Consistency of Fayet-Iliopoulos terms

Clemens Wieck
Dec 17 2013

Outline

I FI terms vs. "impostors"

II The no-go argument by Komargodski & Seiberg

III Discussion

I FI terms vs. "impostors" (mostly [1])

- Genuine FI terms:

From Julian's talk we know that "genuine" FI terms arise as integration constants of the Killing prepotentials.

$$D^a = i X^{aj} \partial_j K + \xi ,$$

by solving the Killing equations on the complex manifold of scalar fields ϕ_j.

↪ For a linearly realized gauge symmetry:

$$X^j{}_a = i (T_a)^j{}_i \phi^i$$

Example: Canonical K, fields ϕ^i charged with q^i under some $U(1)$

$$\hookrightarrow X^i = i q^i \phi^i \Rightarrow D = -\sum_i q^i |\phi^i|^2 \; (+\xi)$$

- Impostors

A D-term may contain an apparent constant which is not a genuine FI term if

a) the constant is actually field-dependent

b) the corresponding $U(1)_{FI}$ symmetry is Higgsed everywhere in field space.

- a) Happens when a $U(1)$ is realized non-linearly, i.e. axionic:

$$X^i_a = i q^i_a \qquad q^i_a = \text{real constant.}$$

↳ classic example, often arising in string theory [2]:

e.g. via Green-Schwarz mechanism in heterotic s.t.

$$K = -\log(S+\bar{S}), \quad X_S = i q_S$$

$$\Rightarrow D = \frac{q_S}{S+\bar{S}} \quad (+\xi)$$

If S is stabilized at some high scale & integrated out, this looks like a constant.

↳ But: integrating out $S+\bar{S}$ supersymmetrically gives a large mass to Im S as well.

↳ Since Im S gets eaten by the $U(1)$ vector multiplet, this must be heavy, too

→ No low-energy FI term.

▶ b) In principle a generalization of a)

↳ basic idea: An FI term associated with a massive vector multiplet can always be redefined away.

- Decompose massive v.m. V_m into massless v.m. & chiral superfield S:

$$V_m = V + S + \bar{S}$$

↳ transformation under $U(1)$:

$$V \to V' = V - (\Lambda + \bar{\Lambda}) \quad ; \quad S \to S' = S + \Lambda$$

(S corresponds to a pure gauge unless V_m is massive everywhere in field space. → S is globally defined as a dynamical field).

- Consider theory with FI term associated with V_m

↳ perform Kähler transformation:

$$K' = K + \alpha(S + \bar{S}), \quad W' = W \cdot e^{-\alpha S}$$

$$\Rightarrow D = iK_i X^i + \xi = iK_i' X^i + \alpha + \xi$$

↳ Get rid of FI term by choosing $\alpha = -\xi$.

⇒ Genuine FI term only exists when gauge symmetry restored at least at one point in field space.

$\boxed{\text{II}}$ The no-go argument by Kom. & Seiberg (mostly [3,4])

Ferrara - Zumino multiplet

- Consider inherent symmetries of globally supersymmetric theories:

	R-symmetry	supersymmetry	gen. coord. transf.
Noether current:	j^μ	$S_{\mu\alpha}$	$T_{\mu\nu}$

↪ Ferrara, Zumino '75: All of these can be combined in a supercurrent supermultiplet

$$\mathcal{J}_\mu \equiv \frac{1}{4}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}\mathcal{J}_{\alpha\dot{\alpha}}.$$

- Theory with unbroken R symmetry & superconf. symmetry:

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = 0$$

⤳ superconformal anomaly structure determined by $\partial^\mu j_\mu,\ \bar{\sigma}^\mu S_{\mu\alpha},\ T \equiv T_\mu^{\ \mu}$

- Theory with maximally broken superconf. symmetry:

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

↪ X: chiral superfield

↪ component expression for $\mathcal{J}_\mu$:

$$\mathcal{J}_\mu = j_\mu + \theta \cdot f(S_{\mu\alpha}) + \bar{\theta}\,\bar{f}(\bar{S}_{\mu\dot{\alpha}}) + \theta\sigma^\mu\bar{\theta}\cdot F(T_{\nu\mu}, T) + \ldots$$

↪ Why is this a useful object?

Coupling to supergravity

$$\text{global supersymmetry} \xrightarrow{\boxed{?}} \text{local supersymmetry}$$

- In a superconformal theory: All currents conserved

  ↪ $\boxed{?}$ = couple supercurrent multiplet to supergravity mult.

$$\int d^4\theta \, J_{\alpha\dot\alpha} H^{\alpha\dot\alpha} \quad , \quad H^{\alpha\dot\alpha} \supset b_\mu, \psi_{\mu\alpha}, e_\mu^m, a_\mu .$$

conf. sugra multiplet.

