On integrability of BFKL and BKP equations in multi-color gauge theories including $N = 4$ SUSY

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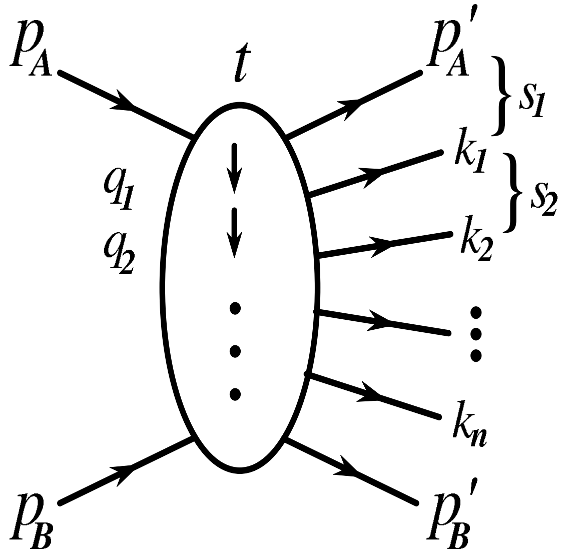
1 Amplitudes in multi-Regge kinematics



$$M^{FKL}_{2\to 2+n} = 2s\, g\, \frac{s_1^{\omega_1}}{q_1^2+m^2}\, gT^{d_1}_{c_2 c_1} C_1^{\mu} e^1_{\mu}\, \frac{s_2^{\omega_2}}{q_2^2+m^2} \ldots gT^{d_n}_{c_{n+1} c_n}\, C_n^{\sigma} e^n_{\sigma}\, \frac{s_{n+1}^{\omega_{n+1}}}{q_{n+1}^2+m^2}\, g\,,$$

$$C_1 = -q_1^{\perp} - q_2^{\perp} - p_A\left(\frac{q_1^2+m^2}{p_A k_1} - \frac{p_B k_1}{p_A p_B}\right) + p_B\left(\frac{q_2^2+m^2}{p_B k_1} - \frac{p_A k_1}{p_A p_B}\right)\,,$$

$$\omega_r = -\frac{g^2 N_c}{16\pi^3}\int \frac{d^2k\,\left(q_r^2+m^2\right)}{\left(k^2+m^2\right)\left(\left(q_r-k\right)^2+m^2\right)}$$

2 Fadin-Kuraev-Lipatov equation (1975)

Total cross-section in the Higgs model at $s \to \infty$ in LLA

$$\sigma_t \sim s^{\Delta}\,,\ \Delta = -\frac{\alpha_s N_c}{2\pi}\,E_0 = \frac{4\alpha_s N_c}{\pi}\,\ln 2$$

FKL equation for the Pomeron state at $t = 0$

$$E f(r) = H f(r)\,,\ H = T(p) + V(r)$$

Kinetic energy related to two Regge trajectories

$$T(p) = \frac{2(|p|^2 + m^2)}{|p|\sqrt{|p|^2 + 4m^2}}\,\ln\frac{\sqrt{|p|^2 + 4m^2} + |p|}{\sqrt{|p|^2 + 4m^2} - |p|}\,,\ |\hat{p}|^2 = -\frac{1}{r}\,\partial\, r\,\partial$$

Potential energy

$$V(r) = -4\,K_0(r\,m) + \frac{N_c^2 + 1}{N_c^2}\,\hat{P}\,,\ \hat{P}\,\phi(p) = \frac{m^2}{|p|^2 + m^2}\int\frac{d^2p'}{\pi}\,\frac{\phi(p')}{|p'|^2 + m^2}$$

Semiclassical solution (Levin, Lipatov, Siddikov (2014))

3 BFKL equation in LLA (1978)

Balitsky-Fadin-Kuraev-Lipatov equation

$$E\,\Psi(\vec{\rho}_1,\vec{\rho}_2) = H_{12}\,\Psi(\vec{\rho}_1,\vec{\rho}_2)\ :\ \Delta = -\frac{\alpha_s N_c}{2\pi}\,E_0 = \frac{4\alpha_s N_c}{\pi}\,\ln 2$$

Holomorphic separability (L. (1988))

$$H_{12} = h_{12}+h_{12}^*\,,\ [h_{12},h_{12}^*] = 0\,,\ E = \epsilon+\epsilon^*\,,\ \epsilon = \psi(m)+\psi(1-m)-2\psi(1)$$

