Computer Exercise Straight line trajectory fit

Physics example: A muon track is measured in four layers of streamer tube detectors at x positions of 4., 5., 6. and 7. (in cm), with a measurement precision for y of 0.5 cm. The goal is to determine its trajectory, assuming it to be a straight line.

Macro StraightLineFit.C, accessible at
www.desy.de/~obehnke/stat/school_apr14/StraightLineFit.C

fits a straight line track trajectory through four measured points.

- Steering parameters in the macro:
  - $xmin$, $xmax$ = Interval of the trajectory displayed
- Output:
  - Histogram $data$ (it's of the type TGraphErrors)
  - Plots are drawn of the
    - fitted histogram with error bands
    - error ellipse of the two fitparameters

Tasks:

a) Run the macro as it is by .x StraightLineFit.C and fill the fit results for $p0$, $p1$, their errors and correlation into the table below

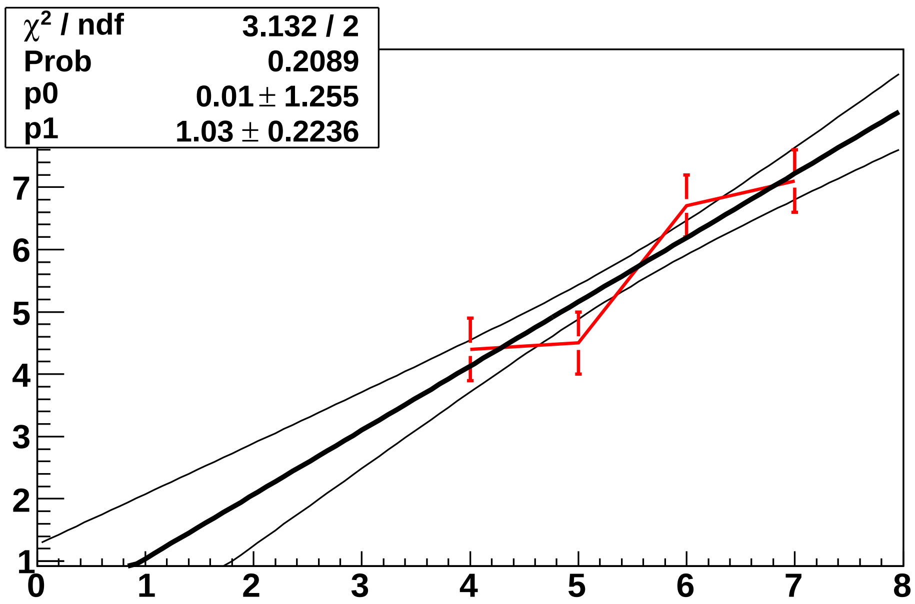
b) Precision of trajectory: Evaluate (by eye) from the shown error bands at which point roughly the trajectory is known best and with which precision (fill the results in the table below)

c) Precision of extrapolated trajectory: Evaluate the precision of the extrapolated trajectory at $x = 100$ (Hint: Change $xmax$ to large value and run the macro again)

