Inflationary Magnetogenesis & Non-Gaussianity

Martin S. Sloth

Based on arXiv:1305.7151,1210.3461,1207.4187 w. Ferreira, Jain and work to appear w. Nurmi

Observations

- For explaining micro-Gauss Galactic magnetic fields, primordial seeds larger than 10^{-20} Gauss required
- Recent claims of a lower bound on magnetic field in the intergalactic space of 10^{-15} Gauss [Neronov, Vovk 2010]

➡ Indication of inflationary magnetogenesis

- Upper bound on primordial magnetic fields of order nano-Gauss from CMB

A little bit of history...

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

in FRW space is conformal inv. ⇒ doesn't feel

expansion

➡ Electromagnetic fields are not amplified by inflation

➡ Breaking of conformal invariance needed

- Consider coupling of EM fields to other fields, which may couple to gravity in a non-conformal invariant way

➡ Production of magnetic fields? [Tuner, Widrow 1988]

Different models

- Dynamical gauge coupling

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}\lambda(\phi)F_{\mu\nu}F^{\mu\nu}$$

[Ratra 1992]

- Coupling to gravity

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha_n}{4}R^n F_{\mu\nu}F^{\mu\nu}\right]$$

same as above, when Φ is the inflaton

- Axial coupling

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\lambda(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

strong constraints from NG and backreaction

- Mass term

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m^2 A_\mu A^\mu\right]$$

  - Negative mass-squared needed for generating enough magnetic fields
  - Generating neg. mass-squared from Higgs mech. ⇒ one needs ghost scalar field with neg. kinetic energy

[Dvali et. al. 2007, Himmetoglu, Contaldi, Peloso 2009]

Magnetogenesis in Ratra-type models

- In Coulomb gauge we have ($A_0 = 0, \quad \partial_i A^i = 0$)

$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\,\lambda(\phi)F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\int d^3x\, d\tau\lambda(\phi)\left(A_i'^2 - \frac{1}{2a^2}(\partial_i A_j - \partial_j A_i)^2\right)$$

- With the magnetic field given by

$$B_i(\tau, \mathbf{x}) = \frac{1}{a}\epsilon_{ijk}\partial_j A_k(\tau, \mathbf{x})$$

- Defining the magnetic power spectrum

$$\left\langle B_i(\tau, \mathbf{k})B^i(\tau, \mathbf{k}')\right\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_B(k)$$

it can be computed from

$$P_B(k) = 2\frac{k^2}{a^4}|A_k(\tau)|^2$$

- Define pump field $S^2(\eta) = \lambda(\phi(\eta))$

- and a canonically normalized vector field $v_i = S(\tau)A_i$

- Such that the quadratic action takes the simple form

$$S_v = \frac{1}{2}\int d\tau d^3x \left[v_i'^2 - (\partial_j v_i)^2 + \frac{S''}{S}v_i^2\right]$$

- The EOM for the mode function $v_k = S(\tau)A_k$ is

$$v_k'' + \left(k^2 - \frac{S''}{S}\right)v_k = 0$$

- With $\lambda(\phi(\tau)) = \lambda_I(\tau/\tau_I)^{-2n}$ the solution normalized to Bunch-Davis vacuum is

$$v_k(\tau) = \frac{\sqrt{\pi}}{2}e^{i\pi(1+n)/2}\sqrt{-\tau}H^{(1)}_{\frac{1}{2}+n}(-k\tau)$$

- which leads to

$$P_B(k) = \frac{1}{\lambda_I}\frac{\pi}{2}\frac{H^4}{k^3}\left(\frac{\tau}{\tau_I}\right)^{2n}(-k\tau)^5 H^{(1)}_{\frac{1}{2}+n}(-k\tau)H^{(2)}_{\frac{1}{2}+n}(-k\tau)$$

➡ For $n > -1/2$ the spectral index of the magnetic power spectrum is

$$n_B = (4 - 2n)$$

- For a scale invariant spectrum, $n=2$, back-reaction remains small
- In this case, with $H \simeq 10^{14}$ GeV, a magnetic field strength of order nano-Gauss an be achieved on Mpc scales

Strong coupling problem

- Adding the EM coupling to the SM fermions

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}\lambda(\phi)F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi\right]$$

