Fast evaluation of one-loop matrix elements with OpenLoops

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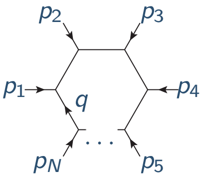
Outline

1. **OpenLoops matrix element generator**
   - Performance
   - Numerical Stability
2. **Sherpa+OpenLoops**
3. **Examples**

OpenLoops

To calculate a one-loop amplitude, we start from Feynman diagrams, factorised into **colour factors**, tensor coefficients, and tensor integrals.



$$= \mathcal{C} \cdot \sum_{r=0}^{R} \mathcal{N}_r^{\mu_1 \dots \mu_r} \cdot \int d^d q \, \frac{q_{\mu_1} \dots q_{\mu_r}}{D_0 \; D_1 \; \dots \; D_{N-1}} \qquad D_i = (q + \textstyle\sum_{\ell=0}^{i} p_\ell)^2 - m_i^2$$

The OpenLoops matrix element generator implements the open loops algorithm for the recursive numerical construction of the coefficients $\mathcal{N}_r^{\mu_1 \dots \mu_r}$ in 4 dimensions, combined with tensor integral reduction (Collier [Denner, Dittmaier, Hofer]) or, alternatively,
OPP reduction (CutTools [Ossola, Papadopoulos, Pittau],
Samurai [Mastrolia, Ossola, Reiter, Tramontano]).

Universal building blocks: connect vertices and propagators around the loop, factorising the loop momenta.

Now also independently implemented in MadLoop

Paradigm shifts in one-loop calculations

“Traditional approach”

amplitude = coefficients $\otimes$ tensor integrals

Algebraic methods, complexity limited by huge expressions (slow generation, large code), fast evaluation possible

“NLO revolution”

amplitude = trees $\otimes$ on-shell reduction

loop information is lost and reconstructed by multiple evaluations of tree amplitudes, easy to automate, compact code possible, but slow evaluation and problems with numerical stability

“NLO counter revolution”

back to the tensor integral representation (retaining loop information), but construct coefficients by a tree-like numerical recursion, up to 2 orders of magnitude speed-up wrt. “trees $\otimes$ on-shell” + numerically stable tensor intgeral evaluation

Performance

process	diags	size/MB	time/ms
$u\bar{u} \to t\bar{t}$	11	0.1	0.27(0.16)
$u\bar{u} \to W^+W^-$	12	0.1	0.14
$u\bar{d} \to W^+g$	11	0.1	0.24
$u\bar{d} \to Zg$	34		0.75
$gg \to t\bar{t}$	44	0.2	1.6(0.7)
$u\bar{u} \to t\bar{t}g$	114	0.4	4.8(2.4)
$u\bar{u} \to W^+W^-g$	198	0.4	3.4
$u\bar{d} \to W^+gg$	144	0.5	4.0
$u\bar{d} \to Zgg$	408		17
$gg \to t\bar{t}g$	585	1.2	40(14)
$u\bar{u} \to t\bar{t}gg$	1507	3.6	134(101)
$u\bar{u} \to W^+W^-gg$	2129	2.5	89
$u\bar{d} \to W^+ggg$	1935	4.2	120
$u\bar{d} \to Zggg$	5274		524
$gg \to t\bar{t}gg$	8739	16	1460(530)

Measured on an i7-3770K (single thread) with gfortran 4.8 -O0, dynamic (ifort static ∼30% faster), tensor integral reduction with Collier.

Colour and helicity summed.

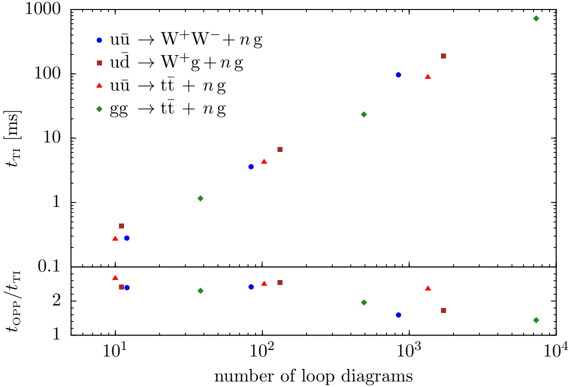
W production includes leptonic decays and non-resonant contributions.

$t\bar{t}$ production numbers in brackets are for massless decays.

$2 \to 4$ runtime range: 10 ms (6 quarks) – 2 s (6 gluons)

Scaling behaviour of runtimes

(i5-750 + ifort, not updated yet to i7-3770K + gfortran)



Upper frame: runtime of a colour and helicity summed matrix element using tensor integrals.

