Infinite Chiral Symmetry in Four and Six Dimensions

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Based on work with
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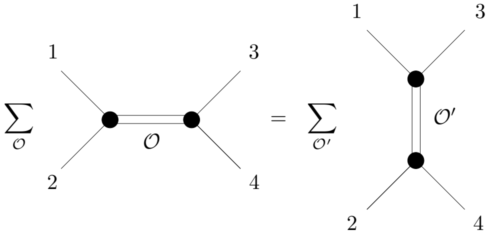
Conformal Bootstrap

Abstract algebra of local operators

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k c_{12k}(x)\mathcal{O}_k(0)$$

subject to unitarity and crossing constraints



Since 2008, successful *numerical* approach in any d.
(Rattazzi Rychkov Tonni Vichi)

Two sorts of questions for the superconformal bootstrap

What is the space of consistent SCFTs in $d \leqslant 6$?

For maximal susy, well-known list of theories.
Is the list complete?

What is the list with less susy?

Can we bootstrap concrete models?

The bootstrap should be particularly powerful for models uniquely cornered by few discrete data.
Only method presently available for finite N, non-Lagrangian theories, such as the $6d$ (2,0) SCFT.

More technically, not clear how much susy can really help.

A natural question:

> *Do the bootstrap equations in $d > 2$ admit a solvable truncation for superconformal theories?*

The answer is Yes for large classes of theories:

(A) Any $d = 4$, $\mathcal{N} \geqslant 2$ or $d = 6$, $\mathcal{N} = (2,0)$ SCFT admits a subsector $\cong$ $2d$ chiral algebra.

(B) Any $d = 3$, $\mathcal{N} \geqslant 4$ SCFT admits a subsector $\cong$ $1d$ TQFT.

Beem Lemos Liendo Peelaers LR van Rees

Beem LR van Rees

Warm-up: $\mathcal{N} = 1$ chiral ring

Chiral operators in an $\mathcal{N} = 1$, $d = 4$ QFT

$$\{\tilde{Q}_{\dot{\alpha}}\,, \mathcal{O}(x)] = 0\,, \quad \dot{\alpha} = \dot{+}, \dot{-}\,, \quad \Leftrightarrow \quad \Delta = \frac{3}{2} r$$

Further define cohomology class $[\mathcal{O}(x)]_{\tilde{Q}}$

$$\mathcal{O}(x) \sim \mathcal{O}(x) + \{\tilde{Q}_\alpha, \dots]\,.$$

From susy algebra $P = \{Q, \tilde{Q}\}$:

$$\frac{\partial}{\partial x}\mathcal{O}(x) = [P, \mathcal{O}(x)] = \{\tilde{Q}, \mathcal{O}'(x)]\,, \quad \mathcal{O}'(x) = \{Q, \mathcal{O}(x)]\,,$$

so $[\mathcal{O}(x)]_{\tilde{Q}}$ is x-independent. Concretely,

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \dots \mathcal{O}_n(x_n) \rangle = \text{constant} \qquad x_i \in \mathbb{R}^4\,.$$

In fact, in an $\mathcal{N} = 1$ superconformal theory, constant $\equiv 0$ since $r_i > 0$.

Meromorphy in $\mathcal{N} = 2$, $d = 4$ SCFTs

Fix a plane $\mathbb{R}^2 \subset \mathbb{R}^4$, parametrized by $(z, \bar{z})$.

Claim : $\exists$ subsector $\mathcal{A}_\chi = \{\mathcal{O}_i(z_i, \bar{z}_i)\}$ with meromorphic

$$\langle \mathcal{O}_1(z_1, \bar{z}_1)\, \mathcal{O}_2(z_2, \bar{z}_2) \ldots \mathcal{O}_n(z_n, \bar{z}_n) \rangle = f(z_i)\,.$$

Rationale: $\mathcal{A}_\chi \equiv$ cohomology of a nilpotent $\mathbb{Q}$,

$$\mathbb{Q} = \mathcal{Q} + \mathcal{S}\,,$$

$\mathcal{Q}$ Poincaré, $\mathcal{S}$ conformal supercharges.