- The physical electric coupling is

$$e_{phys} = e/\sqrt{\lambda(\phi)}$$

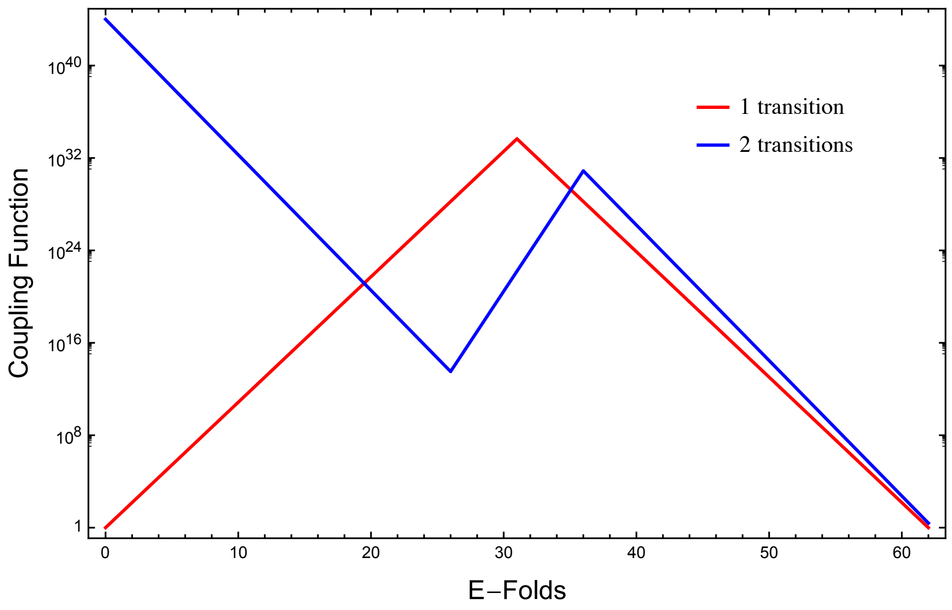
- Since $\sqrt{\lambda} \propto a^n$ then for $n > 0$ the electric coupling decreases by a lot during inflation, and must have been very large at the beginning

➡ QFT out of control initially [Demozzi, Mukhanov, Rubinstein 2009]

- Solutions??? Speculations [Bonvin, Caprini, Durrer 2011, Caldwell, Motta 2012, Bartolo, Matarrese, Peloso, Ricciardone 2012, and more...]

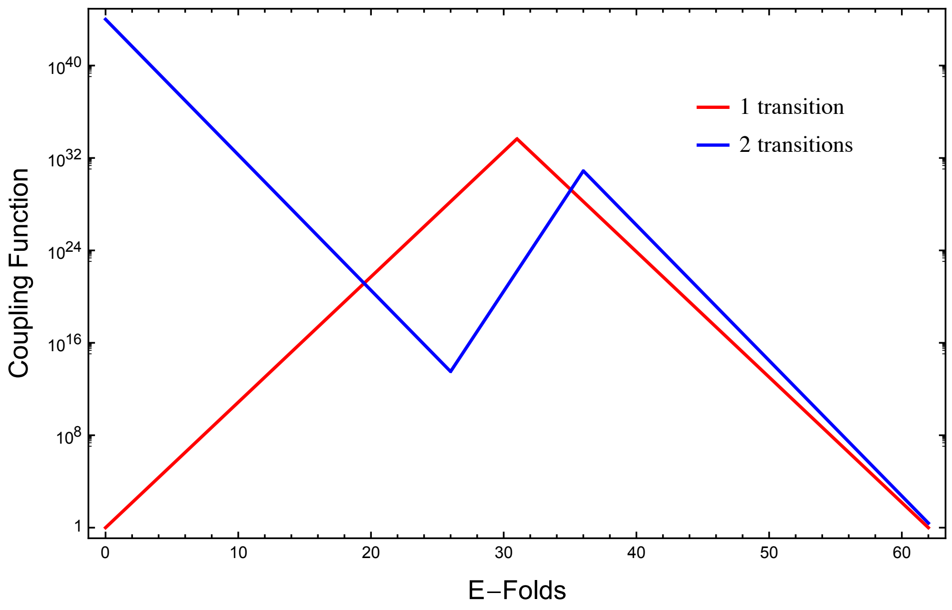
➡ More work required! [Ferreira, Jain, MSS 2013]

