Aspects of the Inhomogeneous Universe

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DESY

1202.1247, 1207.1286, 1209.4326, 1302.0740, 1309.4771

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Outline

- Concepts in main stream cosmology
- Motivation
- Light-Cone Averaging Prescription
- Application: Luminosity-distance (d_L) Redshift (z) Relation in the Concordance Model.

1. Consequences for H_0 and the cosmological constant
2. Probing the Primordial Power Spectrum

Concepts in Cosmology-Geometry

- Metric ($c = 1$ unless specified)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad g_{\mu\nu}^{Minkowski} = \mathrm{diag}(1, -1, -1, -1)$$

- Homogeneous, isotropic background + small perturbations
- Friedmann Robertson Walker metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

- Scale factor $a(t)$, spatial curvature $k = 0, \pm 1$
- Hubble parameter

$$H(t) = \frac{1}{a} \frac{da}{dt}$$

- Hubble constant $H_0 = H(t_0) = 100h\,\mathrm{km/s/Mpc}$, $h \simeq 0.7$, pc=3.26Lyr
- Redshift $1 + z = \lambda(t_0)/\lambda(t) = a(t_0)/a(t)$
- Hubble law $[\mathcal{F} = \mathcal{L}/(4\pi d_L^2),\ q_0 = -\ddot{a}/(aH_0^2)|_{t_0}]$

$$H_0 d_L = cz + \frac{1}{2}(1 - q_0) cz^2 + \dots$$

Concepts in Cosmology-Matter

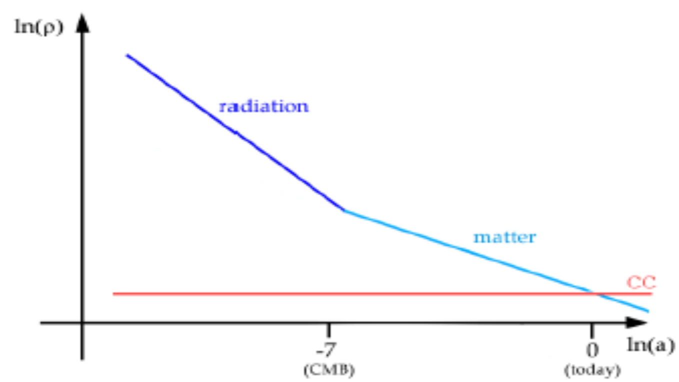
- Ideal fluid with energy density ρ and pressure p

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu}$$

- Energy-momentum 'conservation'

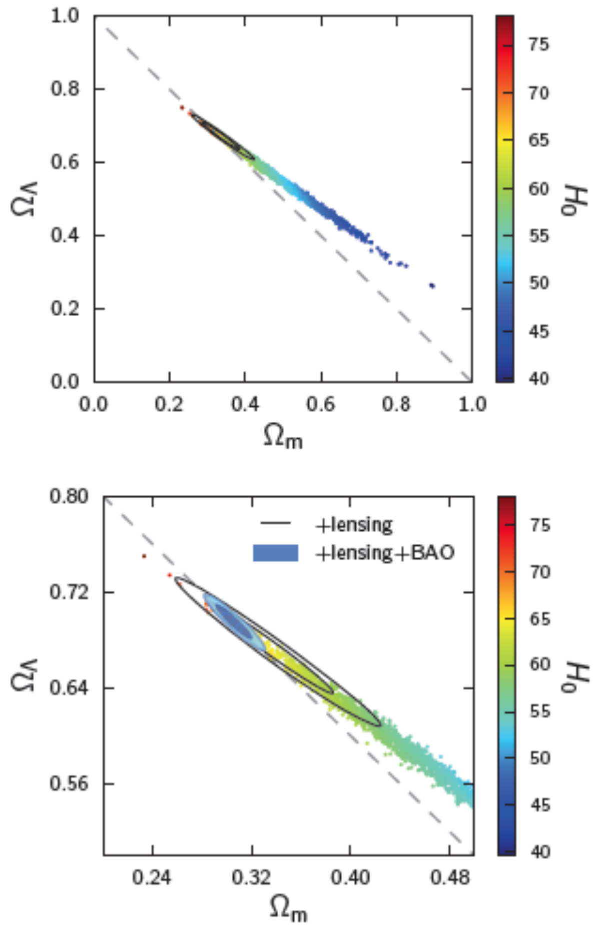
$$D_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad d(a^3\rho)/dt = -p\, da^3/dt$$

- Equation of state $p = \omega\rho$
- Relativistic matter (radiation) $\omega = 1/3 \Rightarrow \rho_r \propto 1/a^4$
- Non-relativistic matter $p \ll \rho$, $\omega \approx 0 \Rightarrow \rho_m \propto 1/a^3$
- Vacuum energy (cos. constant) $\rho_\Lambda = const \Rightarrow \omega = -1,\ T^{\mu\nu}_\Lambda = \rho_\Lambda g^{\mu\nu}$

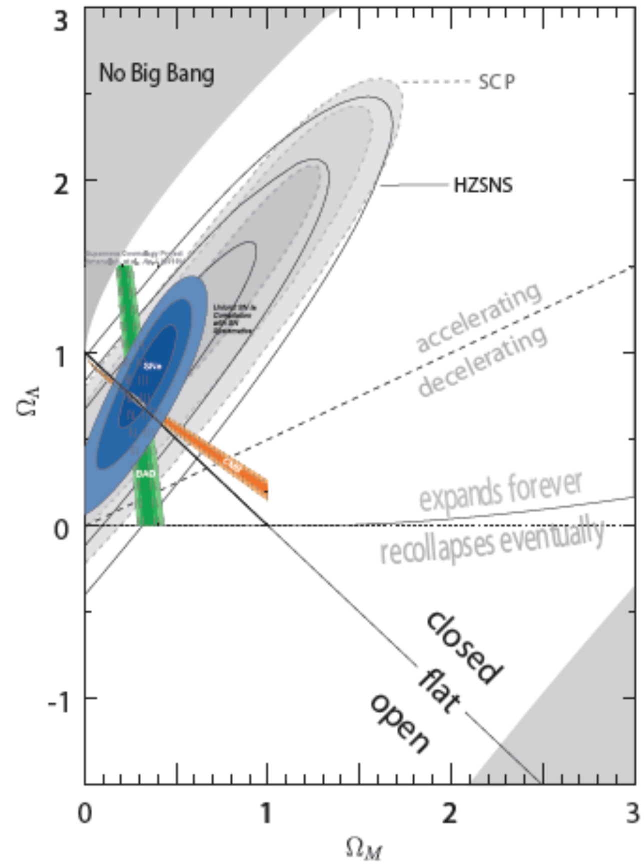


$$\frac{H^2}{H_0^2} = \Omega_{R0}(1+z)^4 + \Omega_{K0}(1+z)^2 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}$$

$$d_L(z) \approx \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}}}$$



Planck XVI 1303.5076



Goobar, Leibundgut 1102.1431

Spatially flat $|\Omega_k| < 0.0067$ (95% C.L. Planck+WP+BAO)

Main Stream Cosmology

- Assume FLRW = homogeneous and isotropic metric.

$\Longrightarrow$ Implicit averaging

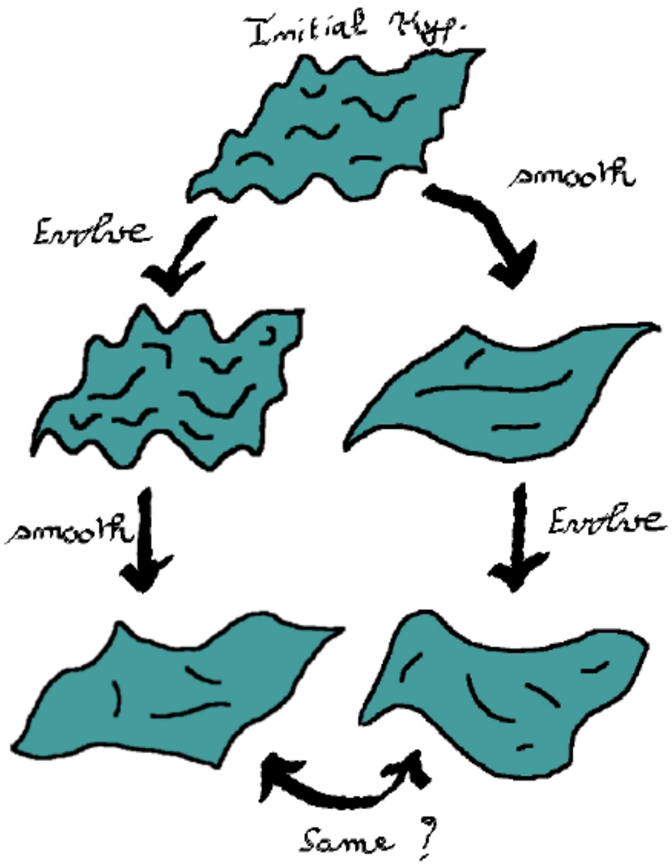
- Modeling the energy momentum tensor as a perfect fluid.
- Pert. give rise to structure, highly non-linear at some scale. Background unchanged.

$\Longrightarrow$ Implicitly neglected the possibility of backreaction.

$\Longrightarrow$ GR is non-linear. Averaging and solving do not commute. Are we getting the correct answer?

Green and Wald 2010

Importance of averaging



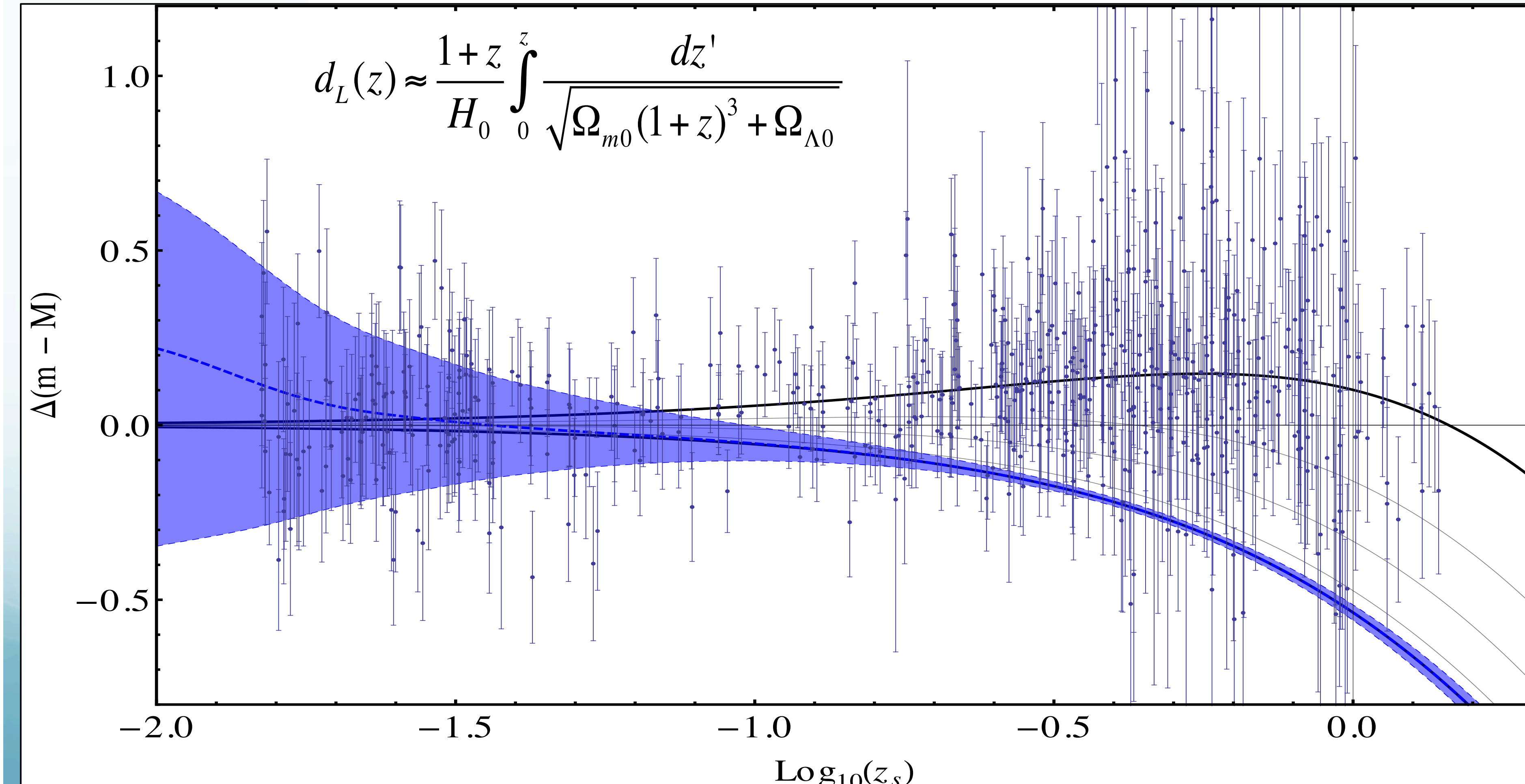
The evolution of an inhomogeneous spacetime after averaging differs from the evolution of its averaged spacetime.