$\bar{z}$ dependence is $\mathbb{Q}$-exact: cohomology classes $[\mathcal{O}(z, \bar{z})]_{\mathbb{Q}} \rightsquigarrow \mathcal{O}(z)$.

Analogous to (but richer than) the $d = 4$, $\mathcal{N} = 1$ chiral ring, where cohomology classes $[\mathcal{O}(x)]_{\tilde{\mathcal{Q}}_{\dot{\alpha}}}$ are x-independent.

Cohomology at the origin: Schur operators

At the origin of $\mathbb{R}^2$, $\mathbb{Q}$-cohomology $\mathcal{A}_\chi$ easy to describe.

$\mathcal{O}(0,0) \in \mathcal{A}_\chi \leftrightarrow \mathcal{O}$ obeys the chirality condition

$$\frac{\Delta - \ell}{2} = R$$

Δ conformal dimension
$\ell = j_1 + j_2$ spin on $\mathbb{R}^2$, with (j_1, j_2) Lorentz spins in $\mathbb{R}^4$
R Cartan generator of $SU(2)_R$ R-symmetry

These are precisely the operators counted by the Schur limit of the supeconformal index. Gadde LR Razamat Yan

Killed by 2 real Poincaré supercharges (out of 8), one $\mathcal{Q}$ and one $\tilde{\mathcal{Q}}$, an intrinsically $\mathcal{N} = 2$ condition.

Schur operators

Multiplet	$\mathcal{O}_{\rm Schur}$	h	Lagrangian "letters"
$\hat{\mathcal{B}}_R$	$\Psi^{11\ldots1}$	0	Q, $\tilde{Q}$
$\mathcal{D}_{R(0,j_2)}$	$\tilde{\mathcal{Q}}^1_{\dot{+}}\,\Psi^{11\ldots1}_{\dot{+}\ldots\dot{+}}$	$R+j_2+1$	Q, $\tilde{Q}$, $\tilde{\lambda}^1_{\dot{+}}$
$\bar{\mathcal{D}}_{R(j_1,0)}$	$\mathcal{Q}^1_{+}\,\Psi^{11\ldots1}_{+\ldots+}$	$R+j_1+1$	Q, $\tilde{Q}$, λ^1_{+}
$\hat{\mathcal{C}}_{R(j_1,j_2)}$	$\mathcal{Q}^1_{+}\,\tilde{\mathcal{Q}}^1_{\dot{+}}\,\Psi^{11\ldots1}_{+\ldots+\,\dot{+}\ldots\dot{+}}$	$R+j_1+j_2+2$	$D_{+\dot{+}}$, Q, $\tilde{Q}$, λ^1_{+}, $\tilde{\lambda}^1_{\dot{+}}$

- $\hat{\mathcal{B}}_R$: Higgs branch chiral ring operators
- $\mathcal{D}_{R(0,j_2)}/\bar{\mathcal{D}}_{R(j_1,0)}$: Additional $\mathcal{N}=1$ (anti-)chiral ring operators. "Hall-Littlewood" chiral ring.
- $\hat{\mathcal{C}}_{R(j_1,j_2)}$: Other less familiar semi-short operators. $\hat{\mathcal{C}}_{0(0,0)}$ is the stress-tensor multiplet, also containing R-symmetry currents.
- Coulomb branch $\frac{1}{2}$ BPS operators (such as $\mathrm{Tr}\,\phi^k$) not Schur.

$$[\mathbb{Q}, \mathfrak{sl}(2)] = 0 \quad \text{but} \quad [\mathbb{Q}, \overline{\mathfrak{sl}(2)}] \neq 0$$