The Sawtooth Model



The Sawtooth Model

- Relax assumption of monotonic coupling function



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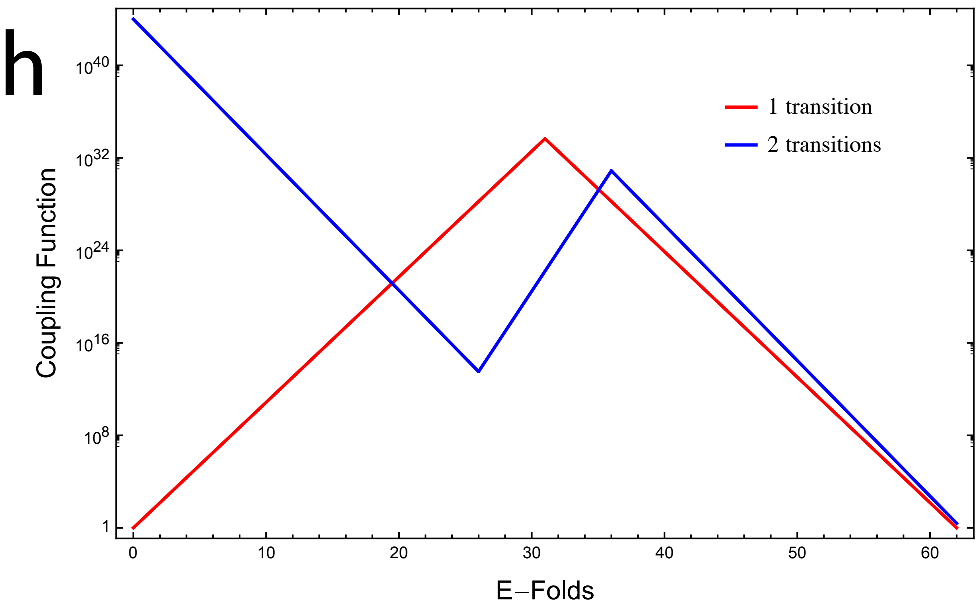
➡ Patch together piecewise sections with steeper slope for more enhancement



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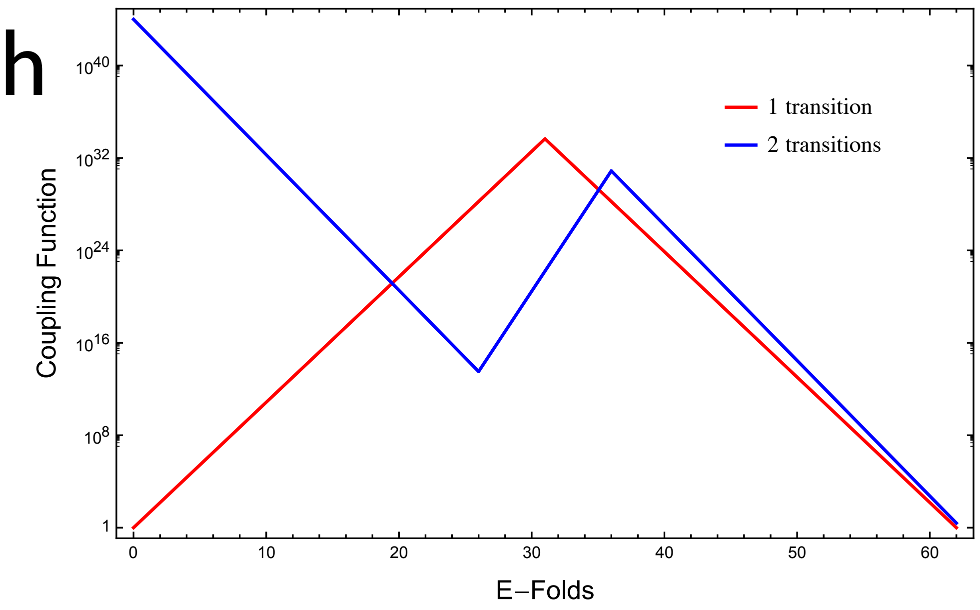

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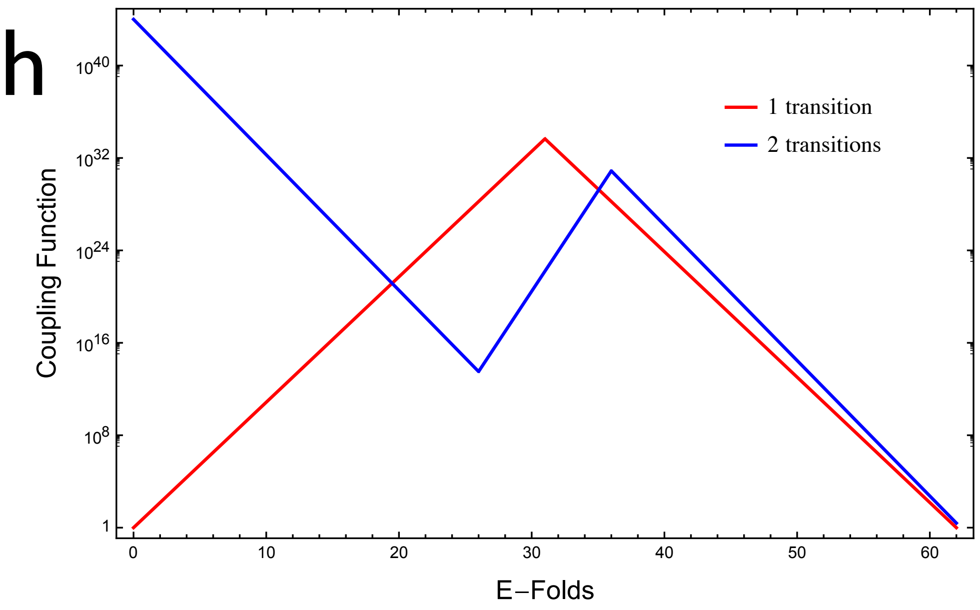

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➡ One might be worried that adding more steep sections, the energy density of the steep sections will add up and prohibit this!

