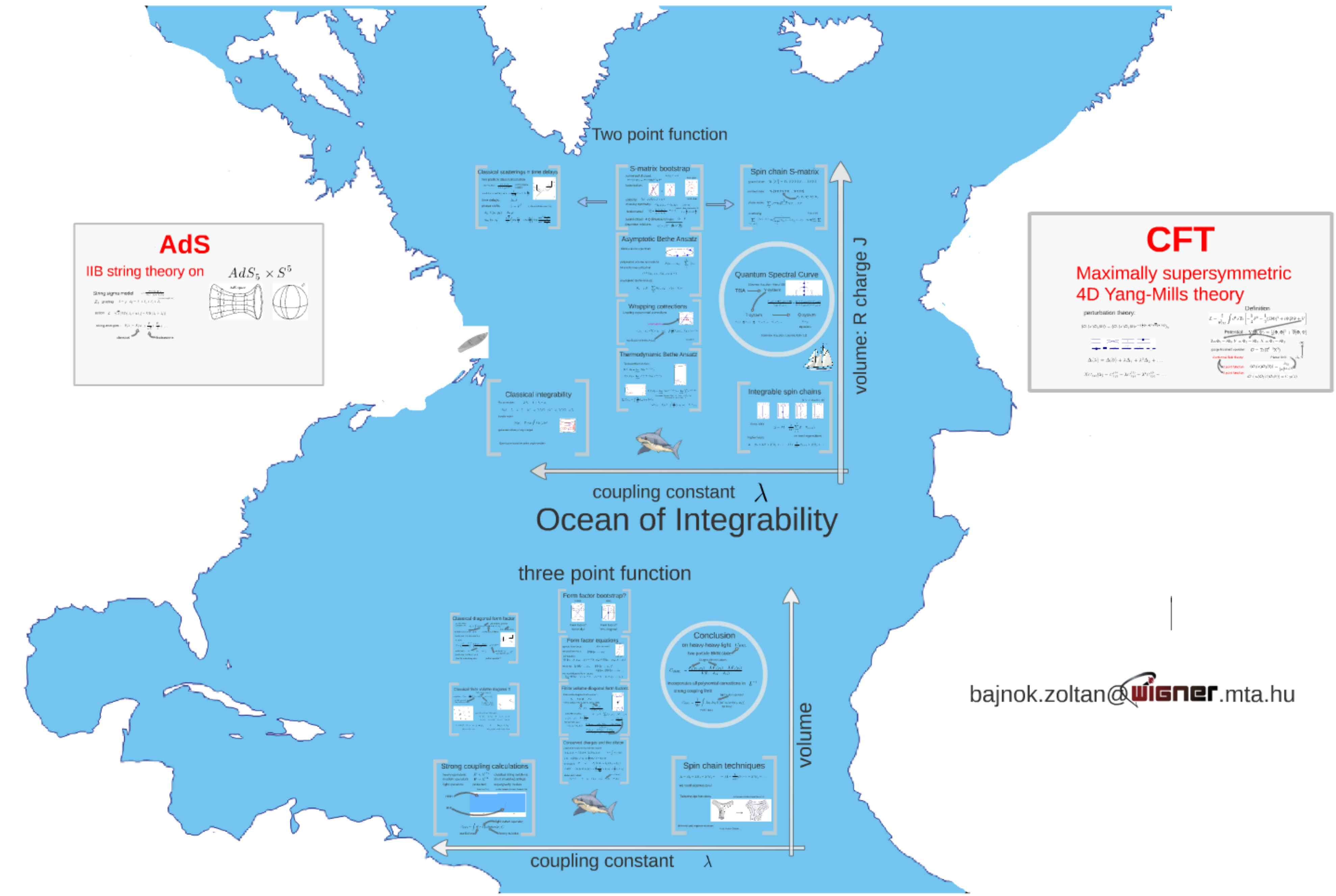
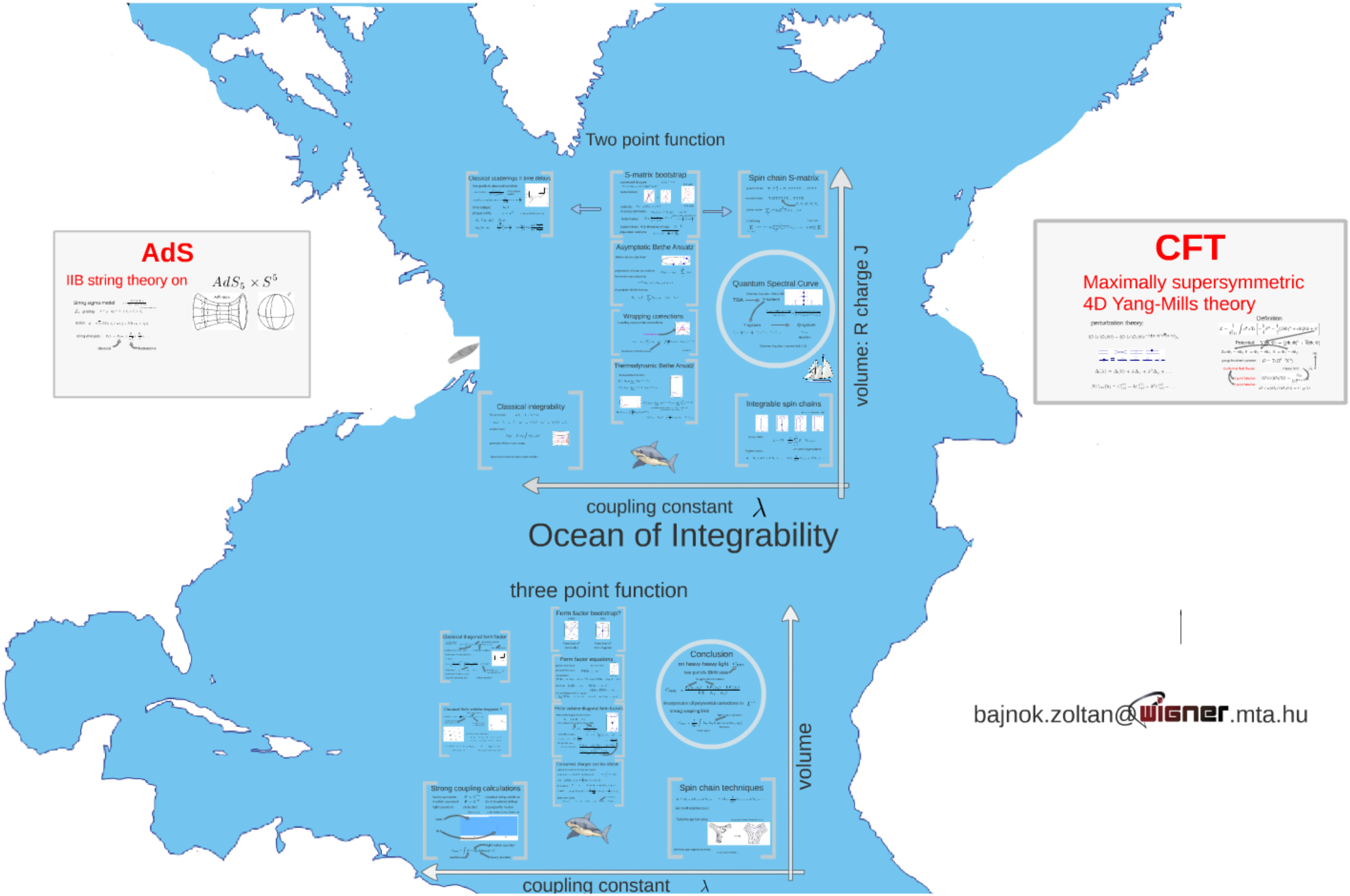


bajnok.zoltan@wigner.mta.hu



AdS
IIB string theory on $AdS_5 \times S^5$
Two point function
Classical scatterings = time delays
S-matrix bootstrap
Spin chain S-matrix
Asymptotic Bethe Ansatz
Quantum Spectral Curve
Wrapping corrections
Thermodynamic Bethe Ansatz
Classical integrability
Integrable spin chains
volume: R charge J
coupling constant λ
Ocean of Integrability
CFT
Maximally supersymmetric
4D Yang-Mills theory
three point function
Form factor bootstrap?
Classical diagonal form factor
Form factor equations
Classical finite volume diagonal?
Finite volume diagonal form factors
Conserved charges and the stress
Conclusion
Strong coupling calculations
Spin chain techniques
volume
coupling constant λ
bajnok.zoltan@wigner.mta.hu

