Worldsheet S matrix for $AdS_3 \times S^3 \times T^4$

Alessandro Sfondrini

based on work in collaboration with

R. Borsato, T. Lloyd, O. Ohlsson Sax & B. Stefański jr.

arXiv:1403.4543, 1406.0453 and work in progress

review: arXiv:1406.2971

GATIS

Plan

1. AdS_3/CFT_2 holography
2. Symmetries of pure-RR $AdS_3 \times S^3 \times T^4$
3. Worldsheet S matrix for pure-RR $AdS_3 \times S^3 \times T^4$
4. Mixed RR/NSNS flux S matrix
5. Conclusions and outlook

Gravity in AdS_3

- Early instance of holography, with Virasoro symmetry.

  [Brown, Henneaux '86]

- Rich black-hole physics.

  [Bañados, Teitelboim, Zanelli '92]

- Very relevant for string-theory black holes.

  [Strominger, Vafa '96]

- Higher-spin theories natural.

  [Gaberdiel, Gopakumar '10] [...]

AdS_3 superstring backgrounds

- Naturally arises from the D1-D5 system.
- Near-horizon geometries with 16 supercharges

$$AdS_3 \times S^3 \times T^4 \qquad \text{and} \qquad AdS_3 \times S^3 \times S^3 \times S^1$$

- Dual gauge theory has adjoint and fundamental matter, and flows to a SCFT with $\mathcal{N} = (4, 4)$ symmetry.
- Can be supported by RR and/or NSNS fluxes.
- Pure-NSNS background gives WZW model and is manageable.

[Maldacena, Ooguri '01]

Integrability

String NLSM is classically integrable for pure-RR and mixed fluxes.

[Babichenko, Stefański, Zarembo '10] [Sundin, Wulff '12] [Cagnazzo, Zarembo '12]

New feature: **massless fundamental modes** in light-cone gauge.

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Massive sector has many similarities with $AdS_5 \times S^5$.
Integrable massive-sector S matrix studied for

- Pure-RR $AdS_3 \times S^3 \times S^3 \times S^1$ [Borsato, Ohlsson Sax, AS '12]
- Pure-RR $AdS_3 \times S^3 \times T^4$ [Borsato, Ohlsson Sax, AS, Stefański, Torrielli '13]
- Mixed-flux $AdS_3 \times S^3 \times T^4$ [Hoare, Tseytlin, Stepanchuk '13]

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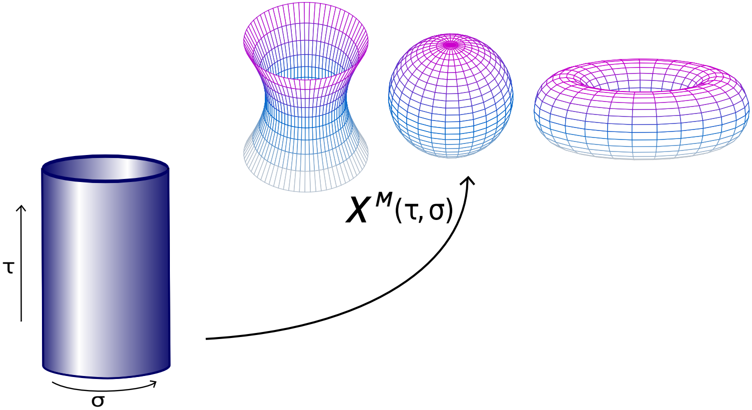
Many perturbative confirmations.

[Sundin,Wulff '12] [Beccaria,Levkovich-Maslyuk,Macorini,Tseytlin '12] [Abbott '13] [Engelund,McKeown,Roiban '13] [Bianchi,Hoare '14] [Babichenko,Dekel,Ohlsson Sax '14] [...]