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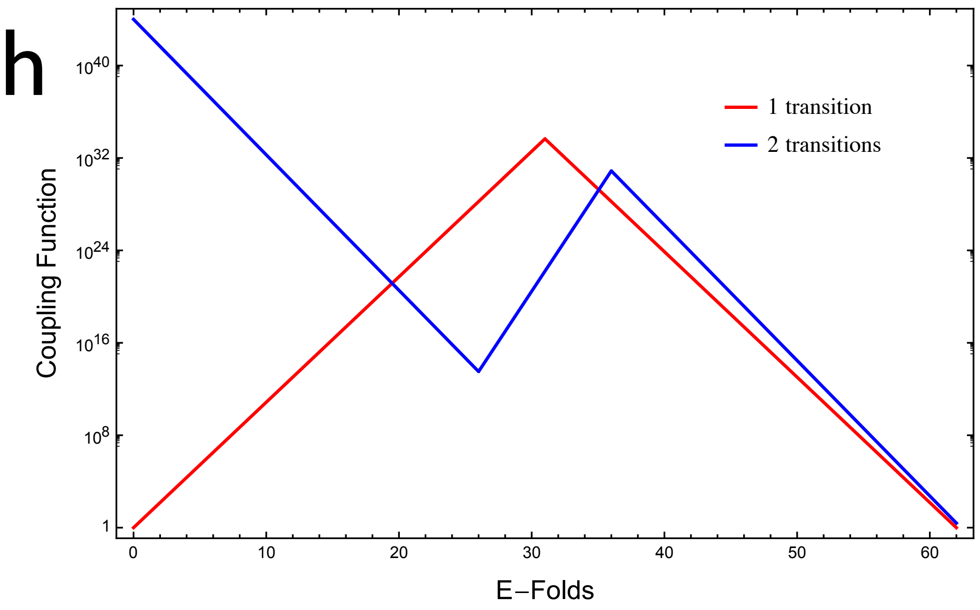
- This turns out not to be true!



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[Ferreira, Jain, MSS 2013]

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- By using the appropriate matching conditions, the dominant solution before the transition matches to the decaying solution after the transition.

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➡ The loss in the electric field spectrum avoids the back reaction problem

➡ The loss in the magnetic spectrum however also implies a too small value of the magnetic field strength at the end of inflation

[Ferreira, Jain, MSS 2013]

Low scale inflationary magnetogenesis

- Even in the monotonic case without strong coupling and back-reaction, the magnetic fields are very strong at the end of inflation

$$\frac{d\rho_B}{d\log k} \approx \frac{\mathcal{F}(n)}{2\pi^2} H^4 (-k\eta)^{4+2n}$$

