Supergravity and the η-problem

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1 The η-problem

The essential question we wish to answer in the following is: How natural is it to have a Lagrangian, which is suitable for inflation? Recall that the crucial requirement for successful inflation was that the potential is flat. For a single-field model this amounts to the following requirements upon the slow-roll parameters

$$\epsilon = M_p^2 \left(\frac{V'}{V}\right)^2 \ll 1 \quad , \quad \eta = \frac{M_p^2}{2}\frac{V''}{V} \ll 1 \tag{1.1}$$

for a sufficiently large region in field space, such that the universe is expanded by at least 60 e-folds. The correct framework to describe the Lagrangian for inflation should be effective field theory (EFT). The scale for the validity of this EFT should be at most $\Lambda_{UV} = M_p$, though it might also be smaller. Now, the principles of EFT tell us to replace the "classical" action by the Wilson action (Burgess [1]: "When computing only low-energy observables there is therefore no fundamental reason to call the microscopic action, S, the classical action instead of the Wilson actions..."). The latter includes operators, which should be Planck-suppressed in our theory, such as

$$\frac{\mathcal{O}_6}{M_p^2} \tag{1.2}$$

unless there is some symmetry, which forbids them. Keep in mind, that in principle all operators should be taken into account that respect the symmetries. Now, let us assume a small field model for the moment with a given potential V, then one may consider $\mathcal{O}_6 = V(\phi)\phi^2$ [1]. Thus,

$$\Delta\eta \equiv \eta' - \eta \approx \frac{1}{2}\frac{\mathcal{O}_6''}{V} = 1 + \eta + \frac{\phi}{M_p}\epsilon \sim \mathcal{O}(1) \ , \tag{1.3}$$

where η' is computed with respect to the corrected potential $V(1 + (\phi/M_p)^2)$. Higher dimensional operators of this form do not pose any problems, since $\phi \ll M_p$ will suppress any corrections to η [2]. However, the story is different for large-field models, where one in principle has a sensitivity to infinitely many contributions from operators at all orders.

[1] Recall that V necessarily has mass dimension 4.

[2] This is also the reason why we do not discuss an "ϵ-problem": Here only dimension 5-operators can potentially induce a large correction and as long as the Lagrangian has a $\mathbb{Z}_2$-symmetry such terms are absent. In large field models, however, one also expects $\epsilon \sim \mathcal{O}(1)$.

Equivalently the problem can be regarded as the inflaton mass being sensitive to radiative corrections. Thus, the problem is similar to the naturalness problem for scalar masses in the Standard Model. The latter is solved by supersymmetry, which makes the former masses natural. For inflation on the other hand, the η-problem appears generically within supersymmetric models as we will see next.
As a remark: One may regard the appearance of the η-problem also as a fortune, since it shows that inflation is sensitive to Planck-scale physics and hence requires knowledge of the appropriate Planck-scale-theory, such as string theory. This allows to test the latter, at least to some degree. Without this sensitivity inflationary model building would be almost independent from UV-physics. One reason why the η-problem is "good news" for string theory in particular, is that the former appears generically within inflation in the context of supergravity, which we will see next.

2 The η-problem for F-term Inflation

Let us examine the η-problem in the context of models of inflation within $\mathcal{N} = 1$ supergravity. We choose the inflaton to be described by a scalar field subject to the F-term potential [2]

$$V = \mathrm{e}^{K/M_p^2}\left[K^{\phi\overline{\phi}} D_\phi W \overline{D_\phi W} - \frac{3}{M_p^2}|W|^2\right] \quad , \tag{2.1}$$

where K and W are the Kähler potential respectively the superpotential and $D_\phi = \partial_\phi + M_p^{-2}(\partial_\phi K)$ the Kähler-covariant derivative. The object $K^{\phi\overline{\phi}}$ denotes the inverse of the Kähler metric. Expanding the Kähler potential $K = K(0) + K_{\phi\overline{\phi}}(0)\,\phi\overline{\phi} + \ldots$ [3] around some chosen origin $\phi = 0$ we obtain

$$\mathcal{L} = -K_{\phi\overline{\phi}}\,\partial\phi\partial\overline{\phi} - V(0)(1 + K_{\phi\overline{\phi}}(0)\frac{\phi\overline{\phi}}{M_p^2} + \ldots) \tag{2.2}$$

The additional factor $K_{\phi\overline{\phi}}(0)$ can be reabsorbed by a field redefinition. Thus, one naturally, i.e. model-independently, expects corrections of order $\Delta\eta \sim 1$ to the flat potential $V(0)$.
From this point on we have the following possibilities to proceed:

1. Cancel the corrections towards η by fine-tuning of the Kähler and superpotential
2. Discard F-term inflationary models. One may try to construct inflaton potentials via D-terms. However, in supergravity these are not independent upon the F-terms and thus, one naively expects the same problem to appear, unless the respective F-term is negligible.
3. Invoke a symmetry, which protects the inflaton from obtaining large radiative corrections to its mass. The problem is thus solved in the spirit of how supersymmetry makes scalar mass terms natural.