- Without supercanf. symmetry: Use chiral compensator formalism

  ↪ cf. Ido's talk.

$$\begin{tikzcd}
\boxed{\text{global theory}}_{(1)} \arrow[r, "\Sigma, \bar\Sigma"] & \boxed{\text{compensated theory}}_{(2)} \arrow[r, "\langle\Sigma\rangle, \langle\bar\Sigma\rangle"] \arrow[d] & \boxed{\text{"frozen" theory}}_{(3)} \arrow[d] \\
 & \text{couple this to sugra} & \text{effectively gauged theory}
\end{tikzcd}$$

↯ Beware: Symmetry structure of theories (1) and (3) might be widely different.

↪ see Sec. $\boxed{III}$

- Remember: conformal scaling properties determined by "Weyl weight" w:

  e.g. $w(V) = 0$, $w(W_\alpha) = \frac{3}{2}$; $w(x,\theta) = (-1, -\frac{1}{2})$

  $\Rightarrow w(K) = 2$, $w(W) = 3$ for conf. symmetry.

- Theory (1) ⟶ Theory (2):

  Introduce chiral compensators $\Sigma, \bar\Sigma$ with $R(\Sigma) = \pm\frac{2}{3}$, $w(\Sigma) = 1$.

a) superpotential: $W_{(1)} \to W_{(2)} = \left(\frac{\Sigma}{\sqrt{3}M_p}\right)^3 \underbrace{W_{(1)}(\tilde{\phi}_i, \tilde{X}_n)}_{\equiv \tilde{W}}$,

where $\phi_i = \left(\frac{\Sigma}{\sqrt{3}M_p}\right)\tilde{\phi}_i$ new chiral fields

$X_n = \left(\frac{\Sigma_n}{\sqrt{3}M_p}\right)^n \tilde{X}_n$ new couplings of mass dim. n.

b) Kähler potential: Decompose $K_{(1)}(\phi, \bar{\phi}, V) = \Sigma_n K_n$

$\hookrightarrow \tilde{K} = K_{(1)}(\tilde{\phi}, \bar{\tilde{\phi}}, V) = \sum_n \left(\frac{\Sigma\bar{\Sigma}}{3M_p^2}\right)^{-n/2} K_n$

$\to K_{(2)} = -\bar{\Sigma}\Sigma \exp\left(-\frac{\tilde{K}}{3M_p^2}\right)$

$\Rightarrow$ $W_{(2)}$ and $K_{(2)}$ are superconformal & R-symmetric!

- Theory (2) $\longrightarrow$ Theory (3): Freeze $\langle\Sigma\rangle = \langle\bar{\Sigma}\rangle = \sqrt{3}M_p$.

- Example: Free theory with FI term

$W = 0 \quad , \quad K = 2\xi V$

$\hookrightarrow J_{\alpha\dot{\alpha}} = 2W_\alpha \bar{W}_{\dot{\alpha}} + \frac{2\xi}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]V$

[or in general: $J_{\alpha\dot{\alpha}} = -K_{i\bar{j}} D_\alpha\phi_i \bar{D}_{\dot{\alpha}}\bar{\phi}_{\bar{j}} + \frac{1}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]K$]

- Couple FI term to Supergravity

- Theory (1) invariant under $U(1)_{FI}$

$\hookrightarrow$ Theory (2) invariant under R + superconf. symmetry
+ invariant under $U(1)'_{FI}$

$\hookrightarrow$ a) $K_{(2)}$ must be invariant under $U(1)'_{FI}$

$\hookrightarrow$ assign charge to Σ: $\underline{Q_{\Sigma,\bar{\Sigma}} = \pm \frac{2\xi}{3M_p^2}}$ (*)

b) $W_{(2)}$ must be invariant under $U(1)'_{FI}$

$\hookrightarrow$ $\tilde{W}$ transforms nontrivially: $\underline{Q_{\tilde{W}} = -\frac{2\xi}{M_p^2}}$.

- Places strong restrictions on <u>Theory (1)</u>:

  All terms in $\tilde{W}$ must have the same $U(1)'_{FI}$ charge

  $\hookrightarrow$ There must be an additional <u>global</u> <u>R symmetry</u> to make this possible.

- Same applies to <u>Theory (2)</u>:

  Theory (1) had local $U(1)_{FI}$ invariance, charges of Φ_i assigned s.t. $W_{(1)}$ invariant.

