (perturbative) order, with well controlled uncertainty. Calculations are by definition *inclusive*.

$\Longrightarrow$ Describes jet (-seed) production.

Combining ME and PS

MEs describe jet production, including correlations between partons.
. . . but break down for soft/collinear partons.
PSs describe jet evolution, including any number of partons.
. . . but cannot describe well-separated partons.

For realistic prediction, we need to combine these complementary approximations.

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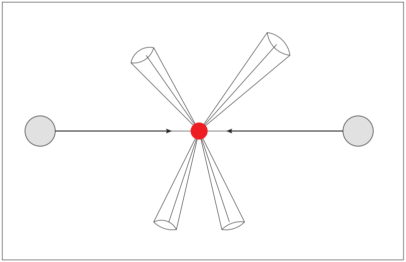

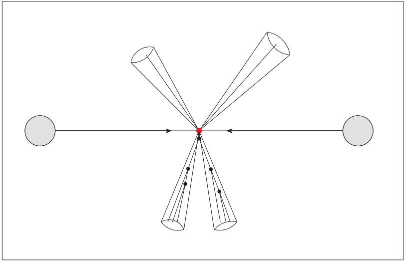

. . . we need to make sure that we use only one calculation for any configuration, i.e. we have to avoid overlaps.

One solution

1. Define two regions of jet resolution
   - ME region: States with all parton separations above a regularisation cut-off ρ_{MS}.
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   $\Rightarrow$ *Fixed-order calculations for n an $n+1$ partons overlap*.

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   ⇒ *Fixed-order calculations for n an $n+1$ partons overlap*.

   To remove this overlap, remember the PS:

$$\mathbf{PS}\left[\sigma^{\mathrm{ME}}_{+0}\right] = \underbrace{\sigma^{\mathrm{PS}}_{+0}}_{\text{exclusive due to Sudakov factor}} + \underbrace{\sigma^{\mathrm{PS}}_{+1}}_{\text{exclusive due to Sudakov factors}} + \underbrace{\sigma^{\mathrm{PS}}_{+\geq 2}}_{\text{inclusive}}$$

   ⇒ Convert the inclusive states of the ME calculation into *exclusive* states by multiplying PS Sudakovs.

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   ⇒ Convert the inclusive states of the ME calculation into *exclusive* states by multiplying PS Sudakovs.
3. Fill in states in the PS region with the parton shower.
4. Combine reweighted calculations. Minimise dependence on ρ_{MS}.

CKKW(-L)[1] merging

After Sudakov reweighting, we are allowed to combine the ME calculations.

However, in the PS region, the shower does more than adding Sudakovs. It e.g. also uses dynamical renormalisation and factorisation scales.

[1] JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) . . .

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$\Rightarrow$ For a smooth transition between states "above" and "below" ρ_{MS}, use the same running scales in the ME calculations as well.

$\Rightarrow$ Reweight MEs with α_{s}- and PDF-ratios.

Then combine the reweighted ME samples.

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⇒ For a smooth transition between states "above" and "below" ρ_{MS}, use the same running scales in the ME calculations as well.

⇒ Reweight MEs with α_{s}- and PDF-ratios.

Then combine the reweighted ME samples. Until recently, this meant:

- ◇ Start the PS on the reweighted ME configuration. Throw away the event if the PS produced another state in the ME region.
- ◇ Add the outcomes of the measurements on all accepted events.

There is a left-over dependence on ρ_{MS}, which is ameliorated by cancellations between the ME (above ρ_{MS}) and the PS real corrections (below ρ_{MS}).

The dependence on ρ_{MS} enters beyond the "accuracy of the shower".

[1] JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) . . .

Illustration

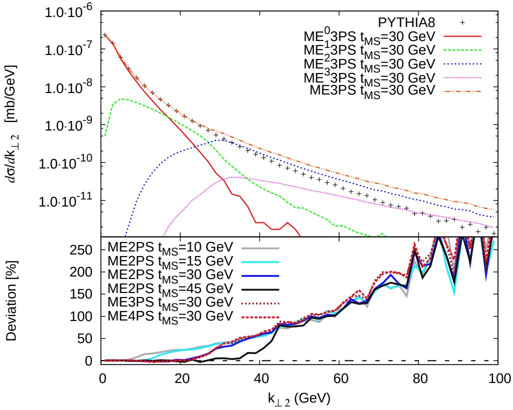


Figure: Separation between the first and second jet for W+jets, when clustering to exactly two jets. The coloured lines show the different reweighted multi-parton MEs. Lower inset shows the improvement compared to the default shower.