Holomorphic Hamiltonian

$$h_{12} = \frac{1}{p_1}(\ln\rho_{12})p_1 + \frac{1}{p_2}(\ln\rho_{12})p_2 + \ln(p_1p_2) - 2\psi(1)$$

Möbius invariant solution (L. (1986))

$$\Psi = \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^m \left(\frac{\rho_{12}^*}{\rho_{10}^*\rho_{20}^*}\right)^{\widetilde{m}}\,,\ m = \gamma+\frac{n}{2}\,,\ \widetilde{m} = \gamma-\frac{n}{2}\,,\ \gamma = \frac{1}{2}+i\nu$$

4 Integrability of the BKP equation

Bartels-Kwiecinski-Praszalowicz equation (1980)

$$E\,\Psi(\vec{\rho}_1, ..., \vec{\rho}_n) = H\,\Psi(\vec{\rho}_1, ..., \vec{\rho}_n)$$

Holomorphic separability at large N_c (L. (1988))

$$H = \frac{1}{2}\,(h + h^*)\,,\;\; h = \sum_{k=1}^{n} h_{k,k+1}\,,$$

Monodromy matrix (L. (1993))

$$t(u) = \prod_{k=1}^{n} L_k = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix},\;\; L_k = \begin{pmatrix} u + \rho_k\, p_k & p_k \\ -\rho_k^2\, p_k & u - \rho_k\, p_k \end{pmatrix}$$

Transfer matrix and integrability (L. (1993))

$$T(u) = tr\, t(u) = A(u) + B(u)\,,\;\; [T(u), T(v)] = [T(u), h] = 0$$

5 Effective action approach

Gluon and Reggeized gluon fields

$$v_\mu(x) = -iT^a v^a_\mu(x)\,,\;\; A_\pm(x) = -iT^a A^a_\pm(x)$$

Local gauge transformations

$$\delta v_\mu(x) = \frac{1}{g}[D_\mu, \chi(x)],\; \delta\psi(x) = -\chi(x)\,\psi(x),\; \delta A_\pm(x) = 0$$

Effective action for reggeized gluons (L., 1995)

$$S = \int d^4x\,\left(L_0 + L_{ind}^{GR}\right)\,,\;\; L_0 = i\bar{\psi}\hat{D}\psi + \frac{1}{2}\,Tr\,G^2_{\mu\nu}$$

$$L_{ind}^{GR} = -\frac{1}{g}\partial_+\,P\exp\left(-\frac{g}{2}\int_{-\infty}^{x^+} v_+(x')d(x')^+\right)\,\partial^2_\sigma A_- + (+\to -)$$

6 Next-to-leading BFKL kernel

Eigenvalue of BFKL kernel in two loops (F., L. (1998))

$$\omega = 4\,G\,\left(2\psi(1) - \psi(M) - \psi(M^*)\right) + 4\,G^2\,\Delta(n,\nu)\,,\;\; G = \frac{g^2 N_c}{16\pi^2}$$

Hermitian separability in $N = 4$ SUSY (K.,L. (2000))

$$\Delta(n,\nu) = \phi(M) + \phi(M^*) - \frac{\rho(M) + \rho(M^*)}{2G/\omega}\,,\;\; M = i\nu + \frac{1+|n|}{2}\,,$$

$$\rho(M) = \beta'(M) + \frac{1}{2}\zeta(2)\,,\; \beta'(z) = \frac{1}{4}\left[\Psi'\Big(\frac{z+1}{2}\Big) - \Psi'\Big(\frac{z}{2}\Big)\right]$$

Maximal transcendentality (K.,L. (2002))

$$\phi(M) = 3\zeta(3) + \Psi''(M) - 4\sum_{k=0}^{\infty}\frac{\beta'(k+1)}{k+M} + 2\beta'(M)\Big(\Psi(1) - \Psi(M)\Big)$$

7 Pomeron and graviton in N=4 SUSY

Eigenvalue of the BFKL kernel in a diffusion approximation

$$j = 2 - \Delta - \Delta\,\nu^2\,,\;\; \gamma = \frac{j}{2} + i\nu$$

AdS/CFT relation for the graviton Regge trajectry

$$j = 2 + \frac{\alpha'}{2}\,t\,,\;\; t = E^2/R^2\,,\;\; \alpha' = \frac{R^2}{2}\,\Delta$$

Large coupling expansion of Δ (KLOV, BPST, CGC, KL, GLSV)

$$\Delta = 2\lambda^{-1/2} + \lambda^{-1} - 1/4\,\lambda^{-3/2} - 2(1+3\zeta_3)\lambda^{-2} + \ldots\,,\;\; \lambda = g^2\,N_c$$