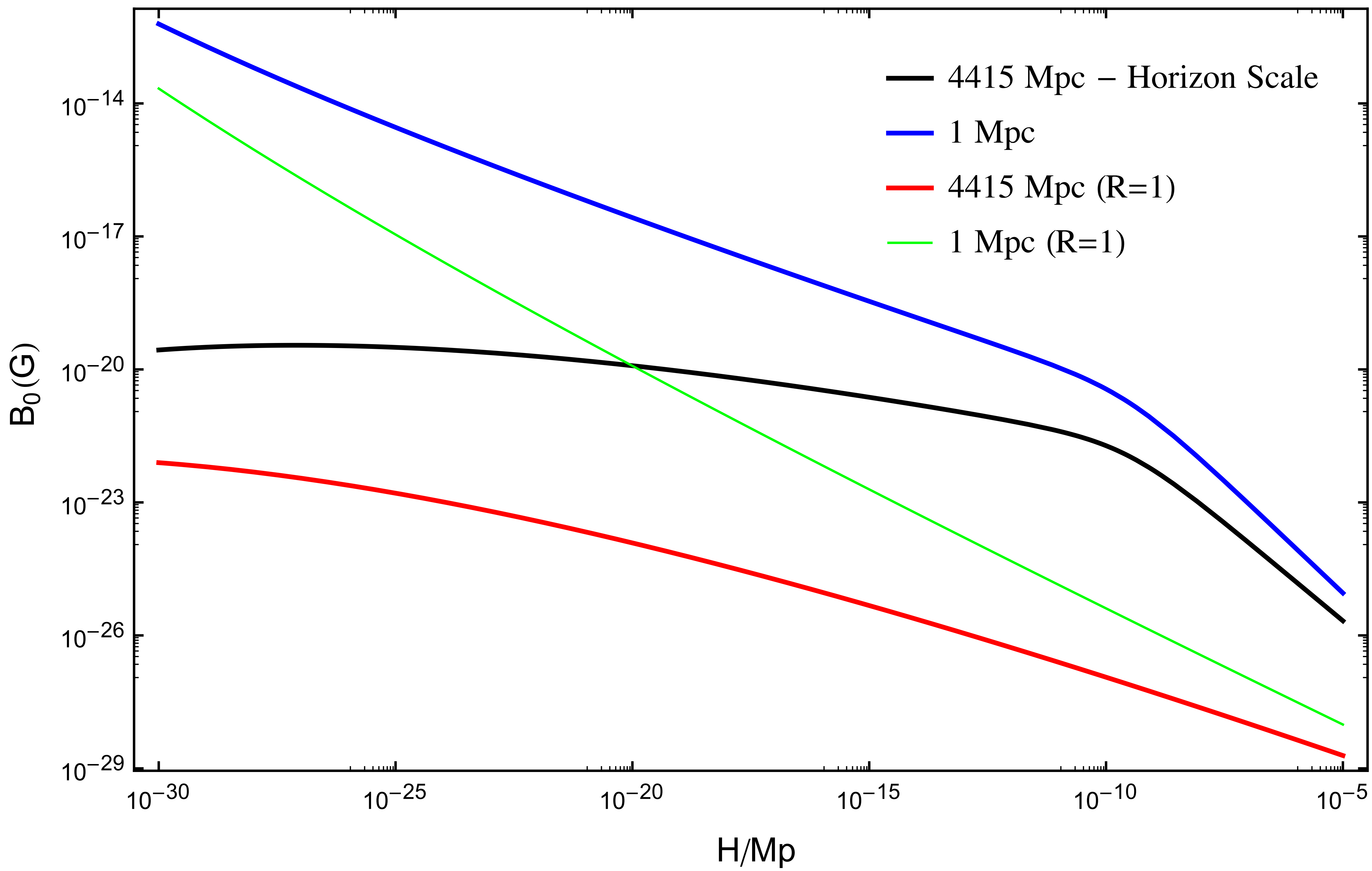
$$1\mathrm{G} \approx 10^{-20}\mathrm{GeV}^2$$

- It is the subsequent post-inflationary evolution where $B^2 \propto 1/a^2$ which dilutes the magnetic field

- Minimize post-inflationary dilution by assuming TeV scale inflation

➡ femto-Gauss magnetic fields on Mpc scale today

[Ferreira, Jain, MSS 2013]



4415 Mpc − Horizon Scale
1 Mpc
4415 Mpc (R=1)
1 Mpc (R=1)
B_0(G)
10^{-14}
10^{-17}
10^{-20}
10^{-23}
10^{-26}
10^{-29}
10^{-30}
10^{-25}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
H/Mp

Gauge field production during inflation

- The issues with inflationary magnetogenesis might prompt us to ask more generally how we can probe more precisely the effect of gauge fields produced during inflation
- The gauge field act as an isocurvature field and induces a curvature perturbation $\zeta_B \propto B^2$ since

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta P_{nad}$$

with

$$\delta P_{nad} = \delta p_B - \frac{\dot{p}}{\dot{\rho}}\delta\rho_B$$

- Scale invariance then implies

$$P_{\zeta_B} \sim 10^{-10} \left(\frac{B_{\rm today}}{10^{-9}{\rm G}}\right)^2 \left(\frac{0.01}{\epsilon}\right)^2 \left(\frac{N_{\rm CMB}}{N_0}\right)^2 (N_0 - N_{\rm CMB})$$

- So $P_{\zeta_B} \lesssim P_\zeta$ implies

$$B_{\rm today} \lesssim 10^{-9}\ {\rm G}$$

Non-Gaussian features of gauge field production during inflation

- Non-Gaussian contributions of the form

$$\langle \zeta_B \zeta_B \zeta_B \rangle \qquad \langle \zeta \zeta_B \zeta_B \rangle \qquad \langle \zeta \zeta \zeta_B \rangle$$

turns out to provide even stronger constraints

- Since $\zeta_B \propto B^2$, they are derived from more fundamental correlations of the form

$$\langle B^2 B^2 B^2 \rangle \qquad \langle \zeta \zeta B^2 \rangle \qquad \langle \zeta B^2 B^2 \rangle$$