  $\hookrightarrow$ promote this into Theory (2) by choosing $\Sigma, \bar{\Sigma}$ to be neutral under $U(1)_{FI}$.

  $\Rightarrow$ Only possible if this $U(1)_{FI}$ does not shift V or transforms $K_{(2)}$

  $\Rightarrow$ $U(1)_{FI}$ is <u>global</u> in Theory 2!

$\lightning$ Banks, Seiberg: There are no global symmetries in quantum gravity

$\hookrightarrow$ <u>FI term inconsistent.</u>

$\boxed{III}$ Discussion (mostly [4, 5])

- Arguments made in [4]

  - [3] links the trouble with FI terms to the breaking of R symmetry, i.e. $\partial_\mu j^\mu \neq 0$ induced by ξ.

    ↪ This is misleading, since the chiral compensator formalism ("old minimal formalism") ~~breaks~~ by construction breaks R. Remember, $R(\Sigma) = \pm\frac{2}{3}$. FI term by itself preserves R.

    ⟶ Do not use chiral comp. form. when R symmetry is exact in Theory (1).

  - In case of exact R symmetry, use linear compensator formalism ("new minimal formalism").

    ↪ Sohnius, West '81

    ↪ conservation equation:

    $$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = L_\alpha \quad , L_\alpha \text{ chiral multiplet}$$

    → $J_{\alpha\dot{\alpha}}$ = linear multiplet (= vector multiplet without $\theta^2, \bar{\theta}^2$ components)

    ↪ introduce compensators for gauging whose vev preserve R symmetry ~~R~~ + make it local.

▸ In linear formalism, ξ does not contribute to $J_{\alpha\dot\alpha}$.

↪ no gauge charge for compensators

↪ no global symmetries appear.

→ FI term consistently coupled to sugra.

• But note that in chiral formalism the appearance of global symmetries is indeed due to the FI term, not the broken R symmetry. With $\xi = 0$, $K_{(1)}$ and $K_{(2)}$ are invariant under $U(1)_{FI} = U(1)'_{FI}$.

▸ In an R-symmetric theory, chiral & linear formalism are related:

$$J^{(c)}_{\alpha\dot\alpha} = J_{\alpha\dot\alpha} + \frac{1}{3}[D_\alpha, \bar D_{\dot\alpha}] \underbrace{\left(K - \frac{3R(\phi)}{2}\phi_i K_i\right)}_{\equiv k}$$

↪ k must have terms which allow $k \neq 0$ for $J^{(c)}_\mu$ and $J^{(L)}_\mu$ to be different — e.g. FI term.

↪ $J^{(c)}_\mu$ may only break $U(1)_{FI}$ if it differs from $J^{(L)}_\mu$.

▸ In contrast to chiral formalism, linear formalism preserves exact symmetry structure going from Theory (1) → Theory (3):

$$U(1)_{FI} \longrightarrow U(1)_{FI}$$

$$U(1)_R^{\text{global}} \longrightarrow U(1)_R^{\text{local}}$$

- <u>Conclusion of [4]</u>: Whether FI term can be consistently coupled to supergravity depends on whether Theory (1) has an exact symmetry or not.

- <u>New (and contradicting) arguments made in [5]</u>:

- The Ferrara-Zumino multiplet J^{μ} is actually <u>ill-defined</u> whenever $\xi \neq 0$.

- This may be remedied in the presence of an exact R symmetry, using the so-called "R-multiplet", which is somewhat analog to the "new minimal" formalism.

  ↳ <u>But</u>: It is stated that in this case, one is left with a global symmetry in Theory (3), contrary to what was said in [4].

  ↳ $\xi \neq 0$ + exact R symmetry is excluded.

- If $\xi \neq 0$ and no exact R symmetry is present, use a non-minimal formalism via "S-multiplet".

  ↳ introduces additional chiral field Y.

  ↳ start with constant $\xi \neq 0$ in Theory (1), end up with Y-dependent ξ in Theory (3). Y Higgses $U(1)_{FI}$ everywhere.

  $\Rightarrow$ FI term not meaningful anymore, cf. Sec. $\boxed{\text{I}}$.

References

[1] F. Zwirner et al., 1110.2174

[2] M. Dine et al., Nucl. Phys. B289 (1987) 589

[3] Z. Komargodski & N. Seiberg, 0904.1159

[4] K.R. Dienes & B. Thomas, 0911.0677

[5] Z. Komargodski & N. Seiberg, 1002.2228

~~[6]~~