Exact expression for the slope of γ at $j=2$ (KLOV, V., Basso)

$$\gamma'(2) = -\frac{\lambda}{24} + \frac{1}{2}\frac{\lambda^2}{24^2} - \frac{2}{5}\frac{\lambda^3}{24^2} + \frac{7}{20}\frac{\lambda^4}{24^4} - \frac{11}{35}\frac{\lambda^5}{24^5} + \ldots = -\frac{\sqrt{\lambda}}{4}\,\frac{I_3(\sqrt{\lambda})}{I_2(\sqrt{\lambda})}$$

8 Elastic BDS amplitude

Regge form of the amplitude at large s/t

$$M_{2\to 2}^{BDS} = \Gamma(t) \left(\frac{-s}{\mu^2}\right)^{\omega(t)} \Gamma(t)$$

Reggeized gluon trajectory

$$\omega(t) = -\frac{\gamma_K(a)}{4} \ln \frac{-t}{\mu^2} + \int_0^a \frac{da'}{a'} \left(\frac{\gamma_K(a')}{4\epsilon} + \beta(a')\right)$$

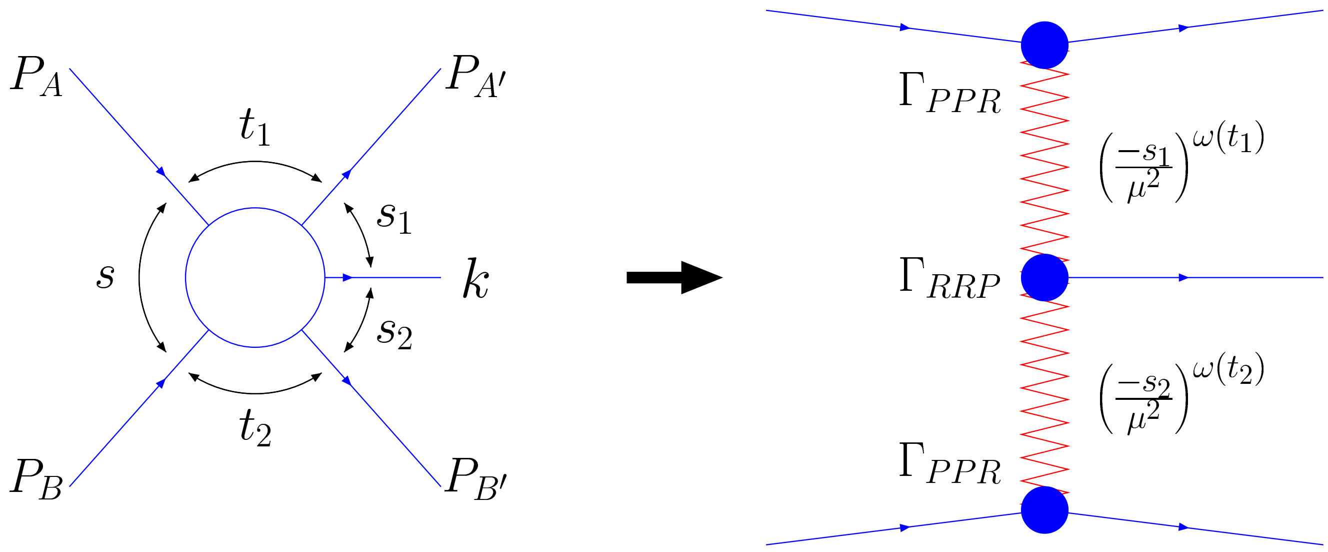
Cusp anomalous dimension

$$\gamma_K(a) = 4a - 4a^2\zeta(2) + 22\zeta(4)a^3 + \ldots, \;\; a = \frac{\alpha_s N_c}{2\pi}$$

Collinear anomalous dimension

$$\beta(a) = -a\zeta_3/2 + a^2(2\zeta_5 + 5\zeta_2\zeta_3/3) + \ldots$$

9 One particle production (BLV)



$$\ln \Gamma^{BDS}_{\kappa=s_1 s_2/s} = -\frac{1}{2}\left(\omega(t_1)+\omega(t_2) - \int_0^a \frac{da'}{a'}\left(\frac{\gamma_K(a')}{4\epsilon}+\beta(a')\right)\right)\ln\frac{-\kappa}{\mu^2} - \frac{\gamma_K(a)}{16}\left(\ln^2\frac{-\kappa}{\mu^2} - \ln^2\frac{-t_1}{-t_2} - \zeta_2\right) - \frac{1}{2}\int_0^a \frac{da'}{a'}\ln\frac{a}{a'}\left(\frac{\gamma_K(a')}{4\epsilon^2}+\frac{\beta(a')}{\epsilon}+\delta(a')\right)$$