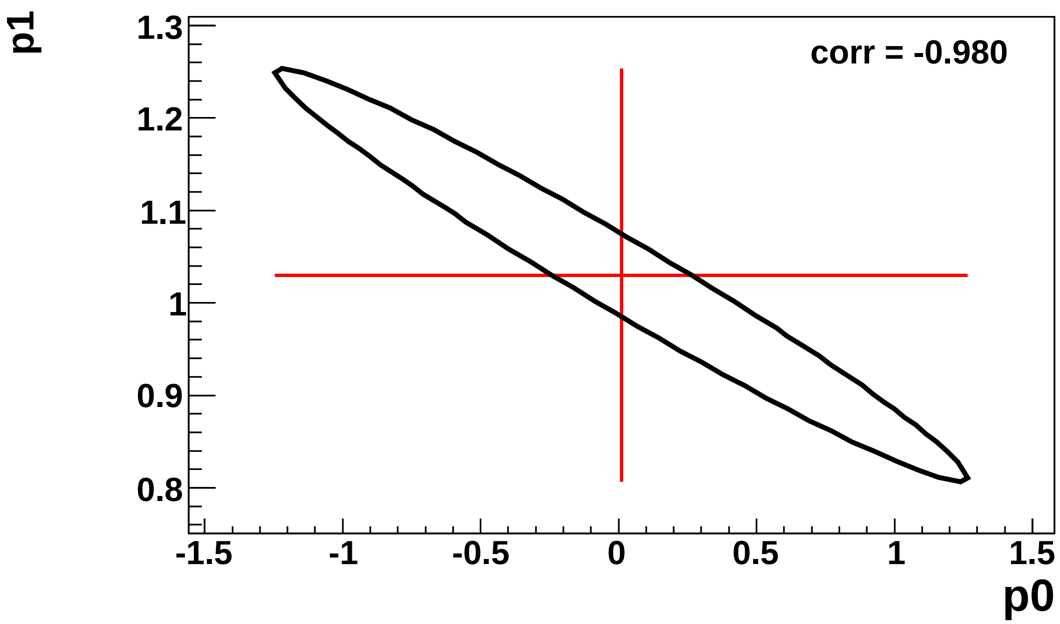
d) Effect of shift of x coordinate origin: Shift all four $xVal$ points in the macro (simply by overwriting by hand) by a constant value -5.5, set $xmin = -4.$ and $xmax = 4.$ and run the macro again. Fill the fit results in the table. Can you explain why the correlation of $p0$ and $p1$ has changed?

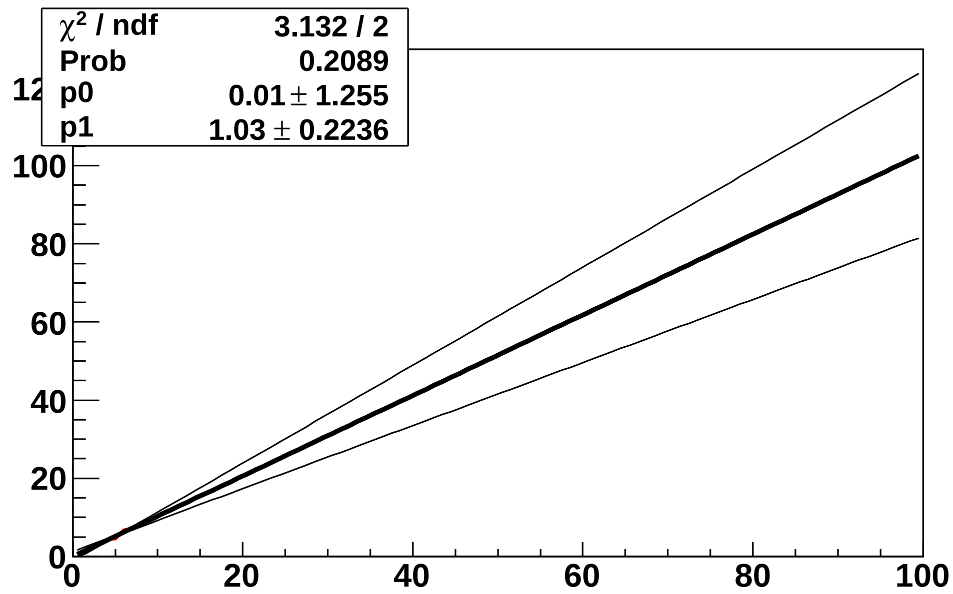
e) Apply a very precise vertex constraint at the origin: Change N to 5 and add a new first point to the measurement points list with $xVal = 0.0$, $xErr = 0.0$, $yVal = 0.0$ and $yErr = 0.0001$ (just by hand). Run the macro again and write down the fitted results in the table. How much are the parameter errors reduced by adding this extra point?