CKKW-L results

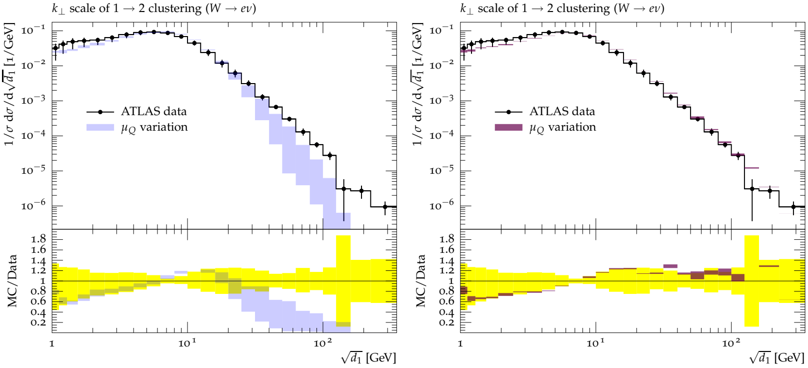


Figure: $k_\perp$-separation between the first and second jet for W+jets, when clustering to exactly two jets. The bands are obtained by varying the PS starting scale in $\mu_Q \in [\frac{1}{2}M_W, 2M_W]$

. . . however

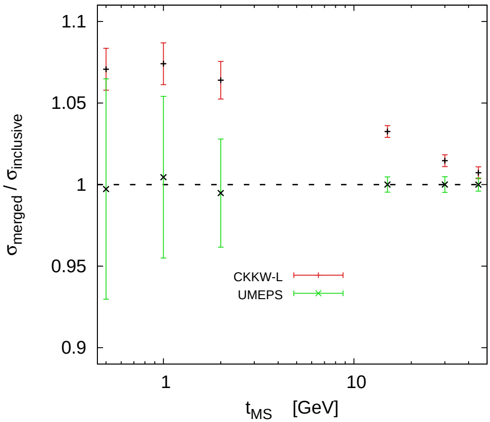


Figure: Ratio of the inclusive cross section after merging, compared to the tree-level inclusive cross section.

The problem with CKKW-L

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).

These are the improvements that we need to describe multiple hard jets!
But they should not invalidate the inclusive (0-jet) cross section!

Traditional approach: Don't use a too small merging scale.

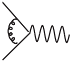
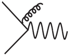
$\rightarrow$ Uncancelled terms numerically not important.

New approach[1]:

Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on ρ_{MS}, thus preserving the inclusive cross section.

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

The KLN theorem states: The sum of virtual and real corrections is IR-finite.

$\sigma\Big($  $\Big) \quad + \quad \int d\sigma\Big($  $\Big) \quad = \quad \text{finite}$

One step back

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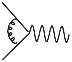
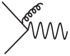
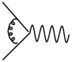
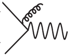
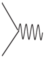
$\sigma\Big($  $\Big) \quad + \quad \int d\sigma\Big($  $\Big) \quad = \quad \text{finite}$

PS unitarity tells us: The sum of approximate PS virtual and real corrections vanishes. . .