Non-Gaussian features of gauge field production during inflation

- Non-Gaussian contributions of the form

So far ignored

$$\langle \zeta_B \zeta_B \zeta_B \rangle \qquad \langle \zeta \zeta_B \zeta_B \rangle \qquad \langle \zeta \zeta \zeta_B \rangle$$

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[Nurmi & MSS, to appear]

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$$\langle B^2 B^2 B^2 \rangle \qquad \langle \zeta \zeta B^2 \rangle \qquad \langle \zeta B^2 B^2 \rangle$$

Non-Gaussianity from cross-correlations

- Consider

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}\lambda(\phi)F_{\mu\nu}F^{\mu\nu}$$

w. direct coupling of magnetic field with the inflaton

➡NG correlation of magnetic field with inflaton field

$$\langle\zeta(k_1)\mathbf{B}(k_2)\cdot\mathbf{B}(k_3)\rangle \qquad \zeta = \frac{H}{\dot{\phi}}\delta\phi$$

[Kamionkowski, Caldwell, Motta (2012), Jain, MSS (2012), Biagetti, Kehagias, Morgante, Perrier, Riotto (2013)]

(Ordinary) Non-Gaussianity

- To leading order, the perturbations are encoded in the two-point function

$$\langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k'}) P_\zeta(k)$$

- A non vanishing three point function

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$$

is a signal of non-Gaussianity

- Introduce dimensionless f_{NL} :

$$f_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle / P_\zeta(k_1) P_\zeta(k_2) \ + \ perm.$$

as a measure of non-Gaussinity

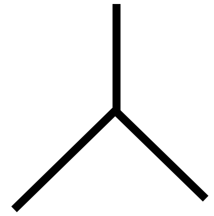
- Similarly

$$\tau_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle / P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_{14}) \ + \ perm.$$

Non-Gaussianity: Single field slow-roll

- Perturbations conserved on super-horizon scales: NG is computed at hoizon crossing

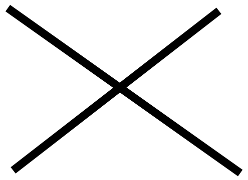
- Bispectrum from 3-point interaction

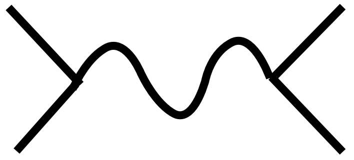


$$f_{NL} \approx \epsilon$$

[Maldacena '02,]

- Trispectrum from connected 4-point interaction and graviton exchange





$$\tau_{NL} \approx \epsilon$$

[Seery, Lidsey, Sloth '06,
Seery, Sloth, Vernizzi '08]

Magnetic non-linearity parameter: b_{NL}

- Let's parametrize $\langle\zeta(k_1)\mathbf{B}(k_2)\cdot\mathbf{B}(k_3)\rangle$ in a similar way

➡ Introduce new magnetic non-linearity parameter: b_{NL}

- Define the cross-correlation bispectrum

$$\langle\zeta(\mathbf{k}_1)\mathbf{B}(\mathbf{k}_2)\cdot\mathbf{B}(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- We then define

$$B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv b_{NL}P_\zeta(k_1)P_B(k_2)$$

$$\langle\zeta(\mathbf{k})\zeta(\mathbf{k}')\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_\zeta(k)$$

$$\langle\mathbf{B}(\mathbf{k})\cdot\mathbf{B}(\mathbf{k}')\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')P_B(k)$$

Local b_{NL}

- In the case where b_{NL} is momentum independent, it takes the *local* form:

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2} b_{NL}^{local} \zeta^{(G)} \mathbf{B}^{(G)}$$

- Compare with *local* f_{NL}, given by

$$\zeta = \zeta^{(G)} + \frac{3}{5} f_{NL}^{local} \left(\zeta^{(G)} \right)^2$$

Two interesting shapes

1. The squeezed limit $k_1 \ll k_2, k_3 = k$

- We obtain a new *magnetic consistency relation*

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(\mathbf{k_3}) \rangle = (n_B - 4)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k)$$

with $b_{NL}^{local} = (n_B - 4)$

- Compare with *Maldacena consistency relation*

$$\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = -(n_s - 1)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_\zeta(k)$$

with $f_{NL}^{local} = -(n_s - 1)$

Two interesting shapes

2. The flattened shape $k_1/2 = k_2 = k_3$

- This is the shape where b_{NL} turns out to be maximized with

$$|b_{NL}| \sim \mathcal{O}(10^3)$$

The magnetic consistency relation

- In terms of the vector field, we have

$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\,\lambda(\phi)F_{\mu\nu}F^{\mu\nu}$$
$$= \frac{1}{2}\int d^3x\,d\tau\,\lambda(\phi)\left(A_i'^2 - \frac{1}{2}(\partial_i A_j - \partial_j A_i)^2\right)$$