To define $\mathbb{Q}$-closed operators $\mathcal{O}(z, \bar{z})$ away from origin,
we twist the right-moving generators by $SU(2)_R$,

$$\widehat{L}_{-1} = \bar{L}_{-1} + \mathcal{R}^- \,, \quad \widehat{L}_0 = \bar{L}_0 - \mathcal{R} \,, \quad \widehat{L}_1 = \bar{L}_1 - \mathcal{R}^+ \,.$$

$$\widehat{\mathfrak{sl}(2)} = \{\mathbb{Q}, \dots\}$$

$\mathbb{Q}$-closed operators are "twisted-translated"

$$\mathcal{O}(z, \bar{z}) = e^{zL_{-1} + \bar{z}\widehat{L}_{-1}} \, \mathcal{O}(0) \, e^{-zL_{-1} - \bar{z}\widehat{L}_{-1}} \,.$$

$SU(2)_R$ orientation correlated with position on $\mathbb{R}^2$. (Cf. Drukker Plefka)

Chirality condition $\frac{\Delta - \ell}{2} - R = 0 \Leftrightarrow \widehat{L}_0 = 0$

By the usual formal argument, the $\bar{z}$ dependence is exact,

$$[\mathcal{O}(z,\bar{z})]_{\mathbb{Q}} \quad \rightsquigarrow \quad \mathcal{O}(z)\ .$$

Cohomology classes define left-moving $2d$ operators $\mathcal{O}_i(z)$, with conformal weight

$$h = R + \ell.$$

They are closed under OPE,

$$\mathcal{O}_1(z)\mathcal{O}_2(0) = \sum_k \frac{c_{12k}}{z^{h_1+h_2-h_k}}\,\mathcal{O}_k(0)\,.$$

$\mathcal{A}_\chi$ has the structure of a $2d$ chiral algebra

Example: free hypermultiplet

Look for states with $\widehat{L}_0 = \frac{\Delta - j_1 - j_2}{2} - R = 0$.
The complex scalars Q and $\tilde{Q}$ fit the bill, since $\Delta = 1$, $R = \frac{1}{2}$, $j_1 = j_2 = 0$.
They are top components of $SU(2)_R$ doublets,

$$Q^{\mathcal{I}} = \begin{pmatrix} Q \\ \tilde{Q}^* \end{pmatrix}, \qquad \tilde{Q}^{\mathcal{I}} = \begin{pmatrix} \tilde{Q} \\ -Q^* \end{pmatrix}.$$

Away from the origin, consider twisted-translated operators

$$q(z,\bar{z}) := Q(z,\bar{z}) + \bar{z}\tilde{Q}^*(z,\bar{z}), \qquad \tilde{q}(z,\bar{z}) := \tilde{Q}(z,\bar{z}) - \bar{z}Q^*(z,\bar{z}) .$$

Elementary exercise:

$$q(z,\bar{z})\,\tilde{q}(0) \sim \bar{z}\,\tilde{Q}^*(z,\bar{z})\,\tilde{Q}(0) \sim \frac{\bar{z}}{z\bar{z}} = \frac{1}{z}.$$

The cohomology classes $[q(z,\bar{z})]_{\mathbb{Q}}$, $[\tilde{q}(z,\bar{z})]_{\mathbb{Q}}$ define a pair of symplectic bosons of weight $h = \frac{1}{2}$, for which $c_{2d} = -1$.
Normal ordered products of $\partial^n q$ and $\partial^n \tilde{q}$ span the entire chiral algebra associated to the free hypermultiplet.

Example: free vector multiplet

The gauginos λ^1_+ and $\tilde{\lambda}^1_{\dot{+}}$ satisfy $\widehat{L}_0 = 0$.