$\sigma_{PS}\Big($  $\Big) \quad + \quad \int d\sigma_{PS}\Big($  $\Big) \quad = \quad 0$

. . . because the virtual corrections are simply $-1\times$ integrals of splitting kernels

$\sigma\Big($  $\Big) \otimes \left(-\int dp_\perp^2 dz d\phi P_{q\to qg}\right) \quad + \quad \int \sigma\Big($  $\Big)\, dp_\perp^2 dz d\phi P_{q\to qg} \quad = \quad 0$

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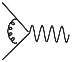
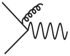
$\sigma\Big($  $\Big) \quad + \int d\sigma\Big($  $\Big) \quad = \quad \text{finite}$

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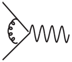
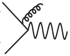
$\sigma_{PS}\Big($  $\Big) \quad + \int d\sigma_{PS}\Big($  $\Big) \quad = \quad 0$

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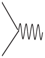
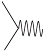
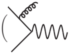
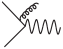
$\sigma\Big($  $\Big) \otimes \Big(-\int dp_\perp^2 dz d\phi P_{q\to qg}\Big) \quad + \int \sigma\Big($  $\Big)\, dp_\perp^2 dz d\phi P_{q\to qg} \quad = \quad 0$

Thus, when including the $+n$-jet calculation, we *improve* the PS approximate virtual corrections by instead subtracting the full integrated $+n$-jet result. . .

$- \int d\sigma\Big($  $\Big) \quad + \int d\sigma\Big($  $\Big) \quad = \quad 0$

. . . and recover the inclusive cross section. $\Longrightarrow$ Unitarised ME+PS merging

This sketch can directly be extended to the case when we have (α_s-, PDF- or Sudakov-) weighted $+n$-jet states ($\widehat{\mathrm{B}}_n$), e.g. two-jet merging:

$$\begin{aligned} \langle \mathcal{O} \rangle \;\; = \;\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 \; - \; \int_s \widehat{\mathrm{B}}_{1\to 0} \; - \; \int_s \widehat{\mathrm{B}}_{2\to 0} \right] \\ & + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 \; - \; \int_s \widehat{\mathrm{B}}_{2\to 1} \right] \\ & + \int \cdots \int \mathcal{O}(S_{+2j}) \; \widehat{\mathrm{B}}_2 \; \Bigg\} \end{aligned}$$

Comments on UMEPS

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$$\begin{aligned}\langle\mathcal{O}\rangle \;\; = \;\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 \; - \; \int_s \widehat{\mathrm{B}}_{1\to 0} \; - \; \int_s \widehat{\mathrm{B}}_{2\to 0} \right] \\ & + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 \; - \; \int_s \widehat{\mathrm{B}}_{2\to 1} \right] \\ & + \int \cdots \int \mathcal{O}(S_{+2j}) \; \widehat{\mathrm{B}}_2 \; \Bigg\}\end{aligned}$$

We can get the integrated version of the real-emission matrix elements by projecting onto an underlying Born configuration. Such configurations are available anyway since we need them to perform the Sudakov weighting.

The "subtract what you add" prescription means that this will produce *counter-events* with negative weight.

NB: UMEPS combines features of CKKW-L and LoopSim.

UMEPS result

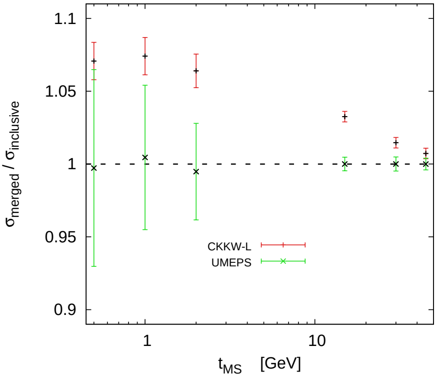


Figure: Ratio of the inclusive cross section after merging, compared to the tree-level inclusive cross section.

⇒ UMEPS preserves the inclusive cross section. However, the statistical error can be larger than in CKKW-L (due to positive and negative weights).

UMEPS results

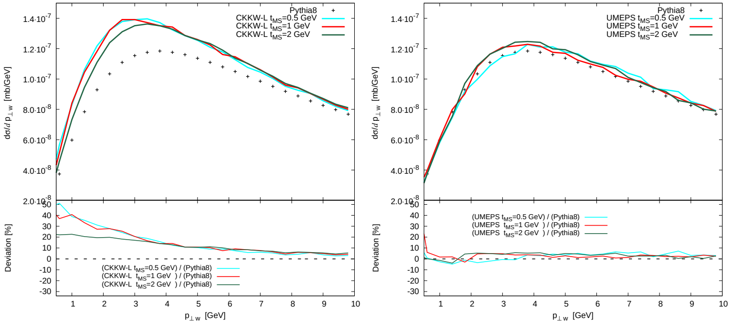


Figure: $p_\perp$ of the W-boson in the Sudakov region (for 2-jet merging, $E_{CM} = 7$ TeV). Lower inset shows the comparison to default PYTHIA8.