CFT

Maximally supersymmetric 4D Yang-Mills theory

perturbation theory:

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \langle \mathcal{O}_1(x)\mathcal{O}_2(0)e^{-i(\frac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi])}\rangle_0$$

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \lambda^2\Delta_2 + \ldots$$

$$NC_{123}(\lambda) = C_{123}^{(0)} + \lambda C_{123}^{(1)} + \lambda^2 C_{123}^{(2)} + \ldots$$

Definition

$$\mathcal{L} = \frac{2}{g_{YM}^2}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

Potential: $\mathrm{V}(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$

$\mathrm{Z}=\Phi_5 + i\Phi_6, Y = \Phi_3 + i\Phi_4, X = \Phi_1 + i\Phi_2$

gauge invariant operator: $\mathcal{O} = \mathrm{Tr}(\mathrm{Z}^{\mathrm{J}-2}\mathrm{X}^2)$

Conformal field theory:

Planar limit: $\lambda = g_{YM}^2 N$ → ∞

2 point function $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \frac{\delta_{12}}{|x|^{2\Delta_1(\lambda)}}$

3 point function $\langle \mathcal{O}_1(\infty)\mathcal{O}_2(1)\mathcal{O}_3(0)\rangle = C_{123}(\lambda)$

Definition

$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\overline{\Psi} \not{D} \Psi + V \right]$$

Potential: $\mathrm{V}(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \overline{\Psi}[\Phi, \Psi]$

$$\mathrm{Z} = \Phi_5 + i\Phi_6, Y = \Phi_3 + i\Phi_4, X = \Phi_1 + i\Phi_2$$

∞

gauge invariant operator: $\mathcal{O} = \mathrm{Tr}(\mathrm{Z}^{\mathrm{J}-2}\mathrm{X}^2)$

Conformal field theory:

Planar limit: $\lambda = g_{YM}^2 N$

2 point function

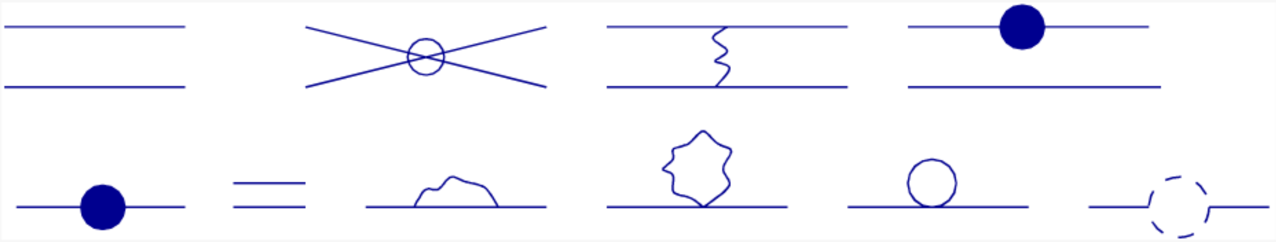
$$\langle \mathcal{O}_1(x) \mathcal{O}_2(0) \rangle = \frac{\delta_{12}}{|x|^{2\Delta_1(\lambda)}}$$

3 point function

$$\langle \mathcal{O}_1(\infty) \mathcal{O}_2(1) \mathcal{O}_3(0) \rangle = C_{123}(\lambda)$$

perturbation theory:

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \langle \mathcal{O}_1(x)\mathcal{O}_2(0)e^{-i(\frac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi])}\rangle_0$$

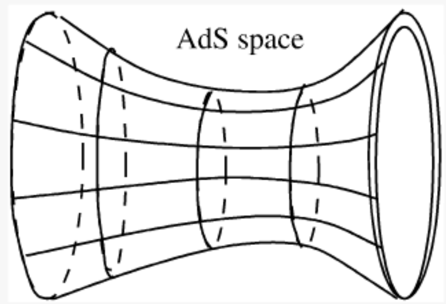


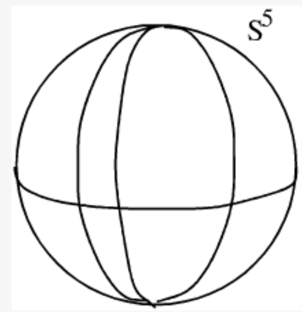
$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \lambda^2\Delta_2 + \ldots$$

$$N C_{123}(\lambda) = C_{123}^{(0)} + \lambda C_{123}^{(1)} + \lambda^2 C_{123}^{(2)} + \ldots$$

AdS

IIB string theory on $AdS_5 \times S^5$





String sigma model $g \in \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$

[Metsaev, Tseytlin '98]

Z_4 grading $J = g^{-1}dg = J_1 + J_2 + J_3 + J_4$

action $\mathcal{L} = \sqrt{\lambda}\,(\mathrm{STr}(J_2 \wedge *J_2) - \mathrm{STr}(J_1 \wedge J_3))$

string energies : $E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$

classical fluctuations

String sigma model

$$g \in \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$$

[Metsaev, Tseytlin '98]

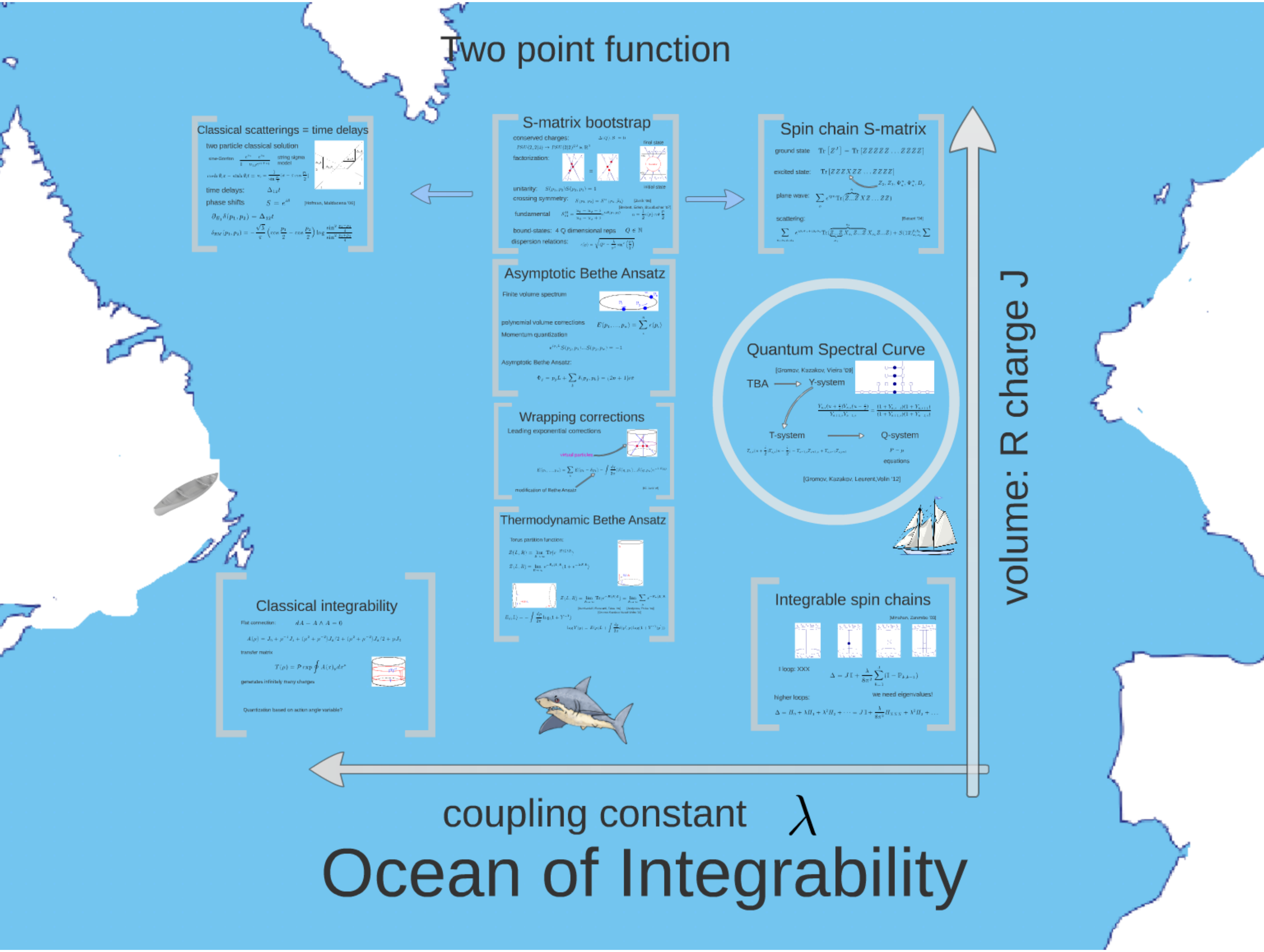
Z_4 grading $\quad J = g^{-1}dg = J_1 + J_2 + J_3 + J_4$