- where the magnetic field power spectrum is

$$P_B(k) = \frac{k^2}{a^4}\left\langle \mathbf{A}(\tau,\mathbf{k})\cdot\mathbf{A}(\tau,-\mathbf{k})\right\rangle$$

- Consider $\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$ in the squeezed limit $k_1 \ll k_2, k_3 = k$

- The effect of the long wavelength mode is to shift the background of the short wavelength modes

$$\lim_{k_1\to 0}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$$
$$= \left\langle\zeta(\tau_I,\mathbf{k}_1)\left\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\right\rangle_B\right\rangle$$

- Since the vector field only feels the background through the coupling λ, all the effect of the long wavelength mode is captured by

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta\ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots$$

- Define pump field $S^2 = \lambda_0$

- and linear Gaussian canonical vector field

$$v_i = S(\tau) A_i^{(G)}$$

- Such that the quadratic action takes the simple form

$$S_v = \frac{1}{2} \int d\tau d^3x \left[v_i'^2 - (\partial_j v_i)^2 + \frac{S''}{S} v_i^2 \right]$$

- Since all the effect of the long wavelength mode is in

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta\ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots$$

➡ One finds

$$\langle A_i(\tau,\mathbf{x}_2)A_j(\tau,\mathbf{x}_3)\rangle_B = \left\langle \frac{1}{\lambda_B}v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\right\rangle$$

$$\simeq \frac{1}{\lambda_0}\langle v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\rangle \quad - \frac{1}{\lambda_0^2}\frac{d\lambda}{d\ln a}\zeta_B\,\langle v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\rangle$$

- Using

$$d\lambda/d\ln a \;=\; \dot{\lambda}/H$$

- and

$$\lim_{k_1\to 0}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3\rangle$$
$$= \left\langle\zeta(\tau_I,\mathbf{k}_1)\,\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3\rangle_B\right\rangle$$

➡ One finds

$$\lim_{k_1\to 0}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$$
$$\simeq -\frac{1}{H}\frac{\dot{\lambda}}{\lambda}\,\langle\zeta(\tau_I,\mathbf{k}_1)\zeta(\tau_I,-\mathbf{k}_1)\rangle_0\,\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle_0$$

Consistency relation

- Expressing it in terms of the magnetic fields

➡Magnetic consistency relation

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle$$
$$= -\frac{1}{H} \frac{\dot{\lambda}}{\lambda} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

- With $\lambda(\phi(\tau)) = \lambda_I (\tau/\tau_I)^{-2n}$

➡One has $b_{NL} = (n_B - 4)$

The full *in-in* QFT calculation

- Perturbing the metric in the ADM formalism

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- with the metric ansatz $h_{ij} = a^2 e^{2\zeta} \left[e^{\gamma}\right]_{ij}$
- and solving for the lapse and shift

$$N = 1 + \frac{1}{H}\dot{\zeta}$$
$$N_i = \partial_i \left(-\frac{1}{H}\zeta + a^2 \epsilon \, \partial^{-2} \dot{\zeta} \right)$$

- It is easy to see that the interaction Hamiltonian

$$H_{\zeta AA} = -\frac{1}{2} \int d^3x \, a^3 \, T^{\mu\nu} \delta g_{\mu\nu}$$

- becomes

$$H_{\zeta AA} = -\frac{1}{2} \int d^3x a^3 \left(\frac{1}{H} \dot{\zeta} T^{00} - \partial_i \left(-\frac{1}{H}\zeta + a^2 \epsilon \partial^{-2} \dot{\zeta} \right) T^{0i} - a^2 \zeta T^{ii} \right)$$

[Caldwell & Motta, 2012]

- However this form of the Hamiltonian is a total derivative to leading order

- After a few partial integration, the leading order term in the slow roll expansion cancels out, and one finds

$$H_{\zeta AA} = -\frac{1}{2}\int d^3x \left(a\dot{\lambda}\frac{1}{H}\zeta\left(\dot{A}_i\dot{A}_i - \frac{1}{2a^2}(\partial_i A_j - \partial_j A_i)^2\right) - \partial_t\left(a^3\frac{1}{H}\zeta T^{00}\right)\right)$$

- This now agrees with what one would obtain in the uniform curvature gauge by expanding the coupling as a function of the inflaton fluctuations

$$\dot{\lambda}\zeta = \frac{d\lambda}{d\phi}\frac{d\phi}{dt}\zeta = -\partial_\phi\lambda H\delta\phi$$