Few questions:

- Can smoothing of structure contribute to an acceleration term (DE)?
  Is there an effect from small scales to large scales ?
  ⇒ Nice way out of the coincidence problem? (T. Buchert)
- Consequences on cosmological parameters? (C. Clarkson, J. Larena)
  Negligible for a physical reason?
- What is the scale of homogeneity in the Universe? $100 Mpc$?
- Fitting Problem : What is the best-fit FLRW model to a given lumpy Universe?
- How Einstein field equations transform after a coarse-graining procedure? How do we average vectors and tensors?

Motivation and "Spoiler"

$$\Delta(m - M) = 5\log_{10}\left[\overline{\langle d_L \rangle}\right] - 5\log_{10}\left[\frac{(2+z_s)z_s}{2H_0}\right].$$



Explanations

- Changing the energy content of the Universe: Cosmological Constant, Dark Energy...

*Challenge: Fine-tuned, coincidence?, fundamental theory?

- Changing Gravity: f(R), scalar-tensor theory...

*Challenge: Solar system tests, fine-tuned, fundamental theory?

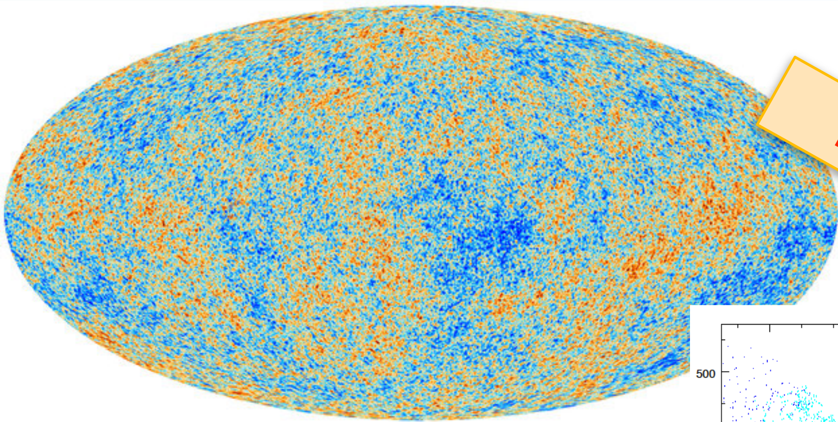
- Changing the metric: void models

*Challenge: Matching with CMB and other probes, fine-tuned in space =>giving up the Copernican principle

- Changing the equations of motion: averaging, smoothing out small scale inhomogeneities...

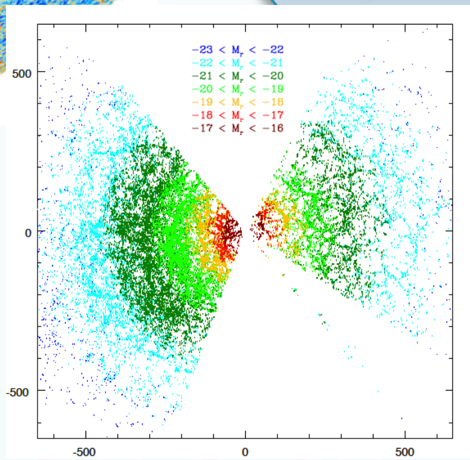
  *Challenge: Proper Averaging, matching with data, magnitude of the effect

LCDM Consistency?



PLANCK

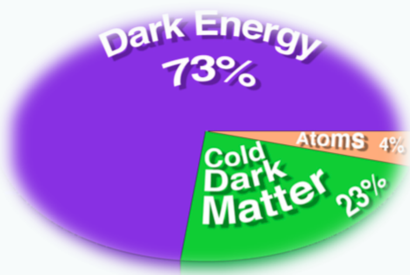
2.73 K



SDSS

Perturbations at Low Redshift

- Measurements of SNIa ➔ Mostly neglected, naively argued as irrelevant ~10^{-10} (Amplitude of the primordial power spectrum)



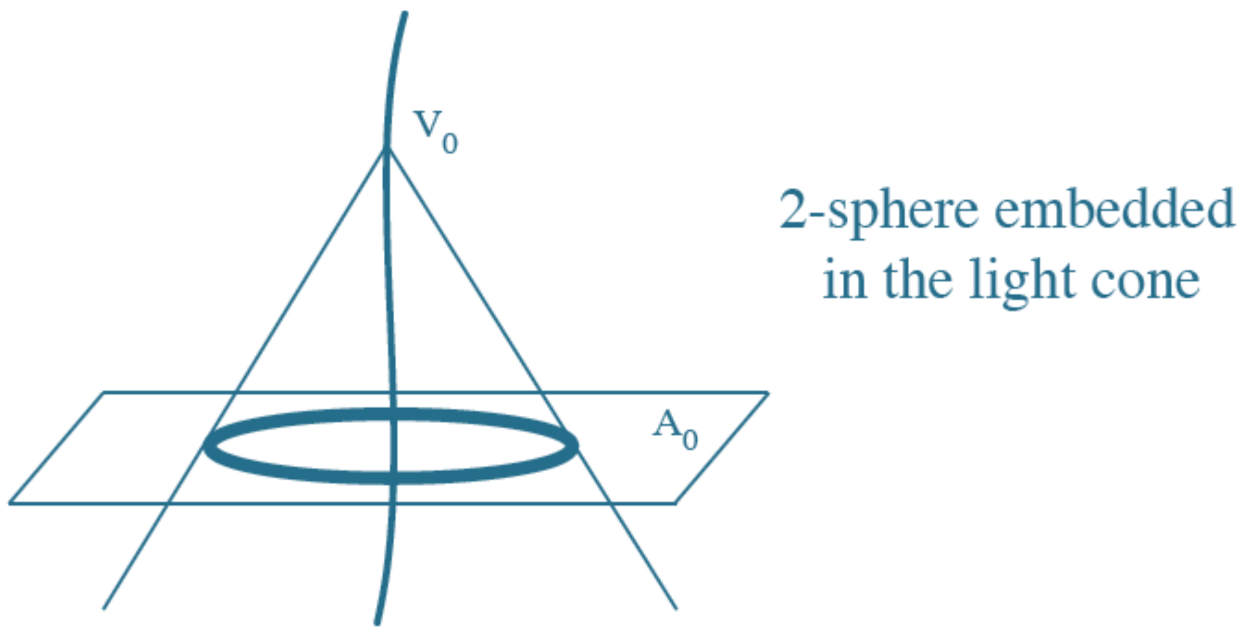
<u>The concordance model of cosmology:</u>

- SN and before PLANCK ~73% CC, H_0 =73.8-/+2.4 km/sec/Mpc

- PLANCK 68% CC, H_0 =67.4-/+1.4 km/sec/Mpc

Do perturbations alter the picture???

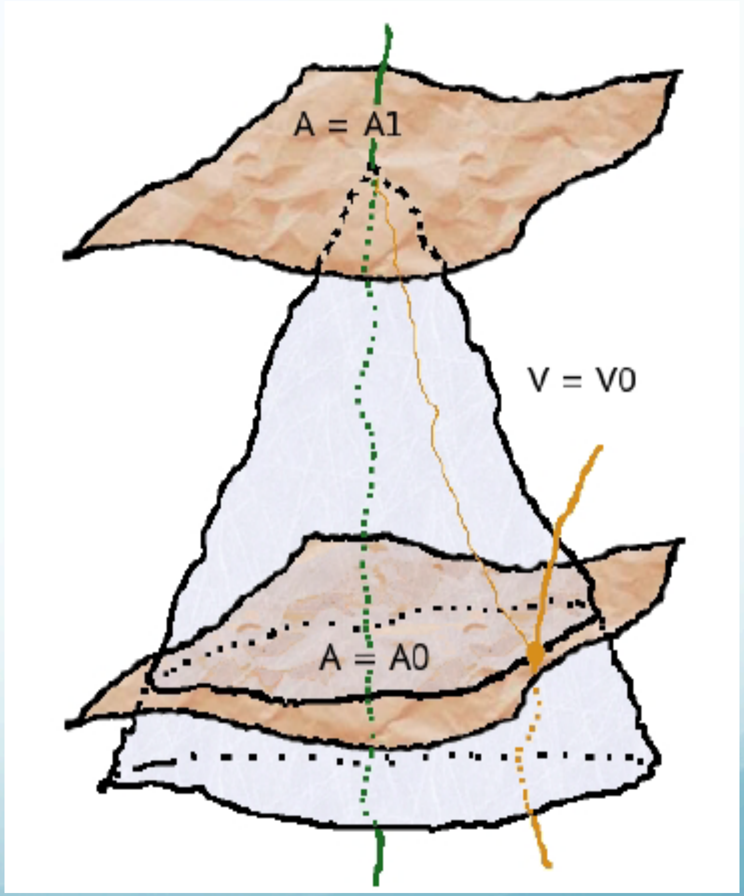
Average d_L on the past LC as a function of redshift



A= redshift, V= light-cone coordinate

LC Averaging

- Useful for interpreting light-like signals in cosmological observations.
- Hyper-surfaces using meaningful physical quantities: Redshift, temperature etc.
- Observations are made on the light-cone. Volume averaging give artefacts and the matching with data is not clear.
- Past attempts: Coley 0905.2442; Rasanen 1107.1176, 0912.3370



Light Cone Averaging 1104.1167

- A-priori - the averaging is a geometric procedure, does not assume a specific energy momentum

$$I(S; V_0, A_0; -) = \int d^4x \sqrt{-g}\, \delta(V_0 - V)\delta(A - A_0)|\partial_\mu V \partial^\mu A| S(x)$$

$$k_\mu \equiv \partial_\mu V$$

$$\langle S \rangle_{V_0, A_0} = \frac{I(S; V_0, A_0; -)}{I(1; V_0, A_0; -)}$$

The prescription is gauge inv., {field reparam. A->A'(A), V->V'(V)} and invariant under general coordinate transformation. A(x) is a time-like scalar, V(x) is null.

This gives a procedure for general space-times. Novelty: Exact treatment of geodesics.

GLC Metric and Averages

$$ds^2_{GLC} = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\tilde{\theta}^a - U^a dw)(d\tilde{\theta}^b - U^b dw).$$

- Ideal Observational Cosmology – Ellis et al.
- Evaluating scalars at a constant redshift for a geodetic observer.

$$I(S, w_0, z) = \int d^2\tilde{\theta}\sqrt{\gamma(w_0, z, \tilde{\theta}^a)}S(w_0, z, \tilde{\theta}^a);$$

$$\langle S \rangle = \frac{I(S, w_0, z)}{I(1, w_0, z)};$$

$$1 + z = \frac{\Upsilon_O}{\Upsilon_S}$$

Detour: - Exact Result - Flux

LC average of flux for any space-time amounts to the area of the 2-sphere! (Nambu-Goto action)

$$\Phi = \frac{L}{4\pi d_L^2}; \quad d_L(z) = (1+z)^2 d_S; \quad d_S^2 \equiv \frac{dS}{d\Omega_O} = \frac{\sqrt{\gamma}}{\sin\tilde{\theta}}$$

$$\langle d_L^{-2}\rangle(w_o, z_s) = (1+z_s)^{-4}\frac{\int dS \frac{d\Omega_0}{dS}}{\int dS} = (1+z_s)^{-4}\frac{\int d\Omega_0}{\int dS} = (1+z_s)^{-4}\frac{4\pi}{\mathcal{A}(w_o, z_s)} \quad ,$$

$$\mathcal{A}(w_o, z_s) = \int_{\Sigma(w_o,z)} d^2\xi\sqrt{\gamma} \quad .$$

$$\sqrt{\gamma} = \rho^2 \sin\theta$$

$$d_s \equiv \rho = \sum_{l,m} a_{lm}(w_0, z_s) Y_{lm}(\theta, \varphi)$$

$$\int d^2\theta\sqrt{\gamma} = \int d^2\theta \rho^2 \sin\theta = \sum_{l,m} |a_{lm}(w_0, z_s)|^2 > a_{00}^2$$

Anisotropies always “mimic” acceleration!

