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⟨ Global Supersymmetry and Inflation ⟩

Reference : "P-term, D-term and F-term Inflation"
by Kallosh and Linde
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Inflation: We need a model where the potential is very flat in a finite interval for slow-roll to be valid, and then leads to a rapid end of inflation while reheating the universe.

Linde, '91

* Hybrid inflation scenario

Inflation is implemented by two fields: one is a slowly varying field during inflation, and the other provides the vacuum energy density which drives inflation.

* Supersymmetry

- SUSY may lead to more natural models of inflation because flat directions are natural.
  ↳ lifted by SUSY breaking, or non-perturbative effects.

- N=1 SUSY $\begin{cases} \text{chiral supermultiplet } (\Phi) : \phi, \psi, F \\ \text{vector} \quad '' \quad (V) : A_\mu, \lambda, D \end{cases}$

F and D are auxiliary fields (not physical).

SUSY action is described by K (Kähler potential) and W (Superpotential)
(dim=2) (dim=3)

- SUSY requires W to be a holomorphic function of chiral superfields.
- SUSY protects W against radiative corrections,
- $U(1)_R$ symmetry: the action does not depend on the phase of W,

Scalar potential reads $V = |F|^2 + D^2$,

where e.o.m leads to $F = \partial_\phi W$, $D = \sum_i q_i |\phi_i|^2$,

⟨Hybrid inflation in SUSY⟩

1. F-term inflation

(Copeland, Liddle, Lyth, Stewart, Wands, '94
Dvali, Shafi, Schaefer, '94

$$\boxed{W = \lambda S(\bar{\Phi}\phi - M^2).}$$

- S is a gauge singlet, and $\phi + \bar{\Phi}$ are vector-like under some gauge group G.
- $U(1)_R$ forbids S^n terms $(n \geq 2)$: W and S carry the same R-charge, while $\bar{\Phi}\phi$ is neutral.

* Scalar potential is given by

$$V = \lambda^2 |S|^2 (|\phi|^2 + |\bar{\Phi}|^2) + \lambda^2 |\bar{\Phi}\phi - M^2|^2 + \underbrace{\text{D-terms}}_{V_D \sim (|\bar{\Phi}|^2 - |\phi|^2)^2}.$$

There is a SUSY minimum at $|\bar{\Phi}| = |\phi| = M$ and $S = 0$.

* If S is located at $|S| \gg M$ in the early universe, ϕ and $\bar{\Phi}$ are fixed at the origin because they obtain large SUSY mass $= \lambda |S|$.

$$V = \lambda^2 M^4.$$

S is exactly flat at tree-level, but the potential is radiatively generated.

$$\Delta V = \frac{\lambda^4 M^4}{8\pi^2} \ln\left(\frac{\lambda^2 |S|^2}{\Lambda^2}\right),$$

because SUSY is broken by $F^S = \lambda M^2$, which provides SUSY breaking soft masses to the scalar components of ϕ and $\bar{\Phi}$.

Hence, $V_{eff} = \lambda^2 M^4 \left[1 + \frac{\lambda^2}{8\pi^2} \ln\left(\frac{\lambda^2 |S|^2}{\Lambda^2}\right) \right]$ at $\bar{\Phi} = \phi = 0$ and $|S| \gg M$.

$V_{eff} \to$ hybrid inflation:

- S slowly rolls down along the valley ($\phi = \bar{\Phi} = 0$, $S \neq 0$), and inflation is driven by the vacuum energy density provided by ϕ an $\bar{\Phi}$ ($V = \lambda^2 M^4$).
- Inflation ends when slow-roll conditions are violated.

  (when S reaches $S = M$, ϕ and $\bar{\Phi}$ become tachyonic and develop a large VEV.)
- Then, damped oscillation about the SUSY minimum occurs. (reheating).