action $\quad \mathcal{L} = \sqrt{\lambda}\,(\mathrm{STr}(J_2 \wedge * J_2) - \mathrm{STr}(J_1 \wedge J_3))$

string energies : $\quad E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$

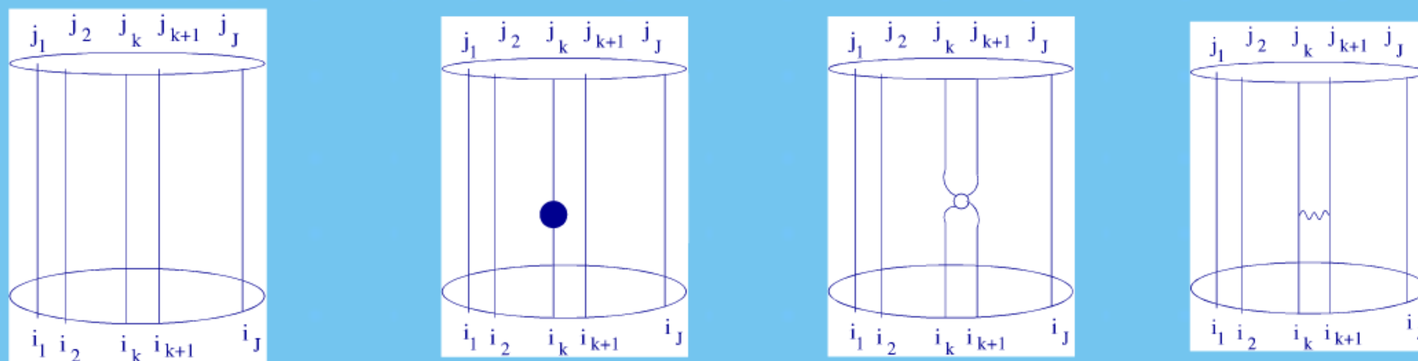
classical

fluctuations


Two point function
Classical scatterings = time delays
two particle classical solution
time delays:
phase shifts
S-matrix bootstrap
conserved charges:
factorization:
unitarity:
crossing symmetry:
fundamental
bound-states: 4 Q dimensional reps
dispersion relations:
Spin chain S-matrix
ground state
excited state:
plane wave:
scattering:
Asymptotic Bethe Ansatz
Finite volume spectrum
polynomial volume corrections
Momentum quantization
Asymptotic Bethe Ansatz:
Wrapping corrections
Leading exponential corrections
virtual particles
modification of Bethe Ansatz
Thermodynamic Bethe Ansatz
Torus partition function:
Quantum Spectral Curve
[Gromov, Kazakov, Vieira '09]
TBA
Y-system
T-system
Q-system
equations
[Gromov, Kazakov, Leurent,Volin '12]
Classical integrability
Flat connection:
transfer matrix
generates infinitely many charges
Quantization based on action angle variable?
Integrable spin chains
1 loop: XXX
higher loops:
we need eigenvalues!
volume: R charge J
coupling constant λ
Ocean of Integrability

Integrable spin chains

[Minahan, Zarembo '03]



I loop: XXX

$$\Delta = J\,\mathbb{I} + \frac{\lambda}{8\pi^2}\sum_{k=1}^{J}\left(\mathbb{I} - \mathbb{P}_{k,k+1}\right)$$

we need eigenvalues!

higher loops:

$$\Delta = H_0 + \lambda H_1 + \lambda^2 H_2 + \cdots = J\,\mathbb{I} + \frac{\lambda}{8\pi^2} H_{XXX} + \lambda^2 H_2 + \ldots$$

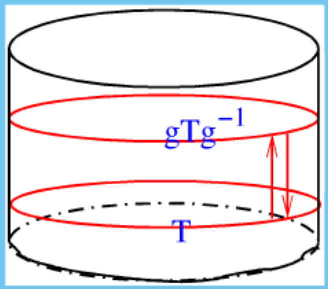
Classical integrability

Flat connection: $$dA - A \wedge A = 0$$

$$A(\mu) = J_0 + \mu^{-1} J_1 + (\mu^2 + \mu^{-2}) J_2/2 + (\mu^2 + \mu^{-2}) J_2/2 + \mu J_3$$

