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DESY workshop
seminar

Non-minimal F-term Inflation

07.01.14

Aim : to introduce a new class of SUGRA inflation models with logarythmic Kähler potentials and non-minimal coupling to gravity.

Plan :
1. Motivation – constructing inflation models in SUGRA
2. Superconformal approach to SUGRA – standard approach to creating SUGRA theories.
3. Canonical Superconformal SUGRA (CSS) – a particular class of models with canonical kinetic terms in the Jordan frame.
4. Building models : logarythmic v power law – including non-minimal coupling
   - 4.1 Symmetry breaking potential
   - 4.2 Quadratic potential
   - 4.3 General potential
5. Summary

References :

[1] Kallosh and Linde 1008.3375
→ main reference for this lecture.

[2] Kallosh, Linde, Rube 1011.5945
→ more on general $W = Sf(\Phi)$

[3] Einhorn and Jones 0912.2718
Ferrara et al 1004.0712
Lee 1004.0712
Ferrara et al 1008.2942

Papers introducing this idea / SUGRA Higgs Inflation.

[4] Linde, Noorbala, Westphal 1101.2652
→ observational consequences

1. Motivation – building inflation models in SUGRA

- inflation requires a flat scalar potential
- in general, the F-term part of the scalar potential is

$$V \propto e^{K} \quad \leftarrow \text{Kähler potential}$$

- simplest Kähler potential: $K \propto \Phi \bar{\Phi}$

$\Rightarrow$ scalar potential not flat enough for inflation.

Possible solutions

(1) search for flat directions

(2) choose a new Kähler potential with shift symmetry:

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2$$

$\Leftrightarrow$ e^K does not depend on $(\Phi - \bar{\Phi})$, which becomes the inflaton.

(3) add a non-minimal coupling to gravity.

$\Leftrightarrow$ this, and the connection to shift symmetry, is the subject of this lecture.

Note: we do inflation calculations in the Einstein frame because then we can use the standard formalism.

2. Superconformal approach to SUGRA

[see also Ido's talk]

Standard approach:
1. write down a super-conformal theory including a "conformal compensator" φ
2. make $\varphi \propto M_{pl}$
3. obtain SUGRA Lagrangian in Einstein frame.

Example:

- $$\mathcal{L} = \sqrt{-g}\left[\underbrace{+}_{\text{note the sign!}}\frac{1}{2}\partial_\mu \underbrace{\varphi}_{\substack{\text{"conformal compensator"} \\ \text{or "Weyl scalar"} \\ \text{or "conformon"}}} \partial_\nu \varphi\, g^{\mu\nu} + \frac{\varphi^2}{12}R(g) - \frac{1}{2}\partial_\mu \underbrace{h}_{\text{scalar}} \partial_\nu h\, g^{\mu\nu} - \frac{h^2}{12}R(g) - \frac{\lambda h^4}{4}\right]$$

↳ this is conformally invariant under
$$g_{\mu\nu} \to g'_{\mu\nu} = e^{-2\sigma(x)} g_{\mu\nu}$$
$$\varphi \to \varphi' = e^{\sigma(x)}\varphi$$
$$h \to h' = e^{\sigma(x)} h$$

- Now fix $\boxed{\varphi = \sqrt{6}\, M_p}$ (gauge fixing)

↳ adding a scale (M_p) breaks conformal invariance
→ we can have Einstein gravity.

↳ note that terms such as $h^2\varphi^2$, $h\varphi^3$ would become large, so must be forbidden. [Decoupling of φ from matter fields is a condition of the theory]

- $$\mathcal{L} = \underbrace{\sqrt{-g}\,\frac{M_p^2}{2}R(g)}_{\text{Einstein gravity}} - \underbrace{\sqrt{-g}\left[\frac{1}{2}\partial_\mu h \partial_\nu h\, g^{\mu\nu} + \frac{h^2}{12}R(g) + \frac{\lambda}{4}h^4\right]}_{\text{conformally invariant matter part.}}$$

[If we switch to Einstein frame, this is not obvious]