	Straight line fit trough four points
Task a)	$p0 = 0.01 \pm 1.255$ $p1 = 1.03 \pm 0.224$ corr = -0.980
Task b)	x-best precision = 5.5 y-error = 0.3
Task c)	y-error($x = 100$) = 22 cm
Task d)	Shifting all x values by -5.5: $p0 = 5.7 \pm 0.25$ $p1 = 1.03 \pm 0.22$ corr = 0.0
Task e)	Adding vertex constraint at $x = 0$: $p0 = 0.0 \pm 0.0001$ $p1 = 1.032 \pm 0.044$ corr = 0.0


χ^2 / ndf 3.132 / 2
Prob 0.2089
p0 0.01 ± 1.255
p1 1.03 ± 0.2236
1 2 3 4 5 6 7
0 1 2 3 4 5 6 7 8

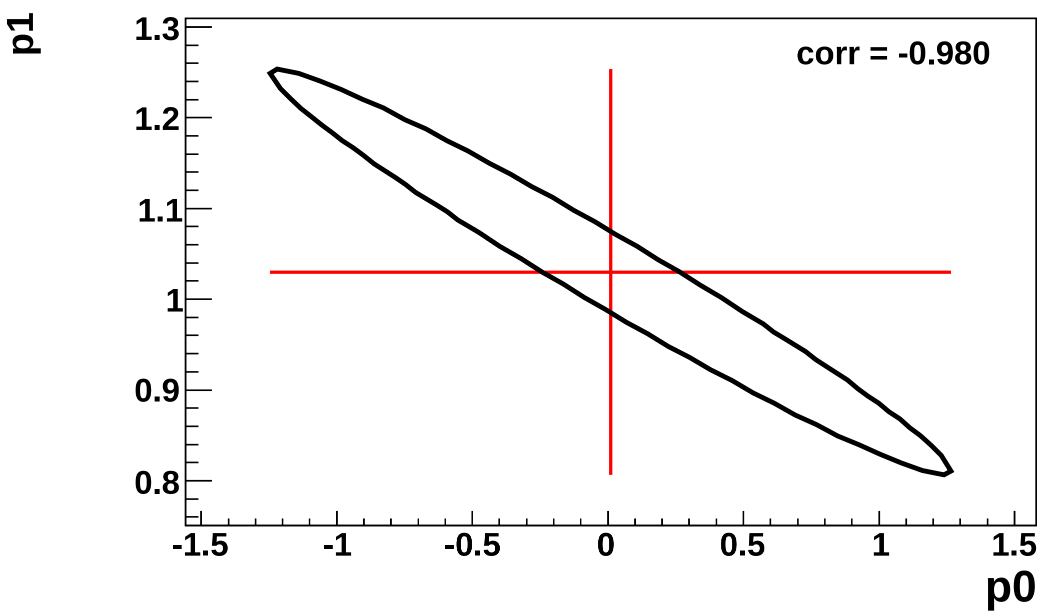
Task 1a


corr = -0.980
p1
1.3 1.2 1.1 1 0.9 0.8
-1.5 -1 -0.5 0 0.5 1 1.5
p0

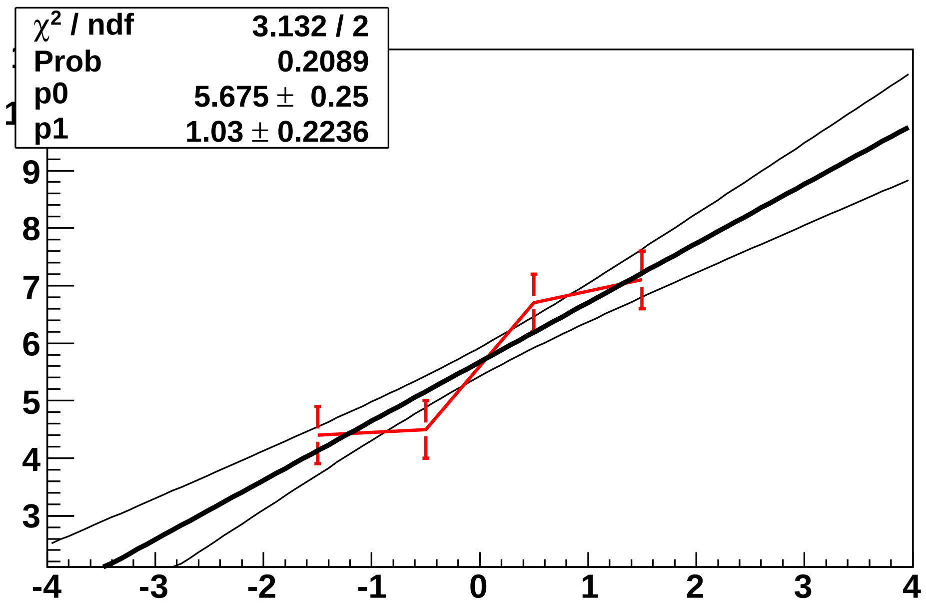


χ^2 / ndf 3.132 / 2
Prob 0.2089
p0 0.01 ± 1.255
p1 1.03 ± 0.2236
0
20
40
60
80
100

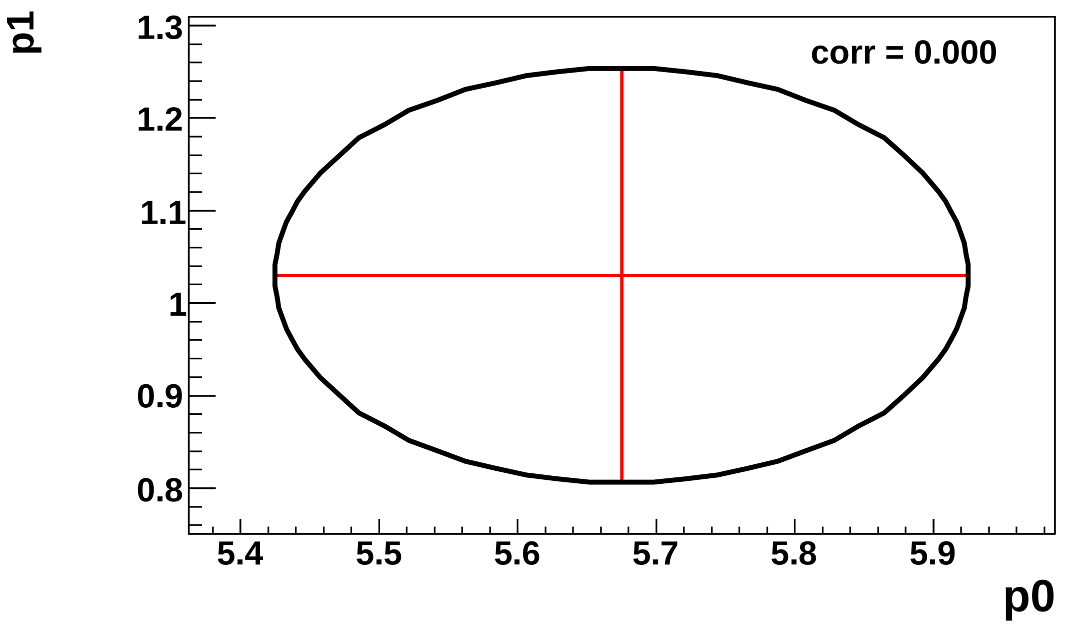
Task 1c

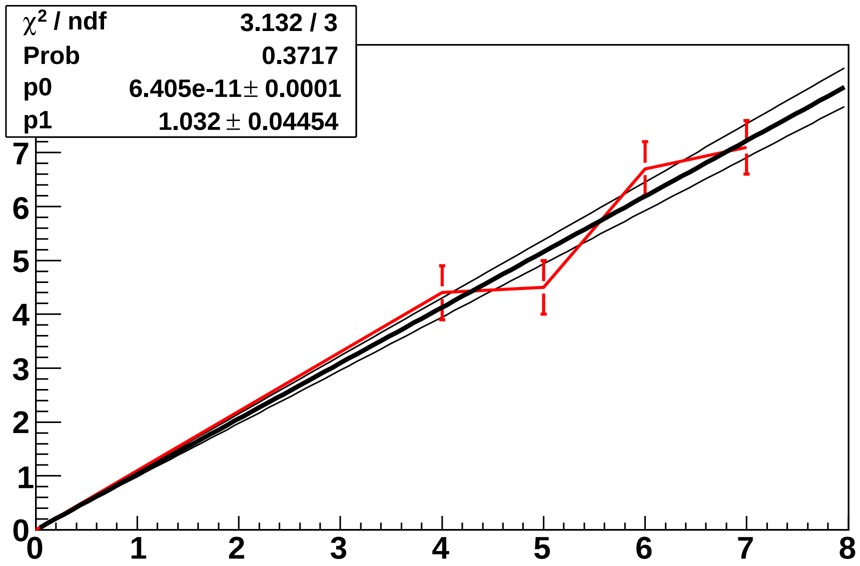


corr = -0.980
p1
p0
1.3
1.2
1.1
1
0.9
0.8
-1.5
-1
-0.5
0
0.5
1
1.5

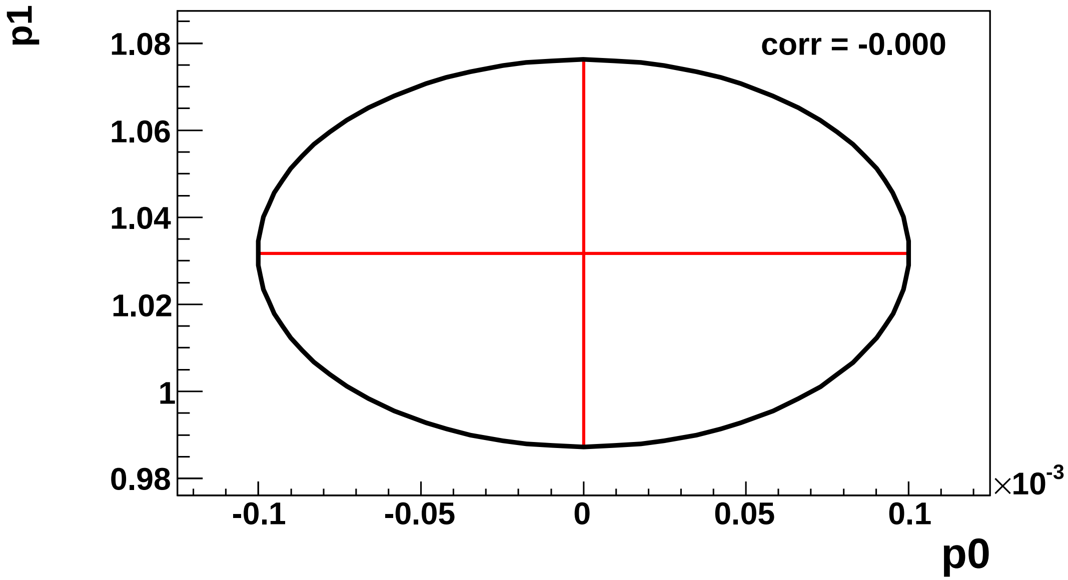

χ^2 / ndf 3.132 / 2
Prob 0.2089
p0 5.675 ± 0.25
p1 1.03 ± 0.2236
9
8
7
6
5
4
3
-4
-3
-2
-1
0
1
2
3
4

Task 1c


corr = 0.000
p1
1.3
1.2
1.1
1
0.9
0.8
5.4
5.5
5.6
5.7
5.8
5.9
p0


χ^2 / ndf 3.132 / 3
Prob 0.3717
p0 6.405e-11 ± 0.0001
p1 1.032 ± 0.04454

Task 1e


corr = -0.000
p0
p1
×10^-3

The covariance matrix $V = \begin{pmatrix} \sigma_i^2 & \rho_{ij}\sigma_i\sigma_j \\ \rho_{ij}\sigma_i\sigma_j & \sigma_j^2 \end{pmatrix}$ of two parameters θ_i and θ_j can be represented by error ellipses (see Fig. from PDG below) The role of the correlation coefficient ρ_{ij}:

- If one shifts θ_i to $\hat{\theta}_i + \sigma_i$ one has to shift θ_j to $\hat{\theta}_j + \rho_{ij}\sigma_j$ to keep the χ^2 increase minimal (stay down in the χ^2 valley)
- When fixing θ_j the error on θ_i is reduced to $\sigma_{inner} = \sqrt{1-\rho^2}\,\sigma_i$