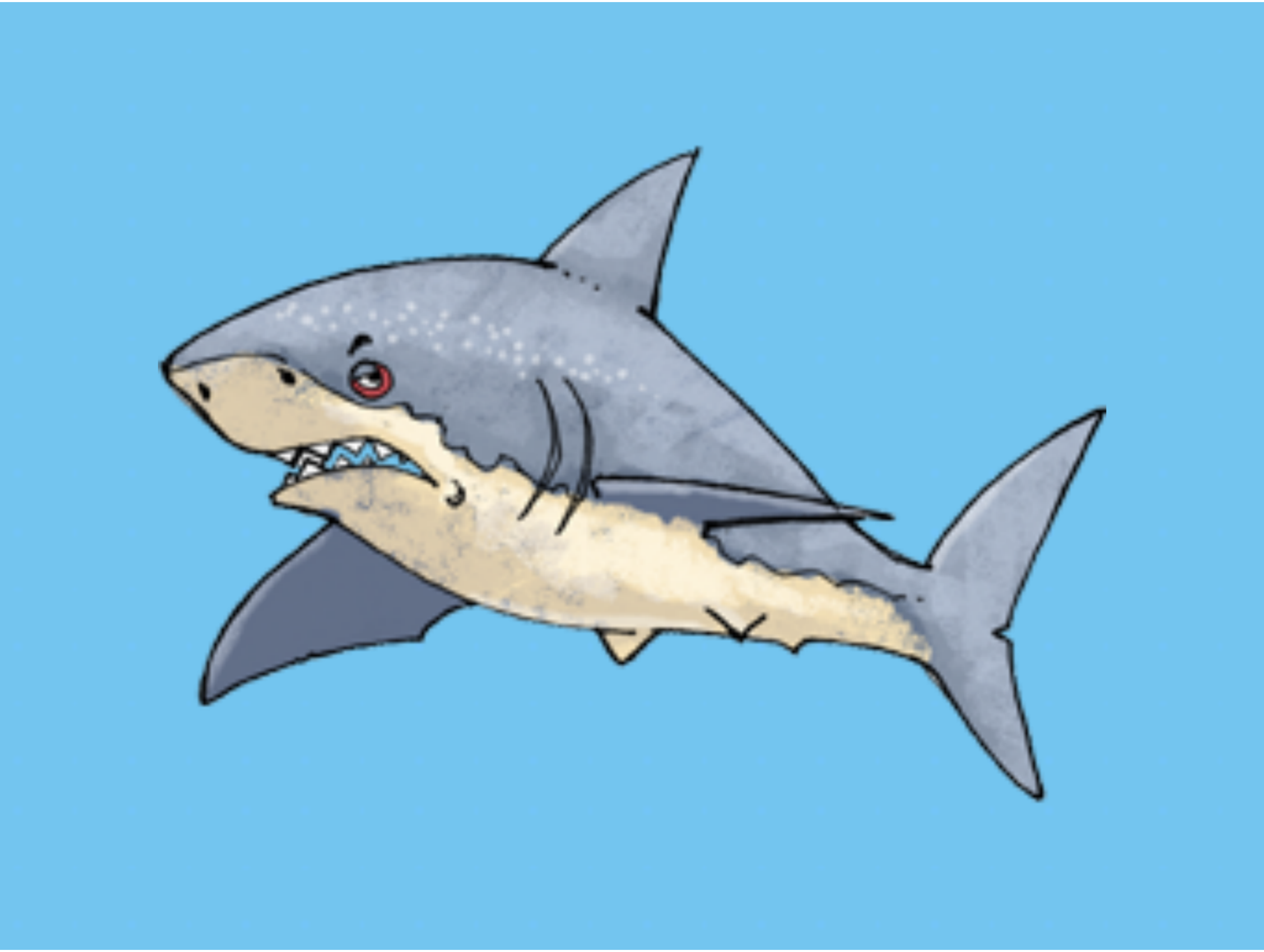
transfer matrix

$$T(\mu) = \mathcal{P} \exp \oint A(x)_\mu dx^\mu$$



generates infinitely many charges

Quantization based on action angle variable?

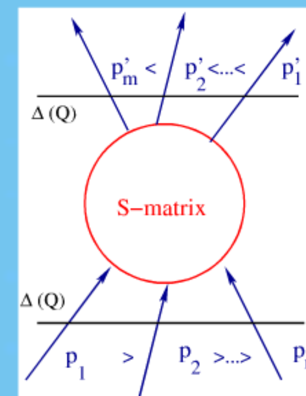


S-matrix bootstrap

conserved charges: $[\Delta(Q), S] = 0$

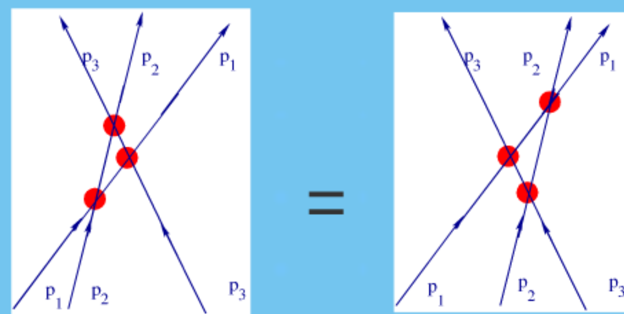
$PSU(2,2|4) \to PSU(2|2)^{\otimes 2} \ltimes \mathbb{R}^3$

final state



initial state

factorization:



unitarity: $S(p_1,p_2)S(p_2,p_1) = 1$

crossing symmetry: $S(p_1,p_2) = S^{c_1}(p_2,\bar{p}_1)$ [Janik '06]

[Beisert, Eden, Staudacher '07]

fundamental $S_{11}^{11} = \frac{u_1 - u_2 - i}{u_1 - u_2 + i} e^{i2\theta(p_1,p_2)}$ $u = \frac{1}{2}\epsilon(p)\cot\frac{p}{2}$

bound-states: 4 Q dimensional reps $Q \in \mathbb{N}$

dispersion relations: $\epsilon(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2}\sin^2\left(\frac{p}{2}\right)}$

Spin chain S-matrix

ground state $\mathrm{Tr}\left[Z^J\right] = \mathrm{Tr}\left[Z\,Z\,Z\,Z\,Z\,\ldots\,Z\,Z\,Z\,Z\right]$

excited state: $\mathrm{Tr}\left[Z\,Z\,Z\,X\,Z\,Z\,\ldots\,Z\,Z\,Z\,Z\right]$

$X \leftarrow Z_2, Z_3, \Psi_a^{\alpha}, \Psi_a^{\dot\alpha}, D_\mu$

plane wave: $$\sum_n e^{ipn}\mathrm{Tr}(\overbrace{Z\ldots Z}^{n}\,X\,Z\,\ldots\,Z\,Z)$$

scattering: [Beisert '04]

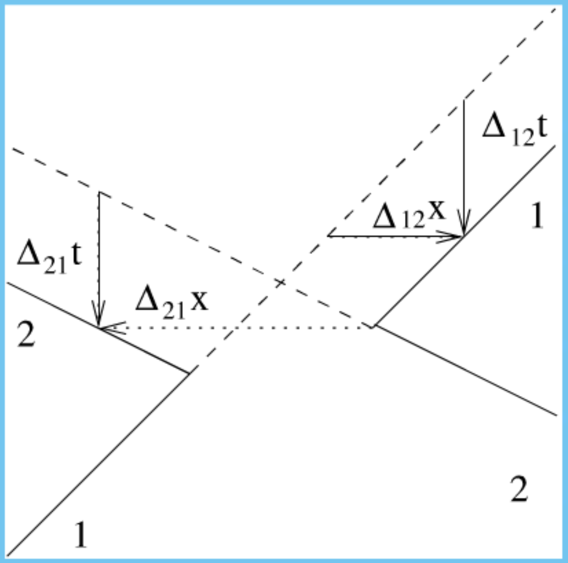
$$\sum_{n_1 n_2 a_1 a_2} e^{ip_1 n_1 + ip_2 n_2}\mathrm{Tr}(\overbrace{\underbrace{Z\ldots Z}_{n_1}\,X_{a_1}\,Z\ldots Z}^{n_2}\,X_{a_2}Z\ldots Z) + S(12)^{b_1 b_2}_{a_1 a_2}\sum$$

Classical scatterings = time delays

two particle classical solution

sine-Gordon $\dfrac{e^{u_1}+e^{u_2}}{1-u_{12}e^{u_1+u_2}}$ string sigma model

$$\cosh\theta_i x - \sinh\theta_i t = u_i = \frac{1}{\sin\frac{p_i}{2}}(\sigma - \tau\cos\frac{p_i}{2})$$



time delays: $\Delta_{12}t$

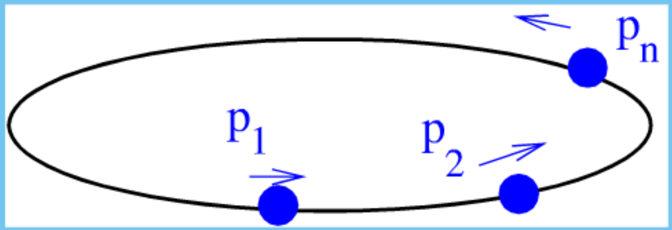
phase shifts $S = e^{i\delta}$ [Hofman, Maldacena '06]

$$\partial_{E_1}\delta(p_1,p_2) = \Delta_{12}t$$

$$\delta_{HM}(p_1,p_2) = -\frac{\sqrt{\lambda}}{\pi}\left(\cos\frac{p_1}{2}-\cos\frac{p_2}{2}\right)\log\frac{\sin^2\frac{p_1-p_2}{4}}{\sin^2\frac{p_1+p_2}{4}}$$