The twisted-translated operators

$$\lambda(z,\bar{z}) := \lambda^1_+(z,\bar{z}) + \bar{z}\lambda^2_+(z,\bar{z})\,, \quad \tilde{\lambda}(z,\bar{z}) := \tilde{\lambda}^1_{\dot{+}}(z,\bar{z}) + \bar{z}\tilde{\lambda}^2_{\dot{+}}(z,\bar{z})$$

give rise in cohomology to chiral fields $\lambda(z)$, $\tilde{\lambda}(z)$, with

$$\tilde{\lambda}(z)\lambda(0) \sim \frac{1}{z^2}\ , \quad \lambda(z)\tilde{\lambda}(0) \sim -\frac{1}{z^2}\ .$$

Setting

$$\tilde{\lambda}(z) := b(z)\ , \quad \lambda(z) := \partial c(z)\ .$$

we recognize a bc ghost system of weights (1, 0), for which $c_{2d} = -2$.

χ : 4d $\mathcal{N} = 2$ SCFT $\longrightarrow$ 2d Chiral Algebra.

- Virasoro enhancement of $\mathfrak{sl}(2)$, with $T(z)$ arising from a component of the $SU(2)_R$ conserved current, $T(z) := [\mathcal{J}_R(z, \bar{z})]_{\mathbb{Q}}$, with

$$c_{2d} = -12\, c_{4d}\,,$$

where c_{4d} is one of the conformal anomaly coefficient.

- Affine symmetry enhancement of global flavor symmetry, with $J(z)$ arising from the moment map operator, $J(z) := [M(z, \bar{z})]_{\mathbb{Q}}$, with

$$k_{2d} = -\frac{k_{4d}}{2}\,.$$

- Guaranteed generators:
Generators of the $4d$ Higgs branch $\Rightarrow$ generators of the chiral algebra.
Higgs branch relations encoded in null states of the chiral algebra!
(Crucial that k_{2d} takes special negative levels).
More generally, generators of HL ring $\Rightarrow$ generators of the chiral algebra.

Consequences for $4d$ physics: new unitarity bounds

Consider the full-fledged $4pt$ correlator of some protected operators.

$$\langle \mathcal{O}_1^{\mathcal{I}_1}(x_1)\mathcal{O}_2^{\mathcal{I}_2}(x_2)\mathcal{O}_3^{\mathcal{I}_3}(x_3)\mathcal{O}_4^{\mathcal{I}_4}(x_4)\rangle\,.$$

The mere existence of our twist implies the superconformal Ward identities: the correlator can be expressed in terms of some unprotected functions $\mathcal{G}_i(z,\bar{z})$, and of some protected meromorphic functions $f_i(z)$.
The chiral algebra precisely captures $f_i(z)$.

Example: $4pt$ correlator of moment maps M, in the adjoint of the flavor group G_F. The $f_i(z)$ are uniquely fixed in terms of the flavor central charge k_{4d}.

Inserting the exact expressions for $f_i(z)$ in the double OPE expansion $\Rightarrow$ general unitarity bounds for k_{4d} valid in any interacting SCFT.

A crucial assumption is that the theory has no higher spin conserved currents.
Maldacena Zhiboedov

G_F		Bound	Representation
SU(N)	$N \geqslant 3$	$k_{4d} \geqslant N$	$\mathbf{N^2-1}_{\text{symm}}$
SO(N)	$N = 4, \ldots, 8$	$k_{4d} \geqslant 4$	$\mathbf{\frac{1}{24}N(N-1)(N-2)(N-3)}$
SO(N)	$N \geqslant 8$	$k_{4d} \geqslant N-4$	$\mathbf{\frac{1}{2}(N+2)(N-1)}$
USp($2N$)	$N \geqslant 3$	$k_{4d} \geqslant N+2$	$\mathbf{\frac{1}{2}(2N+1)(2N-2)}$
G_2		$k_{4d} \geqslant \frac{10}{3}$	$\mathbf{27}$
F_4		$k_{4d} \geqslant 5$	$\mathbf{324}$
E_6		$k_{4d} \geqslant 6$	$\mathbf{650}$
E_7		$k_{4d} \geqslant 8$	$\mathbf{1539}$
E_8		$k_{4d} \geqslant 12$	$\mathbf{3875}$

Table : Unitarity bounds for k_{4d} arising from positivity in non-singlet channels.