Binetruy, Dvali. '96 Halyo, '96

2. D-term inflation

$\boxed{W = \lambda S \bar{\Phi} \phi}$ and $U(1)$ with a Fayet-Illioupous term.

Scalar potential reads

$$V = \lambda^2 |S|^2 (|\phi|^2 + |\bar{\phi}|^2) + \lambda^2 |\bar{\Phi}\phi|^2 + \frac{g^2}{2}\Big(|\phi|^2 - |\bar{\phi}|^2 + \underbrace{\xi}_{\text{FI term}}\Big)^2$$

* There is a SUSY minimum at $S = \phi = 0$ and $|\bar{\Phi}| = \sqrt{\xi}$.

* S is flat along $\phi = \bar{\Phi} = 0$ (at tree-level) : $V = \frac{g^2}{2}\xi^2$

$\Rightarrow$ inflation is driven by $V = \frac{g^2}{2}\xi^2$.

radiative potential $\Delta V(S)$ is generated because the D-term provides SUSY breaking masses to ϕ and $\bar{\Phi}$.

<Potential difficulties of hybrid inflation>

* Supergravity corrections can spoil the slow-roll situation:
Planck-suppressed operators, e.g. $\frac{|S|^4}{M_{pl}^2}$ in the Kähler potential, generate an inflaton mass in F-term inflation because $F^S = \lambda M^2$.

Supergravity corrections are controllable in D-term inflation ($F^S = 0$, $D \neq 0$).

* In supergravity, FI term can arise only from gauged R-symmetry, or as a result of the Green-Schwarz (GS) anomaly cancellation.
The GS mechanism gives rise to moduli-dependent FI term, and thus it is quite important to understand how the string moduli are stabilized.

→ 17. Dec. talk by Wieck. Clemens.

* The reheating temperature should be sufficiently low not to overproduce gravitinos (via scattering process between particles in thermal bath).

⟨Constraints on hybrid inflation⟩

1. Density perturbation

To account for the density fluctuations in the cosmic microwave background, we need

$$\frac{V^{\frac{3}{2}}}{V'} \sim 5\times 10^{-4} \quad \text{(for slow-roll models)}$$

2. Planck results.

- spectral index : $n_s \approx 1 + 2\frac{V''}{V} - 3\left(\frac{V'}{V}\right)^2$.
- tensor-to-scalar ratio : $r = \frac{A_t}{A_s} \approx 8\cdot\left(\frac{V'}{V}\right)^2$,

$n_s = 0.9603 \pm 0.0073$, $r < 0.11$ at 95% C.L.

→ Inflation models with $V'' > 0$ do not fit well the observation.

3. Cosmic strings.

After inflation, ϕ and $\bar{\phi}$ obtain a large VEV, and spontaneous symmetry breaking takes place.

→ Cosmic strings are formed, and lead to perturbations of the metric proportional to the string tension μ.

The density perturbations by cosmic strings are different from the inflationary adiabatic perturbations, and are constrained by CMB and matter power spectrum.

⇝ $G\mu \lesssim 10^{-7}$. $\left(G = \frac{1}{8\pi M_{pl}^2}\right)$.

Kallosh, Linde, 03

⟨D-term inflation⟩

D-term inflation naturally appears in the context of the D3/D7 model of brane inflation. It is based on N=2 SUSY gauge theory where global conformal symmetry SU(2, 2|2) is broken down to N=2 SUSY by adding N=2 FI term, $\vec{\xi}$.

$\underbrace{\vec{\xi} = (\xi_1, \xi_2, \xi_3)}_{\text{determined by the choice of fluxes on the branes.}}$: triplet of SU(2).

The model has one vector multiplet and one charged hypermultiplet.
In N=1 SUSY notations, the potential is written

$$V = 2g^2\left[|S|^2(|\Phi_+|^2 + |\Phi_-|^2) + \left|\Phi_+\Phi_- - \frac{\xi_+}{2}\right|^2\right] + \frac{g^2}{2}\left(|\Phi_+|^2 - |\Phi_-|^2 - \xi_3\right)^2,$$

with $\xi_+ = \xi_1 + i\xi_2$, g = gauge coupling.
Here S comes from the abelian N=2 gauge multiplet, while $\Phi_\pm$ are from the hypermultiplet.