Light-cone strings as a Non-Linear Sigma Model



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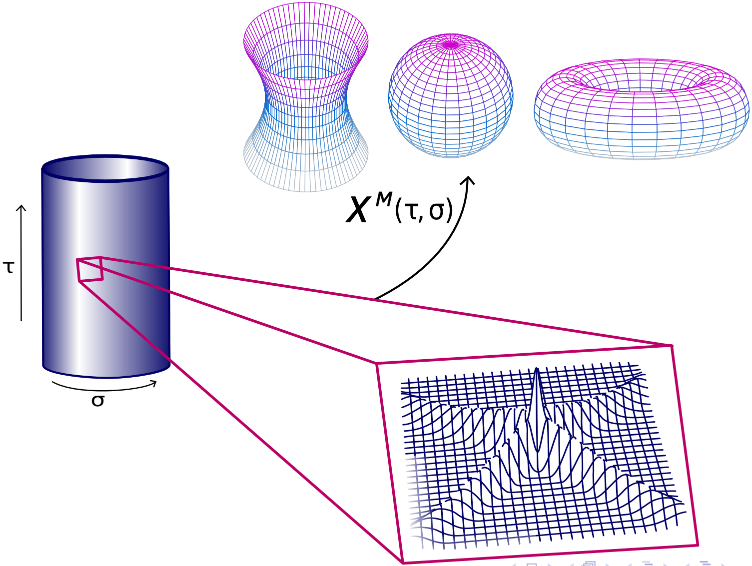

Factorised scattering in two dimensions

Work under the assumption of integrability.

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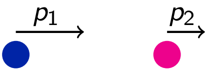

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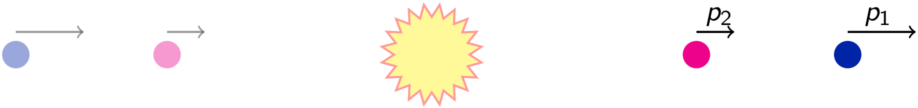

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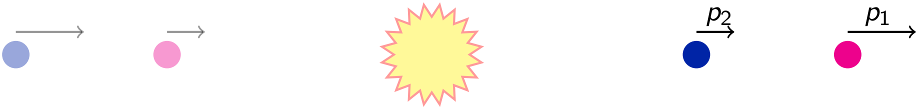

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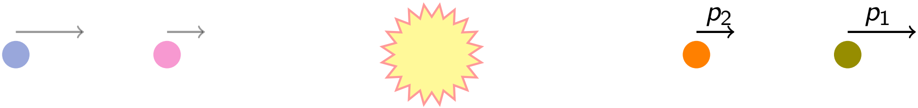
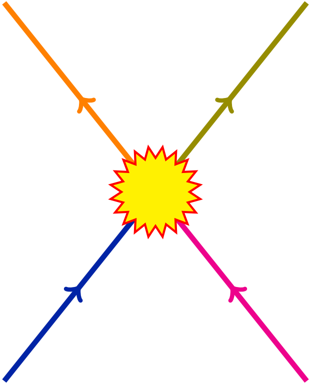
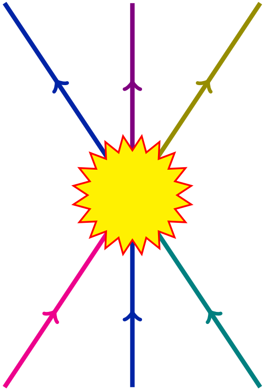


Two-body S matrix



Consistency conditions: Yang-Baxter equation



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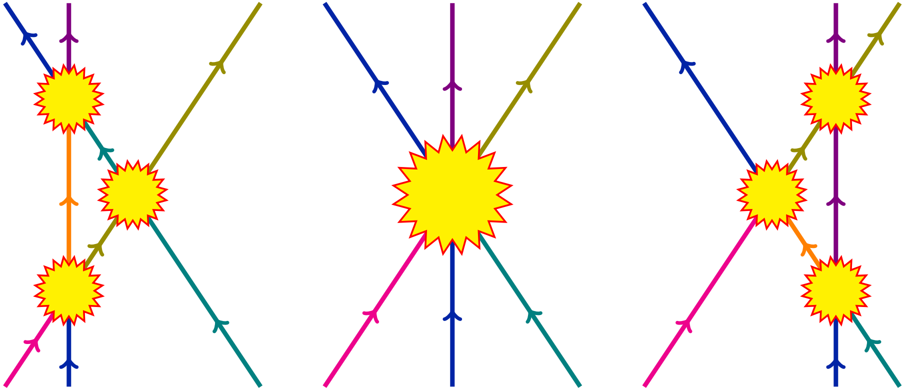