⇒ CKKW-L overshoots for (very) low merging scales due to uncancelled terms.

⇒ UMEPS describes the Sudakov peak nicely.

Is UMEPS enough?

UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

We want to use the full NLO results whenever possible.

Is UMEPS enough?

UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

We want to use the full NLO results whenever possible.

Do NLO multi-jet merging for UMEPS. Basic idea:

- ◇ Subtract approximate UMEPS $\mathcal{O}(\alpha_\mathrm{s})$-terms, add back full NLO.
- ◇ To preserve the inclusive (NLO) cross section, add approximate NNLO.

⇒ UNLOPS[1].

[1] JHEP1303(2013)166 (Leif Lönnblad, SP), contains also NL^3 = CKKW-L@NLO. Similar scheme in JHEP1308(2013)114 (Simon Plätzer)

The UNLOPS method

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \qquad - \int_s \widehat{\mathrm{B}}_{1\to 0} \qquad - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\qquad \widehat{\mathrm{B}}_1 \qquad - \int_s \widehat{\mathrm{B}}_{2\to 1} \qquad \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} \qquad - \int_s \widehat{\mathrm{B}}_{1\to 0} \qquad - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \widehat{\mathrm{B}}_1 \qquad - \int_s \widehat{\mathrm{B}}_{2\to 1} \qquad \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_s^n)$- and $\mathcal{O}(\alpha_s^{n+1})$-terms:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

Or for the case of M different NLO calculations, and N tree-level calculations:

$$\langle\mathcal{O}\rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \Bigg\{ \widetilde{\mathrm{B}}_m + \left[\widehat{\mathrm{B}}_m \right]_{-m,m+1} + \int_s \mathrm{B}_{m+1\to m}$$

$$- \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \left[\int_s \widehat{\mathrm{B}}_{i\to m} \right]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}^{\uparrow}_{i+1\to m} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to m} \Bigg\}$$

$$+ \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{\mathrm{B}}_M + \left[\widehat{\mathrm{B}}_M \right]_{-M,M+1} - \left[\int_s \widehat{\mathrm{B}}_{M+1\to M} \right]_{-M} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i+1\to M} \Bigg\}$$

$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

UNLOPS merging of zero and one parton at NLO:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$
$$+ \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

Or for the case of M different NLO calculations, and N tree-level calculations:

$$\langle\mathcal{O}\rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \Bigg\{ \widetilde{\mathrm{B}}_m + \left[\widehat{\mathrm{B}}_m \right]_{-m,m+1} + \int_s \mathrm{B}_{m+1\to m}$$
$$- \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \left[\int_s \widehat{\mathrm{B}}_{i\to m} \right]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}^{\uparrow}_{i\to m} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to m} \Bigg\}$$
$$+ \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{\mathrm{B}}_M + \left[\widehat{\mathrm{B}}_M \right]_{-M,M+1} - \left[\int_s \widehat{\mathrm{B}}_{M+1\to M} \right]_{-M} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i+1\to M} \Bigg\}$$
$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

This is best done internally in the event generator.
The formula will not be on the exam.

Combine NLO calculations for different jet multiplicities ...

...and add further tree-level calculations on top ...