10 Analyticity of production amplitudes

Steinmann constraints for overlapping channels

$$\Delta_{s_r}\Delta_{s_{r+1}}M_{2\to 2+n}=0$$

Regge representation for planar amplitude $M_{2\to 3}$

$$\frac{M_{2\to 3}}{\Gamma(t_1)\Gamma(t_2)|\Gamma_a|}=c^a_R(-\widetilde{s})^{j_2}(-s_1)^{j_1-j_2}+c^a_L(-\widetilde{s})^{j_1}(-s_2)^{j_2-j_1}\;,\;\;\widetilde{s}=s|k^a_\perp|^2$$

Regge ansatz for planar amplitude $M_{2\to 4}$

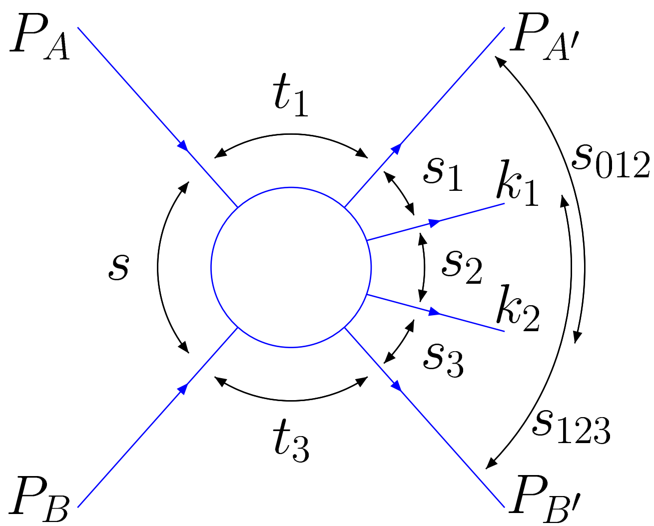
$$\frac{M_{2\to 4}}{\Gamma(t_1)\Gamma(t_2)|\Gamma_a||\Gamma_b|}=c^a_Rc^b_L(-\widetilde{s})^{j_2}(-s_1)^{j_1-j_2}(-s_3)^{j_3-j_2}$$

$$+c^a_Rc^b_R(-\widetilde{s})^{j_3}(-\widetilde{s}_{012})^{j_2-j_3}(-s_1)^{j_1-j_2}+c^a_Lc^b_L(-\widetilde{s})^{j_1}(-\widetilde{s}_{123})^{j_2-j_1}(-s_3)^{j_3-j_2}$$

$$+c^a_Lc^b_R\left(k(-\widetilde{s})^{j_3}(-\widetilde{s}_{012})^{j_1-j_3}(-s_2)^{j_2-j_1}+l(-\widetilde{s})^{j_1}(-\widetilde{s}_{123})^{j_3-j_1}(-s_2)^{j_2-j_3}\right),$$

$$c^a_R=\frac{\sin\pi\omega_{1a}}{\sin\pi\omega_{12}}\;,\;\;c^a_L=\frac{\sin\pi\omega_{2a}}{\sin\pi\omega_{21}}\;,\;\;k=\frac{\sin\pi\omega_1}{\sin\pi\omega_2}\,\frac{\sin\pi\omega_{23}}{\sin\pi\omega_{13}}\;,\;\;l=\frac{\sin\pi\omega_3}{\sin\pi\omega_2}\,\frac{\sin\pi\omega_{21}}{\sin\pi\omega_{31}}$$

11 Regge factorization violation (BLS)



$$M_{2\to4}^{BDS}|_{s_2>0;\, s_1,s_3<0} = \exp\left[\frac{\gamma_K(a)}{4}\, i\pi\, \left(\ln\frac{t_1t_2}{(\vec{k}_1+\vec{k}_2)^2\mu^2} - \frac{1}{\epsilon}\right)\right]$$

$$\times\Gamma(t_1)\left(\frac{-s_1}{\mu^2}\right)^{\omega(t_1)}\Gamma(t_2,t_1)\left(\frac{-s_2}{\mu^2}\right)^{\omega(t_2)}\Gamma(t_3,t_2)\left(\frac{-s_3}{\mu^2}\right)^{\omega(t_3)}\Gamma(t_3)$$

12 Mandelstam cuts in j_2-plane

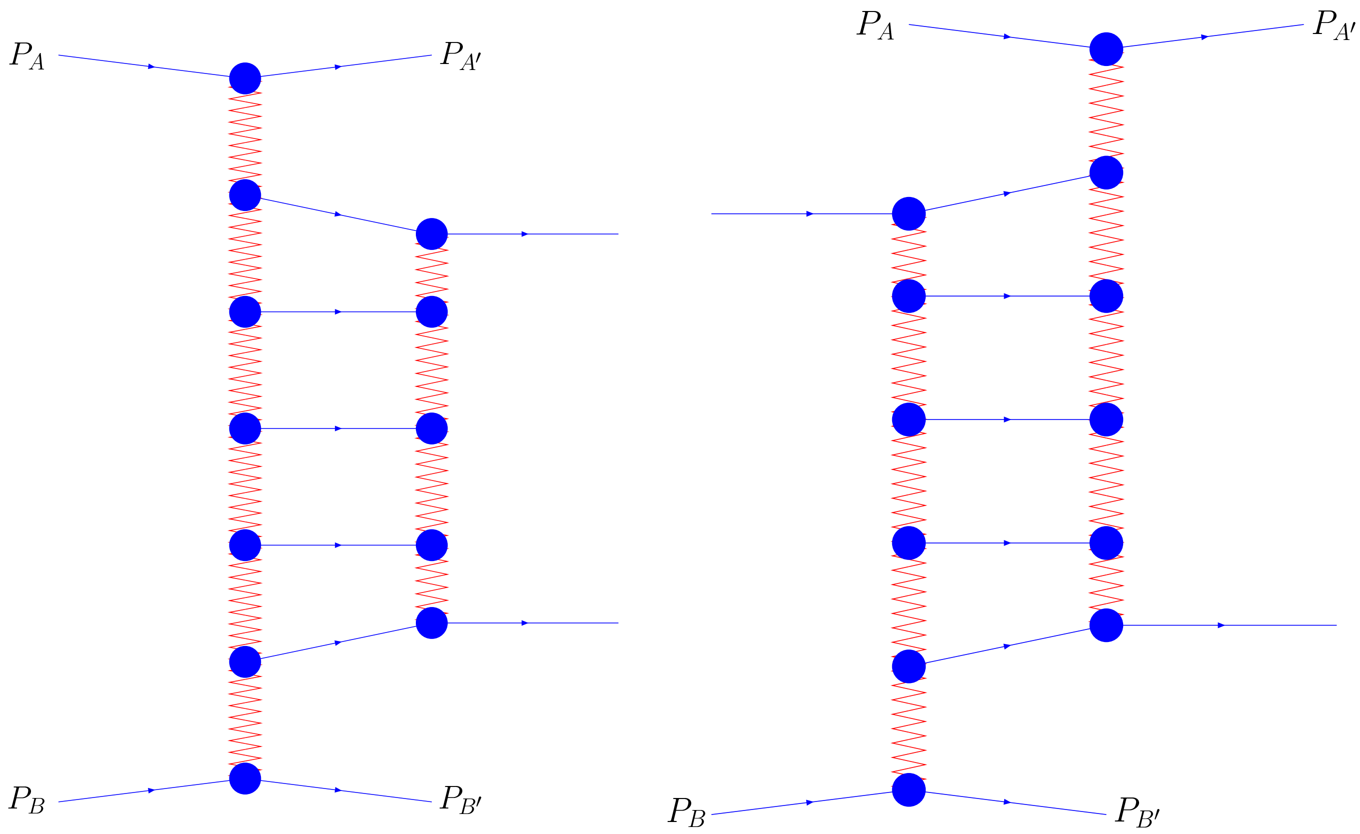


Figure 1: BFKL ladders in $M_{2\to 4}$ and $M_{3\to 3}$

13 Regge ansatz breakdown and cuts

Generating function in the Regge pole ansatz for $A_{2\to 4}$

$$\frac{A_{2\to 4}^{\tau_1\tau_2\tau_3}}{s|s_1|^{\omega_1}|s_2|^{\omega_2}|s_3|^{\omega_3}\Gamma(t_1)\Gamma(t_2)|\Gamma_a||\Gamma_b|} = ...+\left(\tau_1\tau_3\, e^{-i\pi\omega_2}+\tau_1\tau_2\tau_3\right) A+...$$

Non-physical singularities at $s, s_2 > 0;\ s_1, s_3 < 0$ (L. (2010))

$$A = \frac{2\cos\pi\omega_2\ \sin\pi\omega_a\ \sin\pi\omega_b}{i\sin\pi\omega_2}+i\sin\pi(\omega_a+\omega_b)+\cos\pi\omega_{ab}\,,\ \ \omega_{ab}=\frac{\gamma_K}{8}\ \ln|w|^2$$

Remainder function for $A_{2\to 4}$

$$A = R(u_1,u_2,u_3)\, A_{2\to 4}^{BDS}\,,\ \ u_1=\frac{ss_2}{s_{012}s_{123}}\,,\ \ u_2=\frac{s_1t_3}{s_{012}t_2}\,,\ \ u_3=\frac{s_3t_1}{s_{123}t_2}$$

Regge pole and cut contributions (BLS (2008), L. (2010))

$$R\, e^{i\pi\delta} = \cos\,\pi\omega_{ab}+i\int\frac{d\omega}{2\pi i}s_2^{\omega}\, f_\omega\,,\ \ \delta=\frac{\gamma_K}{8}\ \ln\frac{|w|^2}{|1+w|^4}\,,\ \ |w|^2=\frac{u_2}{u_3}$$

14 Amplitude $A_{2\to 4}$ in $N=4$ SUSY

Remainder factor in next-to-leading LLA (F.,L. (2011))

$$R\,e^{i\pi\delta} = \cos\,\pi\omega_{ab} + i\,\frac{a}{2}\sum_{n=-\infty}^{\infty}(-1)^n e^{i\phi n}\int_{-\infty}^{\infty}\frac{|w|^{2i\nu}d\nu}{\nu^2+\frac{n^2}{4}}\,\Phi(\nu,n)\left(\frac{-1}{\sqrt{u_2u_3}}\right)^{\omega^{(8)}(\nu,n)},$$

$$\cos\phi = \frac{1-u_1-u_2-u_3}{2\sqrt{u_2u_3}}\,,\;\;\Phi(\nu,n) = 1-a\left(\frac{E_{\nu n}^2}{2}+\frac{3}{8}\,n^2/(\nu^2+\frac{n^2}{4})^2+\zeta(2)\right)$$

Spectrum of the eigenvalues of the BFKL kernel

$$\omega^{(8)}(\nu,n) = -aE_{\nu,n} - a^2(\epsilon_{\nu n}^{FL}+3\zeta(3))\,,\;\;E_{\nu n} = -\frac{|n|/2}{\nu^2+\frac{n^2}{4}}+2\Re\psi(1+i\nu+\frac{|n|}{2})-2\psi(1)$$

Eigenvalue in the next-to-leading order (F.,L. (2011)

$$\epsilon_{\nu n}^{FL} = -\frac{\Re}{2}\left(\psi''(1+i\nu+\frac{|n|}{2})-\frac{2i\nu\psi'(1+i\nu+\frac{|n|}{2})}{\nu^2+\frac{n^2}{4}}\right)-\zeta(2)\,E_{\nu n}-\frac{1}{4}\,\frac{|n|\left(\nu^2-\frac{n^2}{4}\right)}{\left(\nu^2+\frac{n^2}{4}\right)^3}$$

15 Regge factorization and crossing

Generating function for amplitudes in crossing channels

$$A_{2\to n+1}^{\tau_1...\tau_n} = A + \sum_{r=1}^{n} \tau_r\, A_r + \sum_{r_1<r_2} \tau_{r_1}\tau_{r_2}\, A_{r_1 r_2} + ... + \tau_1\tau_2...\tau_n\, A_{1...n}$$

Regge pole factorization (Weis (1971))

$$\frac{A_{2\to n+1}^{\tau_1...\tau_n}}{s\Gamma(t_1)\Gamma(t_2)|\Gamma_a||\Gamma_b|...} = |s_1|^{\omega_1}\xi_1 V^{1,2}|s_2|^{\omega_2}\xi_2 V^{2,3}...V^{n-1,n}|s_n|^{\omega_n}\xi_n$$

Reggeon vertices and signature factors

$$V^{1,2} = \frac{\xi_{12}}{\xi_1}c_R^a + \frac{\xi_{21}}{\xi_2}c_L^a\,,\;\; \xi_r = e^{-i\pi\omega_r} - \tau_r\,,\;\; \xi_{12} = e^{-i\pi\omega_{12}} + \tau_1\tau_2$$

Necessity of the cuts for canceling the poles $1/\xi_r$ (BKL (2013))

$$A_{2\to n+1}^{\tau_1...\tau_n} \approx \sum_{r=2}^{n-1}\frac{c_r}{\xi_r} + \sum_{r_2=3}^{n-1}\sum_{r_1=2}^{r_2-1}\frac{c_{r_1 r_2}}{\xi_{r_1}\xi_{r_2}} + ...$$

16 Contribution with n reggeized gluons

Amplitude with the n-reggeon exchange

$$A_{2\to 2+r} = \int \prod_{i=1}^{n-1} d^2k_i^\perp \Phi_1(k_1^\perp, ..., k_{n-1}^\perp)\Phi_2(k_1^\perp, ..., k_{n-1}^\perp) \prod_{t=1}^{n} \frac{s^{j(-|k_i^\perp|^2)}}{|k_i^\perp|^2}$$

Impact factors

$$\Phi_1(k_1^\perp, ..., k_{n-1}^\perp) = \int \prod_{i=1}^{n-1} d\alpha_i f(k_1^\perp, \alpha_1, ..., k_{n-1}^\perp, \alpha_{n-1})$$

Conditions for a nonzero contribution (L. (2009))

$$r = 2n-2\,,\ s_1, s_2, ..., s_{n-1} < 0\,,\ s_n > 0\,,\ s_{n+1}, s_{n+2}, ..., s_{2n-1} < 0$$

Production amplitudes in LLA

$$A_{2\to 2n} = \int \prod_{i=1}^{n-1} d^2k_i^\perp d^2k_i^{\perp\prime} \Phi_1(k_1^\perp, ...)\Phi_2(k_1^{\perp\prime}, ...) \int d\omega s^\omega G_\omega(k_1^\perp, ...k_n^{\perp\prime})$$

17 Integrable open spin chain

BKP equation for states in adjoint representation at large N_c

$$E\Psi = H\psi\,,\ \ H = h + h^*\,,\ \ [h, h^*] = 0$$

Dual variables and fixing the Möbius invariance (L. (2009))

$$h = \ln(Z_1^2\partial_1) - 2\psi(1) + \ln\partial_{n-1} + \sum_{k=1}^{n-2} h_{k,k+1}\,,\ \ p_k = Z_{k-1,k}\,,\ \ Z_0 = 0\,,\ \ Z_n = \infty$$

Pair hamiltonian of the open spin chain

$$h_{1,2} = \ln(Z_{12}^2\partial_1) + \ln(Z_{12}^2\partial_2) - 2\ln Z_{12} - 2\psi(1)$$

Integrals of motion and Baxter equation (L. (2009))

$$[\hat{D}(u), h] = 0\,,\ \ \hat{D}(u) = \sum_{r=0}^{n-1} u^r\, \hat{q}_{n-1-k}\,,\ \ Z_r = \rho_r$$

18 Baxter-Sklyanin approach

Sklyanin ansatz for the wave function

$$\Omega = \prod_k Q(\hat{u}_k, \hat{u}_k^*)\, \Omega_0 \,, \quad B(\hat{u}_k) = 0$$

Pseudo-vacuum atate (F.,K. (1995))

$$\Omega_0 = \prod_{l=1}^{n-1} |Z_l|^{-4}$$

Baxter equation for the open spin chain (L. (2009))

$$D(u)Q(u) = (u+i)^{n-1} Q(u+i)\,, \; D(u) = \prod_{l=1}^{n-1} (u - ia_l)\,, \; a_l = i\nu_l + \frac{n_l}{2}$$

Baxter function and holomorphic energies (L. (2009))

$$Q(u) = \prod_{l=1}^{n-1} \frac{\Gamma(-iu - a_l)}{\Gamma(-iu + 1)}\,, \; \epsilon = \sum_{l=1}^{n-1} (\psi(a_l) + \psi(-a_l) - 2\psi(1))$$

19 Three-gluon composite state

Wave function in the coordinate representation

$$\Psi = Z_2^{a_1+a_2} (Z_2^*)^{\widetilde{a}_1+\widetilde{a}_2} \int \frac{d^2y}{|y|^2}\, y^{-a_2}\, (y^*)^{\widetilde{a}_2} \left(\frac{y-1}{y-Z_2/Z_1}\right)^{a_1} \left(\frac{y^*-1}{y^*-Z_2^*/Z_1^*}\right)^{\widetilde{a}_1}$$