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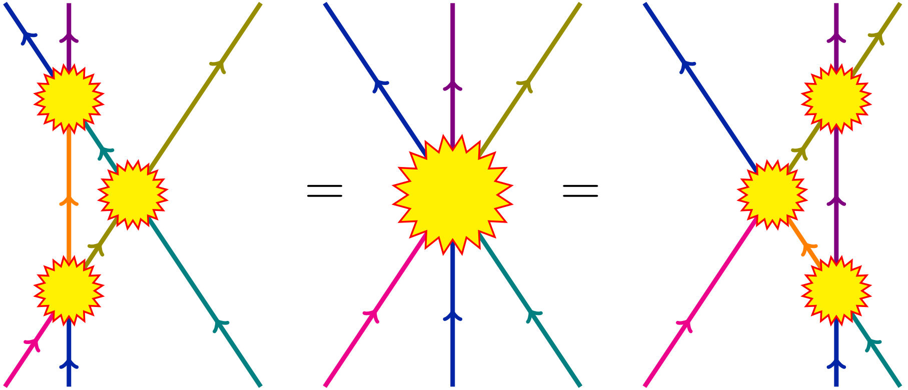

This talk

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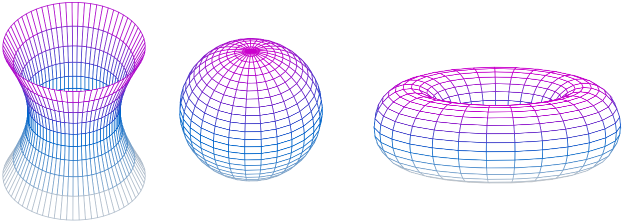
Construct all-loop integrable S matrix, including massless modes.

- Focus first on pure-RR $AdS_3 \times S^3 \times T^4$.
- Use light-cone gauge.
- Will not use coset formulation.
- Focus on off-shell symmetries.
- Discuss extension to mixed-flux background.

Plan

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$: symmetries

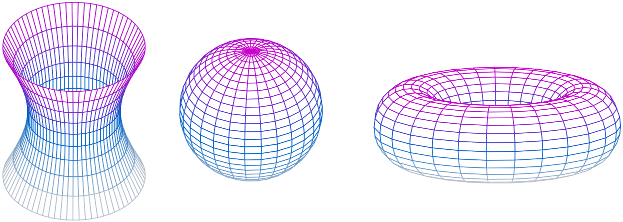


Superisometry algebra

$$\mathfrak{psu}(1,1|2)_{\mathsf{L}} \oplus \mathfrak{psu}(1,1|2)_{\mathsf{R}} \oplus \mathfrak{u}(1)^4$$

$\longrightarrow$ 16 supercharges

$AdS_3 \times S^3 \times T^4$: symmetries



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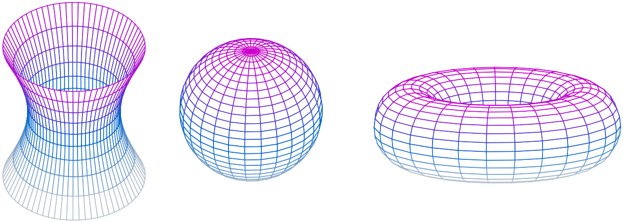

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$\longrightarrow$ 16 supercharges $\qquad \mathfrak{so}(4) = \mathfrak{su}(2)_\bullet \oplus \mathfrak{su}(2)_\circ$

Light-cone gauge symmetry algebra

Light-cone gauge breaks isometries to $\mathfrak{psu}(1|1)^4_{\text{c.e.}} \oplus \mathfrak{so}(4)$.

On-shell we have $\mathbf{P}|\text{state}\rangle = 0$ and the algebra

$$\begin{aligned} \{\mathbf{Q}_{\text{L}\,a}, \overline{\mathbf{Q}}_{\text{L}}{}^{b}\} &= \tfrac{1}{2}\delta_a{}^b(\mathbf{H}+\mathbf{M}), & \{\mathbf{Q}_{\text{L}\,a}, \mathbf{Q}_{\text{R}}{}^{b}\} &= 0, \\ \{\mathbf{Q}_{\text{R}}{}^{a}, \overline{\mathbf{Q}}_{\text{R}\,b}\} &= \tfrac{1}{2}\delta^a{}_b(\mathbf{H}-\mathbf{M}), & \{\overline{\mathbf{Q}}_{\text{L}}{}^{a}, \overline{\mathbf{Q}}_{\text{R}\,b}\} &= 0, \end{aligned}$$

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The off-shell central extension is

$$\mathbf{C} = i\,\frac{h}{2}\left(e^{i\,\mathbf{P}} - 1\right),$$

where we did not use the coset, but only fermion redefinitions.