- Using the *in-in* formalism for evaluating the expectation value at some time τ_I

$$\langle\Omega|\,\mathcal{O}(\tau_I)\,|\Omega\rangle = \langle 0|\,\bar{\mathrm{T}}\left(e^{i\int_{-\infty}^{\tau_I} d\tau H_I}\right)\mathcal{O}(\tau_I)\mathrm{T}\left(e^{-i\int_{-\infty}^{\tau_I} d\tau H_I}\right)|0\rangle$$

- One obtains

$$\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle = \frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta^{(0)}_{k_1}(\tau_I)|^2|A^{(0)}_{k_2}(\tau_I)||A^{(0)}_{k_3}(\tau_I)|$$
$$\times\left[\left(\mathbf{k}_2\cdot\mathbf{k}_3+\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^3}{k_2^2k_3^2}\right)k_2k_3\tilde{\mathcal{I}}^{(1)}_n+2(\mathbf{k}_2\cdot\mathbf{k}_3)^2\tilde{\mathcal{I}}^{(2)}_n\right].$$

[Jain, MSS 2012]

$$\tilde{\mathcal{I}}^{(1)}_2 = \frac{1}{(k_2k_3)^{3/2}k_t^2}\times\left[-k_1^3-2k_1^2(k_2+k_3)-2k_1(k_2^2+k_2k_3+k_3^2)-(k_2+k_3)(k_2^2+k_2k_3+k_3^2)\right]$$

$$\tilde{\mathcal{I}}^{(2)}_2 = \frac{1}{(k_2k_3)^{5/2}k_t^2}\times\left[(k_1+k_2)^2(-3k_1^3-3k_1^2k_2-k_2^3)+(k_1+k_2)(-9k_1^3-6k_1^2k_2-2k_2^3)k_3\right.$$
$$+(-9k_1^3-6k_1^2k_2-2k_1k_2^2-2k_2^3)k_3^2$$
$$\left.-2(2k_1^2+k_1k_2+k_2^2)k_3^3-2(k_1+k_2)k_3^4-k_3^5+3k_1^3k_t^2(\gamma+\ln(-k_t\tau_I))\right]$$

$$k_t = k_1+k_2+k_3$$

The flattened shape

- The integral contains a growing log

➡ The effect maximal, when the coefficient of the log is maximal

➡ The correlation is maximal in flattened shape

$$k_1 = 2k_2 = 2k_3$$

- In this case

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle \simeq -96 \ln(-k_t \tau_I) P_\zeta(k_1) P_B(k_2)$$

- For the largest scales exiting the horizon about *60* e-folds before the end of inflation, so $\ln(-k_t \tau_I) \sim 60$

➡ $$\left| b_{NL}^{flat} \right| \sim 5760 \quad !$$

[Jain, MSS 2012]

The squeezed limit

- In the squeezed limit, $k_1 \ll k_2, k_3 = k$, the integrals simplify significantly, and we have

$$\tilde{\mathcal{I}}_n^{(1)} = -\pi \int^{\tau_I} d\tau J_{n-1/2}(-k\tau_I) Y_{n-1/2}(-k\tau_I) \qquad \tilde{\mathcal{I}}_n^{(2)} = \tilde{\mathcal{I}}_{n+1}^{(1)}$$

- For integer values of n, one can show that

$$\tilde{\mathcal{I}}_n^{(1)} = -(n - 1/2)/k^2$$

- which gives

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

- which gives a local type non-linearity parameter

$$b_{NL}^{local} = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I}$$

consistent with the magnetic consistency relation [Jain, MSS 2012]

Induced non-Gaussianity

- The large b_{NL} in the flattened shape implies that $\langle\zeta\zeta_B\zeta_B\rangle$ becomes large in this shape

[Nurmi & MSS, to appear]

➡This provides much stronger bounds than the previous known result $\langle\zeta_B\zeta_B\zeta_B\rangle$

[Barnaby, Namba & Peloso, 2012; Lyth & Karciauskas, 2013; Fujita & Yokoyama,2013]

Conclusions

- If the magnetic consistency relation is violated it will rule out an important class of models for magnetogenesis
- The consistency relation is an important theoretical tool for consistency check of calculations
- The new b_{NL} parameter can be very large in the flattened limit and has interesting phenomenological implications
- We have found new strong bounds on the amount of inflation in the λF^2 models

Inflationary Magnetogenesis
&
Non-Gaussianity

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Based on arXiv:1305.7151,1210.3461,1207.4187 w. Ferreira, Jain
and work to appear w. Nurmi