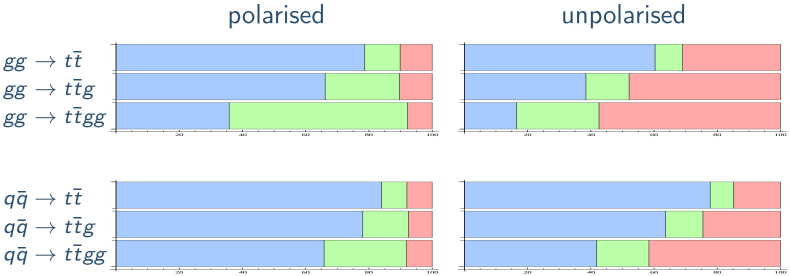
Lower frame: ratio of the runtime using OPP and tensor integrals.

- almost linear behaviour with the number of diagrams.

Helicity sums

single helicity: time for tensor reduction $\gg$ time for coefficients

full helicity sum: time for tensor reduction $\approx$ time for coefficients



fractions of total runtime for scalar integrals, tensor reduction, coefficients

Full helicity sums cost only a factor ~ 2

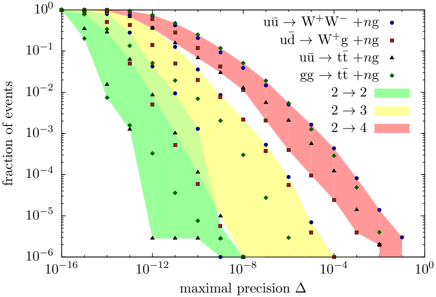
Runtime fractions suggest limited potential for algorithmic improvements.

Numerical Stability

numerical precision
using tensor integrals
in double precision

11-15 digits on average,
1 permille with <5 digits
in the worst 2 → 4 case
for well separated particles

$\sqrt{s} = 1\,\text{TeV}$, $p_T > 50\,\text{GeV}$, $\Delta R_{ij} > 0.5$, 10^6 points/process



- Numerically stable in double precision almost everywhere.
- "Suspicious" points are detected on-the-fly and rescued if possible.
- Critical kinematics: decaying particles can be aligned with the beam: in $pp \to \ell\ell\nu\nu j$ a fraction of $O(10^{-4}\text{-}10^{-5})$ of the points is unstable.
- In NNLO real-virtual corrections, MC integration in soft regions is stable down to $10^{-4}\sqrt{s}$ in double precision.

Sherpa+OpenLoops

Full automation of NLO simulations:
combine OpenLoops with Monte Carlo event generators.

Tool of choice: **Sherpa** [Gleisberg et al. '09], provides

- IR subtraction, real emission, phase space integration
- parton shower and MC@NLO matching [Höche, Krauss, Schönherr, Siegert '12]
- MEPS@NLO multi-jet merging [Höche, Krauss, Schönherr, Siegert '13]
- Hadronisation, underlying event, . . .

Sherpa+OpenLoops is steered by standard Sherpa runcards,
matrix element generation is completely transparent to the user.

Process libraries for ATLAS and CMS

The Sherpa-OpenLoops interface is included in the recent **Sherpa 2.0** release.

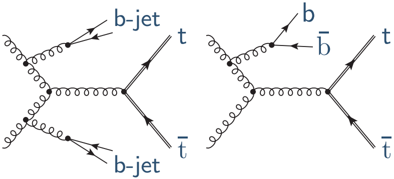
- Libraries for a wide range of processes are available to the ATLAS and CMS Monte Carlo working groups.

W/Z	γ	jets	HQ pairs	single-top	Higgs
$V+3j$	$\gamma+3j$	$3(4)j$	$t\bar{t}+1j$	$tb+1j$	$(H+2j)$
$VV+1(2)j$	$\gamma\gamma+1(2)j$		$t\bar{t}V+0(1)j$	$t+1(2)j$	$VH+1j$
$gg\to VV+1j$	$V\gamma+1(2)j$		$b\bar{b}V+0(1)j$	$tW+0(1)j$	$t\bar{t}H$
$VVV+0(1)j$					$qq\to Hqq+0(1)j$

- Validated process-by-process (>100 partonic channels).
- All contributing 1-loop diagrams, full colour.
- Off-shell leptonic W/Z decays (complex masses).

Please give us your feedback,
we'll be happy to assist if any problems occur

Example 1: $t\bar{t}H(\to b\bar{b})$ backgrounds

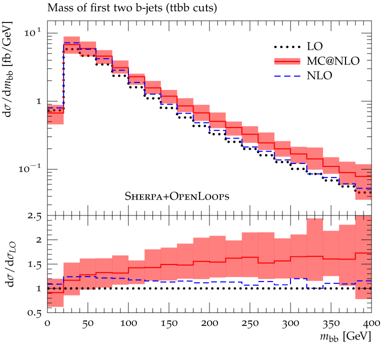


MC@NLO matching for $t\bar{t}b\bar{b}$ with massive b quarks

[Cascioli, PM, Moretti, Pozzorini, Siegert '13]

- Background to $t\bar{t}H(\to b\bar{b})$
- Signal/background $\sim 10\%$
- m_b regulates collinear singularities; NLO description of collimated $b\bar{b}$ pairs (previously $t\bar{t}g$ + PS $g \to b\bar{b}$)
- MC@NLO: $> 30\%$ effect in the signal region ($m_{b\bar{b}} > 100\,\mathrm{GeV}$)
- MC@NLO excess disappers when $g \to b\bar{b}$ PS splittings are disabled $\to$ important effect of second splitting

with massless b quarks: [Bredenstein, Denner, Dittmaier, Pozzorini '08, '09, '10; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09]



Example 2: $WW + 0{,}1$ jets

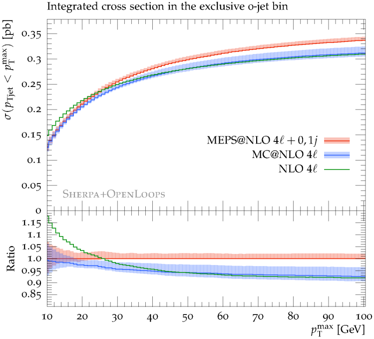
[Cascioli, Höche, Krauss, P. M., Pozzorini, Siegert]

Background to $H \to WW^*$ in exclusive jet bins.

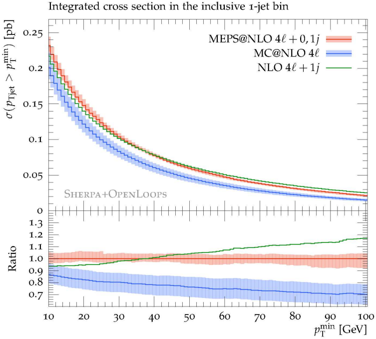
Study the impact of parton shower, loop^2, and jet merging.

Exclusive observables, jet vetoes → need resummation.

MEPS@NLO: NLO accurate in 0- and 1-jet bin.



Integrated cross section in the exclusive 0-jet bin

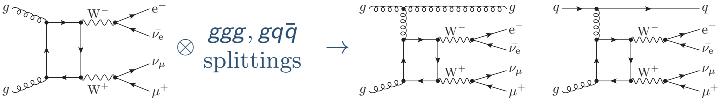


Integrated cross section in the inclusive 1-jet bin

1-jet bin (right): 20-30% deficit of MC@NLO (LO accurate only), up to 20% excess of fixed order NLO in the tail (scale not adapted to jet p_T).

At loop^2-level the gluon fusion channel $gg \to 4\ell$ opens,
a finite and gauge invariant subset of NNLO contributions.

Sizable contribution in the signal region.
Parton shower introduces $gq/g\bar{q}/q\bar{q}$ channels via $g \to q\bar{q}$ splittings



→ must be included in matrix elements for consistent merging.

Loop^2 introduces genuine NNLO shape distorsions
→ use to estimate extrapolation uncertainties from control to signal region (cannot be done by scale variations).

σ_S/σ_C	NLO	MC@NLO	MEPS@NLO	MEPS@NLO+LOOP^2	$\delta_{S/C}$
0-jets	$0.615\,^{-0.1\%}_{-0.1\%}$	$0.622\,^{-0.7\%}_{+0.1\%}\,^{+0.2\%}_{-0.4\%}$	$0.624\,^{+0\%}_{-0.3\%}\,^{+0.5\%}_{-0\%}$	$0.632\,^{-0.3\%}_{+0.5\%}\,^{+0.2\%}_{+0.3\%}$	1.3%
1-jet	$0.339\,^{+1.4\%}_{-3.4\%}$	$0.326\,^{-2.3\%}_{-0.1\%}\,^{+1.2\%}_{+0.1\%}$	$0.331\,^{+0.5\%}_{-2.1\%}\,^{+1.5\%}_{-0\%}$	$0.338\,^{-0.4\%}_{-1.8\%}\,^{+1.8\%}_{+0.1\%}$	2.1%

Summary

OpenLoops generator for one-loop amplitudes

- Automatic, fast, numerically stable.
- Virtual corrections to 2 → 4 processes (plus decays) are no more a serious bottleneck

Sherpa+OpenLoops

- Full automation of NLO simulations, including MC@NLO matching to parton shower and MEPS@NLO multi-jet merging
- Uses standard Sherpa runcards

Be aware that the new NLO tools might reveal unexpected structures in simulations, which must be understood
→ don't use the tools too blindly.