These bounds are saturated by the SCFTs on $D3$ branes probing the F-theory singularities of type $H_1, H_2, D_4, E_6, E_7, E_8$, whose Higgs branches are one-instanton moduli spaces.

When the bounds are saturated, certain states become null in the affine Lie algebra. These nulls are interpreted in $4d$ as the "Joseph relations" on the moment map

$$(M \otimes M)|_{\mathcal{I}_2} = 0 \ , \qquad \mathrm{Sym}^2(\mathbf{adj}) = (2\,\mathbf{adj}) \oplus \mathcal{I}_2 \ .$$

In the singlet channel, the stress-tensor also contributes, and positivity implies a bound involving the conformal and flavor anomalies,

$$\frac{\dim G_F}{c_{4d}} \geqslant \frac{24h^\vee}{k_{4d}} - 12 \ .$$

When the bound is saturated,

$$c_{2d} = c_{Sugawara} = \frac{k_{2d} \dim G_F}{k_{2d} + h^\vee} \ .$$

For $\mathcal{N} = 4$, $d = 4$ SCFTs, unitarity and no HS currents imply

$$a = c \geqslant \frac{3}{4} \,,$$

saturated by $\mathcal{N} = 4$ SYM with gauge group $SU(2)$.

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Gauging prescription

Start with $4d$ SCFT $\mathcal{T}$, with flavor symmetry G_F.
We can generate a new SCFT $\mathcal{T}_G$ by gauging $G \subset G_F$, provided $\beta_G = 0$.

If we already know the chiral algebra $\chi[\mathcal{T}]$, can we find $\chi[\mathcal{T}_G]$?

Extra $4d$ vector multiplet $\Rightarrow$ extra $(b^A c_A)$ ghost system, in the adjoint of G.
We must also restrict to gauge singlets.
This is the correct answer at zero gauge coupling. But at finite coupling, some states are lifted and the chiral algebra must be smaller.

Elegant prescription to find quantum chiral algebra. Pass to the cohomology of

$$Q_{\mathrm{BRST}} := \oint \frac{dz}{2\pi i}\, j_{\mathrm{BRST}}(z)\,, \quad j_{\mathrm{BRST}} := c_A \left[J^A - \frac{1}{2} f^{AB}{}_C\, c_B b^C \right],$$

where J^A is the G affine current of $\chi[\mathcal{T}]$.

$Q^2_{\mathrm{BRST}} = 0$ precisely when the $\beta_G = 0$, which amounts to $k_{2d} = -2h^\vee$.

By this prescription, we can in principle find $\chi[\mathcal{T}]$ for any Lagrangian SCFT $\mathcal{T}$.

Some non-trivial examples

By low level-calculations of BRST cohomology and guesswork, we find that in some interesting cases the chiral algebra is finitely generated:

- $SU(2)$ gauge theory with $N_f = 4 \Rightarrow \mathfrak{so}(8)_{-2}$ AKM algebra.
- E_6 SCFT $\Rightarrow (\mathfrak{e}_6)_{-3}$ AKM algebra.
- $\mathcal{N} = 4$ SYM with gauge group $G \Rightarrow \mathcal{N} = 4$ super $\mathcal{W}$-algebra, with generators given by chiral primaries of dimensions $\{h_i = \frac{r_i+1}{2}\}$, where $\{r_i\}$ are the exponents of G.
  (So for $G = SU(2)$, simply the $\mathcal{N} = 4$ algebra.)

In these example, the simplest guess (generators coming from HL ring) works.
There are examples with additional generators.

In fact in general we don't know whether the chiral algebra need to be finitely generated.

Chiral algebras of class $\mathcal{S}$

Beem Peelaers LR van Rees, to appear

SCFTs of class $\mathcal{S}$: labelled by decorated Riemann surfaces $\mathcal{C}_{g,s}$.

By passing to the chiral algebra, we obtain
a generalized TQFT valued in chiral algebras.