GLC Metric

- FLRW -0th order. We shall use up to 2nd order.

$$w = r + \eta, \qquad \tau = t, \qquad \Upsilon = a(t), \qquad U^a = 0,$$
$$\gamma_{ab} d\theta^a d\theta^b = a^2(t) r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

- τ can be identified as the time coordinate in the synchronous gauge of arbitrary space-time.

$$g^{t\mu}_{SG} = \{-1, \vec{0}\} = -\left[\partial_\tau + \Upsilon^{-1}(\partial_w + U^a \partial_a)\right] X^\mu = -u^\nu \partial_\nu X^\mu = -\frac{dX^\mu}{d\lambda},$$

IBD et al. '12

$$d_L^{FLRW}(z_s) = (1+z_s) a_0 \int_{\eta_s}^{\eta_0} d\eta = (1+z_s) \int_0^{z_s} \frac{dz}{H(z)}$$
$$= \frac{1+z_s}{H_0} \int_0^{z_s} dz \left[\sum_n \Omega_{n0} (1+z)^{3(1+w_n)}\right]^{-1/2}$$

Averaged d_L at Constant Redshift

- Ethrington's Reciprocity Law, for any spacetime:

$$\Phi = \frac{L}{4\pi d_L^2}; \quad d_L(z) = (1+z)^2 d_S; \quad d_S^2 \equiv \frac{dS}{d\Omega_O} = \frac{\sqrt{\gamma}}{\sin\tilde{\theta}}$$

SPT:

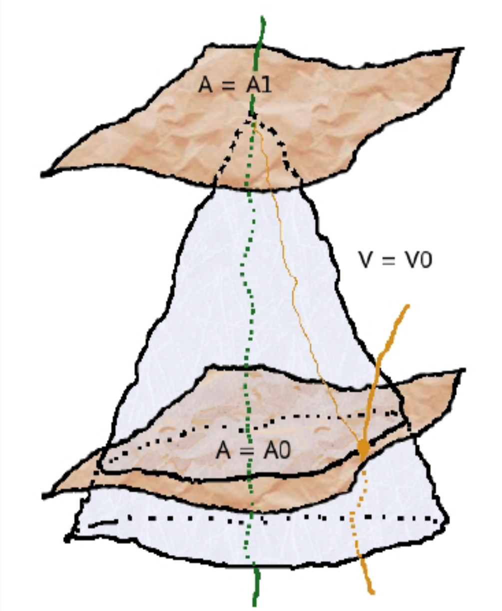
$$\delta \equiv \frac{\rho}{\rho_0} - 1 = \frac{2}{3}\frac{a}{\Omega_{m0}H_0^2}\nabla^2\Psi$$

$$\langle d_L \rangle_{w_0,z} = (1+z)^2 \frac{\int d^2\theta \sqrt{|\gamma(w_0,\tau(z,w_0,\theta^a),\theta^a)|}\, d_s(w_0,\tau(z,w_0,\theta^a),\theta^a)}{\int d^2\theta \sqrt{|\gamma(w_0,\tau(z,w_0,\theta^a),\theta^a)|}},$$

Measure of Integration

Fluctuations in scalar

The Perturbed Quantities



- EFE gives Poisson eq. that connects the density contrast and the gravitational potential:

$$\delta \equiv \frac{\rho}{\rho_0} - 1 = \frac{2}{3}\frac{a}{\Omega_{m0} H_0^2}\nabla^2\Psi$$

- Both the area distance and the measure of integration are expressed in terms of the gravitational potential and its derivatives. Vector and tensor pert. do not contribute.

$$d_L = d_L^{(0)}[1 + d_L^{(1)}(\Psi, \partial\Psi \ldots) + d_L^{(2)}(\Psi, \partial\Psi \ldots) + \ldots]$$

$$\int d^2\tilde{\theta}\sqrt{\gamma} = \int d\Omega[1 + \mu^{(1)}(\Psi, \partial\Psi \ldots) + \mu^{(2)}(\Psi, \partial\Psi \ldots) + \ldots]$$

The Optimal Observable - Flux

- LC average of flux for any space-time is the area of the 2-sphere! (Nambu-Goto action)
- d_L or μ are more biased

$$\langle d_L^{-2}\rangle(w_o, z_s) = (1+z_s)^{-4}\frac{\int dS \frac{d\Omega_0}{dS}}{\int dS} = (1+z_s)^{-4}\frac{\int d\Omega_0}{\int dS} = (1+z_s)^{-4}\frac{4\pi}{\mathcal{A}(w_o, z_s)} \quad ,$$

$$\mathcal{A}(w_o, z_s) = \int_{\Sigma(w_o,z)} d^2\xi\sqrt{\gamma} \quad .$$

$$\langle\{d_L\}\rangle = d_L^{FLRW}[1+f_d(z)]$$

$$\Phi \sim \langle\{d_L^{-2}\}\rangle = (d_L^{FLRW})^{-2}[1+f_\Phi(z)]$$

$$f_\Phi(z) = \int \frac{dk}{k} P_k[\tilde{f}_{\Phi(1,1)}(k,z) + \tilde{f}_{\Phi 2}(k,z)]$$

Interpretation & Analysis

- d_L is a stochastic observable – mean, dispersion, skewness...

$$f_\Phi(z) = \left[\widetilde{f}_{1,1}(z) + \widetilde{f}_2(z)\right] \int_0^\infty \frac{dk}{k} \left(\frac{k}{\mathcal{H}_0}\right)^2 \mathcal{P}(k),$$

In the flux - The dominant contribution are Doppler terms $\sim k^2$

- Any other function of d_L gets also k^3 contributions – lensing contribution, dominates at large redshift, $z>0.3$

- In principle the upper limit can be infinite. In practice, until where do we trust our spectrum? Linear treatment $k<0.1$-1 Mpc^{-1} and non-linear treatment $k<30h$ Mpc^{-1}

No Divergences!

Interpretation & Analysis

- Superhorizon scales are subdominant.
- At small enough scales, the transfer function wins.
- At intermediate scales, the phase space factor competes with the initial small amplitude.
- In the non-linear regime ($\delta \sim 1$) we use a fit from simulations. (Takahashi et al. 1208.2701)

$$f_\Phi(z) = \left[\tilde{f}_{1,1}(z) + \tilde{f}_2(z)\right] \int_0^\infty \frac{dk}{k} \left(\frac{k}{\mathcal{H}_0}\right)^2 \mathcal{P}(k),$$

$$P_k = P_{prim.} T^2$$

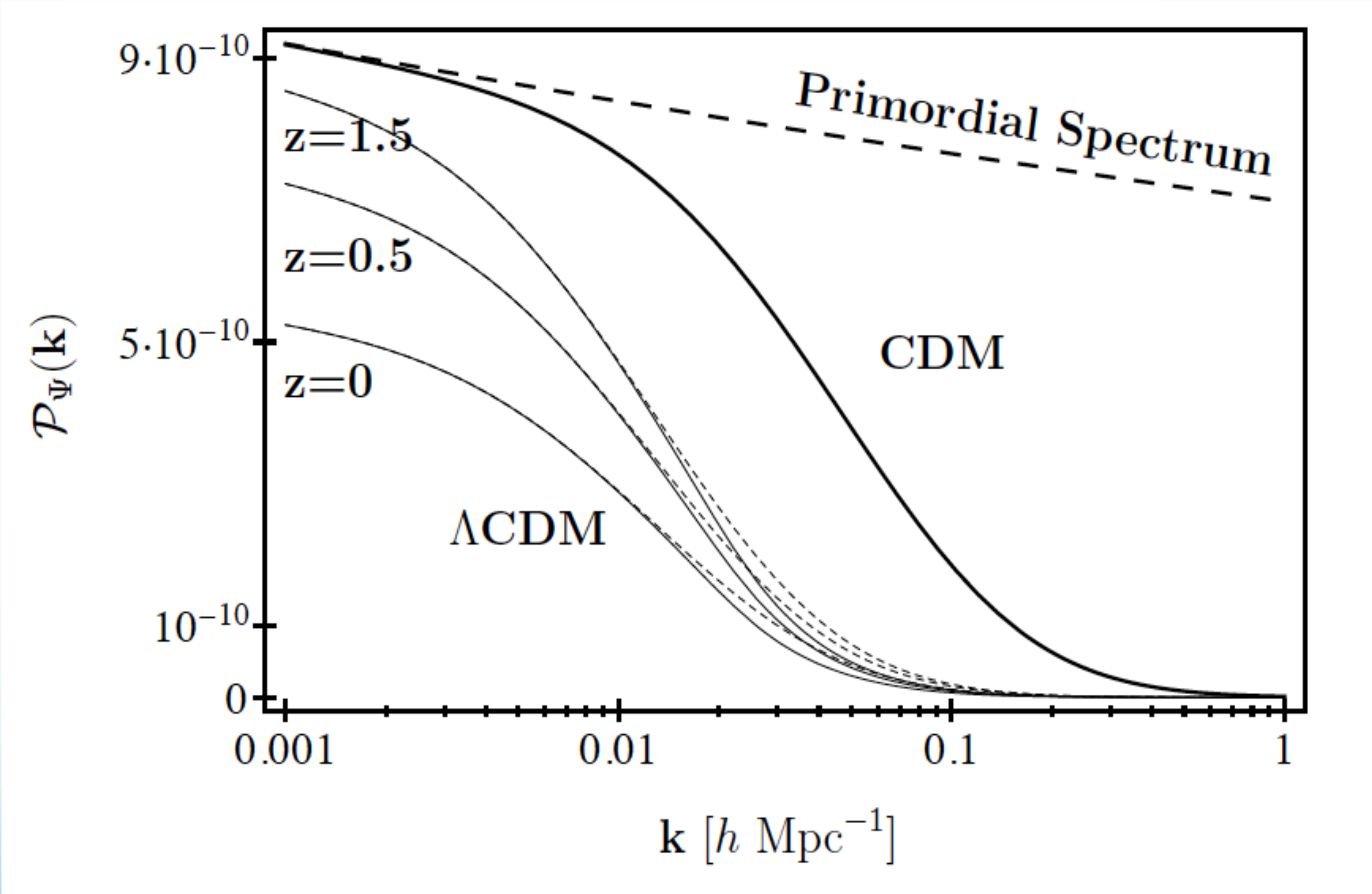
$$\sigma\left(\frac{d_L^{-2}}{d_L^{-2\,FLRW}}\right) \approx \sqrt{\int \frac{dk}{k} P_k \left(\frac{k}{H_0}\right)^3 h(z)}$$

$$T^2(k << k_{eq}) \sim 1$$

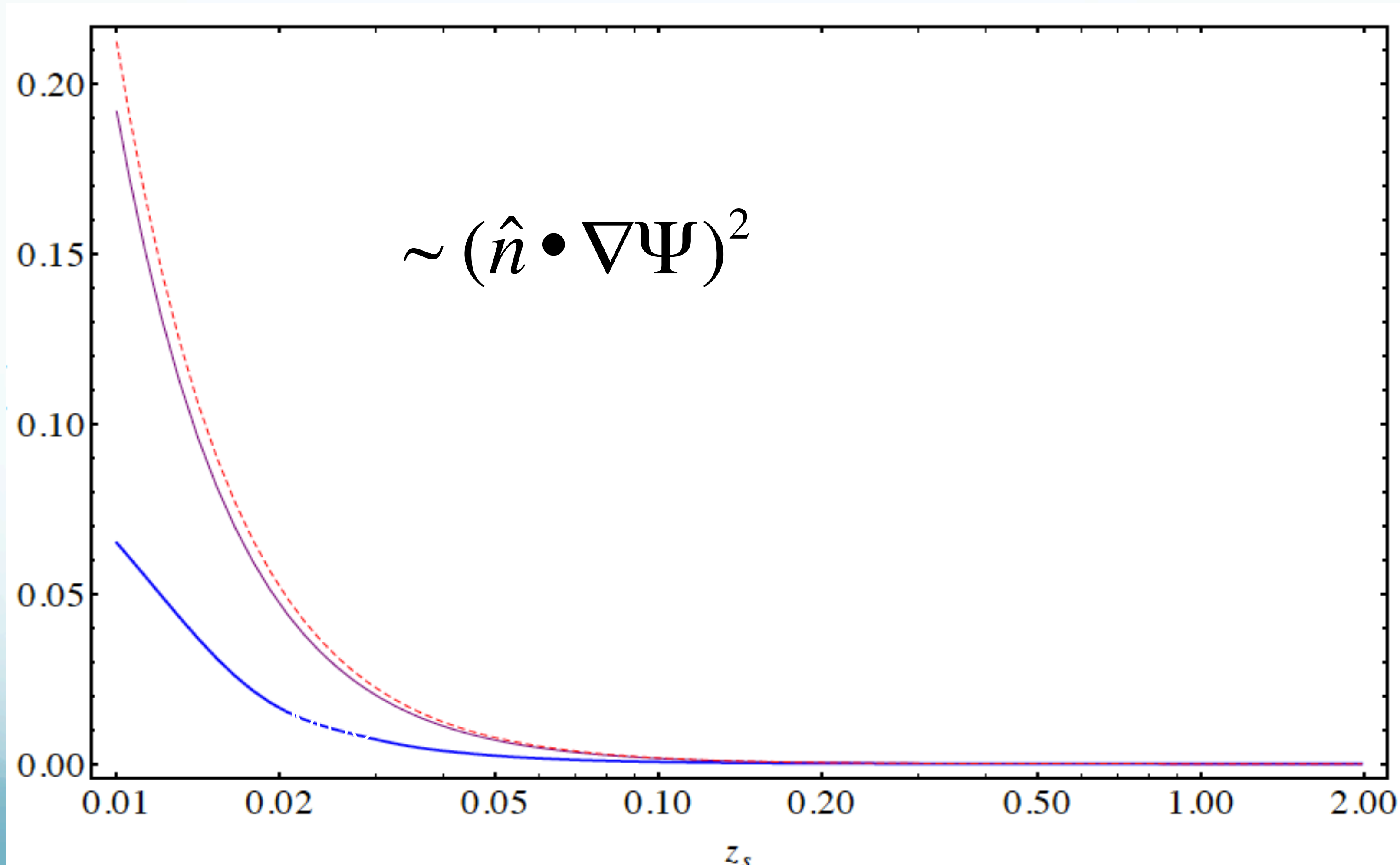
$$T^2(k >> k_{eq}) \sim \ln^2 k \big/ k^4$$

$$Integrand \sim A \times T^2 \times \left(\frac{k}{H_0}\right)^p$$

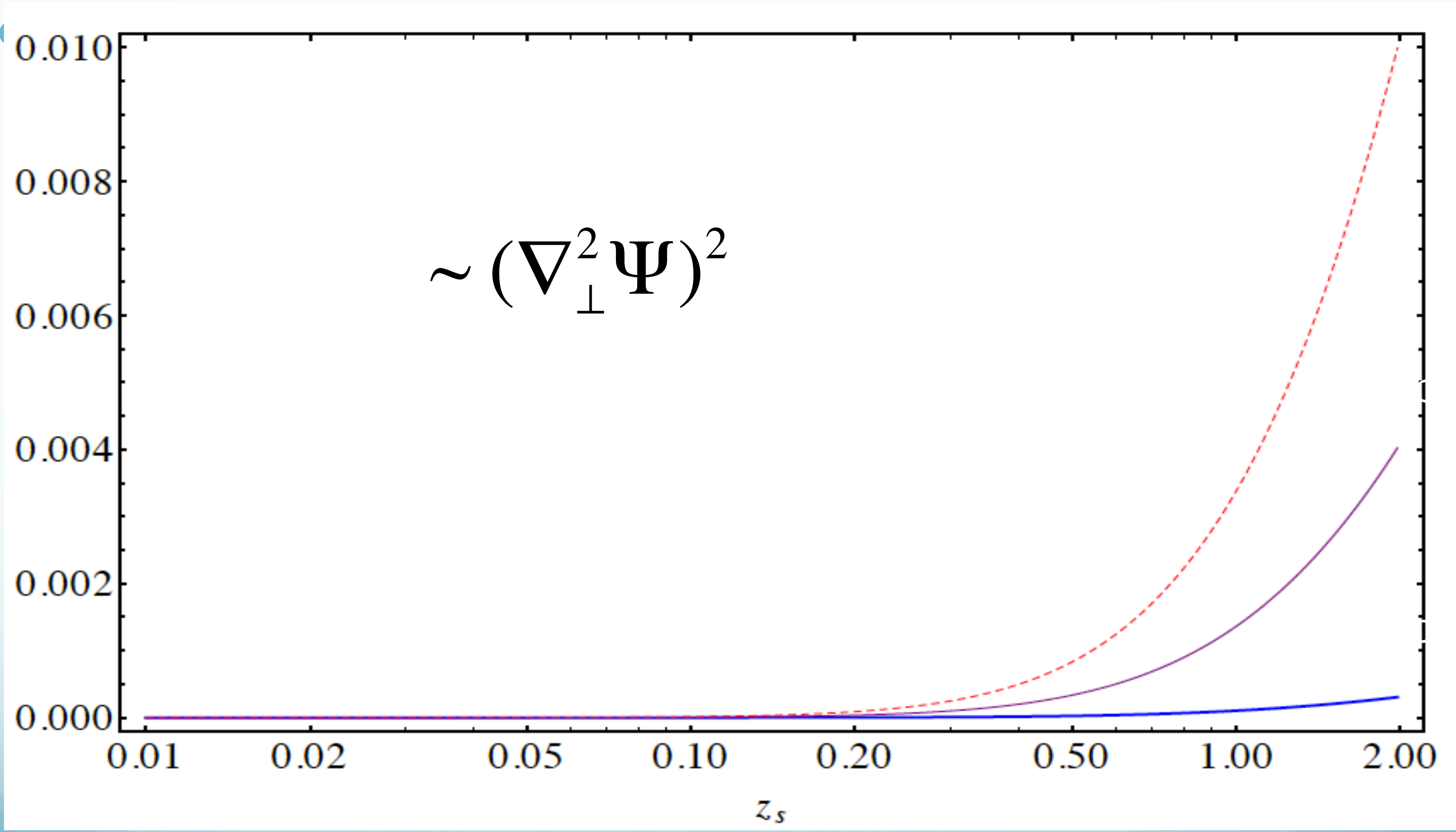
Linear PS of the grav. potential



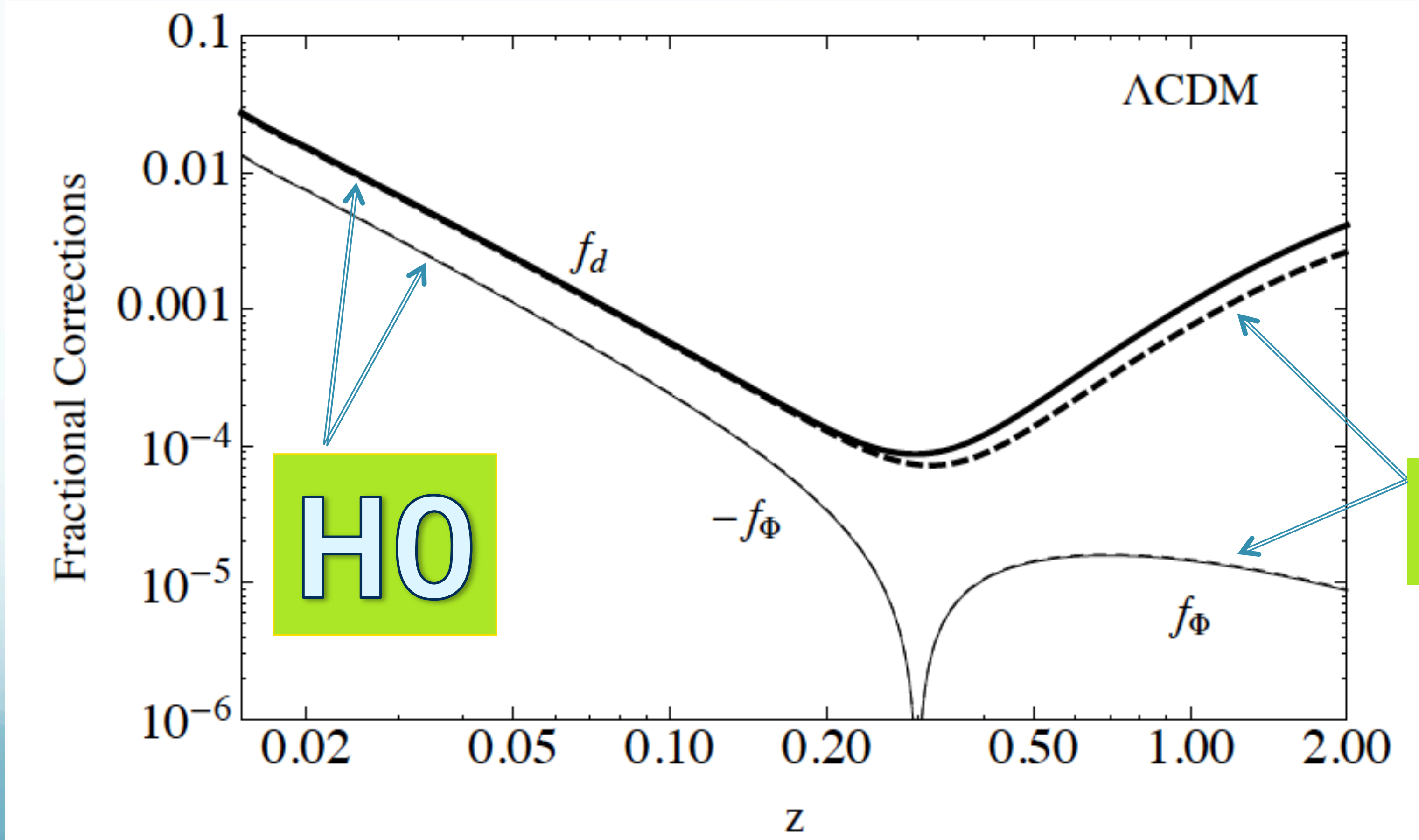
“Doppler²” term in Flux (CDM)



Lensing^2 Term – Not in Flux (CDM)

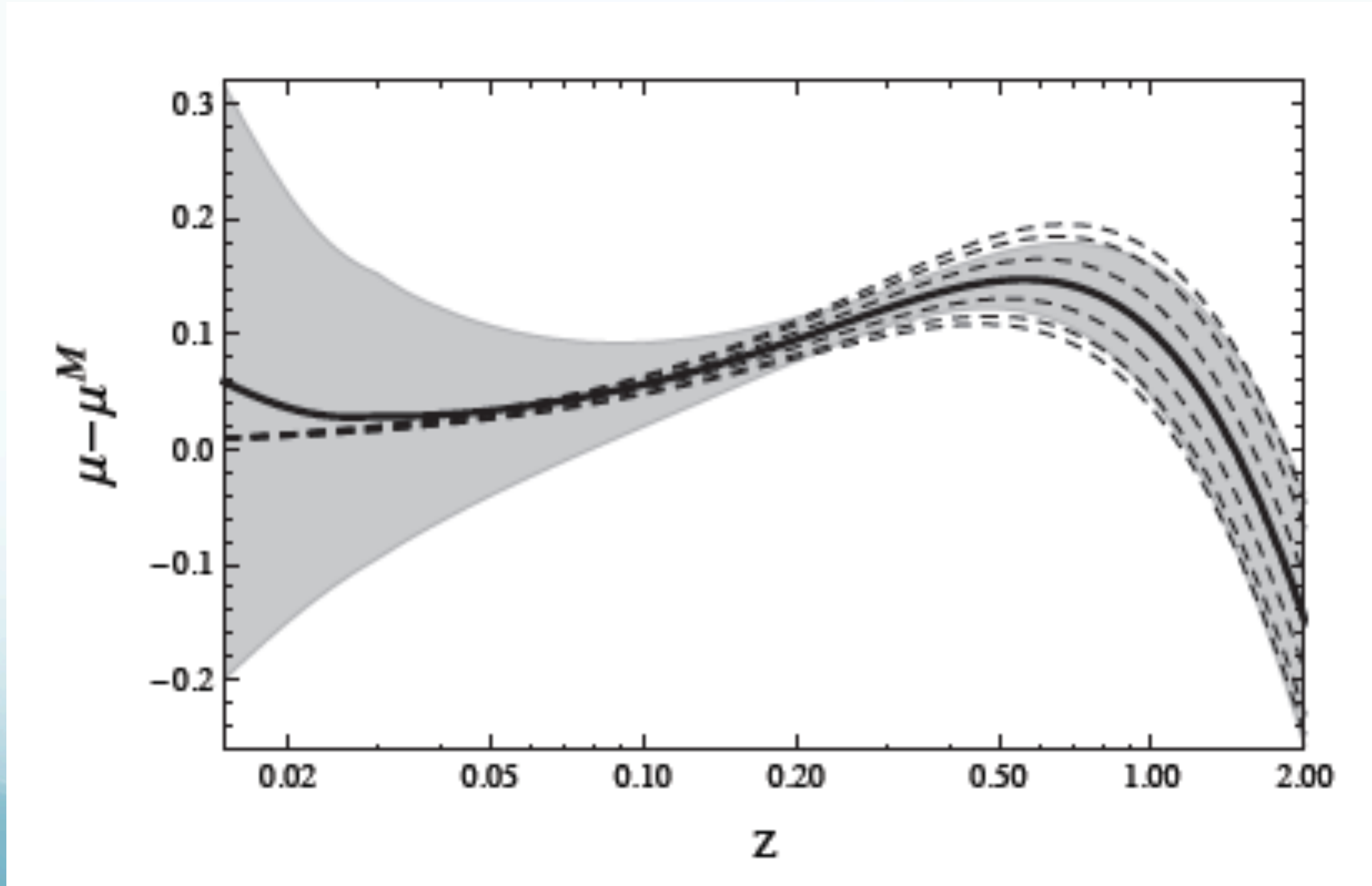


Fractional corrections to the Flux and d_L, k_{UV}=10h,30h Mpc^{-1}



Distance Modulus Average and Dispersion, at z<0.1 Doppler dominates, at z>0.3, Lensing dominates.

$$\mu - \mu^{M} = 5Log_{10}\left[\left\langle \bar{d}_{L} \right\rangle\right] - 5Log_{10}\left[\frac{(2+z_s)z_s}{2H_0}\right]$$



Bottom Line – Part 1

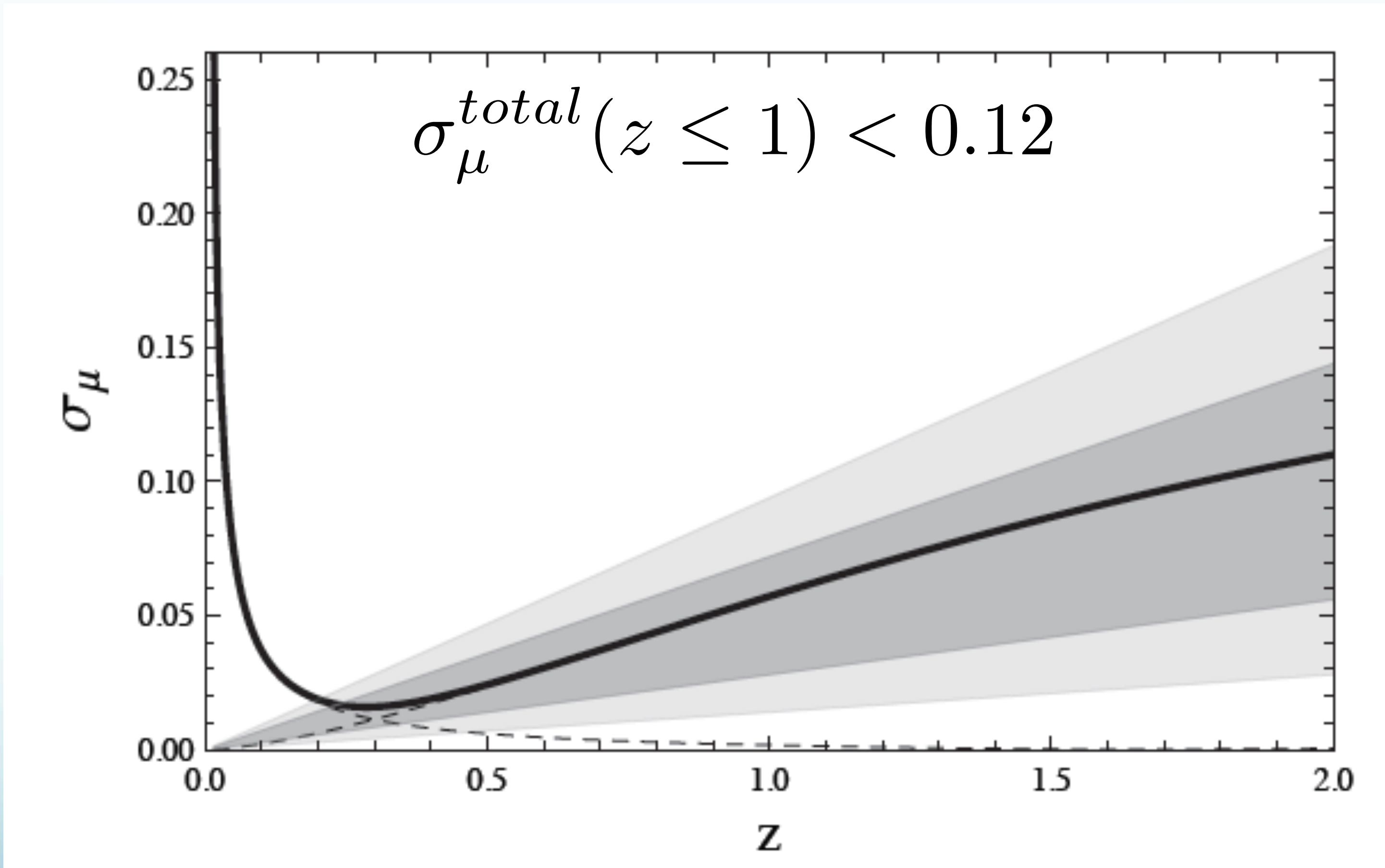
- Flux is the optimal observable. Different bias or "subtraction" mechanisms, in order to extract cosmological parameters.
- Inhomogeneities do not affect the measured CC at an observable level. They do increase the dispersion – see next topic. *1302.0740*
- Inhomogeneities do affect the Hubble parameter at and its dispersion at a percent level. 1311.xxxx?

General Lessons

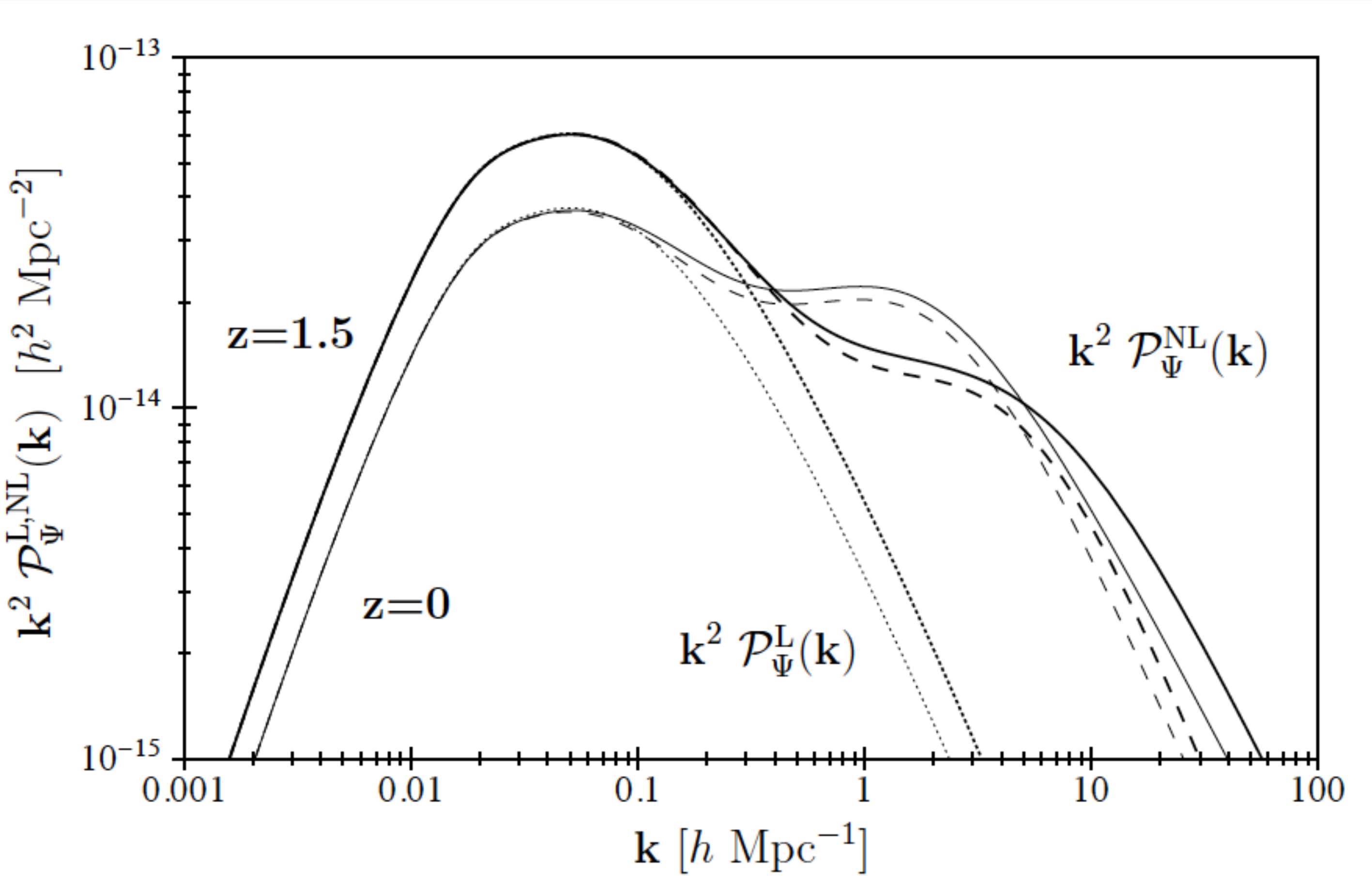
- Unlike volume averages: No divergences
- The contribution from inhomogeneities is several orders of magnitude larger than the naïve expectations due to the large phase space factor.
- Our approach is useful whenever dealing with information carried by light-like signals travelling along our past light-cone.

New Probe: Lensing Dispersion

IBD, Kalaydzhan 1309.4771



Closer Look on the Lensing Integrand



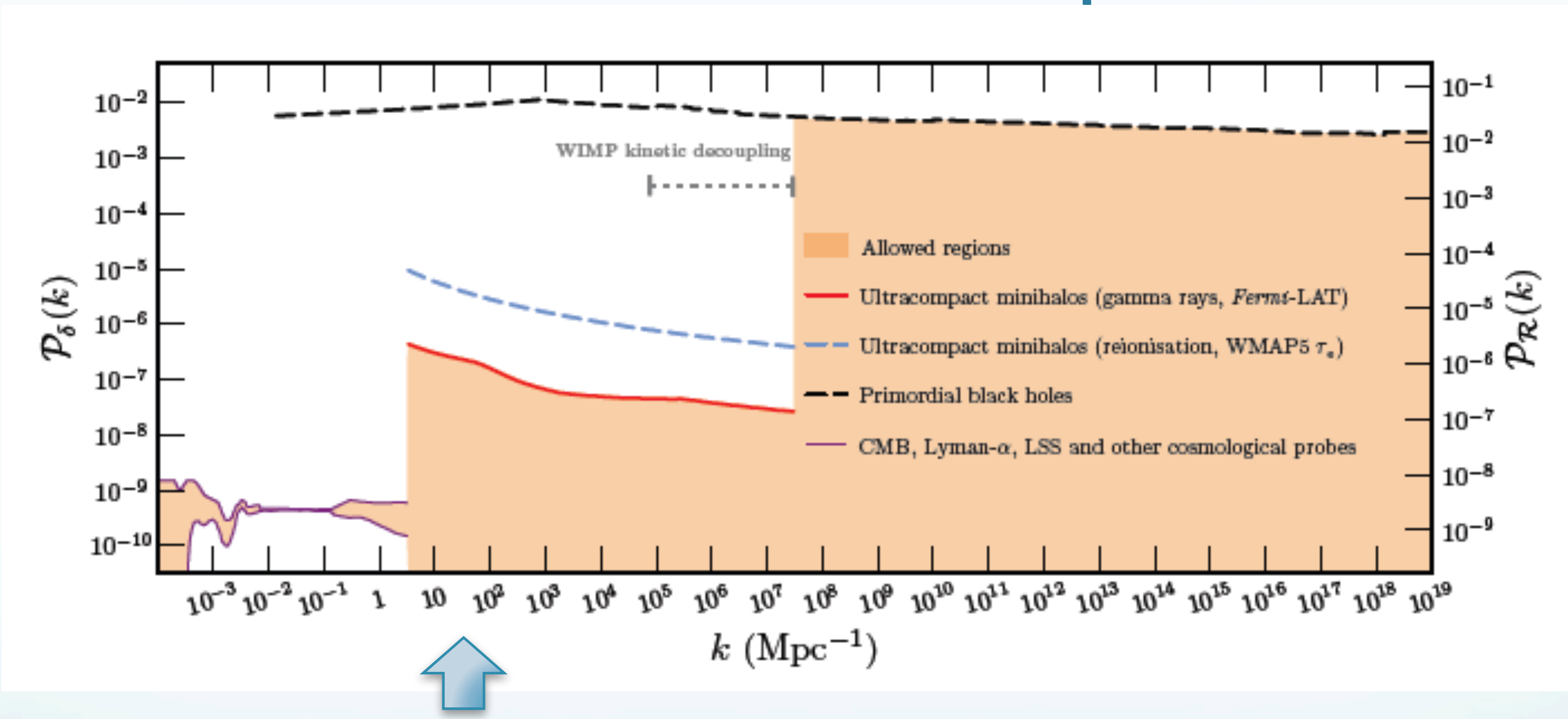
Lensing Dispersion

- Current Data up to z~1.3
- Most Conservative Approach: Constrains late time power spectrum and/or numerical simulations.

$$\sigma_{\mu}^{lens} \approx \sqrt{\int_{H_0}^{k_{UV}} \frac{dk}{k} P_{NL}(k,z)\left(\frac{k}{H_0}\right)^3 h(k,z)}$$

- h(k,z) incorporates the dependence on the EOS parameter w(z) etc.
- The dispersion can be used to constrain EOS, σ_8 primordial power spectrum etc.

Primordial Power Spectrum



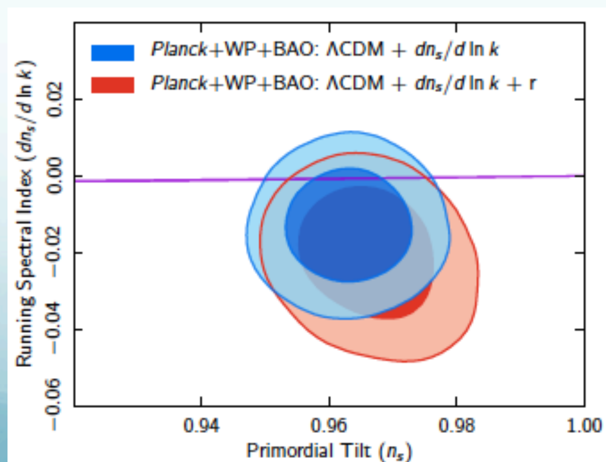
- Many inflationary models predict enhanced spectra (Particle production, features, several inflationary epochs etc.)
- Even with PLANCK, Ly-alpha – measurements only up to k=1 Mpc^{-1}

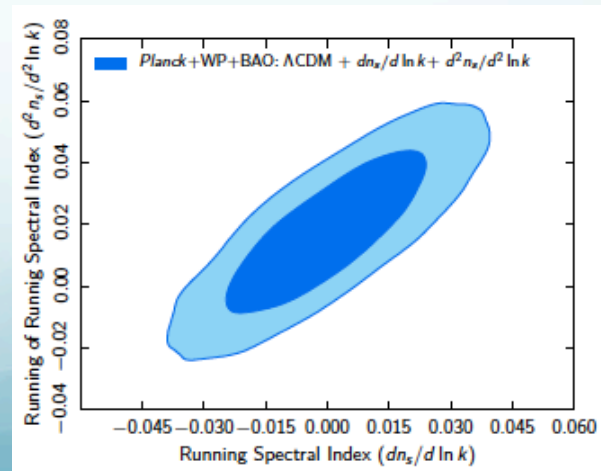
Planck Measurements

- Even with PLANCK, Ly-alpha, etc. only probe ~8.5 e-folds out of 60.
- Few observables, huge degeneracy between models.

$$P_{prim.}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s(k_0)-1+1/2\alpha(k_0)\ln(k/k_0)+1/6\beta(k_0)(\ln(k/k_0))^2}$$

The power spectrum is the actual observable and we need to measure it for as many e-folds possible!





Lensing Dispersion of SNIa

- Model dependent- LCDM and HaloFit model. (Smith et al. 2003, Takahashi et al. 1,2 2012)
- An overall upper bound on dispersion. Any enhancement of the power spectrum will increase the dispersion. WHAT'S THE CORRECT SPECTRUM??

$$\sigma_\mu^{total}(z \leq 1) < 0.12$$

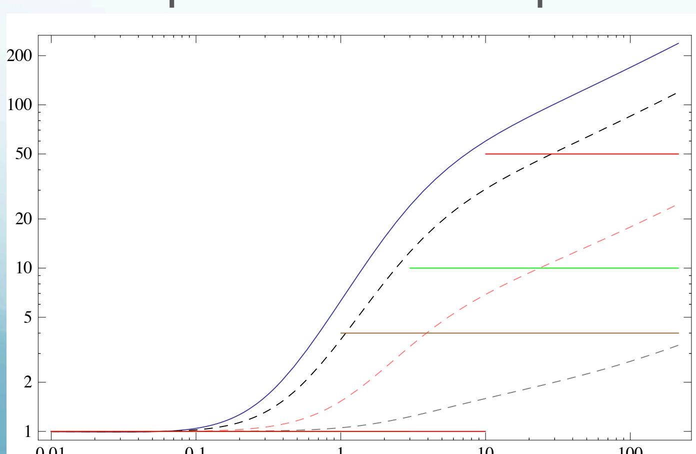
$$\sigma_\mu(z) \simeq 0.7\sqrt{\int \frac{dk}{k} P_\Psi(k,z) \left(\frac{k}{H_0}\right)^3 \tilde{\Delta}\eta(z)^3}$$

$$\tilde{\Delta}\eta(z) = \int_0^z \frac{dy}{\sqrt{\Omega_{m0}(1+y)^3 + \Omega_{\Lambda 0}}}$$

$$\sigma_\mu(z=1) \simeq 0.47\sqrt{\int dk k^2 P_\Psi(k,1)} \equiv 0.47\sqrt{T_2(P_\Psi)}$$

Power spectrum

- HaloFit cannot be trusted for large running or running of running.
- Treat $F(k,z)=P_{NL}/P_L$ as a transfer function, at z=1.
- 2 methods: 1) step function 2) interpolated function. Both extremely underestimating.
- kUV=320h. kUV=30h degrades results but still cuts out parameter space allowed by PLANCK.

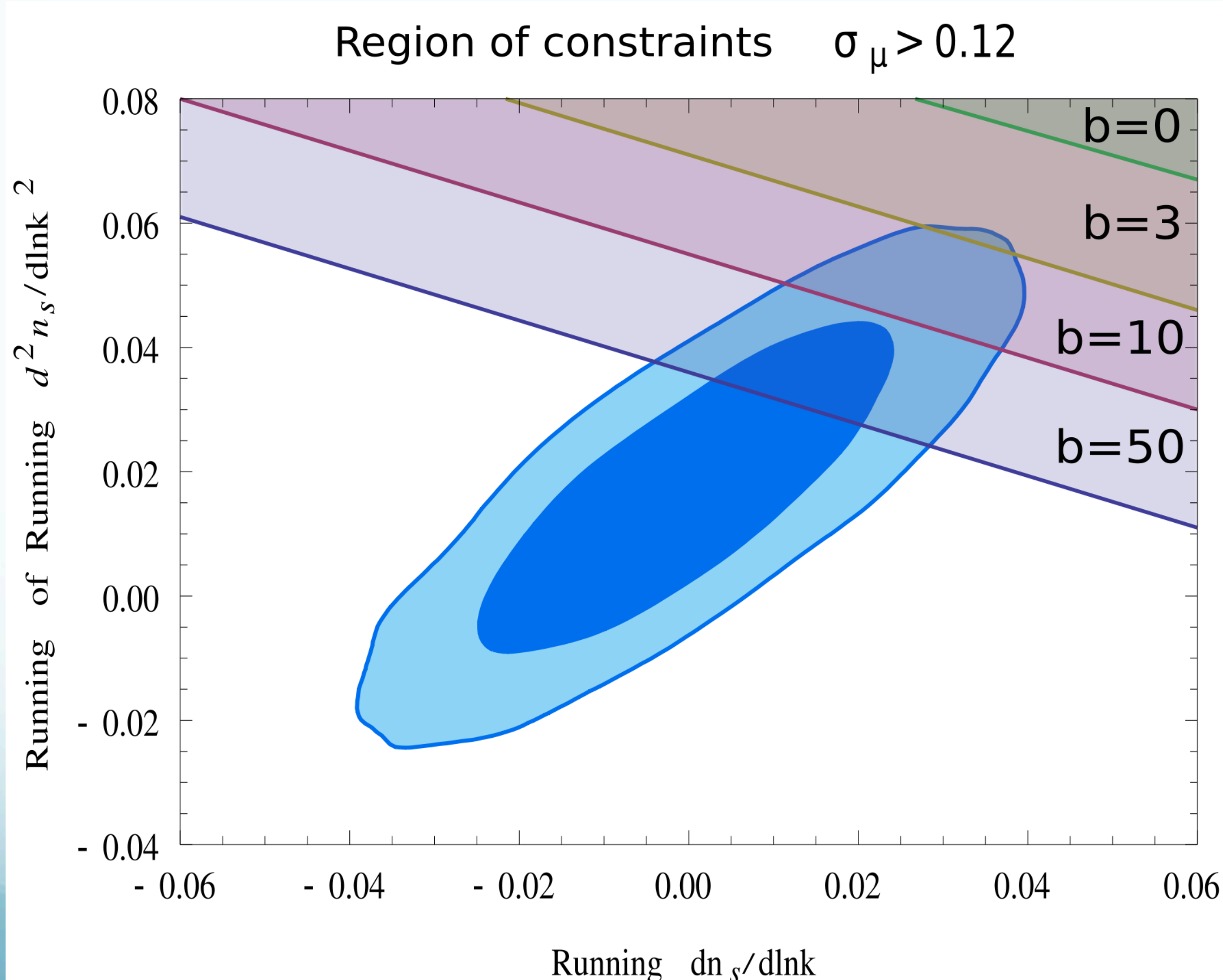


$$P_{prim.}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s(k_0)-1+1/2\alpha(k_0)\ln(k/k_0)+1/6\beta(k_0)(\ln(k/k_0))^2}$$

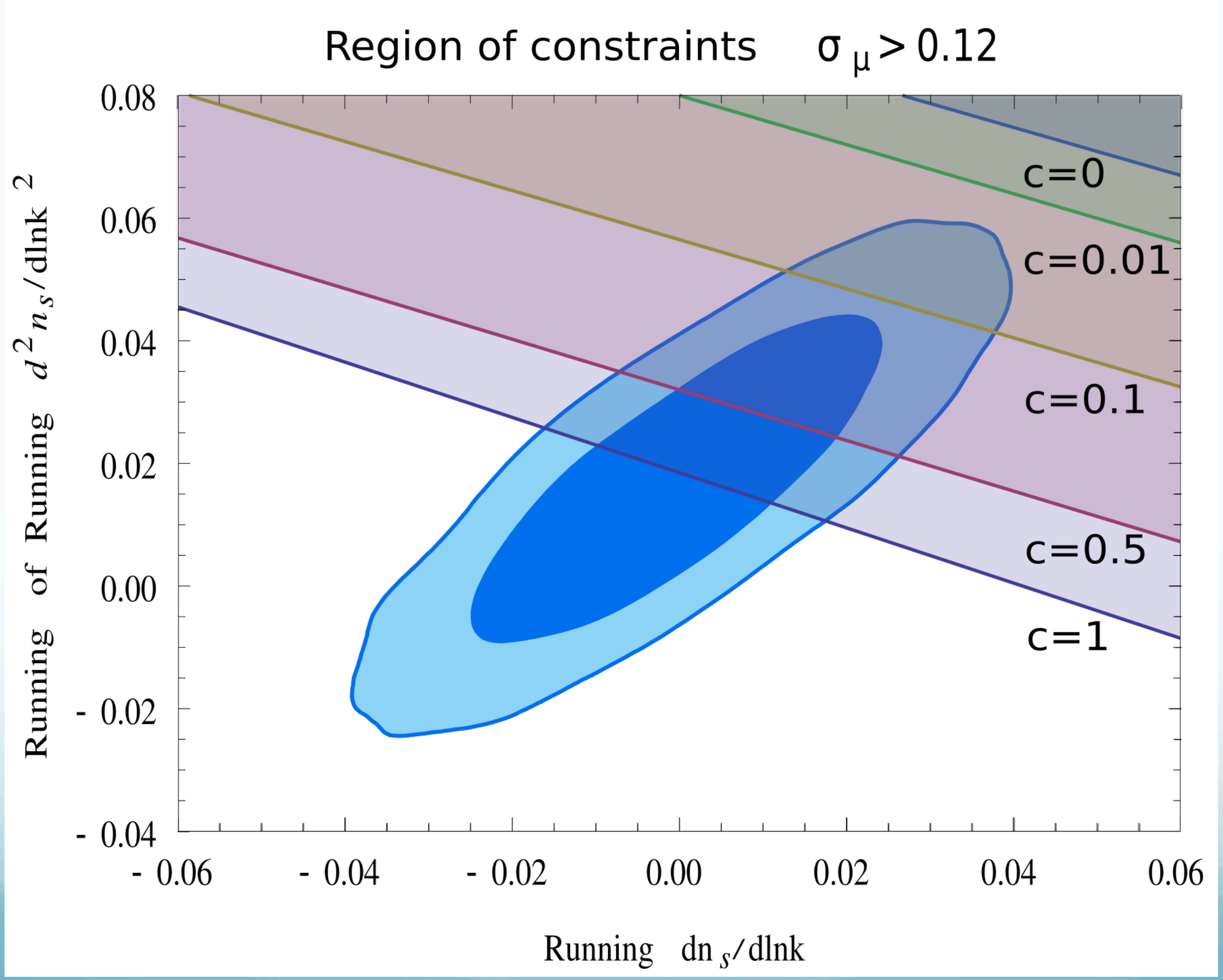
$$T_2(P) = \int_{H_0}^{k_{UV}} dk\, k^2 P_L(1 + b\,\Theta(k - k_{NL}))$$

$$T_2^*(P) = \int_{H_0}^{k_{UV}} dk\, k^2 P_L(k)(1 - c + cF^*(k, z, z^*))$$

$1+b\,H(k-k_{NL})$


Region of constraints σ μ > 0.12
0.08
0.06
0.04
0.02
0.00
- 0.02
- 0.04
- 0.06
- 0.04
- 0.02
0.00
0.02
0.04
0.06
b=0
b=3
b=10
b=50
Running of Running d 2 n s /dlnk 2
Running dn s /dlnk

1-c+c F*(k,z,z*)


Region of constraints σ μ > 0.12
c=0
c=0.01
c=0.1
c=0.5
c=1
0.08
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- 0.04
- 0.06
- 0.04
- 0.02
0.00
0.02
0.04
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Running of Running d²n_s/dlnk²
Running dn_s/dlnk

Conclusions - part 2

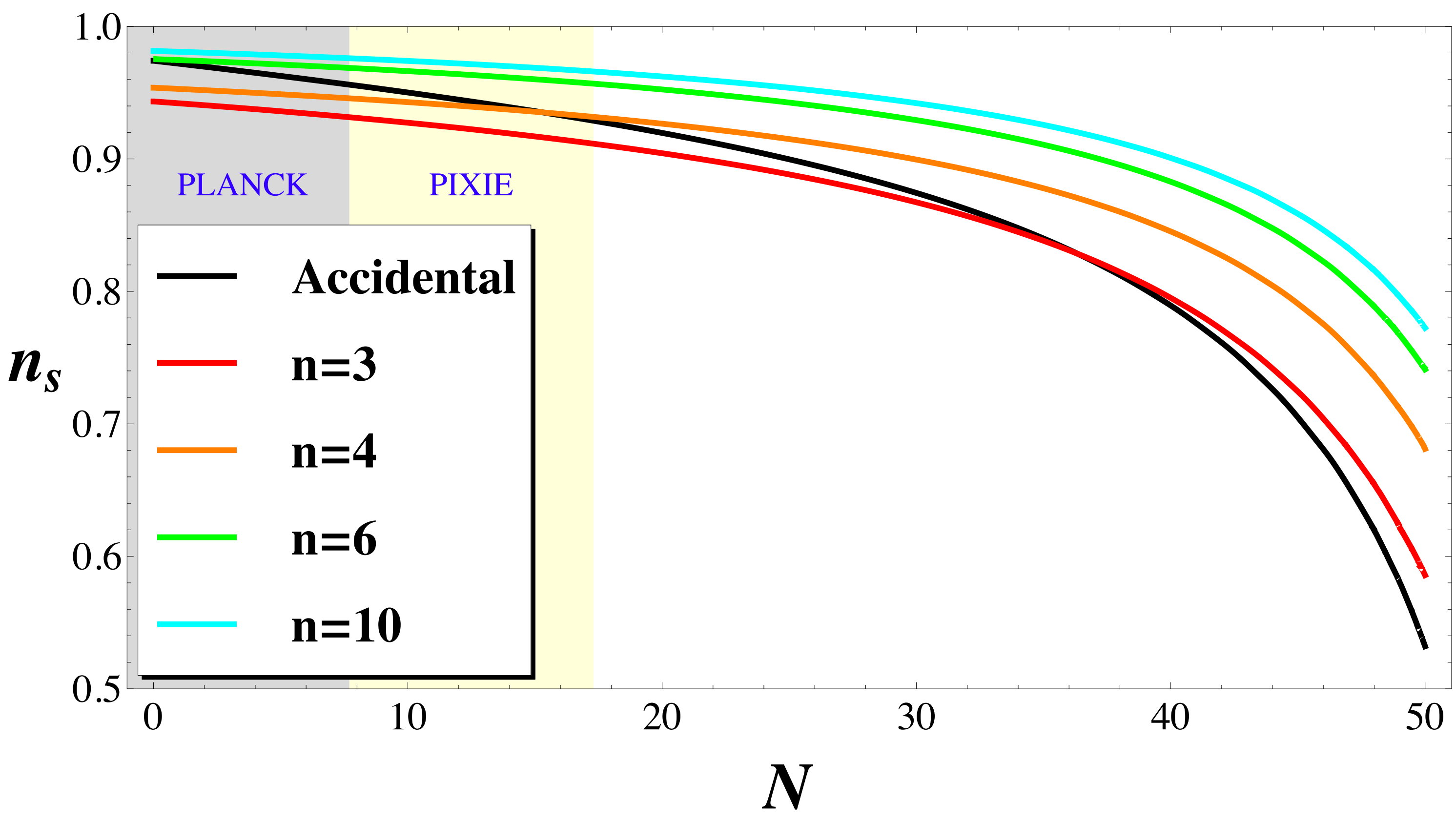
Lensing of SNIa – Model dependent, current data! With conservative estimates already rules out a big region of parameter space.