Asymptotic Bethe Ansatz

Finite volume spectrum



polynomial volume corrections

$$E(p_1, ..., p_n) = \sum_i^n \epsilon(p_i)$$

Momentum quantization

$$e^{ip_jL} S(p_j, p_1)...S(p_j, p_n) = -1$$

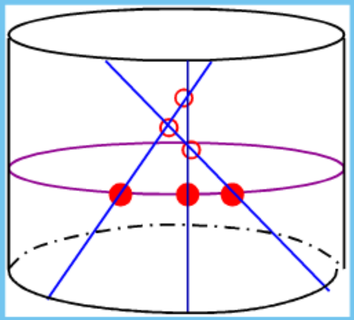
Asymptotic Bethe Ansatz:

$$\Phi_j = p_j L + \sum_k \delta(p_j, p_k) = (2n+1)i\pi$$

k

Wrapping corrections

Leading exponential corrections



virtual particles

$$E(p_1,...,p_n) = \sum_k E(p_k + \delta p_k) - \int \frac{dq}{2\pi} (S(q,p_1)...S(q,p_n) e^{-LE(q)}$$

modification of Bethe Ansatz

[BZ, Janik '08]

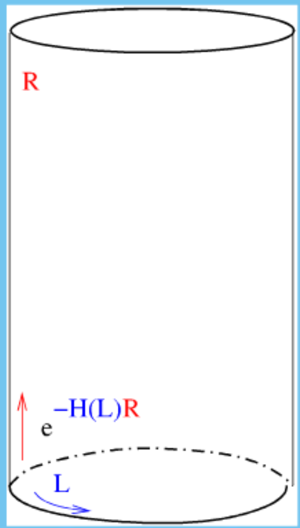
Thermodynamic Bethe Ansatz

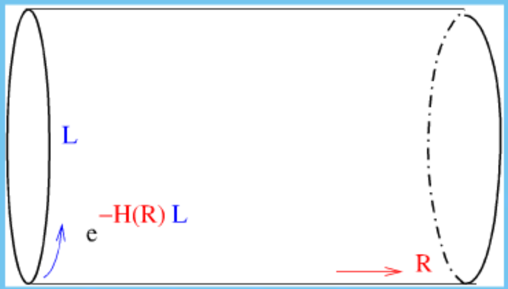
Thermodynamic Bethe Ansatz

Torus partition function:

$$Z(L,R) = \lim_{R\to\infty} \mathrm{Tr}(e^{-H(L)R})$$

$$Z(L,R) = \lim_{R\to\infty} e^{-E_0(L)R}(1+e^{-\Delta E\, R})$$





$$Z(L,R) = \lim_{R\to\infty} \mathrm{Tr}(e^{-H(R)L}) = \lim_{R\to\infty} \sum_n e^{-E_n(L)R}$$

[Bombardelli, Fioravanti, Tateo '09] [Arutyunov, Frolov '09]

[Gromov,Kazakov, Kozak,Vieira '10]

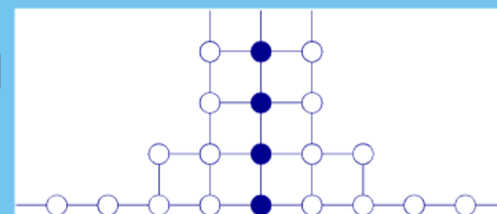
$$E_0(L) = -\int \frac{dp}{2\pi} \log(1+Y^{-1})$$

$$\log Y(p) = E(p)L + \int \frac{dp}{2\pi}\delta(p',p)\log(1+Y^{-1}(p'))$$

Quantum Spectral Curve

[Gromov, Kazakov, Vieira '09]

TBA → Y-system



$$\frac{Y_{a,s}(u+\frac{i}{2})Y_{a,s}(u-\frac{i}{2})}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s-1})(1+Y_{a,s+1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

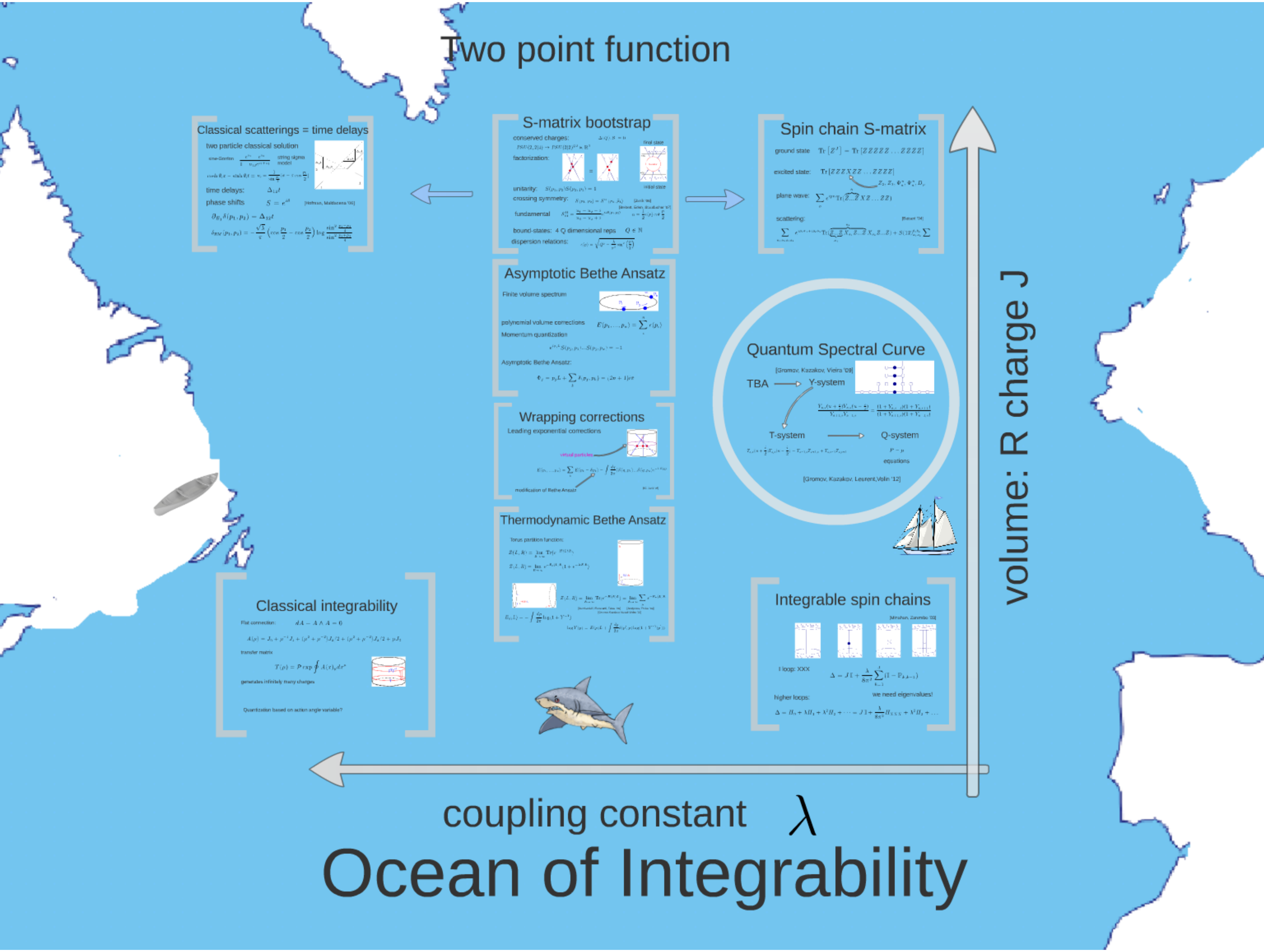
T-system → Q-system

$$T_{a,s}(u+\frac{i}{2})T_{a,s}(u-\frac{i}{2}) = T_{a-1,s}T_{a+1,s} + T_{a,s-1}T_{a,s+1}$$