3. Canonical superconformal SUGRA (CSS)

- standard method: fix $-\frac{1}{6}\mathcal{N}(X,\bar{X})R \Rightarrow \frac{1}{2}M_p^2 R$

- CSS method: fix $-\frac{1}{6}\mathcal{N}(X,\bar{X})R \Rightarrow -\frac{1}{6}\Phi(z,\bar{z})R$ — we don't use this notation to avoid confusion.

$$= \frac{1}{2}\Omega^2(z,\bar{z})R$$ [different notation]

complex scalar fields

- now consider a particular class of theories:
  - $K = -3\log\Omega^2(z,\bar{z})$ [or $\Omega^2(z,\bar{z}) = \exp\left(-\frac{K}{3}\right)$]

  - $\mathcal{L}_J = \underbrace{\sqrt{-g_J}\, 3\Omega^2_{\alpha\bar{\beta}}\, \partial_\mu z^\alpha \partial_\nu \bar{z}^{\bar{\beta}}\, g_J^{\mu\nu}}_{\text{(kinetic)}} + \underbrace{\sqrt{-g_J}\,\frac{1}{2}\Omega^2 R}_{\text{(gravity)}} + \ldots$

  - choose $\Omega^2 = 1 - \frac{1}{3}\left(\delta_{\alpha\bar{\beta}} z^\alpha z^{\bar{\beta}} + J(z) + \bar{J}(\bar{z})\right)$

    arbitrary holomorphic.

    → then we see that $\Omega^2_{\alpha\bar{\beta}} = \frac{-\delta_{\alpha\bar{\beta}}}{3}$

    → $\mathcal{L}_J = \sqrt{-g_J}\left(-\frac{1}{2}\partial_\mu z\, \partial_\nu \bar{z}\, g_J^{\mu\nu}\right) + \ldots$

    i.e. kinetic terms are canonical in Jordan frame.

  - important implication: with this K and Ω,

    $V_J = |\partial W|^2$ - same as global SUSY [when $J(z) = 0$ i.e. matter superconformal symmetry is preserved]

    → e^K, $-3|W|^2$ etc all cancel

- when $J(z) \neq 0$, potential is still simple if $J(z)$ doesn't depend on S and

$\Rightarrow$ NMSSM or more complicated theory

$W = S f(\Phi)$, considering potential at $S=0$,

arbitrary holomorphic.

then $V_J = |\partial_S W|^2 = |f(\Phi)|^2$

[then must stabilise S!]

$$V_E\Big|_{S=0} = \frac{V_J}{\Omega^4}$$

This is the CSS theory:
- canonical kinetic terms
- conformal coupling to gravity
- $V = V_{global}$

- particular example

$$J(z) = -\frac{3\chi}{4}\bar{\Phi}^2$$

χ constant

$$\Rightarrow \Omega^2(\Phi, S; \bar{\Phi}, \bar{S}) = 1 - \frac{1}{3}\left(\Phi\bar{\Phi} + S\bar{S}\right) + \frac{\chi}{4}\left(\Phi^2 + \bar{\Phi}^2\right)$$

$\Rightarrow$ use $K = -3\log\Omega^2$ gives

$$K = -3\log\left(1 - \frac{1}{3}(\Phi\bar{\Phi} + S\bar{S}) + \frac{\chi}{4}(\Phi^2 + \bar{\Phi}^2)\right)$$

$\Rightarrow$ usually decompose:

$$S = \frac{1}{\sqrt{2}}(s + i\alpha) \quad ; \quad \Phi = \frac{1}{\sqrt{2}}(\phi + i\beta)$$

- re-write Ω in different form to make it easier to see the effect of χ.

$$\Rightarrow \Omega^2 = \left[1 - \frac{S\bar{S}}{3} + \frac{1}{12}\left(1 + \frac{3}{2}\chi\right)[\Phi - \bar{\Phi}]^2 - \frac{1}{12}\left(1 - \frac{3}{2}\chi\right)(\Phi + \bar{\Phi})^2\right]$$

- now consider 3 cases:

① $\chi = 0$ ② $\chi = \pm 2/3$ ③ $\chi \neq 2/3$

$\boxed{\chi = 0}$

$\Rightarrow \Omega^2 = 1 - \frac{1}{3}[\Phi\bar{\Phi} + S\bar{S}]$

$\Rightarrow$ conformal coupling to gravity $\left(\xi = -\frac{1}{6}\right)$

$\boxed{\chi = +2/3}$

$\Rightarrow \Omega^2 = 1 - \frac{S\bar{S}}{3} + \frac{1}{6}[\Phi - \bar{\Phi}]^2$

$\Rightarrow$ • $(\Phi + \bar{\Phi})$ is minimally coupled to gravity

• K does not depend on $(\Phi + \bar{\Phi})$

$\Rightarrow$ use $\Phi = \frac{1}{\sqrt{2}}(\phi + i\beta)$

$\Leftrightarrow$ ϕ is minimally coupled to gravity

$\Rightarrow$ if $S = 0 = \beta$ during inflation,

then • K = 0 along trajectory

• kinetic terms canonical

$\Rightarrow$ if $W = S f(\Phi)$,

then $V_J(\phi) = |f(\phi/\sqrt{2})|^2 = V_E(\phi)$

$\boxed{\chi \neq 2/3}$

$\Rightarrow \Omega^2 = 1 - \frac{S\bar{S}}{3} + \frac{1}{2}\left(\frac{1}{6} + \frac{\chi}{4}\right)(\Phi - \bar{\Phi})^2 + \frac{1}{2}\left(-\frac{1}{6} + \frac{\chi}{4}\right)(\Phi + \bar{\Phi})^2$

equivalent to non-minimal coupling

$\frac{\xi}{2}\phi^2 R$ with $\xi = -\frac{1}{6} + \frac{\chi}{4}$

$\Rightarrow$ does the potential change?

$V_J = |f|^2$ $\qquad [S = 0 = \beta]$

$$V_E = \frac{|f|^2}{(1+\xi\phi^2)^2}$$

[Note that ϕ is only canonically normalised in the Jordan frame, NOT in the Einstein frame!]

where we do calculations.

Compare to power law Kähler potential

- $K = \delta_{\alpha\bar\beta} z^\alpha \bar z^{\bar\beta} + J(z) + \bar J(\bar z) + \dots$

$$= (\Phi\bar\Phi + S\bar S) - \frac{3\chi}{4}(\Phi^2 + \bar\Phi^2) + \dots$$

[actually an expansion of $K = -3\log(\dots)$ at small Φ, S]

- use same assumptions as before:
  - $\partial_S J = 0$
  - $W = S f(\Phi)$
  - S stabilised @ $S = 0$

$$\Rightarrow V_E\Big|_{S=0} = e^{\Phi\bar\Phi - \frac{3\chi}{4}(\Phi^2+\bar\Phi^2)} |f(\Phi)|^2$$

$\boxed{\chi = 2/3}$ $\quad \xi = 0 \Rightarrow$ shift symmetry

$$K = S\bar S - \tfrac{1}{2}(\Phi - \bar\Phi)^2$$

- during inflation, $S = 0$; $\Phi = \bar\Phi \Rightarrow K = 0$

$$\Rightarrow V_E\Big|_{S=0} = |f(\Phi)|^2 \qquad \text{same as } K = -3\log\Omega^2$$

$\boxed{\chi \neq 2/3}$. no simple interpretation in terms of non-minimal coupling

- $V_E\big|_{S=0} = e^{-3\xi\phi^2} |f(\Phi)|^2 \neq \log K$ potential

$\Rightarrow$ difficult to achieve inflation.