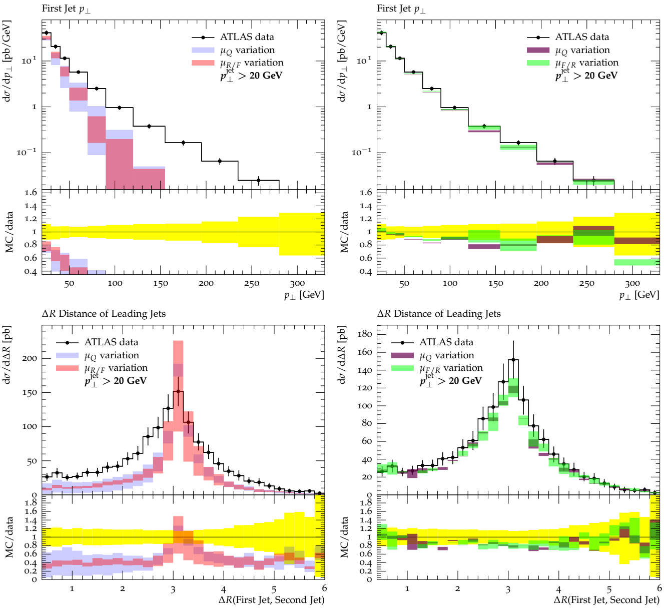
...and have one single inclusive sample with X+(0,..., M)@NLO and X+(M+1,...,N)@LO

Aim: Use NLO for as many multiplicities as possible,
then use LO for more jets,
and only then use PS.

$\Rightarrow$ Reduce μ_F, μ_R dependence due to NLO input,
reduce μ_Q dependence because ME's fill most of the phase space.

...and do this in a process-independent way.

UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO merged results (H+jets)

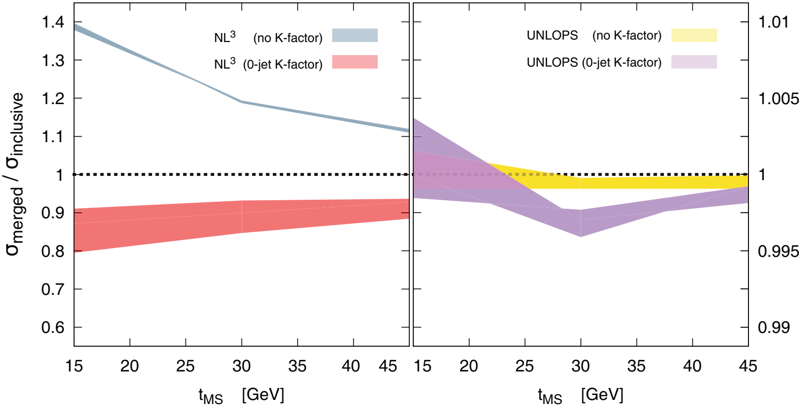
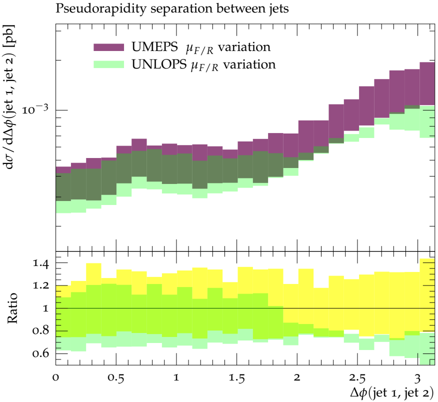


Figure: Ratio of the inclusive cross section for gg→H after merging (H+0)@NLO, (H+1)@NLO and (H+2)@LO, compared to the NLO inclusive cross section.

⇒ NL^3 (=CKKW-L@NLO) has problems for processes with large, loop-driven NLO corrections.

NLO merged results (squarks+jets)



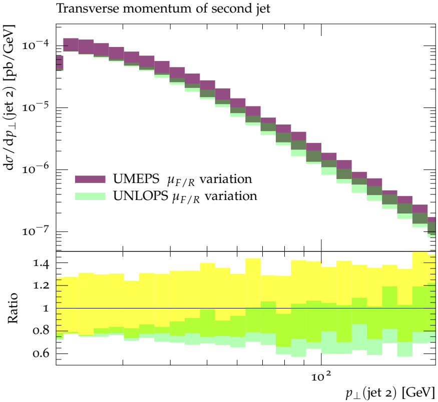


Figure: $\Delta\phi_{12}$ and $p_{\perp 2}$ for u-squark pair production ($m_{\widetilde{u}} = 500$ GeV, $m_{\chi_0} = 500$ GeV, BR($\widetilde{u} \rightarrow u\chi_0$) ≈ 1) after merging (squarks+0)@NLO[1], (squarks+1)@LO and (squarks+2)@LO.