- We expect numerical simulations to give confirm our results and even derive better bounds.
- A likelihood analysis is necessary for confirmation of the effect.
- Future detection will allow a very narrow region.
- A novel probe for cosmology!

Open Issues/Future Prospects

1. Using lensing to constrain other observables and other spectra.
2. More accurate calculation of the lensing effect.
3. Matching the back-reaction effect to other probes: CMB, LSS
4. Applying LC averaging to cosmic shear, BAO, kSZ, strong lensing etc.
5. Other applications, averaging of EFE ....Many open theoretical and pheno. problems.

Example: "string vs. field theory"



$$V(\phi) = \Lambda^4(1 - a_1\phi - a_n\phi^n)$$

1309.0529, IBD, S. Jing
A. Westphal and C. Wieck

Conclusions - part 1

- Flux is the optimal observable. Different bias or "subtraction" mechanisms, in order to extract cosmological parameters.
- Irreducible Scatter - The dispersion is large ~ 2-10% ΛCDM, of the critical density depending on the spectrum.
- The effect is too SMALL AND has the WRONG z dependence to simulate observable CC! It DOES affect H_0 at a measurable level!

Prescription Properties

- Dynamical properties: Generalization of Buchert-Ehlers commutation rule:

$$\frac{\partial}{\partial A_0}\langle S\rangle_{V_0,A_0} = \left\langle \frac{k\cdot\partial S}{k\cdot\partial A}\right\rangle_{V_0,A_0} + \left\langle \frac{\nabla\cdot k}{k\cdot\partial A}S\right\rangle_{V_0,A_0} - \left\langle \frac{\nabla\cdot k}{k\cdot\partial A}\right\rangle_{V_0,A_0}\langle S\rangle_{V_0,A_0}$$

- For actual physical calculations, use EFE/ energy momentum tensor for evaluation. Example: which gravitational potential to use in evaluating the d_L-z relation.

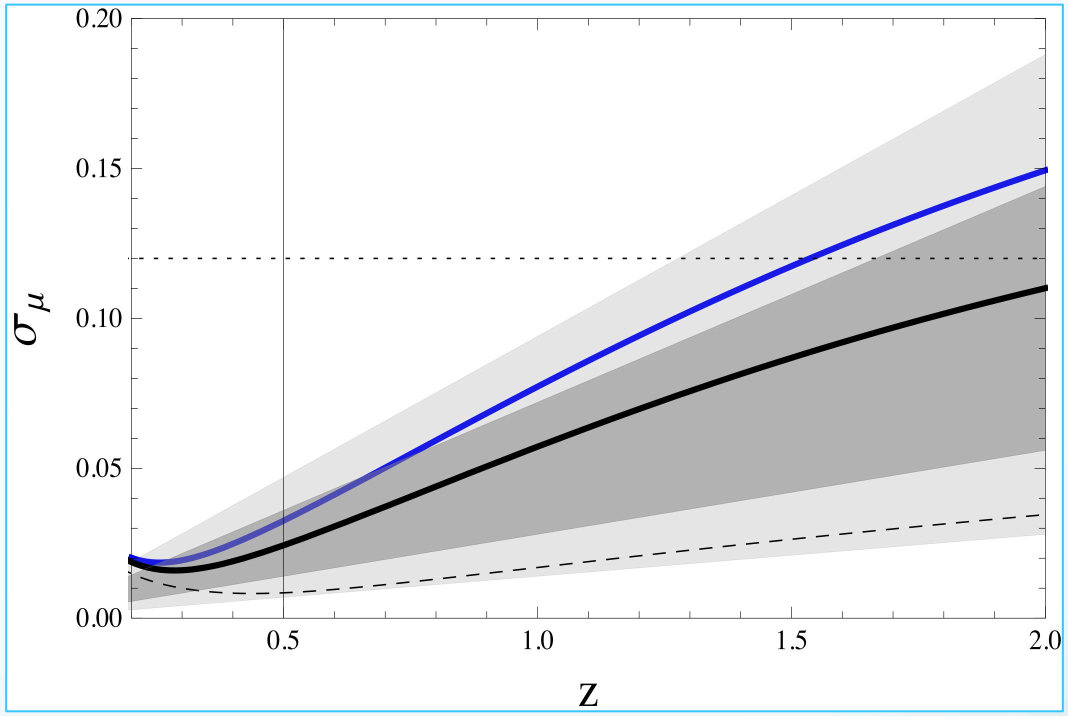
- Averages of different functions give different outcome

$$\left\langle \overline{F}(S)\right\rangle \neq F\left\langle \overline{S}\right\rangle$$

Lensing Dispersion

- Current data has SN up to z~1.3
- Statistical Analysis: dispersion<0.12 in mag. (March et al. 2011)
- Extended halofit, Inoue & Takahashi 2012, up to k=320h Mpc^{-1}

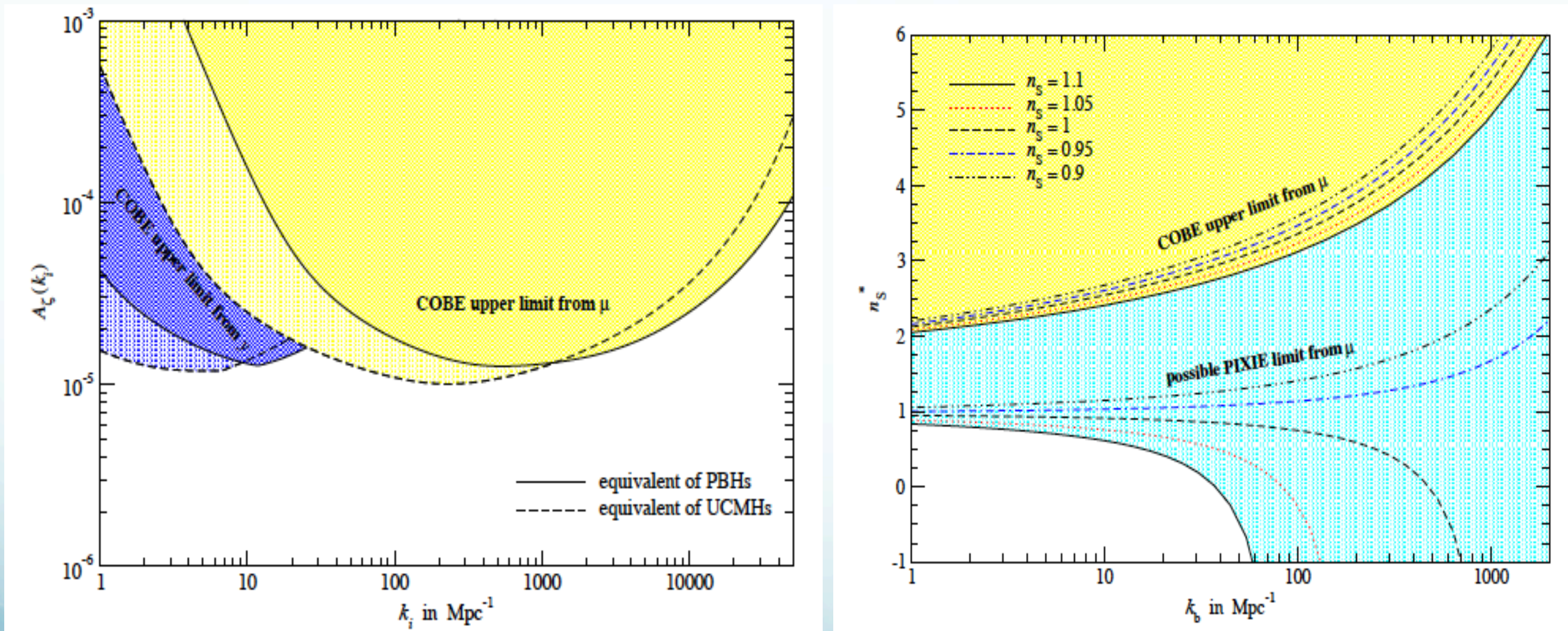
⟹ Can constrain the amplitude on scales 1<k<30h to roughly 10^{-7} and better.



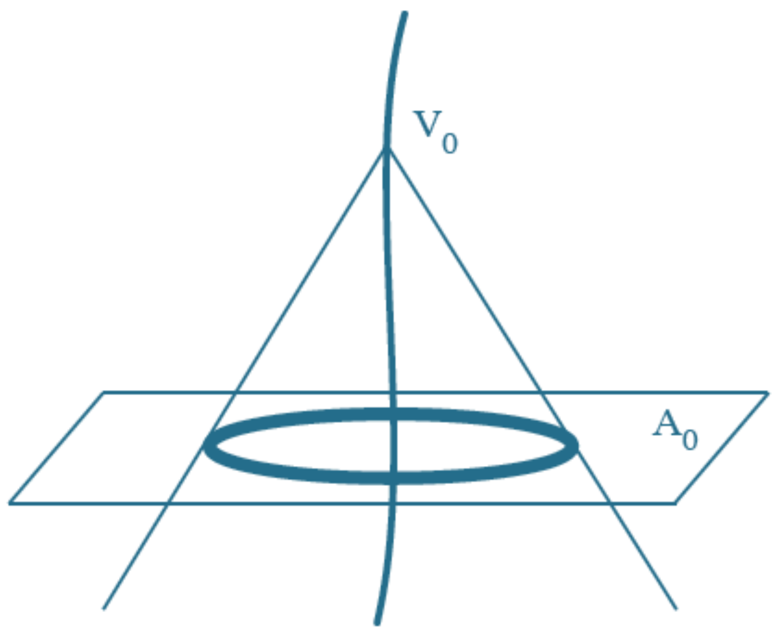
$$\sigma_\mu^{lens} \approx \sqrt{\int_{H_0}^{k_{UV}} \frac{dk}{k} P_{NL}(k,z) \left(\frac{k}{H_0}\right)^3 h(k,z)}$$

Constraints from Spectral Distortions (deviations from BB spectrum)

- Chluba, Erickcek, IBD 2012:



$\langle d_L \rangle(z)$, $\langle f(d_L) \rangle(z)$



A= redshift, V= light-cone coordinate

Statistical Properties

- In principle, we can now calculate <d_L>(z) to first order in the gravitational potential ~ void model. $\overline{\Psi}=0,\ \overline{\Psi^2}\neq 0$
- In order not to resort to a specific realization we need LC+statistical/ensemble average. If perturbations come from primordial Gaussian fluc. (inflation)

$$\{\langle d_L\rangle\} = d_L^{(0)}\left[1+\{\langle \mu^{(1)} d_L^{(1)}\rangle\}+\{\langle d_L^{(2)}\rangle\}+\ldots\right]$$

$$(Var\frac{d_L}{d_L^{FLRW}}) = \{\langle (d_L^{(1)})^2\rangle\}$$

Functions of d_L

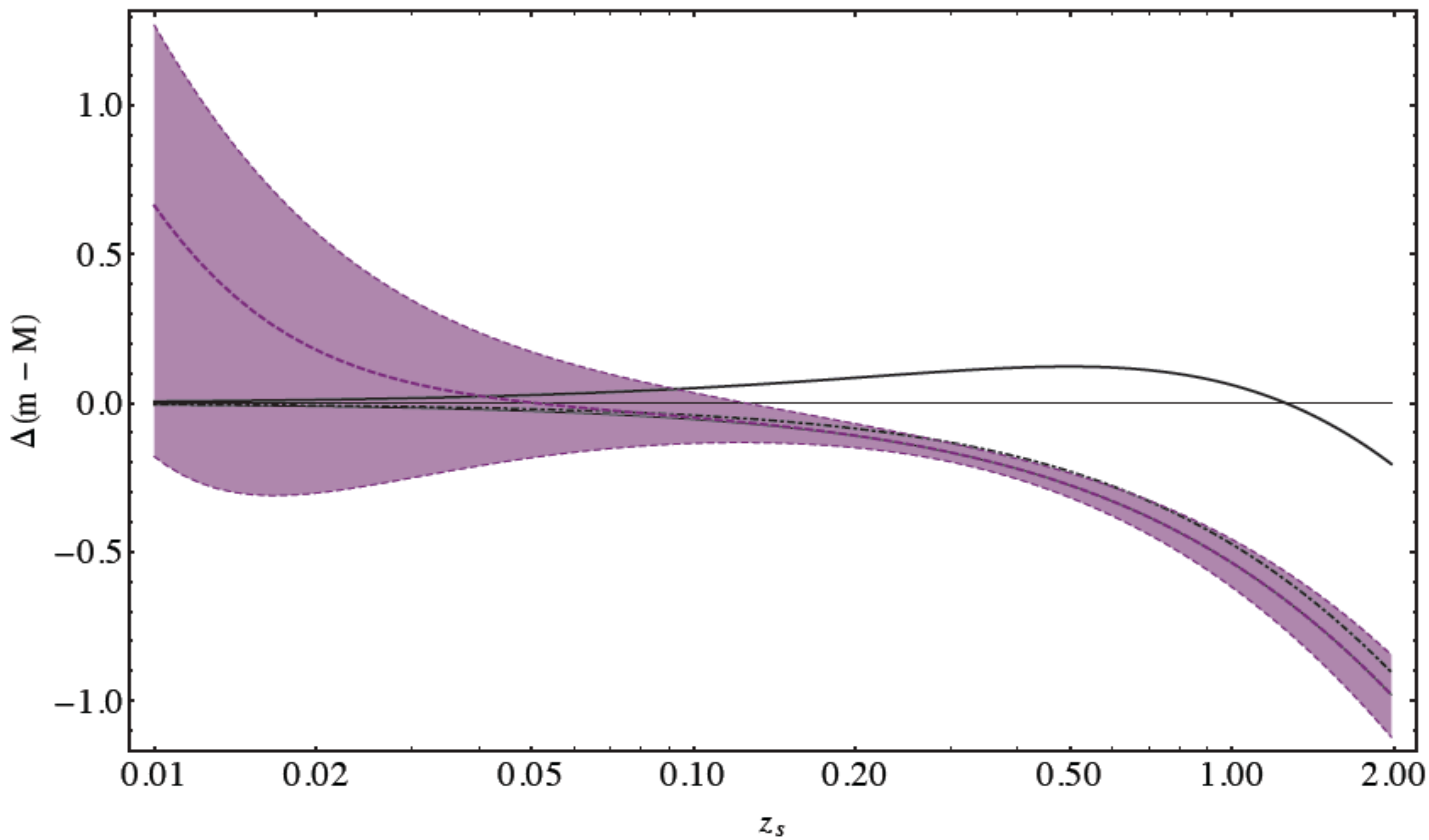
- Standard pert. theory: the gravitational potential, density contrast etc. are gaussian random variables. $\overline{\Psi} = 0, \ \overline{\Psi^2} \neq 0$
- Overbars and {...} denote ensemble average, <..> denote LC average.

$$\langle \{F(S)\} \rangle \neq F \langle \{S\} \rangle$$

- Averages of different functions of scalars receive different contributions.

$$\Phi \sim \langle \{ d_L^{-2} \} \rangle = (d_L^{FLRW})^{-2} [1 + f_\Phi(z)]$$
$$\langle \{ d_L \} \rangle = d_L^{FLRW} [1 + f_d(z)]$$

k_{max}=1 Mpc^{-1}



GLC to FLRW NG 1st Order

$$g^{\mu\nu}_{NG} = a^{-2}(\eta)\,\mathrm{diag}\left(-1+2\Phi, 1+2\Psi, (1+2\Psi)\gamma_0^{ab}\right) .$$

$$\begin{aligned}
\tau &= \int_{\eta_{in}}^{\eta} d\eta' a(\eta')\left[1+\Psi(\eta', r, \theta^a)\right] \ , \\
w &= \eta_+ + \int_{\eta_+}^{\eta_-} dx\, \hat{\Psi}(\eta_+, x, \theta^a) \ , \\
\tilde{\theta}^a &= \theta^a + \frac{1}{2}\int_{\eta_+}^{\eta_-} dx\, \hat{\gamma}_0^{ab}(\eta_+, x, \theta^a) \int_{\eta_+}^{x} dy\, \partial_b \hat{\Psi}(\eta_+, y, \theta^a) \ ,
\end{aligned}$$

$$\hat{\Psi}(\eta_+, \eta_-, \theta^a) \equiv \Psi(\eta, r, \theta^a)$$

$$\hat{\gamma}_0^{ab}(\eta_+, \eta_-, \theta^a) \equiv \gamma_0^{ab}(\eta, r, \theta^a) = diag(r^{-2}, r^{-2}\sin^{-2}\theta)$$

$$\eta_\pm = \eta \pm r$$

LC Calculation and LCDM

- Pure FLRW:

$$d_L^{FLRW}(z_s) = (1+z_s)a_0 \int_{\eta_s}^{\eta_0} d\eta = (1+z_s)\int_0^{z_s} \frac{dz}{H(z)}$$
$$= \frac{1+z_s}{H_0}\int_0^{z_s} dz \left[\sum_n \Omega_{n0}(1+z)^{3(1+w_n)}\right]^{-1/2}$$

$$\frac{d_L(z_s,\theta^a)}{(1+z_s)a_0\Delta\eta} \equiv \frac{d_L(z_s,\theta^a)}{d_L^{FLRW}(z_s)} = 1 - \Psi(\eta_s, \eta_0-\eta_s, \theta^a) + 2\Psi_{\rm av} + \left(1 - \frac{1}{\mathcal{H}_s \Delta\eta}\right) J - J_2.$$

- Perturbed:

Statistical Properties

$$\left\{\left\langle d_L \right\rangle\right\} = d_L^{(0)}\left[1 + \left\{\left\langle \mu^{(1)} d_L^{(1)} \right\rangle\right\} + \left\{\left\langle d_L^{(2)} \right\rangle\right\} + \ldots\right]$$

- In principle, we can now calculate <d$_L$>(z) to first order in the gravitational potential ~ void model. $\overline{\Psi} = 0,\ \overline{\Psi^2} \neq 0$
- In order not to resort to a specific realization we need LC+statistical/ ensemble average. If perturbations come from primordial Gaussian fluc. (inflation)

$$\mu = \sum_i \mu_i, \qquad \sigma = \sum_i \sigma_i,$$

$$\langle S \rangle_\Sigma = \frac{\int_\Sigma d^2\mu\, S}{\int_\Sigma d^2\mu}$$

$$d^2\mu = (d^2\mu)^{(0)}(1+\mu), \qquad S = S^{(0)}(1+\sigma),$$

BR of Statistical and LC Averaging

- The mean of a scalar:

$$\overline{\langle S/S^{(0)}\rangle} = 1 + \overline{\langle\sigma_2\rangle} + \mathrm{IBR}_2 + \overline{\langle\sigma_3\rangle} + \mathrm{IBR}_3 + \dots$$

$$\begin{aligned}
\mathrm{IBR}_2 &= \overline{\langle\mu_1\sigma_1\rangle} - \overline{\langle\mu_1\rangle\langle\sigma_1\rangle}, \\
\mathrm{IBR}_3 &= \overline{\langle\mu_2\sigma_1\rangle} - \overline{\langle\mu_2\rangle\langle\sigma_1\rangle} + \overline{\langle\mu_1\sigma_2\rangle} - \overline{\langle\mu_1\rangle\langle\sigma_2\rangle} - \overline{\langle\mu_1\rangle\langle\mu_1\sigma_1\rangle} + \overline{\langle\mu_1\rangle\langle\mu_1\rangle\langle\sigma_1\rangle},
\end{aligned}$$

=> Effects are second order, but we have the full backreaction of the inhomogeneities of the metric at this order!

- The variance to leading order: $\mathrm{Var}[S/S^{(0)}] = \overline{\langle\sigma_1^2\rangle}.$

Dominant Terms

$$J = I_+ - I_r.$$

$$I_+ = \int_{\eta_+^s}^{\eta_-^s} dx\, \partial_+ \Psi(\eta_s^+, x, \theta^a) = \Psi_s - \Psi_o - 2\int_{\eta_s}^{\eta_0} d\eta\, \partial_r \Psi(\eta, r, \theta^a),$$

$$I_r = \int_{\eta_{in}}^{\eta_s} d\eta \frac{a(\eta)}{a(\eta_s)} \partial_r \Psi(\eta, r_s, \theta^a) - \int_{\eta_{in}}^{\eta_0} d\eta \frac{a(\eta)}{a(\eta_0)} \partial_r \Psi(\eta, 0, \theta^a).$$

$$I_r = (\vec{v}_S - \vec{v}_0) \cdot \hat{n}$$

$$\vec{v}_{s,o} = -\int_{\eta_{in}}^{\eta_{s,o}} d\eta' \frac{a(\eta')}{a(\eta_{s,0})} \vec{\nabla} \Psi(\eta', r, \theta^a)$$

- Doppler effect due to the perturbation of the geodesic.

- The Lensing Term:

$$J_2 = \frac{1}{\eta_0 - \eta_s} \int_{\eta_s}^{\eta_0} d\eta \frac{\eta - \eta_s}{\eta_0 - \eta} \left[\partial_\theta^2 + \cot\theta\, \partial_\theta + (\sin\theta)^{-2} \partial_\phi^2\right] \Psi(\eta', \eta 0 - \eta', \theta^a) \equiv \frac{1}{\eta_0 - \eta_s} \int_{\eta_s}^{\eta_0} d\eta \frac{\eta - \eta_s}{\eta_0 - \eta} \Delta_2 \Psi$$

LC Calculation and LCDM

- Pure FLRW:

$$d_L^{FLRW}(z_s) = (1+z_s)a_0 \int_{\eta_s}^{\eta_0} d\eta = (1+z_s) \int_0^{z_s} \frac{dz}{H(z)}$$
$$= \frac{1+z_s}{H_0} \int_0^{z_s} dz \left[\sum_n \Omega_{n0} (1+z)^{3(1+w_n)} \right]^{-1/2}$$

- Perturbed:

$$\frac{d_L(z_s,\theta^a)}{(1+z_s)a_0\Delta\eta} \equiv \frac{d_L(z_s,\theta^a)}{d_L^{FLRW}(z_s)} = 1 - \Psi(\eta_s, \eta_0 - \eta_s, \theta^a) + 2\Psi_{\rm av} + \left(1 - \frac{1}{\mathcal{H}_s \Delta\eta}\right) J - J_2.$$

- Comparing by defining an effective redshift and averaging at constant redshift and w=w0

$$\frac{a(\bar{\eta}_s^{(0)})}{a(\eta_o)} = \frac{1}{1+z}$$

$$w_0 = \eta_+^s - 2\Delta\eta\Psi_{\rm av} = \eta_0,$$

$$\int_{\eta_+^s}^{\eta_-^s} dx\, \hat{\Psi}(\eta_+^s, x, \theta^a) = -2 \int_{\eta_s}^{\eta_0} d\eta'\, \Psi(\eta', \eta_0 - \eta', \theta^a) \equiv -2\Delta\eta\Psi_{\rm av}.$$

Power Spectrum

- We use the WMAP7 best fit value and the transfer function of Eisenstein & Hu 1997 for CDM.
- We are interested in the overall magnitude so we neglect the baryonic oscillations.

$$P_\Psi(k) \equiv \frac{k^3}{2\pi^2}|\Psi_k|^2 = \left(\frac{3}{5}\right)^2 \Delta_R^2 T(k)^2, \quad \Delta_R^2 = A\left(\frac{k}{k_0}\right)^{n_s-1}$$

- Only subhorizon fluc. $H_0<k$. Superhorizon fluc. are subdominant.
- No UV ($k \rightarrow \infty$) or IR ($z \rightarrow 0$, $k \rightarrow 0$) divergences.

Non-Trivial Averages

- Off-Center LTB
- Anisotropic Models (Except Kantowski-Sachs)
- More general metrics.
- Perturbed FLRW

Application: calculating the averaged luminosity – distance redshift relation

Past attempts: Vanderveld et al. – post Newtonian, Barausse et al., Kolb et al. – SG superhorizon, Pyne at al.,

Averaged d_L at Constant Redshift

- We write the GLC exact results in terms of 2nd order in standard cosmological perturbation theory (SPT) in the Poisson Gauge.
- Novelty: In principle, exact treatment of the geodesic equations and the averaging hyper-surface for any spacetime and any DE model, as long as the geodesic equation is unchanged.
- Previous attempts are limited to perturbations about FLRW and had to solve order by order: Vanderveld et al. – post Newtonian, Barausse et al. 2005, Kolb et al. 2006 – SG superhorizon, Pyne at al 2005…
- Rebuttal : Hirata et al., Geshnizjani et al.

Universe Composition Today

$$\frac{H^2}{H_0^2} = \Omega_{R0}(1+z)^4 + \Omega_{K0}(1+z)^2 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}; \quad \Phi = \frac{L}{4\pi d_L^2}$$

- CC/DE becomes relevant only at z~1, Coincidence Problem?
- Based on CMB, LSS and SNIa observations.

Do perturbations alter the picture???