[3] In general one would expect the expansion to include also terms linear in ϕ and $\overline{\phi}$ as well as those quadratic in both fields. However such terms can always be absorbed by a Kähler transformation $K \to K + H + \overline{H}$, since they are holomorphic resp. anti-holomorphic and the coefficients multiplying the respective terms have to be related via complex conjugation to ensure the reality of K.

Let us illustrate the above possibilities via concrete models:
Regarding 1.): A simple example of how the η-problem is solved was provided in [3, 4]. Here one assumes

$$K = \frac{\Phi\overline{\Phi}}{M_p^2} \quad \text{and} \quad W = c(\Phi - \Phi_0)^2 \tag{2.3}$$

for the Kähler and superpotential. If we split $\Phi = (\chi + i\varphi)/\sqrt{2}$, then χ and φ have canonical kinetic terms and the respective scalar potential has a minimum at $(\chi, \varphi) = (\sqrt{2}\Phi_0, 0)$. Given that $\Phi_0 = 1$, then the potential has a small plateau region for $|\varphi| \ll 1$, where the potential effectively behaves like

$$V(\varphi) = c^2(1 - \sqrt{2}\varphi^3 + \mathcal{O}(1)\varphi^4) \ . \tag{2.4}$$

This potential is flat around the saddle-point $\varphi = 0$. If one starts with a small field value then this model is capable of supporting 60 e-folds of slow-roll inflation. However, this crucially depends upon two parameters, which therefore have to be finely tuned: $\Phi_0 = 1$ as well as φ_{in} close to 0.

Regarding 2.): D-term inflationary models exist, where the respective F-terms vanish along the inflationary trajectory (cf. Jeong's talk two weeks ago), such as D-term hybrid inflation:

$$K = \Phi\overline{\Phi} + X\overline{X} + S\overline{S} \quad \text{and} \quad W = \lambda S \Phi X \tag{2.5}$$

where the fields carry $U(1)$-charges $q_S = 0$, $q_\Phi = -q_X = 1$. The respective potential reads

$$V = V_F + V_D \ , \quad \text{where} \tag{2.6}$$

$$V_D = g^2(\xi - |X|^2 + |\Phi|^2)^2 \tag{2.7}$$

is the potential induced by the D-term with an FI-term $\xi > 0$. Now, S will be the inflaton and X the waterfall field. Successful hybrid inflation invokes a very flat potential $V = g^2\xi^2 > 0$ for large $|S|$ such that $X = \Phi = 0$. Employing a 1-loop Coleman-Weinberg potential this flatness can be lifted to produce a slow-roll regime along the inflationary trajectory with $X = \Phi = 0$. All F-terms as well as the superpotential are suppressed here.

Regarding 3.): In the following we will present two models for F-term inflation in supergravity, which realize the third possibility. The respective symmetry in the former models is a shift symmetry, where

$$\phi \to \phi + a \tag{2.8}$$

for some constant value a. Any model, which respects this symmetry, must have a perfectly flat inflaton potential, excluding operators, which come with derivatives of fields only. The main idea now will be to break this symmetry a little, such that the approximate symmetry controls the expansion of the mass of the inflaton. However, since we know that even Planck-suppressed operators give relevant corrections, and the latter are expected to arise from integrating out Planck-scale degrees of freedom, the shift symmetry must still be approximately valid in the UV-regime. However, one may wonder, whether it is even possible to realize such a restrictive symmetry in the appropriate UV-theory. Fortunately realizations of shift symmetries exist within string theory [5]. On the other hand the models we present below have not been embedded within string theory yet, where yet means 2007, when [5] was written.

3 Natural Chaotic Inflation in Supergravity

Using the above ideas it is possible to construct a natural[4] model for chaotic inflation [6]. Generically it is difficult to obtain such models from the scalar potential in eq. 2.1, since as we saw we are generically facing an η-problem. More concretely due to the exponential factor one typically has a very steep potential. Roughly the slow-roll conditions for the scalar ϕ would already be spoiled for super-Planckian values. The proposed shift symmetry can be of help here. In particular, given some chiral multiplet Φ[5], we can demand that the Kähler potential K possesses a shift symmetry of the kind

$$\Phi \to \Phi + iM_g C \quad , \quad \text{where } \; C \in \mathbb{R} \; , \tag{3.1}$$

such that necessarily $K(\Phi, \overline{\Phi}) = K(\Phi + \overline{\Phi})$. Thus, the imaginary part of the scalar field appearing within the multiplet is not affected by any exponential factor in the respective potential. To break the shift symmetry one may introduce a superpotential of the kind

$$W = m\Phi^2 \; , \tag{3.2}$$

where the parameter m controls the breaking. Since in the limit $m \to 0$ the shift symmetry is restored a small m is natural in t'Hooft's sense. However, the respective scalar potential is not suitable for chaotic inflation, since it is not bounded below and so inflation never ends. This statement also holds if W is a generic monomial in Φ.

To dissolve this problem, in [6] an additional chiral multiplet X is introduced. A superpotential of the form

$$W = mX\Phi \tag{3.3}$$

possesses a $U(1)_R$-symmetry, where X has charge $q = -2$ and Φ is uncharged, as well as a $\mathbb{Z}_2$-symmetry. This is also respected by a Kähler potential of the form

$$K(\Phi, \overline{\Phi}, X, \overline{X}) = K((\Phi + \overline{\Phi})^2, X\overline{X}) \; . \tag{3.4}$$

In the following we will neglect contributions to the Kähler potential induced by the breaking of the shift symmetry, which is justified as long as m is sufficiently small. For simplicity let

$$K = \frac{1}{2}(\Phi + \overline{\Phi})^2 + X\overline{X} \; , \tag{3.5}$$

then we obtain

$$\begin{aligned} V = & m^2 \exp\left(\chi^2 + |X|^2\right) \\ & \times \left[\frac{1}{2}(\chi^2 + \varphi^2)(1 + |X|^4) + |X|^2\left(1 - \frac{1}{2}(\chi^2 + \varphi^2) + \chi^2 \; \left(2 + \chi^2 + \varphi^2\right)\right)\right] \end{aligned} \tag{3.6}$$

as well as canonical kinetic terms. Here we set $M_p = 1$ for the sake of brevity. Furthermore we performed a split of the field Φ into real and imaginary parts as follows

$$\Phi \equiv \frac{1}{\sqrt{2}}\left(\chi + i\varphi\right) \; . \tag{3.7}$$

[4]In this case this refers to t'Hooft's notion of naturalness.

[5]In the following we will denote chiral multiplets and the scalar fields appearing within their decomposition with the same symbol. In particular all fermionic terms are neglected throughout.

We can assume that $|\chi|, |X| \lesssim \mathcal{O}(1)$ since their respective potential contains a factor e^K. Thus, their amplitude will quickly decrease, even if they initially had a large field value [6]. Now, once $|\chi|, |X| \ll \mathcal{O}(1)$, then the potential effectively behaves as

$$V \simeq \frac{1}{2}m^2(\varphi^2 + \chi^2 + 2|X|^2) + \frac{1}{2}m^2\varphi^2\chi^2 \sim \frac{1}{2}m^2\varphi^2 \quad . \tag{3.8}$$

Chaotic inflation assigns the inflaton field a "generic" initial value according to [7]

$$V(\varphi(0)) \sim \frac{1}{2}m^2\varphi(0)^2 \sim 1 \quad \Rightarrow \quad \varphi(0) \sim m^{-1} \quad . \tag{3.9}$$

Now we have to have $\varphi(0) \gg 1$ in order to provide the minimal required number of e-folds. Thus chaotic inflation happens, if $m \ll 1$. This statement is natural due to the existence of the shift symmetry. Furthermore φ as well as X will satisfy the slow-roll conditions, while χ effectively has a large mass since $\varphi \gg 1$. During 60 e-folds of inflation the X field evolves from values $\mathcal{O}(1)$ towards $X \sim m$ at the end of inflation. The spectral index as well as the scalar-to-tensor ratio are dominated by the contribution w.r.t. φ and hence effectively behave like the respective values of single-field $m^2\varphi^2$ slow-roll inflation. From the normalization of the density fluctuations[7] with respect to the CMB measurements one obtains

$$m \simeq 10^{-5} \tag{3.10}$$

Lastly let us make a remark on corrections towards the Kähler potential. One may have terms such as $(\Phi + \overline{\Phi})^{2n}$ or $|X|^{2n}$. The terms of the first kind generically enhance the steepness of the potential for the χ field unless there is some fine-tuning of the coefficients multiplying these monomials. The second class of terms can render $|X|$ even more massive than the mass scale of the inflaton φ [8]. Furthermore one may have terms induced by the breaking of the shift-symmetry such as

$$\Delta K = |m\Phi|^2 \quad . \tag{3.11}$$

However, the above term is negligible as long as $\varphi \lesssim m^{-1}$.

4 Natural Inflation in Supergravity

The second model we discuss will provide a realization of natural inflation in the context of supergravity [5]. Natural inflation is associated with a potential of the form

$$V(\phi) = \Lambda^4 \left[1 \pm \cos\left(\frac{\phi}{f}\right)\right] \quad , \tag{4.1}$$

where $\Lambda \sim m_{GUT}$ [8]. The inflaton here is given by a pseudo Nambu-Goldstone boson (PNMB), whose shift symmetry is explicitly broken[9]. The scale f can either be of order M_p, such that

[6]One may suspect that for large field values the additional fields may start to cause inflation themselves. However, this is never the case as can be seen from the slow-roll parameter $\epsilon_\chi \simeq 4(2 + \chi^2 + \chi^{-2})$, which is always too large. For X one has a similar situation.

[7]Note that $\delta\rho/\rho \sim m\varphi^2$.

[8]This is interesting, if one wants to suppress non-Gaussian signatures in the power spectrum later on.

[9]Examples of such PNMBs are axions with a symmetry breaking scale f. The above potential is then generated by instanton-effects.

inflation occurs at the maximum of the potential, or be larger, in which case V is flat at the minimum.
Now, the model in [5] assumes a shift symmetric Kähler potential

$$K = \frac{1}{4}(\Phi + \overline{\Phi})^2 \quad , \tag{4.2}$$

which differs from the respective potential of the last section only by a factor. Furthermore we introduce a shift symmetry-breaking, non-perturbative[10] superpotential of the type

$$W = w + B\exp(-b\,\Phi) \quad \text{, where} \quad w, B, b \in \mathbb{R} \tag{4.3}$$

Let us again split the field into real and imaginary parts

$$\Phi = \chi + i\varphi \quad , \tag{4.4}$$

then we obtain a scalar potential of the form

$$V(\chi,\varphi) = V_1(\chi) - V_2(\chi)\cos(b\varphi) \quad , \quad \text{where} \tag{4.5}$$
$$V_1(\chi) = \mathrm{e}^{\chi(\chi-2b)}B^2\left[-3 + 2(\chi-b)^2 + \mathrm{e}^{2b\chi}\frac{w^2}{B^2}\left(-3+2\chi^2\right)\right] \tag{4.6}$$
$$V_2(\chi) = \mathrm{e}^{\chi(\chi-b)}2Bw\left[3 + 2\chi(b-\chi)\right] \tag{4.7}$$

Now this potential has a supersymmetric minimum, such that $V_{min} < 0$, implying AdS. Since one usually wants to have that $\Phi > 0$ at that minimum, then w has to be a small parameter. Thus, we have to lift the potential up in order to ensure that $V_{min} = 0$. The uplifting can be performed by adding a model which provides sufficient susy-breaking [11]. The resulting potential has the shape of a valley, which is flat along the φ-direction, as long as b is sufficiently small. Again a smallness of b is natural, since in the limit $b \to 0$ the shift-symmetry of the superpotential is restored. In fact in that situation we roughly retain the model of chaotic inflation from the last section, since now the cosine can be expanded and at leading order we obtain an $m^2\varphi^2$ term in the potential. Furthermore the shift symmetry controls all higher order corrections due to the smallness of b.
Moreover we can match our result to the shape of the natural inflation potential from eq. 4.1 by assuming that the minimum of the scalar potential V lies at $(\chi_0, \varphi_0) = (\chi_0, 0)$, such that

$$V(\varphi) = V_2(\chi_0)(1 - \cos(b\varphi)) \tag{4.8}$$

along the flat direction. Comparison with eq. 4.1 yields

$$V_2(\chi_0) = \Lambda^4 \sim m_{GUT}^4 \quad \text{and} \quad b = f^{-1} \quad . \tag{4.9}$$

Alternatively to a small b one can choose b larger such that inflation occurs at the maximum. In any case the normalization of density perturbations enforces that $wBb^2 \sim 10^{-12}$ [5]. One may choose $w \sim 10^{-4}$ to satisfy this relation. However, a small w seems to be unnatural. It can be made natural by R-symmetry breaking. Otherwise one has to fine-tune this parameter.

[10] One may imagine this to be the first term appearing within an instantonic series.

[11] In [5] the uplifting is done by the means of the KKLT model [9] with all respective fields at the minimum. This also implies a rescaling of Φ, but which can be absorbed into a redefinition of the parameters w and B.

5 Conclusions

Let us briefly recall the main points of the above talk:

- The η-problem is a challenge for building viable models of inflation. It appears generically within models of inflation in supergravity.
- On the other hand its occurence is also fortunate, since it shows that Planck-scale physics couple to the dynamics of the inflation.
- A solution in the context of supergravity can be provided in a way that preserves naturalness by imposing a shift-symmetric Kähler potential and a superpotential, which explicitly breaks the former symmetry. Furthermore the superpotential should include a small parameter, that protects the inflaton from large corrections towards its mass. This is either done by refering to a fundamental theory or by fine-tuning.

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