Fourier transformation

$$\Psi(\vec{Z}_1, \vec{Z}_2) = \int d^2p_1\, d^2p_2\, \exp(i\vec{p}_1\vec{Z}_1)\, \exp(i\vec{p}_2\vec{Z}_2)\, \Psi(\vec{p}_1, \vec{p}_2)\,, \;\; E = E(a_1) + E(a_2)$$

Baxter-Sklyanin representation (L. (2009))

$$\Psi^t(\vec{p}_1, \vec{p}_2) = P^{-a_1-a_2}\, (P^*)^{-\widetilde{a}_1-\widetilde{a}_2} \int du d\widetilde{u}\, u\, \widetilde{u}\, Q(u, \widetilde{u}) \left(\frac{p_1}{p_2}\right)^u \left(\frac{p_1^*}{p_2^*}\right)^{u^*}$$

Baxter function

$$Q(u, \widetilde{u}) = \frac{\Gamma(-u)\,\Gamma(-\widetilde{u})}{\Gamma(1+u)\,\Gamma(1+\widetilde{u})}\, \frac{\Gamma(u-a_1)\,\Gamma(u-a_2)}{\Gamma(1-\widetilde{u}+\widetilde{a}_1)\,\Gamma(1-\widetilde{u}+\widetilde{a}_2)}\,, \quad \int du d\widetilde{u} \equiv \int d\nu \sum_n$$

20 Integrable spin chain deformation

Kinematics at a non-zero t-channel temperature

$$\rho_r = x_r + iy_r\,,\;\; 0 < y_r < 1/T\,,\;\; p_r^y = 2\pi k\,,\;\; k = 0, \pm 1, \pm 2, \ldots$$

BFKL Hamiltonian for $T \neq 0$ (de Vega, Lipatov (2005))

$$h = \sum_{s=1,2} \left(\psi\left(1 + i\frac{p_s}{2\pi T}\right) + \psi\left(1 - i\frac{p_s}{2\pi T}\right) - 2\psi(1) + \frac{2}{p_s}\ln(2\sinh(\pi T \rho_{12}))\, p_s \right)$$

Integrable spin chain with the Möbius group generators

$$M_z = \partial\,,\;\; M_+ = e^{\rho}\partial\,,\;\; M_- = e^{-\rho}\partial$$

Confining potential energy (anti-Meissner effect) (d V.,L. (2013))

$$\lim_{\rho_{12}\to\infty} V(\rho_{12}) \sim |\rho_{12}|$$

21 Semiclassical limit of BFKL kernels

Divergency of the Fredholm constraint for the integral kernel

$$\int d^2pd^2p'|K(p^2,p'^2,(p-p')^2)|^2 < \infty\,,$$

$$\lim_{p-p'\to 0} |K_{BFKL}(p^2,p'^2,(p-p')^2)|^2 \sim \big(\omega(p^2)\delta^2(p-p')\big)^2$$

Semiclassical prediction for the BFKL spectrum in $N=4$ SUSY

$$\lim_{|m|\to\infty} \omega^{(0)}(n,\nu) = -\gamma_K(a)\ln|m|\,, \quad \lim_{|m|\to\infty} \omega^{(8)}(n,\nu) = -\frac{1}{2}\gamma_K(a)\ln|m|$$

Ansatz by Basso et al. for the adjoint BFKL eigenvalue

$$\omega_n^{(8)}(u) = \int_0^\infty \frac{dt}{t}\left(\cos ut\, e^{-nt/2}\left(\frac{\gamma_-(2gt)}{e^t-1} - \frac{\gamma(-2gt)}{2}\right) - \frac{J_0(2gt)\gamma(-2gt)}{e^t-1}\right),$$

$$\nu_n(u) = u - \int_0^\infty \frac{dt}{2t}\left(\sin ut\, e^{-nt/2}\left(\frac{\gamma_+(2gt)}{e^t-1} + \frac{\gamma(2gt)}{2}\right)\right),\quad \gamma(t) = \gamma_+(t)+\gamma_-(t)$$

22 Discussion

1. Remarkable properties of the BFKL and BKP equations
2. Integrability of the BKP equation at large N_c
3. Effective action and loop corrections at high energies
4. Pomeron and reggeized graviton in N=4 SUSY
5. Analytic properties of high energy amplitudes
6. Breakdown of the Regge pole ansatz and Mandelstam cuts
7. Integrability of the equations for adjoint reggeon states
8. BFKL equation at large conformal weights and γ_K