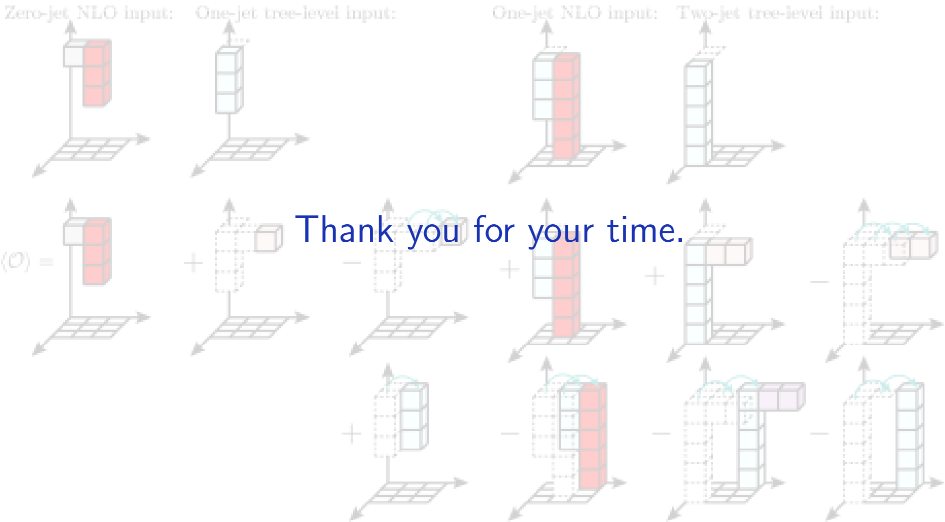
[1] arXiv:1305.4061 (Gavin, Hangst, Krämer, Mühlleitner, Pellen, Popenda, Spira)

. . . or try something else yourself!

. . . delicious Cornflakes@NLO.

Summary

- To describe data, we need to infuse parton showers with matrix elements.
- CKKW-L tree-level merging is included in PYTHIA8.
  But it does not work for arbitrary small merging scales.
- UMEPS tree-level merging is included in PYTHIA8.
  UMEPS almost cancels the merging scale dependence.
  But it's not NLO.
- Two NLO merging schemes are implemented in PYTHIA8:
  NL^3 and UNLOPS.
- UNLOPS is our preferred choice.
- All merging schemes in PYTHIA8 run on LHEF input, e.g. POWHEG-BOX or MADEVENT input.



Thank you for your time.

Back-up.

CKKW(-L) merging

In traditional ME+PS approaches, there is a left-over dependence on ρ_{MS}, which is ameliorated by cancellations between the ME (above ρ_{MS}) and the PS real corrections (below ρ_{MS}). E.g.

$$d\sigma^{\mathrm{ME+PS}} = f_0(\rho_0)\Big\{ \left|\mathcal{M}_{S_{+1,me}}\right|^2 d\Phi_1^{\mathrm{ME}} \alpha_{\mathrm{s}}(\rho_1)\Theta(\rho(S_{+1,me}) - \rho_{\mathrm{MS}})\frac{f_1(\rho_1)}{f_0(\rho_1)}\Pi_{S_{+0}}(\rho_0,\rho_1)\Pi_{S_{+1}}(\rho_1,\rho_c)$$

$$+ \left|\mathcal{M}_{S_{+0,me}}\right|^2 d\Phi_0^{\mathrm{ME}} P(z)\, d\rho_1 dz_1 \alpha_{\mathrm{s}}(\rho_1)\Theta(\rho_{\mathrm{MS}} - \rho(S_{+1,ps}))\frac{f_1(\rho_1)}{f_0(\rho_1)}\Pi_{S_{+0}}(\rho_0,\rho_1)\Pi_{S_{+1}}(\rho_1,\rho_c)\Big\}$$

The merging scale dependence vanishes if the red terms are equal, because the blue Θ-functions would add to one.

