

# IGST 2014: Concluding Talk

... → IGST 2008 → ... → IGST 2010 → ... → IGST 2014 → ...

Same general aims:

- understand quantum gauge theories at any coupling:  
vacua, dimensions, correlators, amplitudes, hidden symmetries, ...
- understand quantum string theories in curved backgrounds  
related to AdS/CFT, gauge/string duality
- develop new non-perturbative methods  
uncover unifying relations between different subjects

## Collection of subjects:

- integrable string sigma models and AdS/CFT

Gromov, Cavaglia, Banjok, Komatsu, Sfondrini, Vicedo, Frolov

- scattering amplitudes

Basso, Lipatov, Sever, Lukowski, Sprenger, Lipstein

- aspects of supersymmetric gauge theory

Pasquetti, Nekrasov, Rastelli, Yamazaki, Mitev, Bissi

- new methods and applications

Bazhanov, Dorigoni, Goncharov, Flauger

from Summary talk at IGST 2010:

## Directions and Open Problems

- algebraic ideas should bring some fruit:  
bridge the gap between SYM and SM (beyond Y/Yangian ? .... )
- reformulation of TBA? Analytic solution at strong coupling
- Pohlmeyer reduction – TBA for a Lorentz-invariant system ?  
solution of generalized SG-type models
- integrability of string sigma-model: further implications  
for WL's / amplitudes, correlation functions, ...
- generalizations:  $AdS_n \times M^k$  models, ( $\beta$ -) deformations, ...
- less susy, non-critical strings, non-planar, ...

## Some IGST 2014 observations:

- importance of **deformation**: gives new perspective or regularization  
[Vicedo, Frolov, Nekrasov, Bazhanov, Lukowski, Pasquetti,...]
- new remarkable **non-perturbative** results  
quantum spectral curve and null WL OPE bringing fruit  
[Gromov, Basso, Sever, ...]
- new exact results from integrability/localization call for better  
understanding **non-perturbative** 4d/2d QFT and developing new tools  
[Nekrasov, Bissi, Bazhanov, Dorigoni, ...]
- one should not forget about **physical** applications  
[Flauger, Lipatov, ...]

## String sigma models

- which are physically interesting?
- which are integrable/solvable? perturb around integrable case?
- solvable examples with gauge-theory (“QCD string”) connection?
- formal approach: start with simplest most symmetric cases and consider **integrable deformations** [Vicedo, Frolov]
- deformations with hidden quantum group symmetry  
e.g.  $U_q(\mathfrak{psu}(2, 2|4))$  – good motivation
- potentially important lessons:
  - do not trust too much point-particle /supergravity limit – string sees hidden (super)symmetries, may avoid apparent singularities,
  - role of other (non-local, non-abelian, duality) relations
  - place of Pohlmeyer reduction (generalized SG) theory in full picture?
  - use of complexification / double of  $\mathfrak{psu}(2, 2|4)$ ?
- quantum group symmetry on gauge theory side?

$AdS_2 \times S^2 \times T^6$  superstring: closely related supercoset

$$\frac{PSU(1, 1|2)}{SO(1, 1) \times SO(2)}$$

[Berkovits, Bershadsky, Hauer, Zhukov, Zwiebach 99]

[Sorokin, AT, Wulff, Zarembo 11]

embedding to IIB string: D3-D3-D3-D3 [Klebanov, AT 96]

$$ds^2 = -(1 + \rho^2)dt^2 + \frac{d\rho^2}{1 + \rho^2} + (1 - r^2)d\varphi^2 + \frac{dr^2}{1 - r^2} + dT^6$$

$$F_5 = \Omega_2(AdS_2) \wedge \Omega_3(T^6) + *, \quad \Omega_3 = \text{Re}(dz^1 \wedge dz^2 \wedge dz^3)$$

effective 4d theory:

$$L = e^{-2\Phi} [R + 4(\partial_m \Phi)^2] - \frac{1}{4} F_{mn} F^{mn} + \dots$$

Bertotti-Robinson:  $AdS_2 \times S^2$  with  $\Phi = 0$  and

$$F_2 = \sqrt{2}(d\rho \wedge dt + dr \wedge d\varphi)$$

## Deformed $AdS_2 \times S^2$ metric

deformed supercoset action leads to 4d metric

$$ds_A^2 + ds_S^2 = \frac{1}{1 - \kappa^2 \rho^2} \left[ -(1 + \rho^2) dt^2 + \frac{d\rho^2}{1 + \rho^2} \right]$$
$$+ \frac{1}{1 + \kappa^2 r^2} \left[ +(1 - r^2) d\varphi^2 + \frac{dr^2}{1 - r^2} \right]$$

- $ds_S^2$  = “sausage” model [Fateev, Onofri, Zamolodchikov 93]

deformation of  $S^2$  stable under RG flow:

$$ds^2 = f(y)(d\varphi^2 + dy^2), \quad \frac{\partial f}{\partial t} \sim R(f) \sim e^{-f} \partial_y^2 f$$

$$ds^2 = \frac{dy^2 + d\varphi^2}{\cosh^2 y + \kappa^2 \sinh^2 y} = \frac{1}{1 + \kappa^2 r^2} \left[ (1 - r^2) d\varphi^2 + \frac{dr^2}{1 - r^2} \right]$$

- curvature of deformed  $AdS_2$  singular at  $\rho \rightarrow \kappa^{-1}$

$$R = 4(1 + \kappa^2) \left[ -\frac{1}{1 - \kappa^2 \rho^2} + \frac{1}{1 + \kappa^2 r^2} \right]$$

Full deformed background [Lunin, Roiban, AT]

$U(1) \times U(1)$  invariant background?

A non-trivial 4d solution of

$$\int d^4x \sqrt{G} [e^{-2\Phi} (R + 4\partial^m \Phi \partial_m \Phi) - \frac{1}{2} \partial^m C \partial_m C - \frac{1}{4} F^{mn} F_{mn}]$$

direct lift to 10d type IIB solution as  $M_\kappa^2 \times T^6$ :

$F_1 = dC$  or  $F_3$  and  $F_2 = dC_1$  - reduction of  $F_5$

$$e^\Phi = [(1 - \kappa^2 \rho^2)(1 + \kappa^2 r^2)]^{-1/2} X^{1/2}(\rho, r)$$

$$C = 2a^{-1} X^{-1/2}(\rho, r) [\kappa \sqrt{1 + \kappa^2 a^2} \rho r - \sqrt{1 - a^2}]$$

$$C_1 = \sqrt{2} X^{-1/2}(\rho, r) [\sqrt{1 + \kappa^2 a^2} (\rho dt + r d\varphi) + \kappa \sqrt{1 - a^2} (r dt - \rho d\varphi)]$$

$$X = 1 + \kappa^2 a^2 (r^2 - \rho^2) - 2\kappa \sqrt{(1 - a^2)(1 + \kappa^2 a^2)} r \rho + \kappa^2 \rho^2 r^2$$

solution for any  $\kappa$  and  $a$

corresponds to supercoset for

$$a = a(\kappa) = \eta = \frac{1}{\sqrt{1 + \kappa^2} + 1}$$

- ★ metric is direct product but RR fields + dilaton are **not**
- ★ dilaton is also singular at  $\rho \rightarrow \kappa^{-1}$  :  $e^\Phi \rightarrow \infty$
- ★  $\sqrt{G}e^{-2\Phi}$  = regular:  
singularity “resolved” by formal T-duality in  $t$ , cf.

$$dr^2 + r^2 d\phi^2, \quad e^\Phi = 1 \quad \leftrightarrow \quad dr^2 + \frac{1}{r^2} d\tilde{\phi}^2, \quad e^{\tilde{\Phi}} = \frac{1}{r}$$

## Solving integrable string sigma models

- symmetry-based l.c. gauge S-matrix approach for  $AdS_3 \times S^3 \times T^6$

[Sfondrini]

importance of understanding massless modes – still at the beginning

[cf. flat space, Flauger;  $S^5$ : Basso]

comparison with perturbative string theory? requires resummation?

e.g. “dressed” propagator  $\rightarrow e \sim 2h \sin \frac{p}{2h}$  dispersion relation

several yet undetermined phases – need alternative approach?

- quantum spectral curve approach [Gromov; Cavaglia]

classical action  $\rightarrow$  Lax  $\rightarrow$  spectral curve  $\rightarrow$  quantum spectral curve

new version of integrability bootstrap – by-pass S-matrix

assumptions? 2d conformal invariance, quantum integrability, ...

extra conditions? normalizations / definitions of couplings?

how to determine  $h(\lambda)$  “intrinsically”?

similar solution for other models  $AdS_n \times S^n \times T^{10-2n}$ ?

## 3-point correlators at strong coupling: [Banjok, Komatsu]

- semiclassical limit:

HHL – use of formfactors

HHH – use of wave functions on classical one-cut solutions

- need understand string vertex operators

or is there an alternative formalism for correlators?

encode integrability data into string field theory?

## Null Wilson loops / amplitudes [Basso, Sever, Sprenger]

great progress but  
go beyond OPE / special limits ?  
what is quantum integrable system behind TBA for WL area?  
precise mathematical formulation of  
“excitations of GKP string” or “flux tube” ?

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# IGST 2015



- dates: 13-17 July 2015
- venue: King's College, Strand, London, UK
- organizers:  
N. Drukker, N. Gromov, A. Tseytlin
- <http://strongcoupling.org/igst>