[Arutyunov, Plefka, Frolov, Zamaklar '06] [Alday, Arutyunov, Frolov '05]

Representations

Perturbatively, we find 8+8 fundamental excitations:

4+4 massive particles	4+4 massless particles
transverse directions of $AdS_3 \times S^3$	flat T^4 directions

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At $\mathbf{P} = 0$, we have that $\mathbf{M}$ plays the role of mass.

Representations: massive modes

$m = +1$

$m = -1$

$|Y^{\mathrm{L}}\rangle$

$|Z^{\mathrm{R}}\rangle$

$|\eta^{\mathrm{L}1}\rangle$ $|\eta^{\mathrm{L}2}\rangle$ $|\eta^{\mathrm{R}}_{1}\rangle$ $|\eta^{\mathrm{R}}_{2}\rangle$

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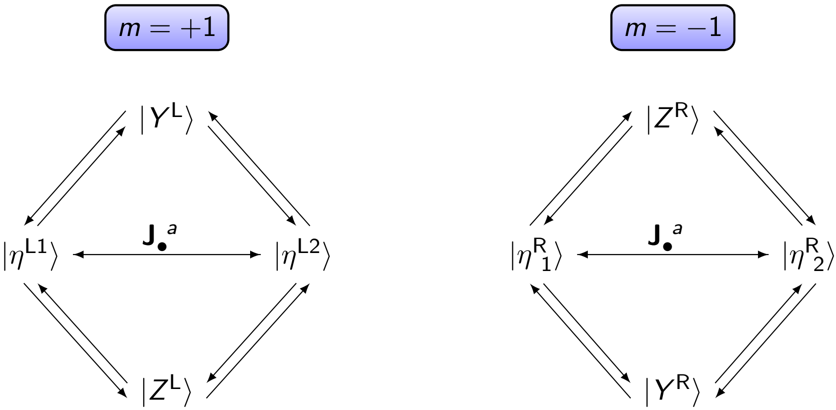

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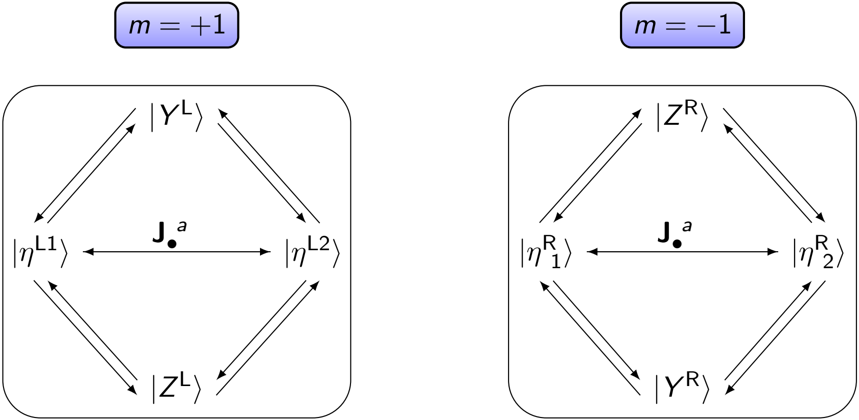

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$|\chi^1\rangle$ $|\chi^2\rangle$

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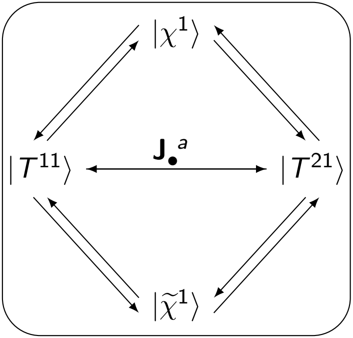

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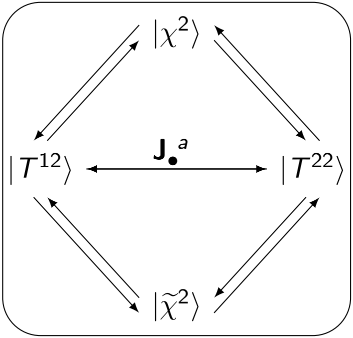

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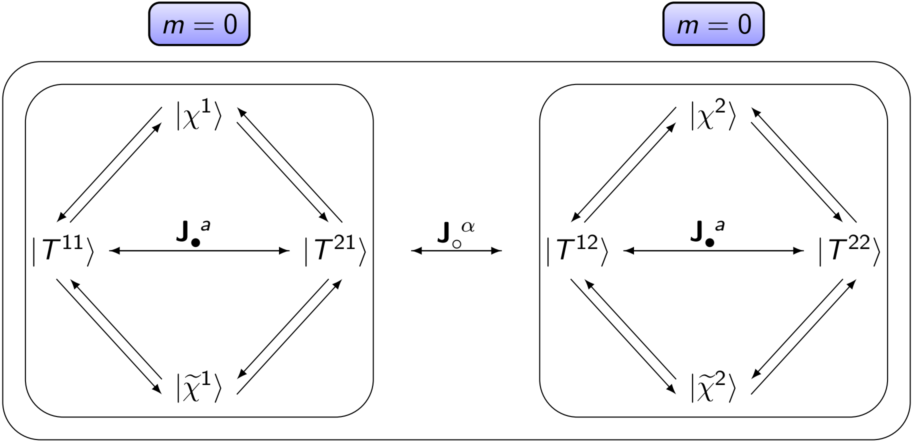

Dispersion relation

From the shortening condition

$$\mathbf{H}^2 = \mathbf{M}^2 + 4\,\mathbf{C}\overline{\mathbf{C}},$$

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Could the mass get quantum corrections? [Gross, Neveu '74]

Masslessness and symmetries

If yes, then we would have e.g.

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Off-shell symmetry algebra, again

Consider $\mathfrak{psu}(1|1)^2_{\text{c.e.}}$

$$\{\mathbf{q}_{\mathsf{L}}, \bar{\mathbf{q}}_{\mathsf{L}}\} = \tfrac{1}{2}(\mathbf{h} + \mathbf{m}), \qquad \{\mathbf{q}_{\mathsf{L}}, \mathbf{q}_{\mathsf{R}}\} = \mathbf{c},$$
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Short fundamental representations $(\phi|\psi)$ of $\mathfrak{psu}(1|1)^2_{\text{c.e.}}$ are:

	$m > 0$	$m < 0$
$\lvert\text{h.w.}\rangle = \lvert\phi\rangle$	ρ_{L}	ρ_{R}

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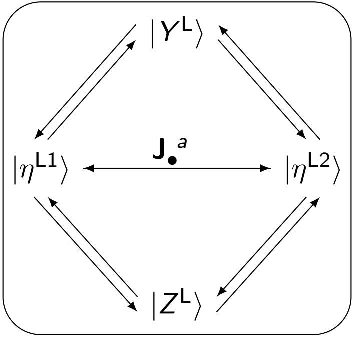
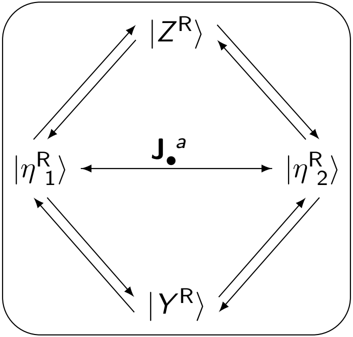
Representations: massive modes

$m = +1$



$\rho_L \otimes \rho_L$

$m = -1$



$\rho_R \otimes \rho_R$

Representations: massless modes

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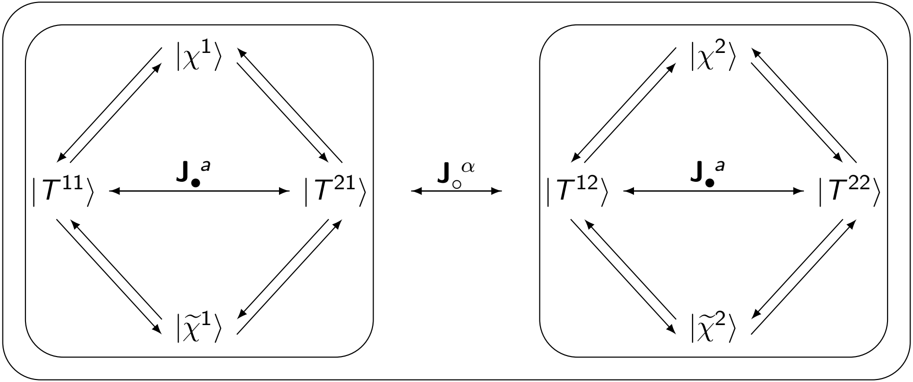

$\rho_{\text{L}} \otimes \tilde{\rho}_{\text{L}} \cong \rho_{\text{R}} \otimes \tilde{\rho}_{\text{R}}$ $\qquad\qquad$ $\rho_{\text{L}} \otimes \tilde{\rho}_{\text{L}} \cong \rho_{\text{R}} \otimes \tilde{\rho}_{\text{R}}$

Plan

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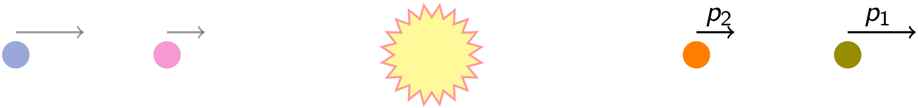
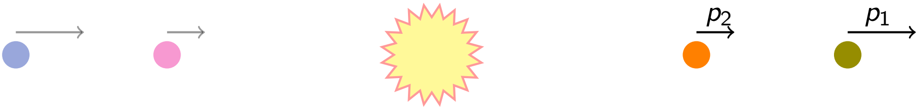
Back to factorised scattering

Recall the picture



Back to factorised scattering

Recall the picture



Massless scattering is usually problematic!

[Zamolodchikov, Zamolodchikov '92] [Zamolodchikov, Fendley, Saleur '93]

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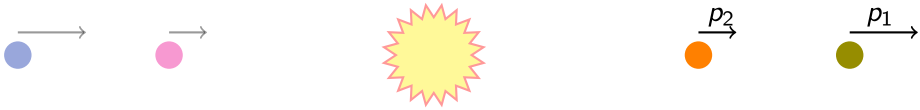

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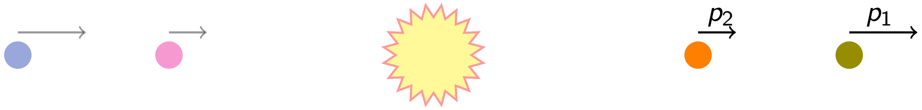

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$$v_{\text{rel}}(p) = \pm c = \frac{\partial E}{\partial p}, \qquad v(p) = \pm 2h \frac{\partial \sin \frac{p}{2}}{\partial p} = \pm h \cos \frac{p}{2}.$$

Finding the worldsheet S matrix

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We construct each block using $\mathfrak{psu}(1|1)^2_{\text{c.e.}}$ invariance.

For every pair of irreps $\rho_{\mathsf{L}}, \rho_{\mathsf{R}}, \tilde{\rho}_{\mathsf{L}}, \tilde{\rho}_{\mathsf{R}}$, we find an essentially unique $\mathbf{S}$:

$$\mathbf{S}^{\mathsf{LL}}, \qquad \mathbf{S}^{\mathsf{RR}}, \qquad \mathbf{S}^{\mathsf{L\tilde{L}}}, \qquad \mathbf{S}^{\mathsf{R\tilde{L}}}, \qquad \dots$$

[Borsato, Ohlsson Sax, AS '12]

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Massive-massive scattering

$$\mathbf{S}^{\bullet\bullet} = \begin{pmatrix} \mathbf{S}^{\mathrm{LL}} \otimes \mathbf{S}^{\mathrm{LL}} & \mathbf{S}^{\mathrm{RL}} \otimes \mathbf{S}^{\mathrm{RL}} \\ \mathbf{S}^{\mathrm{LR}} \otimes \mathbf{S}^{\mathrm{LR}} & \mathbf{S}^{\mathrm{RR}} \otimes \mathbf{S}^{\mathrm{RR}} \end{pmatrix}$$

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$$\mathbf{S}^{\bullet\circ} = \quad \left[\left(\mathbf{S}^{\mathrm{LL}} \otimes \mathbf{S}^{\mathrm{L}\tilde{\mathrm{L}}}\right) \oplus \left(\mathbf{S}^{\mathrm{RL}} \otimes \mathbf{S}^{\mathrm{R}\tilde{\mathrm{L}}}\right)\right]^{\oplus 2}$$

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The massless S matrix is

$$\mathbf{S}^{\circ\circ} = \begin{pmatrix} \sigma_1 & \sigma_2 \\ \sigma_3 & \sigma_4 \end{pmatrix} \otimes \left(\mathbf{S}^{\mathsf{LL}} \otimes \mathbf{S}^{\tilde{\mathsf{L}}\tilde{\mathsf{L}}}\right).$$

Massless-massless scattering

The massless S matrix is

$$\mathbf{S}^{\circ\circ} = \quad \sigma^{\circ\circ}\, \mathbf{S}_{\mathfrak{su}(2)} \;\otimes \big(\mathbf{S}^{\mathsf{LL}} \check{\otimes} \mathbf{S}^{\tilde{\mathsf{L}}\tilde{\mathsf{L}}}\big),$$

due to $\mathfrak{su}_{\circ}(2)$ invariance.

The pure-RR S matrix

- We expect a non-relativistic massless dispersion.

  [Sundin, in progress]

- We have determined the matrix part of **S**, up to five dressing factors. This is compatible with unitarity, Yang-Baxter, crossing.

- The massive dressing factors $\sigma^{\bullet\bullet}$, $\widetilde{\sigma}^{\bullet\bullet}$ have been proposed.

  [Borsato, Ohlsson Sax, AS, Stefański, Torrielli '13]

- The mixed-mass ones $\sigma^{\bullet\circ}, \sigma^{\circ\bullet}$ and the massless-massless one $\sigma^{\circ\circ}$ should still be investigated.

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Mixed-flux background

Adding NS5 branes and strings to the D1-D5 system results in a mixed-flux background.

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The NLSM Lagrangian is modified

$$\mathcal{L} = -\frac{h}{2}\left(\gamma^{\alpha\beta} G_{\mu\nu}(X) + q\, \varepsilon^{\alpha\beta} B_{\mu\nu}(X)\right) \partial_\alpha X^\mu\, \partial_\beta X^\nu + \text{fermions},$$

$q = 0$ pure RR $\qquad$ $q = 1$ pure NSNS

The classical theory is integrable.

[Cagnazzo, Zarembo '12]

Massive sector S matrix

The massive sector S matrix has been conjectured.

[Hoare, Tseytlin '13] [Hoare, Tseytlin, Stepanchuk '13]

Strategy: only need to deform the representations from $q = 0$.

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Irrep	**M**	**C**
L	$+1$	$\frac{ih}{2}\left(e^{i\,\mathbf{P}} - 1\right)$
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$\longrightarrow$ deform the Zhukovski variables $x^{\pm}$:

$$x^{\pm}(p) \to \begin{cases} x^{\pm}(p;+q) & \text{(irrep L)} \\ x^{\pm}(p;-q) & \text{(irrep R)} \end{cases}$$

Massless sector

We can repeat the symmetry analysis for the massless sector:

Irrep	$\mathbf{M}$	$\mathbf{C}$
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$\longrightarrow$ S matrix should follow from deformed Zhukovski variables.

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We argued that the mass is protected by $\mathfrak{su}(2)_\circ$ and crossing.

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After the crossing transformation

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Mixed-flux S matrix

- Matrix part seems to follow from deformation of Zhukovski variables.

- Dressing phases can be more complicated (even for massive sector).

[Babichenko, Dekel, Ohlsson Sax '14]

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Integrability is a very good language for studying AdS_3/CFT_2.

Natural developments

- The remaining dressing factors.
- The asymptotic spectrum: how is $\mathcal{N} = (4, 4)$ realised?
- Compare with finite-gap equations. [Lloyd, Stefański '13]
- Also, play around with winding.
- Find bound state spectrum and S matrix.
- Yangian and secret symmetries. [Pittelli, Torrielli, Wolf '14]
- Repeat all this for $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ and deformations.

The bigger picture

- Investigate the relation between integrability and CFT approach using mixed-flux as interpolation and taking $q \to 1$.

  [Zamolodchikov, Fendley, Saleur '90s ??] [Bazhanov, Lukyanov, Zamolodchikov '90s ??]

- Relate integrability to the symmetric-product orbifold CFT dual.

  [Pakman, Rastelli, Razamat '10]

- Gain new insight in higher-spin theories on AdS_3.

  [Gaberdiel, Gopakumar '14]

- Study black-hole backgrounds by integrability. [David, Sadhukhan '11]