$P-\mu$

equations

[Gromov, Kazakov, Leurent,Volin '12]

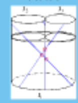


Two point function
Classical scatterings = time delays
two particle classical solution
time delays:
phase shifts
S-matrix bootstrap
conserved charges:
factorization:
unitarity:
crossing symmetry:
fundamental
bound-states: 4 Q dimensional reps
dispersion relations:
Spin chain S-matrix
ground state
excited state:
plane wave:
scattering:
Asymptotic Bethe Ansatz
Finite volume spectrum
polynomial volume corrections
Momentum quantization
Asymptotic Bethe Ansatz:
Wrapping corrections
Leading exponential corrections
virtual particles
modification of Bethe Ansatz
Thermodynamic Bethe Ansatz
Torus partition function:
Quantum Spectral Curve
[Gromov, Kazakov, Vieira '09]
TBA
Y-system
T-system
Q-system
equations
[Gromov, Kazakov, Leurent,Volin '12]
Classical integrability
Flat connection:
transfer matrix
generates infinitely many charges
Quantization based on action angle variable?
Integrable spin chains
[Minahan, Zarembo '03]
1 loop: XXX
higher loops:
we need eigenvalues!
volume: R charge J
coupling constant λ
Ocean of Integrability

three point function

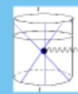
Form factor bootstrap?

HHH



Form factor?
Not really!

HHL



Form factor?
Yes: diagonal

Classical diagonal form factor

Conjecture: $F_2 = \int du_1 du_2 \, \mathcal{V}[\text{solution}(u_1,u_2)] - \dots$

light vertex operator; moduli space of solution; heavy 2pt solution; 1pt subtraction

works well for sine-Gordon

dilaton

$$V = \left(\frac{x^2+z^2}{z}\right)^{-1}\left[\frac{\partial x\bar\partial x + \partial z\bar\partial z}{z^2} + \partial X^K\bar\partial X^K\right]$$

AdS metric $ds^2 = z^{-2}(dz^2 + dx^2)$ — embedding coordinates on S^5

works well for the dilaton

Use for something else! Vertex operator?

Form factor equations

generic form factor [Klose, McLoughlin]

crossed form factor $\langle 0|\mathcal{V}|p_1,\dots,p_n\rangle$

permutation

$$\langle 0|\mathcal{V}|p_1,\dots,p_i,p_{i+1},\dots,p_n\rangle = S(p_i,p_{i+1})\langle 0|\mathcal{V}|p_1,\dots,p_{i+1},p_i,\dots,p_n\rangle$$

crossing

$$\langle p|\mathcal{V}|p_1,\dots,p_n\rangle = \langle 0|\mathcal{V}|p_1,\dots,p_n,\bar p\rangle + \langle p|p_n\rangle\langle 0|\mathcal{V}|p_1,\dots,p_{n-1}\rangle$$

we need diagonal form factors:

$$\lim_{\epsilon\to 0}\langle 0|\mathcal{V}|p_1,\dots,p_n,\bar p_n+\epsilon,\dots,\bar p_1+\epsilon\rangle = F_n(p_1,\dots,p_n)$$

Classical finite volume diagonal ff

Conjecture: $C_{HHL} = \frac{1}{\text{Vol}}\int du_1 du_2 \, \mathcal{V}[\text{solution}(u_1,u_2)]$

light vertex operator; moduli space of finite volume solution; heavy 2pt finite volume solution

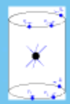
periodicity in the moduli space:

$$(u_1,u_2) \to (u_1,u_2) + (L-\Delta_{12}x,\ \Delta_{21}x)$$
$$(u_1,u_2) \to (u_1,u_2) + (-\Delta_{12}x,\ L-\Delta_{21}x)$$

$$\text{Vol} = L(L-\Delta_{12}x-\Delta_{21}x) = \rho_2 \quad \text{if} \quad -\Delta_{12}x \leftrightarrow \phi_{12}$$

sine-Gordon : OK — string sigma model: renormalize L

Finite volume diagonal form factors



finite volume diagonal matrix element

$$\langle I_1,\dots,I_n|\mathcal{V}|I_1,\dots,I_n\rangle$$

normalization: Kronecker vs. Dirac delta

$$|I_1,\dots,I_n\rangle = \frac{|p_1,\dots,p_n\rangle}{\sqrt{\rho_n(p_1,\dots,p_n)}}$$

density of states:

$$\Phi_j = p_j L + \sum_k \delta(p_j,p_k) = (2I_j+1)i\pi$$

$$\rho_n(p_1,\dots,p_n) = \det\left[\frac{\partial\Phi_j}{\partial p_k}\right] \qquad \phi_{kl} = \frac{\partial\delta(p_k,p_l)}{\partial p_k}$$

two particle case

$$_L\langle p_2,p_1|\mathcal{V}|p_1,p_2\rangle_L = \frac{F_2(p_1,p_2)+\rho_1(p_1)F_1(p_2)+\rho_1(p_2)F_1(p_1)}{\rho_2(p_1,p_2)} = \frac{F_2(p_1,p_2)+L\,F_1(p_2)+L\,F_1(p_1)}{L(L+\phi_{12}+\phi_{21})}$$

Conserved charges and the dilaton

Diagonal form factors of conserved charges

$$Q|p_2,p_1\rangle_L = (o(p_1)+o(p_2))|p_2,p_1\rangle_L \qquad Q = \int_0^L \mathcal{V}(x,t)dx$$

$$_L\langle p_1,p_2|\mathcal{V}|p_2,p_1\rangle_L = \frac{1}{L}(o(p_1)+o(p_1))$$

form factors: $F_1 = o_1 \qquad F_2 = (o_1+o_2)(\phi_{12}+\phi_{21})$

Dilaton: $_L\langle p_1,p_2|\mathcal{D}|p_2,p_1\rangle_L = \frac{1}{L}\frac{d}{d\frac{\sqrt{\lambda}}{\pi}}(\epsilon_1+\epsilon_2) = \frac{1}{L}(\epsilon_1'+\epsilon_2')$

dilaton form factors: $\quad \psi_{ij} = \delta(p_i,p_j)'$

$$F_1 = \epsilon' \qquad F_2 = (\epsilon_1'+\epsilon_2')(\phi_{12}+\phi_{21}) + \psi_{12}\partial_1\epsilon_2 + \psi_{21}\partial_2\epsilon_1$$

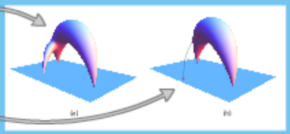
Strong coupling calculations

heavy operators:	$E \propto \lambda^{1/2}$	classical string solutions
medium operators:	$E \propto \lambda^{1/4}$	short (massive) strings
light operators:	protected	supergravity modes

[Zarembo '10] [Costa,Monteiro,Santos, Zoakos '10]

HHH

HHL



$$C_{HHL} = \int d^2\sigma \, \mathcal{V}[\text{solution}(\sigma,\tau)]$$

light vertex operator; worldsheat; heavy solution

Conclusion

on heavy-heavy-light C_{HHL}

two particle BMN state

Diagonal form factors

$$C_{HHL} = \frac{F_2(p_1,p_2)+L\,F_1(p_2)+L\,F_1(p_1)}{L(L+\phi_{12}+\phi_{21})}$$

incorporates all polynomial corrections in L^{-1}

strong coupling limit

$$C_{HHL} = \frac{1}{\text{Vol}}\int du_1 du_2 \, \mathcal{V}[\text{solution}(u_1,u_2)]$$

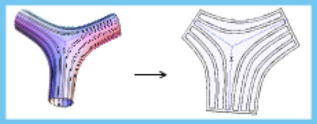
light vertex operator; 2pt heavy; moduli space

Spin chain techniques

$$\Delta = H_0 + \lambda H_1 + \lambda^2 H_2 + \dots = J\,\mathbb{I} + \frac{\lambda}{8\pi^2}H_{XXX} + \lambda^2 H_2 + \dots$$

we need eigenvectors!

