Computer Exercise Straight line trajectory fit

Physics example: A muon track is measured in four layers of streamer tube detectors at x positions of 4., 5., 6. and 7. (in cm), with a measurement precision for y of 0.5 cm. The goal is to determine its trajectory, assuming it to be a straight line.

 $www.desy.de/\tilde{o}behnke/stat/school\_apr14/StraightLineFit.C$ Macro StraightLineFit.C, accessible at

fits a straight line track trajectory through four measured points.

• Steering parameters in the macro:

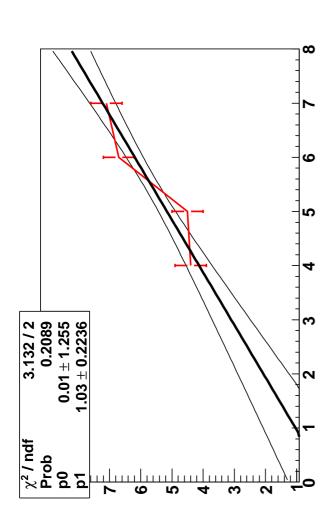
-xmin, xmax = Interval of the trajectory displayed

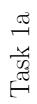
- Output:
- Histogram data (it's of the type TGraphErrors)
- Plots are drawn of the
- \* fitted histogram with error bands
- \* error ellipse of the two fitparameters

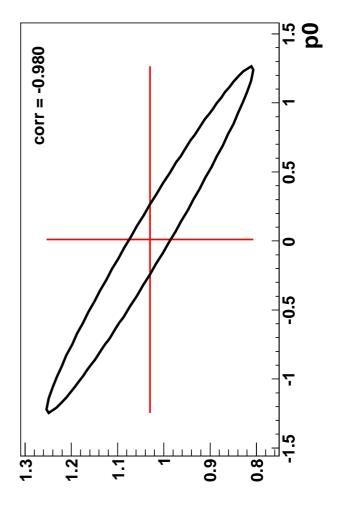
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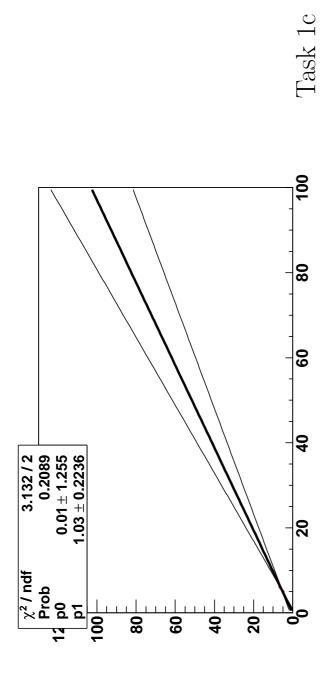
- a) Run the macro as it is by .x StraightLineFit.C and fill the fit results for  $p0,\ p1,$  their errors and correlation into the table below
- b) Precision of trajectory: Evaluate (by eye) from the shown error bands at which point roughly the trajectory is known best and with which precision (fill the results in the table below)
- c) Precision of extrapolated trajectory: Evaluate the precision of the extrapolated trajectory at x=100(Hint: Change xmax to large value and run the macro again)
- d) Effect of shift of x coordinate origin: Shift all four xVal points in the macro (simply by overwriting by hand) by a constant value -5.5, set xmin = -4. and xmax = 4. and run the macro again. Fill the fit results in the table. Can you explain why the correlation of p0 and p1 has changed?
- e) Apply a very precise vertex constraint at the origin: Change N to 5 and add a new first point to by hand). Run the macro again and write down the fitted results in the table. How much are the the measurement points list with xVal = 0.0, xErr = 0.0, yVal = 0.0 and yErr = 0.001 (just parameter errors reduced by adding this extra point?