Thus it corresponds to N=1 model with

$$W = \underbrace{\sqrt{2}g}_{\text{due to N=2 SUSY.}}\, S\left(\Phi_+\Phi_- - \frac{\xi_+}{2}\right), \quad \text{and} \quad D = |\Phi_+|^2 - |\Phi_-|^2 - \xi_3,$$

* The potential has a SUSY minimum at $S=0$ and $|\Phi_\pm|^2 = \frac{\xi \pm \xi_3}{2}$ with $\xi = \sqrt{\xi_i^2}$,
At the minimum, $V=0$.

* de-Sitter valley

Along $\Phi_+ = \Phi_- = 0$ and $|S| \gg S_c \equiv \sqrt{\xi/2}$, the potential is given by

$$V = \frac{1}{2} g^2 \xi^2,$$

and S is a flat direction at tree-level.

SUSY is broken, and leads to the mass splitting $M_\pm^2 = 2g^2(|S|^2 \pm \xi/2)$ for $\Phi_\pm$,
Consequently, the potential of S is radiatively generated,

$$\Delta V = \frac{g^4 \xi^2}{16\pi^2} \ln\left(\frac{|S|^2}{S_c^2}\right), \quad \text{at one-loop.}$$

Let us now couple D-term model to N=1 SUGRA to examine the effect of supergravity correction during inflation.
This can be realized by making one SUSY local.

Taking the minimal Kähler potential, $K = |S|^2 + |\Phi_+|^2 + |\Phi_-|^2$, one obtains.

$$V_{eff} = \frac{g^2 \xi^2}{2} \left[1 + \frac{g^2}{8\pi^2} \ln\left(\frac{|S|^2}{S_c^2}\right) + \frac{f}{2} \frac{|S|^4}{M_{pl}^4} + O(|S|^6) \right],$$

at $\Phi_+ = \Phi_- = 0$ and $|S| > S_c$.

Here $f = \frac{\xi_1^2 + \xi_2^2}{\xi^2}$ lies between 0 and 1.

$\Rightarrow$ The inflation trajectory takes place at $\Phi_+ = \Phi_- = 0$.

- P-term model unifies the F-term and D-term N=1 SUSY models, under the condition that the Yukawa and gauge couplings are related as

$$\lambda^2 = 2g^2.$$

(1) D-term inflation.

P-term model with $\xi_1 = \xi_2 = 0$ coincides with the case of D-term inflation with

$$W = \lambda S \Phi_+ \Phi_- \quad \text{and} \quad D = |\Phi_+|^2 - |\Phi_-|^2 - \xi_3,$$

under the condition. $\lambda^2 = 2g^2$.

In this case, $f = 0$.

(2) F-term inflation

The potential for F-term inflation is recovered taking $\xi_1 = 2M^2$ and $\xi_2 = \xi_3 = 0$, for which

$$W = \lambda S (\Phi'_+ \Phi'_- - M^2) \quad \text{and} \quad D = |\Phi'_+|^2 - |\Phi'_-|^2,$$

under constraint $\lambda^2 = 2g^2$.

F-term inflation has $f = 1$.

Note that Supergravity makes a significant difference between F and D-term models. : f-dependence disappears in the limit $M_{pl} \to \infty$.

In fact, in the absence of SUGRA corrections, the F-term model and D-term model with $\lambda^2 = 2g^2$ are equivalent.