4. Building models – logarythmic v. power law

We will compare various W, K and χ choices. Some will make better inflation models! For orientation, I give a table of possible models:

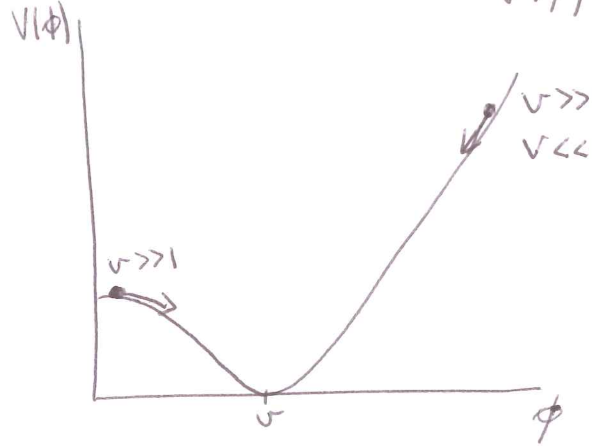
	$W = -\lambda S\left(\Phi^2 - \frac{v^2}{2}\right)$ (symmetry breaking)	$W = m S \Phi$ (quadratic)	$W = S f(\Phi)$ (general)
$K = -3\log\left[1 + \frac{1}{6}(\Phi - \bar{\Phi})^2 - \frac{1}{3} S\bar{S} + \zeta \frac{(S\bar{S})^2}{3}\right]$ $\boxed{\chi = 0}$	(A) requires $\zeta > 1/3$, $v \gg 1$	(e) $\zeta > 1/3$	(i)
$K = -3\log\left[1 + \frac{1}{6}(\Phi - \bar{\Phi})^2 - \frac{1}{3} S\bar{S} + \zeta\frac{(S\bar{S})^2}{3} + \frac{\chi(\Phi^2 - \bar{\Phi}^2)}{4}\right]$ $\boxed{\chi \neq 0}$	(B)	(f) $\lvert\chi\rvert \lesssim 10^{-2}$	
$K = S\bar{S} - \frac{1}{2}[\Phi - \bar{\Phi}]^2 - \zeta(S\bar{S})^2$ $\boxed{\chi = 0}$	(c)	(g) $\zeta \gtrsim 0.1$	
$K = S\bar{S} - \frac{1}{2}[\Phi - \bar{\Phi}]^2 - \zeta(S\bar{S})^2 + \chi(\Phi^2 + \bar{\Phi}^2)$ $\boxed{\chi \neq 0}$	(d) requires $\chi \lesssim 3\times10^{-3}$	(h) $\chi \lesssim 10^{-3}$	

For each case, we should:

1. calculate inflationary potential $V(\phi)$
2. check stability in S and β directions, $m_s^2 > 0$; $m_\beta^2 > 0$
3. compute predictions for Planck.
4. for single field-like model, require $m_s > H$, $m_\beta > H$

case A : $W = -\lambda S\left(\Phi^2 - \frac{v^2}{2}\right)$; $\xi = 0$; $K = -3\log[\cdots]$

- $\frac{\xi (S\bar{S})^2}{3}$ in K is for stability of S. ⟵ [THIS IS A REALLY IMPORTANT POINT]
- for $S = 0 = \beta$, find $V(\phi) = \frac{\lambda^2(\phi^2 - v^2)^2}{4}$



$v \gg 1$ ⟶ last 60 e-folds near quadratic minimum

$v \ll 1$ ⟶ $\sim\phi^4$, so ruled out by Planck.

- stability of β

$$V(\phi, \beta, S=0) = \frac{9\lambda^2\left(\phi^4 - 2\phi^2(v^2-\beta^2) + (v^2+\beta^2)\right)}{4(3-\beta^2)^2}$$

→ minimum @ $\beta = 0$

→ $$m_\beta^2 = \frac{\lambda^2}{3}\left(3(\phi^2+v^2) + \underbrace{(\phi^2-v^2)^2}_{\to\, 4H^2}\right)$$

$m_\beta^2 > 0 \Rightarrow$ stable

$m_\beta > H \Rightarrow$ no fluctuations; rolls to $\beta = 0$ quickly.

- stability of S

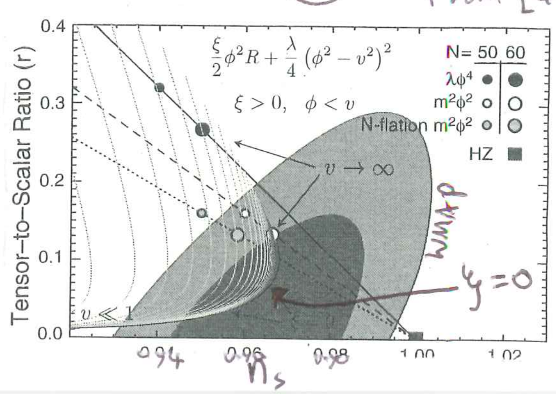
$$M_S^2 = \frac{\lambda^2}{6}\left([6\xi - 1](\phi^2+v^2)^2 + 12\phi^2\right)$$

→ stable if $\xi > 1/6$

→ heavy if $\xi > 1/3$

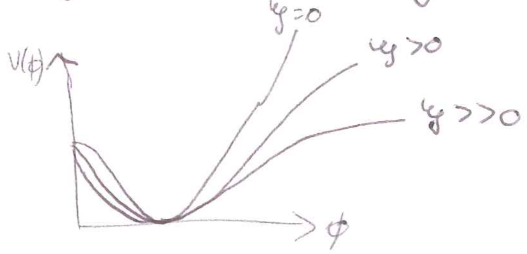
CASE B

From [4]



<u>case B</u>: $W = -\lambda S\left(\Phi^2 - \frac{v^2}{2}\right)$; $\xi \neq 0$; $K = -3\log[\cdots]$

- $V(\phi) = \frac{\lambda^2}{4} \frac{(\phi^2 - v^2)^2}{(1+\xi\phi^2)^2}$



- ▫ $\xi \ll 1$ → get slight modifications to case A
- ▫ $\xi \gg 1$, small v → similar to Higgs Inflation in SUGRA.
- ▫ $\xi \gg 1$, large v → inflation at <u>small</u> ϕ

- non-SUGRA results for generic ξ can be applied [see figs on pages 9 and 10]
- lots of parameter space is covered.

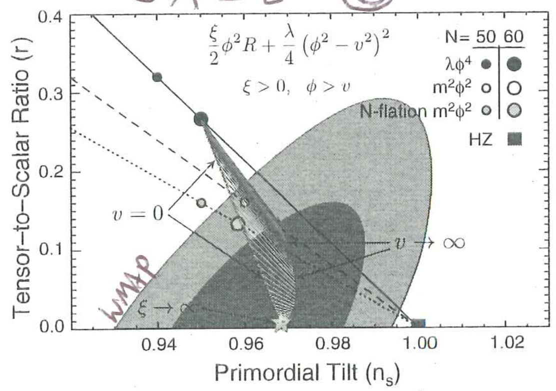
<u>case C</u>: $W = -\lambda S\left(\Phi^2 - \frac{v^2}{2}\right)$; $\xi = 0$; $K = S\bar{S} + \cdots$

- $V(\phi) = \frac{\lambda^2}{4}(\phi^2 - v^2)^2$
- $m_\beta^2 = 6H^2 + \lambda(\phi^2 + v^2)$ ⇒ stabilised
- $m_s^2 = 12\xi H^2 + 2\lambda\phi^2$

  ↳ stable

  ↳ heavy if $\xi \gtrsim 0.1$

CASE B



NB. WMAP contours!! From [4]

- observational consequences same as A

<u>case D</u>: $W = -\lambda S\left(\Phi^2 - \frac{v^2}{2}\right)$; $\xi \neq 0$; $K = S\bar{S} + \cdots$

- $V(\phi) = e^{-3\xi\phi^2} \frac{\lambda^2}{4}(\phi^2 - v^2)^2$
- for 60 e-foldings, require $|3\xi\phi^2| \lesssim 1$ for $|\phi| \lesssim 10$

  ⇒ $|\xi| \lesssim 3\times 10^{-3}$ → fine tuned.

case (e) : $W = mS\Phi$; $\xi = 0$; $K = -3\log[\cdots]$

- $V(\phi) = \frac{m^2\phi^2}{2}$
- $m_\beta^2 = 4H^2 + m^2 \Rightarrow$ stable
- $m_s^2 = 2(6\xi - 1)H^2 + m^2 \Rightarrow$ stable if $\xi > 1/6$
  heavy if $\xi > 1/3$

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case (f) : $W = mS\Phi$; $\xi \neq 0$; $K = -3\log[\cdots]$

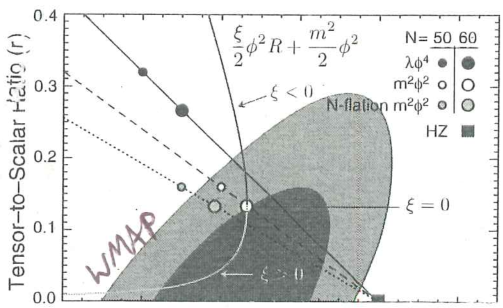
- $V(\phi) = \frac{m^2\phi^2}{2}\frac{1}{(1+\xi\phi^2)^2}$
- $\xi \ll 1$ gives flat maxima at $\phi \simeq \frac{1}{\sqrt{\xi}} \gg 1$

  $\Leftrightarrow$ inflation at $\phi \sim \frac{1}{\sqrt{\xi}}$ (near flat maxima)

  or at $\phi < \frac{1}{\sqrt{\xi}}$ (near quadratic minima)

  $\Rightarrow$ phenomenology requires $|\xi| \lesssim 10^{-2}$

CASE (f)



from [4]

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case (g) : $W = mS\Phi$; $\xi = 0$; $K = S\bar{S} + \cdots$

- $V(\phi) = \frac{m^2\phi^2}{2}$
- $m_\beta^2 = 6H^2 + m^2 \Rightarrow$ stable, heavy
- $m_s^2 = 12\xi H^2 + m^2 \Rightarrow$ stable
  $\Rightarrow$ heavy if $\xi \gtrsim 0.1$

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case (h) : $W = mS\Phi$; $\xi \neq 0$; $K = S\bar{S} + \cdots$

- $V(\phi) = \frac{m^2\phi^2}{2} e^{-3\xi\phi^2}$
- only viable if $\xi \lesssim 10^{-3}$.

case ①: general potential $W = Sf(\Phi)$; $\zeta =$; $K = -3\log[\ ]$

- $V(\phi) = \left| f\left(\frac{\phi}{\sqrt{2}}\right) \right|^2$
- $m_s^2 = 2(6\zeta - 1)H^2 + \left(f'\left(\frac{\phi}{\sqrt{2}}\right) \right)^2$

$\rightarrow$ $S=0$ is stable for $\zeta > 1/6$

- $m_\beta^2 = \frac{4}{3}\left(f\left(\frac{\phi}{\sqrt{2}}\right) \right)^2 + \left(f'\left(\frac{\phi}{\sqrt{2}}\right) \right)^2 - f\left(\frac{\phi}{\sqrt{2}}\right) f''\left(\frac{\phi}{\sqrt{2}}\right)$

↳ if $\phi \gg 1$, (e.g. chaotic inflation) $\frac{4}{3}f^2 = 4H^2$ dominates. $\Rightarrow$ stable

↳ if $\phi \ll 1$, $V(\Phi, S=0) = \frac{|f(\Phi)|^2}{(1-\beta^2/3)^2}$, $f(\phi) = 0$ is minimum.

5. Summary

- introduced models with

$$K = -3\log\left[1 + \frac{(\Phi - \bar{\Phi})^2}{6} - \frac{1}{3}S\bar{S} + \frac{\zeta(S\bar{S})^2}{3} + \frac{\chi(\Phi^2 - \bar{\Phi}^2)}{4} \right]$$

- $\zeta(S\bar{S})^2/3$ needed for stabilisation
- $\chi = 0 \Rightarrow$ conformal coupling ; $\chi = 2/3 \Rightarrow$ minimal coupling
- $\chi \neq 0, 2/3 \Rightarrow$ non-minimal coupling
- if stable, $W = Sf(\Phi)$ gives inflation with
  $V(\phi) = |f(\phi/\sqrt{2})|^2$.
- large part of Planck allowed parameter space is covered.
- but no sign of ζ in string theory.