$$
\begin{aligned}
w_n =& \frac{x_n^+ f_n^+(x_n^+,\rho_n)}{x_n^+ f_n^+(x_n^+,\mu_F)} \frac{x_n^- f_n^-(x_n^-,\rho_n)}{x_n^- f_n^-(x_n^-,\mu_F)} \\
&\times \prod_{i=1}^{n} \left[\frac{\alpha_s(\rho_i)}{\alpha_s(\mu_R)} \frac{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+,\rho_{i-1})}{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+,\rho_i)} \frac{x_{i-1}^- f_{i-1}^-(x_{i-1}^-,\rho_{i-1})}{x_{i-1}^- f_{i-1}^-(x_{i-1}^-,\rho_i)} \Pi_{S_{+i-1}}(x_{i-1},\rho_{i-1},\rho_i) \right] \\
\widehat{\mathrm{B}}_n =& \mathrm{B}_n w_n
\end{aligned}
$$

$$
\int_s \widehat{\mathrm{B}}_{n\to m} = \left[\prod_{a=m+1}^{n-1} \int d\rho_a dz_a d\varphi_a \Theta(\rho_{\mathrm{MS}} - \rho_a) \right] \int d\rho_n dz_n d\varphi_n \mathrm{B}_n w_n
$$

$$
\langle \mathcal{O} \rangle = \sum_{n=0}^{N} \int d\phi_0 \int \dots \int \mathcal{O}(S_{+nj}) \left\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \right\} .
$$

In CKKW-L, w_n contains an additional factor $\Pi_{S_{+n}}(x_n, \rho_n, \rho_{\mathrm{MS}})$.
UMEPS induces this through $\int_s \widehat{\mathrm{B}}_{n+1\to n}$ instead.

NNLO with UNLOPS

Note that in UNLOPS, the lowest-multiplicity cross section is *not* reweighted. Terms entering due to PS weights are $\mathcal{O}(\alpha_s^2)_{\text{PS}} \times \mathcal{O}(\alpha_s^1)_{\text{ME}}$ and $\mathcal{O}(\alpha_s^0)_{\text{PS}} \times \mathcal{O}(\alpha_s^2)_{\text{ME}}$

$$\begin{aligned}\langle\mathcal{O}\rangle \;=\; \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(& \widetilde{\mathsf{B}}_0 - \int_s \widetilde{\mathsf{B}}_{1\to 0} + \int_s \mathsf{B}_{1\to 0} - \left[\int_s \widehat{\mathsf{B}}_{1\to 0}\right]_{-1,2} \\ & - \int_s \mathsf{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathsf{B}}_{2\to 0} \Bigg) \\ & + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathsf{B}}_1 + \left[\widehat{\mathsf{B}}_1\right]_{-1,2} - \left[\int_s \widehat{\mathsf{B}}_{2\to 1}\right]_{-2} \right) \\ & + \int\!\!\int \mathcal{O}(S_{+2j})\, \widehat{\mathsf{B}}_2 \end{aligned}$$

NNLO with UNLOPS

It is simple to remove this single $\mathcal{O}(\alpha_s^2)$-term. Subtracting the term, it is possible to replace the lowest multiplicity cross section by the full NNLO result

$$\begin{aligned}\langle\mathcal{O}\rangle \;=\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left(\widetilde{\widetilde{\mathrm{B}}}_0 - \left[\int_s \widehat{\mathrm{B}}_{1\to 0}\right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 0}\right]_{-2} \right) \\ & + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1\right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1}\right]_{-2} \right) \\ & + \int\!\!\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\}\end{aligned}$$

$\Rightarrow$ The cross section formula becomes simpler again. The inclusive cross section is now

$$\int d\phi_0 \mathcal{O}(S_{+0j}) \left(\widetilde{\widetilde{\mathrm{B}}}_0 + \int_s \mathrm{B}_{2\to 0} \right) + \int d\phi_0 \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \int_s \mathrm{B}_{2\to 1} \right)$$

which is just the NNLO result.

We need an NNLO generator to produce $\widetilde{\widetilde{\mathrm{B}}}_0$ or $\overline{\overline{\mathrm{B}}}_0$ (for $gg \to$H, $\overline{\overline{\mathrm{B}}}_0$ is only dependent on the H-rapidity spectrum).