They are related by a change of variables:
$$\begin{cases} \Phi'_+ = \frac{1}{\sqrt{2}}(\Phi_+ + \Phi_-^*), \\ \Phi'_- = \frac{1}{\sqrt{2}}(\Phi_+^* - \Phi_-). \end{cases}$$

<Applications in Cosmology of N=2 SUSY with D-term inflation>

Let us use the canonically quantized fields, $\phi_\pm = \sqrt{2}\,\Phi_\pm$ and $s = \sqrt{2}\,S$.
Then, $|\phi_\pm|^2 = \xi \pm \xi_3$ at the SUSY minimum of the potential,
and the bifurcation point reads $s_c = \sqrt{\xi}$.

The potential along $\phi_\pm = 0$ and $s > s_c$ is written,

$$V = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\left(\frac{|s|^2}{s_c^2}\right) + f\frac{|s|^2}{8} + \cdots\right),$$

(1) D-term case : $f = 0$.

From the Friedmann eq. $H^2 = \frac{V}{3}$, one has $H = \frac{g\xi}{\sqrt{6}}$. $\Rightarrow$ $a(t) = a(0)\cdot\exp\left(\frac{g\xi t}{\sqrt{6}}\right)$ (the scale factor)

One can also find the value of s s.t. the universe inflates e^N times when it rolls from s_N to the bifurcation point :

$$s_N^2 = \xi + \frac{g^2 N}{2\pi^2}.$$

* Constraint from density perturbation

Density perturbation on the scale of the present cosmological horizon is produced at $s \sim s_N$ with $N \sim 60$:

$$\frac{V^{\frac{3}{2}}}{V'} = \frac{2\sqrt{2}\pi^2\xi}{g}\,s_N \underset{\text{we need}}{\sim} 5\times 10^{-4}, \quad \text{where } N \sim 60.$$

Then, there are two different regimes depending on g.

① $\frac{g^2 N}{2\pi^2} \gg \xi$: we need $\xi \simeq 10^{-5}$ and $g \gtrsim 2\times 10^{-3}$. ($\xi$ closed to M_{GUT}^2),
In this case, the spectral index is given by $n_s = 1 - \frac{1}{N} \sim 0.98$.

② $\frac{g^2 N}{2\pi^2} \ll \xi$: we need $\xi \sim 10^{-3} g^{2/3}$ and $g \ll 2\times 10^{-3}$,
In this regime, one obtains an exactly flat spectrum of density perturbations
$n_s = 1$.

* Constraint from cosmic strings

The string tension is given by $\mu = 2\pi\xi$, and $G\mu \lesssim 10^{-7}$ requires

$$\xi \lesssim 4\times 10^{-7}.$$

This cannot be satisfied in the regime of $\frac{g^2 N}{2\pi^2} \gg \xi$.

To resolve the problem, one may modify superpotential so that spontaneous $U(1)$ breaking occurs at the stage of inflation.

Then cosmic strings are diluted by inflation.

e.g. shifted hybrid inflation

$$W = \lambda S\left(\Phi_+\Phi_- - M^2 - \frac{1}{M_S^2}(\Phi_+\Phi_-)^2\right).$$

$\rightsquigarrow$ the scalar potential has a shifted inflationary valley for $\frac{1}{4} > \frac{M^2}{M_S^2} > \frac{1}{6}$, along $S \neq 0$ and $|\Phi_\pm| = M_S/\sqrt{2}$,

(2) F-term ($f=1$), and a general case with $f \neq 0$.

The additional term (S^4) does not change the amplitude of string perturbations, and the value of V during inflation,

However, it increases V', making inflationary perturbations somewhat smaller,

Let us see the behavior of the potential at $S \sim S_N$.

For $g \gtrsim 2\times 10^{-3}$, which is the region where running spectral index $n_s < 1$ can be obtained, we have $S_N^2 \sim \frac{g^2 N}{2\pi^2}$.

Then, in F-term case with $f=1$, supergravity corrections become important for the description of the last 60 e-folds of inflation.

Again, one needs some mechanism to suppress the contribution of cosmic strings.

* How to construct a successful inflation model?