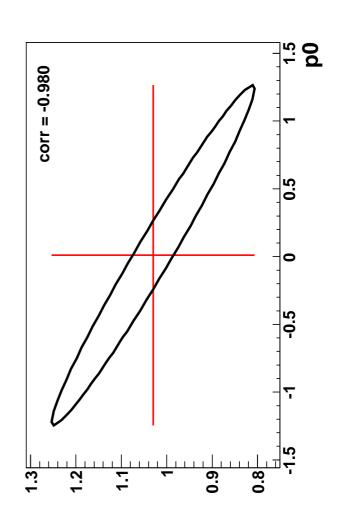
$p0 = 0.01 \pm 1.255$ Task a) $p1 = 1.03 \pm 0.224$ corr = -0.980Task b) $x$ -best precision = 5.5Task b) $y$ -error = 0.3Task c) $y$ -error( $x = 100$ ) = 22 cmTask d) $p0 = 5.7 \pm 0.25$ $p1 = 1.03 \pm 0.22$ Task e) $p1 = 1.03 \pm 0.22$ Task e) $p1 = 1.03 \pm 0.22$ Task e) $p0 = 0.0 \pm 0.0011$ Task e) $p1 = 1.032 \pm 0.044$ corr = 0.0Corr = 0.0Task e) $p1 = 1.032 \pm 0.044$		Straight line fit trough four points
$p1 = 1.03 \pm 0.224$ corr = -0.980 x-best precision = 5.5 y-error = 0.3 y-error = 0.3 y-error(x = 100) = 22 cm y-error(x = 100) = 22 cm p0 = 5.7 \pm 0.25 p0 = 5.7 \pm 0.25 p1 = 1.03 \pm 0.22 corr = 0.0 p1 = 1.03 \pm 0.22 corr = 0.0 p1 = 1.032 \pm 0.044 p1 = 1.032 ± 0.044 corr = 0.0		$p0 = 0.01 \pm 1.255$
corr = -0.980         x-best precision = 5.5         y-error = 0.3         y-error = 0.3         y-error (x = 100) = 22 cm         y-error(x = 100) = 22 cm         p0 = 5.7 ± 0.25         p1 = 1.03 ± 0.22         corr = 0.0         Adding vertex constraint at $x =$ p0 = 0.0 ± 0.0001         p1 = 1.032 ± 0.044         corr = 0.0	Task a)	$p1 = 1.03 \pm 0.224$
x-best precision = 5.5         y-error = 0.3         y-error(x = 100) = 22 cm         y-error(x = 100) = 22 cm         Shifting all x values by -5.5: $p0 = 5.7 \pm 0.25$ $p1 = 1.03 \pm 0.22$ corr = 0.0         Adding vertex constraint at $x =$ $p0 = 0.0 \pm 0.0001$ $p1 = 1.032 \pm 0.044$ corr = 0.0 $p1 = 1.032 \pm 0.044$ corr = 0.0		corr = -0.980
$y$ -error = 0.3 $y$ -error( $x = 100$ ) = 22 cm $y$ -error( $x = 100$ ) = 22 cm         Shifting all x values by -5.5: $p0 = 5.7 \pm 0.25$ $p1 = 1.03 \pm 0.22$ $p1 = 1.03 \pm 0.22$ corr = 0.0 $p0 = 0.0 \pm 0.001$ $p1 = 1.032 \pm 0.044$ $p0 = 0.0 \pm 0.001$ $p1 = 1.032 \pm 0.044$ corr = 0.0		x-best precision = 5.5
$y$ -error( $x = 100$ ) = 22 cm         Shifting all x values by -5.5: $p0 = 5.7 \pm 0.25$ $p1 = 1.03 \pm 0.22$ corr = 0.0         Adding vertex constraint at $x =$ $p0 = 0.0 \pm 0.0001$ $p1 = 1.032 \pm 0.044$ corr = 0.0         corr = 0.0		y-error = 0.3
Shifting all x values by -5.5: $p0 = 5.7 \pm 0.25$ $p1 = 1.03 \pm 0.22$ corr = 0.0 Adding vertex constraint at $x =$ $p0 = 0.0 \pm 0.0001$ $p1 = 1.032 \pm 0.044$ corr = 0.0 corr = 0.0	Task c)	y-error $(x=100)=22$ cm
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$p1 = 1.03 \pm 0.22$ corr = 0.0 Adding vertex constraint at $x =$ $p0 = 0.0 \pm 0.0001$ $p1 = 1.032 \pm 0.044$ corr = 0.0	Tack d	$p0 = 5.7 \pm 0.25$
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Adding vertex constraint at $x = p0 = 0.0 \pm 0.0001$ $p1 = 1.032 \pm 0.044$ corr = 0.0		corr = 0.0
		Adding vertex constraint at $x = 0$ :
	Tack o	$p0 = 0.0 \pm 0.0001$
corr = 0.0		$p1 = 1.032 \pm 0.044$
		corr = 0.0



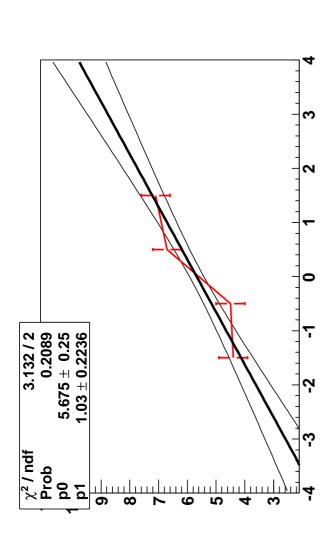




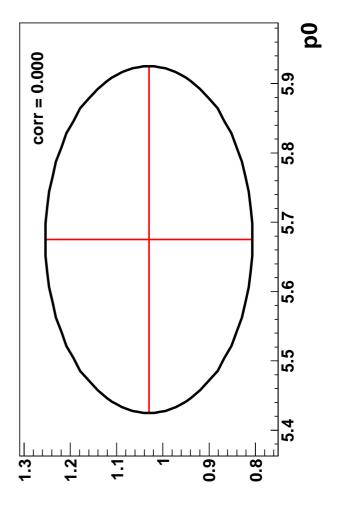


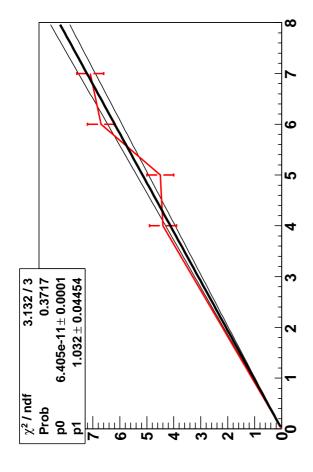


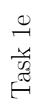
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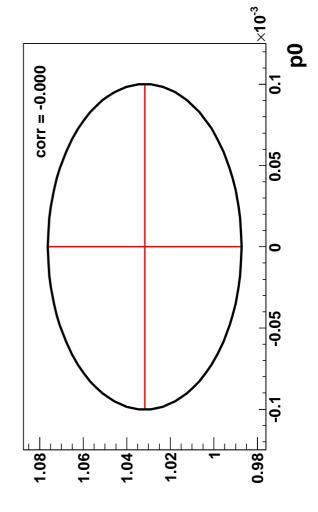


Task 1c









7

 $\sigma_i^2 \quad \rho_{ij}\sigma_i\sigma_j$ of two parameters  $\theta_i$  and  $\theta_j$  can be represented by error ellipses (see Fig. from PDG below) The role of the correlation coefficient The covariance matrix V = $ho_{ij}$ :

- If one shifts  $\theta_i$  to  $\hat{\theta}_i + \sigma_i$  one has to shift  $\theta_j$  to  $\hat{\theta}_j + \rho_{ij}\sigma_j$  to keep the  $\chi^2$  increase minimal (stay down in the  $\chi^2$  valley)
- When fixing  $\theta_j$  the error on  $\theta_i$  is reduced to  $\sigma_{inner} = \sqrt{1 \rho^2} \sigma_i$