Generalized S-duality gets mapped to the associativity of this TQFT.

Rich story that connects 4d physics with non-trivial 2d algebraic structures. For example, Higgsing of the flavor symmetry gets mapped to quantum Drinfeld-Sokolov reduction.

χ_6 : 6d (2,0) SCFT $\longrightarrow$ 2d Chiral Algebra.

- Global $\mathfrak{sl}(2) \to$ Virasoro, indeed $T(z) := [\mathcal{O}_{\mathbf{14}}(z, \bar{z})]_{\mathbb{Q}}$, with $\mathcal{O}_{\mathbf{14}}$ the stress-tensor multiplet superprimary,

$$c_{2d} = c_{6d}$$

in normalizations where c_{6d} (free tensor) $\equiv 1$.

- All $\frac{1}{2}$-BPS operators $(\Delta = 2R)$ are in $\mathbb{Q}$ cohomology.

Generators of the $\frac{1}{2}$-BPS ring $\to$ generators of the chiral algebra.

- Some semi-short multiplets also play a role.

Chiral algebra for $(2,0)$ theory of type A_{N-1}

One $\frac{1}{2}$-BPS generator each of dimension $\Delta = 4, 6, \ldots 2N$

$$\Downarrow$$

One chiral algebra generator each of dimension $h = 2, 3, \ldots N$.

Most economical scenario: these are all the generators.
Check: the superconformal index computed by Kim[3] is reproduced.

$$\mathcal{I}(q,s) := \mathrm{Tr}(-1)^F q^{E-R} s^{h_2+h_3} \ .$$

$$\mathcal{I}(q,s;n) = \prod_{k=1}^{n} \prod_{m=0}^{\infty} \frac{1}{1-q^{k+m}} = \mathrm{PE}\left[\frac{q+q^2+\cdots+q^n}{1-q}\right] ,$$

Plausibly a unique solution to crossing for this set of generators.

- The chiral algebra of the A_{N-1} theory is $\mathcal{W}_N$, with

$$c_{2d} = 4N^3 - 3N - 1 \, .$$

Generalization to D & E

Moduli space of vacua for $(2,0)$ theory of type $\mathfrak{g}$

$$\mathcal{M}_{\mathfrak{g}} = (\mathbb{R}^5)^{r_{\mathfrak{g}}}/W_{\mathfrak{g}} \ ,$$

$\frac{1}{2}$-BPS ring freely generated by $\mathcal{O}_i$, $i = 1, \ldots, r_{\mathfrak{g}} \leftrightarrow$ Casimir invariants of $\mathfrak{g}$.

- For the $(2,0)$ SCFT of type $\mathfrak{g}$, the chiral algebra is $\mathcal{W}_{\mathfrak{g}}$, with

$$c_{2d}(\mathfrak{g}) = 4 d_{\mathfrak{g}} h^{\vee}_{\mathfrak{g}} + r_{\mathfrak{g}} \ .$$

Connection with AGT? $c_{2d}(\mathfrak{g})$ matches Toda central charge for $b = 1$,

$$c_{\text{AGT}} = r_{\mathfrak{g}} + \left(b + \frac{1}{b}\right)^2 d_{\mathfrak{g}} h_{\mathfrak{g}} \ , \quad b^2 = \epsilon_1/\epsilon_2 \, .$$

Half-BPS 3pt functions of $(2,0)$ SCFT

OPE of $\mathcal{W}_{\mathfrak{g}}$ generators $\Rightarrow$ half-BPS 3pt functions of SCFT.

Let us check the result at large N.