Tailoring 3pt functions [Escobedo,Gromov,Sever,Vieira '10-]



determinant representations [Foda, Kostov, Serban, ...]

volume

coupling constant λ

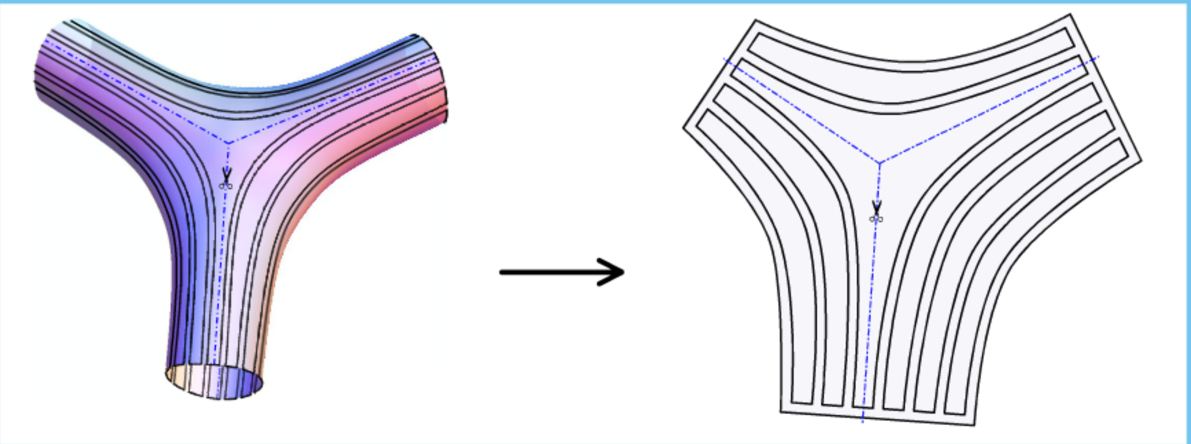
Spin chain techniques

$$\Delta = H_0 + \lambda H_1 + \lambda^2 H_2 + \dots = J\,\mathbb{I} + \frac{\lambda}{8\pi^2} H_{XXX} + \lambda^2 H_2 + \dots$$

we need eigenvectors!

Tailoring 3pt functions

[Escobedo,Gromov,Sever,Vieira '10-]



determinant representations

[Foda, Kostov, Serban, ...]

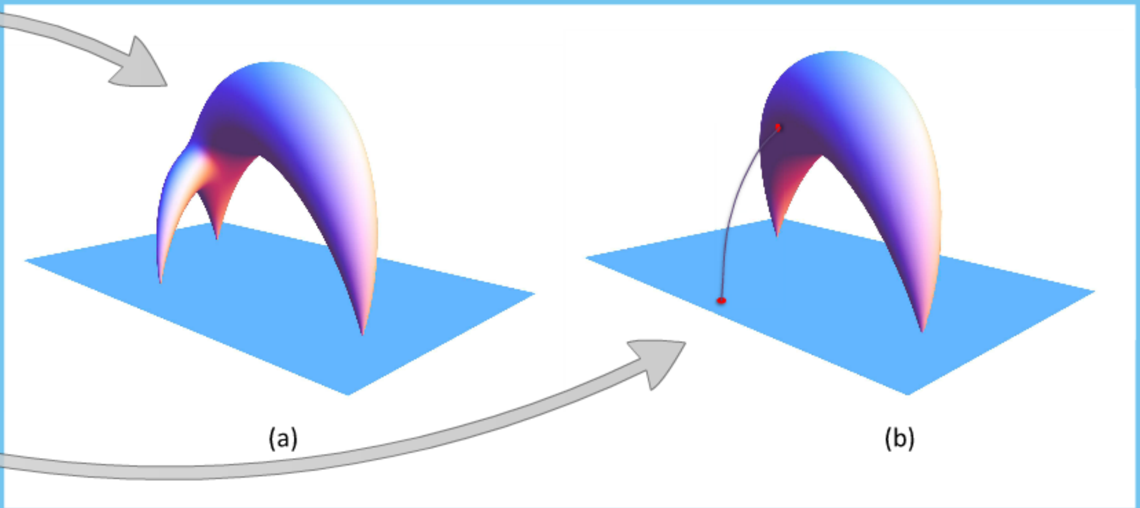
Strong coupling calculations

heavy operators:	$E \propto \lambda^{1/2}$	classical string solutions
medium operators:	$E \propto \lambda^{1/4}$	short (massive) strings
light operators:	protected	supergravity modes

[Zarembo '10] [Costa,Monteiro,Santos, Zoakos '10]

HHH

HHL



light vertex operator

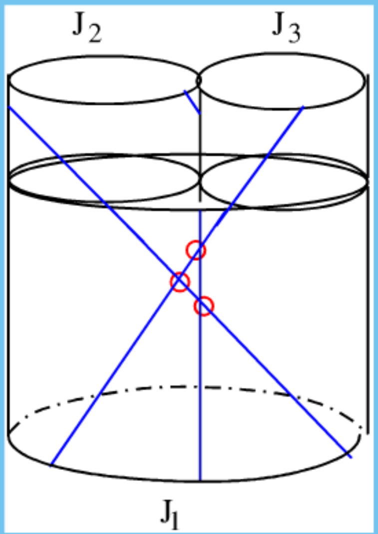
$$C_{HHL} = \int d^2\sigma \; \mathcal{V}[\mathrm{solution}(\sigma,\tau)]$$

worldsheat

heavy solution

Form factor bootstrap?

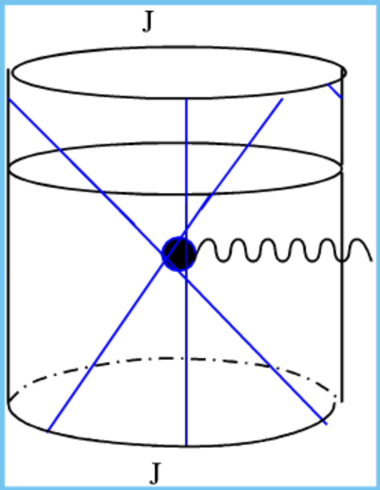
HHH



Form factor?

Not really!

HHL

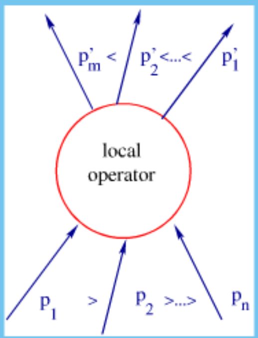


Form factor?

Yes: diagonal

Form factor equations

final state



initial state

generic form factor [Klose, McLoughlin]

crossed form factor $\langle 0|\mathcal{V}|p_1,\ldots,p_n\rangle$

permutation

$$\langle 0|\mathcal{V}|p_1,..,p_i,p_{i+1},..,p_n\rangle = S(p_i,p_{i+1})\langle 0|\mathcal{V}|p_1,..,p_{i+1},p_i,..,p_n\rangle$$

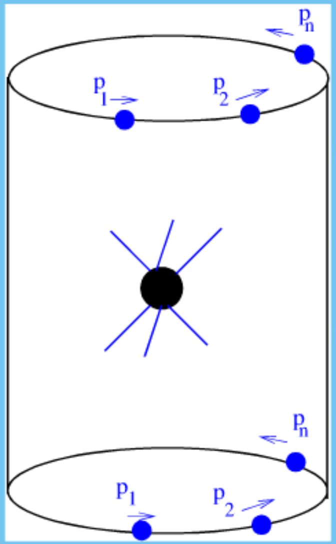
crossing $\langle p|\mathcal{V}|p_1,..,p_n\rangle = \langle 0|\mathcal{V}|p_1,..,p_n,\bar{p}\rangle$

$$+\langle p|p_n\rangle\langle 0|\mathcal{V}|p_1,..,p_{n-1}\rangle$$

we need diagonal form factors:

$$\lim_{\epsilon\to 0}\langle 0|\mathcal{V}|p_1,..,p_n,\bar{p}_n+\epsilon,\ldots,\bar{p}_1+\epsilon\rangle \quad = F_n(p_1,\ldots,p_n)$$

Finite volume diagonal form factors



finite volume diagonal matrix element

$$\langle I_1, ..., I_n | \mathcal{V} | I_1, .., I_n \rangle$$

normalization: Kronecker vs. Dirac delta

$$|I_1, ..., I_n\rangle = \frac{|p_1, ..., p_n\rangle}{\sqrt{\rho_n(p_1, ..., p_n)}}$$

densitity of states:

$$\Phi_j = p_j L + \sum_k \delta(p_j, p_k) = (2I_j + 1) i\pi$$

$$\rho_n(p_1, ..., p_n) = \det \left[\frac{\partial \Phi_j}{\partial p_i} \right]$$

$$\phi_{kl} = \frac{\partial \delta(p_k, p_l)}{\partial p_k}$$

two particle case

$$_L\langle p_2, p_1 | \mathcal{V} | p_1, p_2 \rangle_L = \frac{F_2(p_1, p_2) + \rho_1(p_1) F_1(p_2) + \rho_1(p_2) F_1(p_1)}{\rho_2(p_1, p_2)}$$

$$= \frac{F_2(p_1, p_2) + L\, F_1(p_2) + L\, F_1(p_1)}{L(L + \phi_{12} + \phi_{21})}$$

Conserved charges and the dilaton

Diagonal form factors of conserved charges

$$Q|p_2,p_1\rangle_L=(o(p_1)+o(p_2))|p_2,p_1\rangle_L \qquad Q=\int_0^L \mathcal{V}(x,t)dx$$

$$_L\langle p_1,p_2|\mathcal{V}|p_2,p_1\rangle_L=\frac{1}{L}(o(p_1)+o(p_1))$$

form factors: $F_1=o_1 \qquad F_2=(o_1+o_2)(\phi_{12}+\phi_{21})$

Dilaton: $_L\langle p_1,p_2|\mathcal{D}|p_2,p_1\rangle_L=\frac{1}{L}\frac{d}{d\frac{\sqrt{\lambda}}{\pi}}(\epsilon_1+\epsilon_2)\ =\frac{1}{L}(\epsilon_1'+\epsilon_2')$

dilaton form factors:

$\psi_{ij}=\delta(p_i,p_j)'$

$$F_1=\epsilon' \qquad F_2=(\epsilon_1'+\epsilon_2')(\phi_{12}+\phi_{21})+\psi_{12}\partial_p\epsilon_2+\psi_{21}\partial_p\epsilon_1$$

Classical diagonal form factor

[ZB, Janik, Wereszczynski]

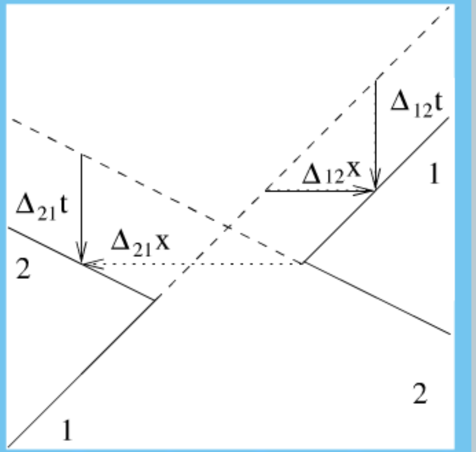
Conjecture: $$F_2 = \int du_1 du_2 \, \mathcal{V}[\text{solution}(u_1, u_2)] - \dots$$

light vertex operator

moduli space of solution

heavy 2pt solution

1pt subtraction



works well for sine-Gordon

dilaton

$$\mathcal{V} = \left(\frac{x^2+z^2}{z}\right)^{-4}\left[\frac{\partial x \bar{\partial} x + \partial z \bar{\partial} z}{z^2} + \partial X^K \bar{\partial} X^K\right]$$

AdS metric $ds^2 = z^{-2}(dz^2 + dx^2)$

embedding coordinates on S^5

works well for the dilaton

Use for something else!

Vertex operator?

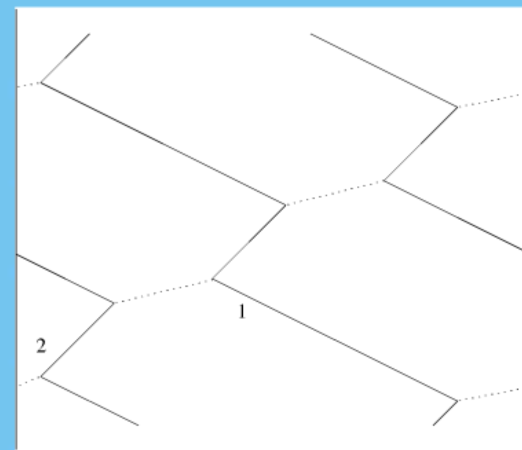
Classical finite volume diagonal ff

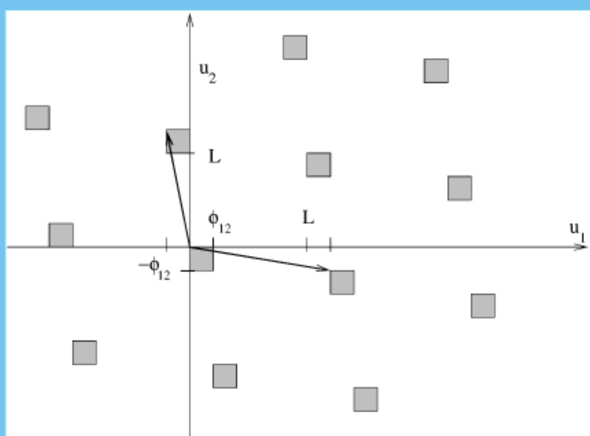
light vertex operator

Conjecture: $C_{HHL} = \frac{1}{\mathrm{Vol}} \int du_1 du_2 \mathcal{V}[\mathrm{solution}(u_1, u_2)]$

moduli space of
finite volume solution

heavy 2pt finite
volume solution





periodicity in the moduli space:

$$(u_1, u_2) \to (u_1, u_2) + (L - \Delta_{12} x, -\Delta_{21} x)$$

$$(u_1, u_2) \to (u_1, u_2) + (-\Delta_{12} x, L - \Delta_{21} x)$$

$$\mathrm{Vol} = L(L - \Delta_{12} x - \Delta_{21} x) = \rho_2 \quad \text{if} \quad -\Delta_{12} x \leftrightarrow \phi_{12}$$

sine-Gordon : OK

string sigma model: renormalize L

Conclusion

on heavy-heavy-light C_{HHL}

two particle BMN state

Diagonal form factors

$$C_{HHL} = \frac{F_2(p_1, p_2) + L\, F_1(p_2) + L\, F_1(p_1)}{L(L + \phi_{12} + \phi_{21})}$$

incorporates all polynomial corrections in L^{-1}

strong coupling limit

light vertex operator

$$C_{HHL} = \frac{1}{\text{Vol}} \int du_1 du_2 \mathcal{V}[\text{solution}(u_1, u_2)]$$

2pt heavy

moduli space