$W_{N\to\infty}$ with $c_{2d} \sim 4N^3 \rightarrow$ a *classical* W-algebra.
(Gaberdiel Hartman, Campoleoni Fredenhagen Pfenninger)

We find

$$C(k_1,k_2,k_3) = \frac{2^{2\alpha-2}}{(\pi N)^{\frac{3}{2}}}\Gamma\left(\frac{\alpha}{2}\right)\left(\frac{\Gamma\left(\frac{k_{123}+1}{2}\right)\Gamma\left(\frac{k_{231}+1}{2}\right)\Gamma\left(\frac{k_{312}+1}{2}\right)}{\sqrt{\Gamma(2k_1-1)\Gamma(2k_2-1)\Gamma(2k_3-1)}}\right)$$

$k_{ijk} \equiv k_i + k_j - k_k$, $\alpha \equiv k_1 + k_2 + k_3$,
in precise agreement with calculation in $11d$ sugra on $AdS_7 \times S^4$!
(Corrado Florea McNees, Bastianelli Zucchini)

$1/N$ corrections in W_N OPE $\Rightarrow$ quantum M-theory corrections.

Codimension-two defects of the $(2,0)$ theory harbor $\mathcal{N}=2$, $d=4$ SCFTs. They are labelled by $\mathfrak{sl}(2)$ embeddings into ADE and have flavor group G_F equal to the commutant of this embedding.

Claim: the chiral algebra of these defect SCFTs is the ADE affine Lie algebra at the *critical level* $k_{2d} = -h^\vee$ for maximal flavor, and its quantum DS reduction for reduced flavor.

Connection to geometric Langlands?

Reminiscent of Alday-Tachikawa's generalization of AGT: Wrapping defects on $\mathcal{C}$, the partition functions display affine $\mathfrak{g}$ invariance at level

$$k_{2d} = -h^\vee - \frac{1}{b^2} \ .$$

Outlook

- From physical expectations about $4d$ SCFTs, interesting purely mathematical conjectures about chiral algebras.
  Develop cohomological tools to prove them.
- For a given SCFT $\mathcal{T}$, develop systematic tools to characterize $\chi[\mathcal{T}]$ in terms of generators.
- Classification of SCFTs related to classification of "special" chiral algebras.
- Add non-local operators.
  Particularly interesting in $d = 6$: a derivation of AGT?
- For Lagrangian theories, relation to localization?

Outlook: numerical boostrap

The chiral algebra determines the protected part of correlators: crucial information to set up the numerical bootstrap for the non-protected part. Some highlights:

- Stress-tensor 4pt function in $d = 4$, $\mathcal{N} = 4$. Beem LR van Rees
  Recently extended to higher BPS operators. Alday Bissi
- Stress tensor 4pt function in$d = 6$, $(2, 0)$ theory.
  Beem Lemos LR van Rees, to appear
- Exploration of landscape of $\mathcal{N} = 2$ SCFTs,
  especially non-Lagrangian ones.
  Beem Lemos Liendo LR van Ress, to appear

Intriguing interplay of striking mathematical physics and numerical experimentation.

Bootstrap of stress-tensor multiplet 4pt in $(2,0)$

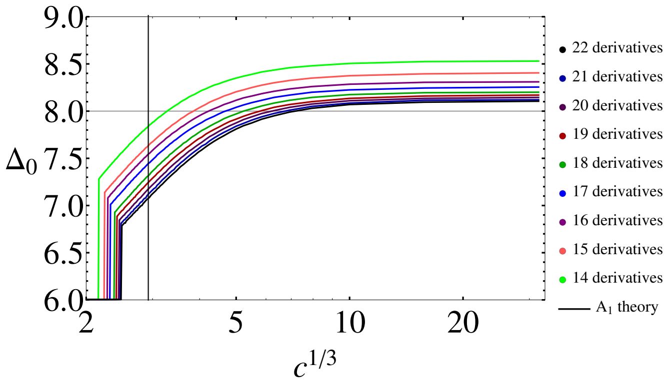

Figure : Upper bound for the dimension Δ_0 of the leading-twist unprotected operator of spin $\ell = 0$, as a function of the anomaly c. Within numerical errors, the bound at large c agrees with the dimension (=8) of the "double-trace" operator : $\mathcal{O}_{\mathbf{14}}\mathcal{O}_{\mathbf{14}}$: