

# Recent advancements in Higgs boson pair production

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**Andreas Papaefstathiou**



**Physik-Institut,  
Universität Zürich**

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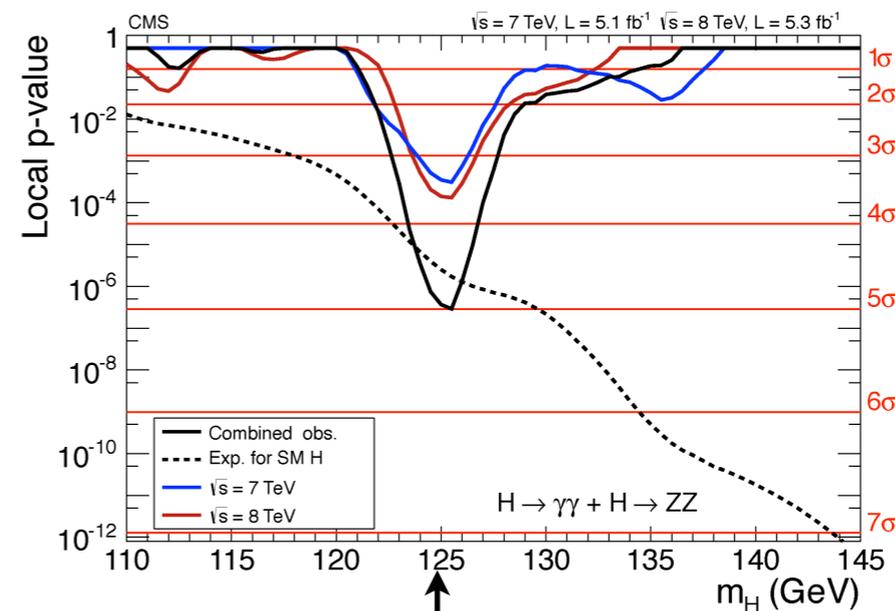
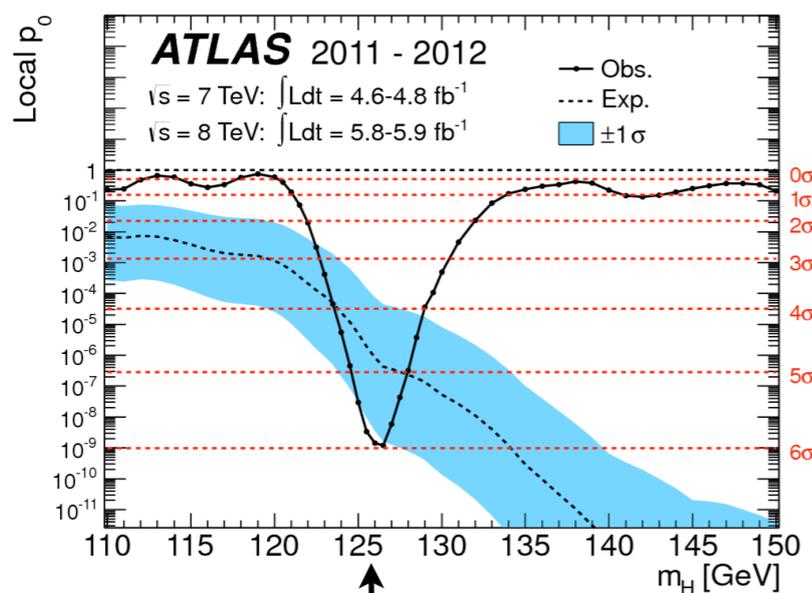
**QCD@LHC2014  
Suzdal, 25-29 August 2014**

# the plan

- motivation,
- **hh (ggF)** at leading order,
- brief overview: search strategies,
- **hh (ggF)** at higher orders,
- Monte Carlo event generation,
- outlook+conclusions.

# motivation

- the Higgs boson has been discovered at the LHC (already “old” news):



(p-values at  
**ATLAS &  
CMS**)

$\sim 125 \text{ GeV}$

- natural next step for LHC: measure its couplings, compare to SM expectations.

# Higgs self-couplings

- part of this venture: self-couplings of the Higgs boson:

$$\mathcal{L} \supset -\frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2v} (1 + \delta) h^3 - \frac{m_h^2}{8v^2} (1 + \tilde{\delta}) h^4$$

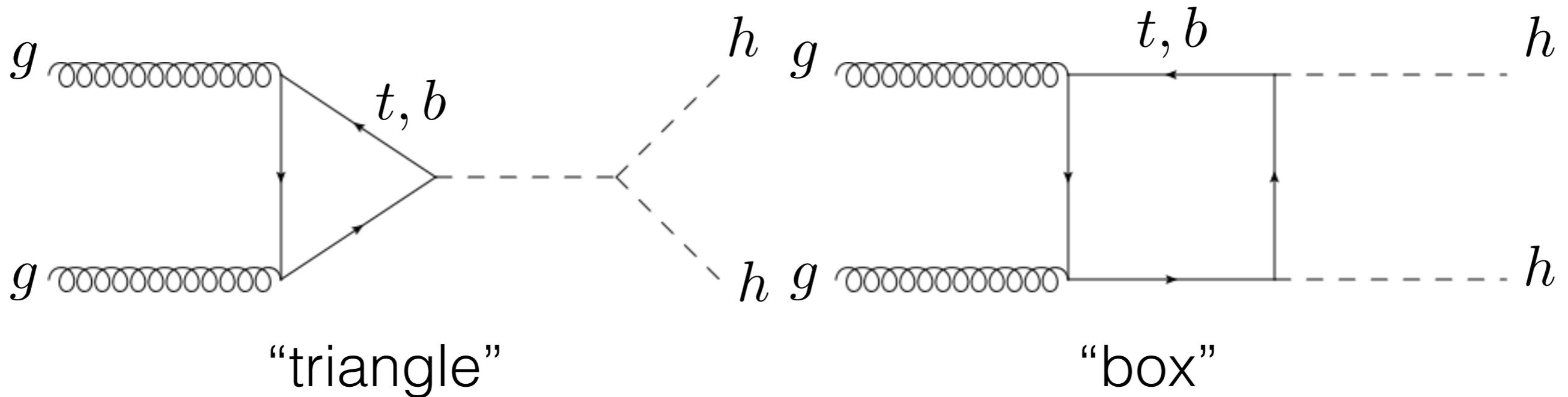
→  $\delta, \tilde{\delta}$  : possible deviations from the SM.

- can be probed @ LHC: only (?) via multi-Higgs boson production: **hh** and **hhh** final states.
- however: quartic coupling **cannot** be probed at the LHC: triple Higgs production cross section is tiny!

$$\sigma(pp \rightarrow hhh @ 14 \text{ TeV}) \sim 0.04 \text{ fb} \Rightarrow 120 \text{ events at } 3000 \text{ fb}^{-1}$$

# Higgs boson pair production (LO)

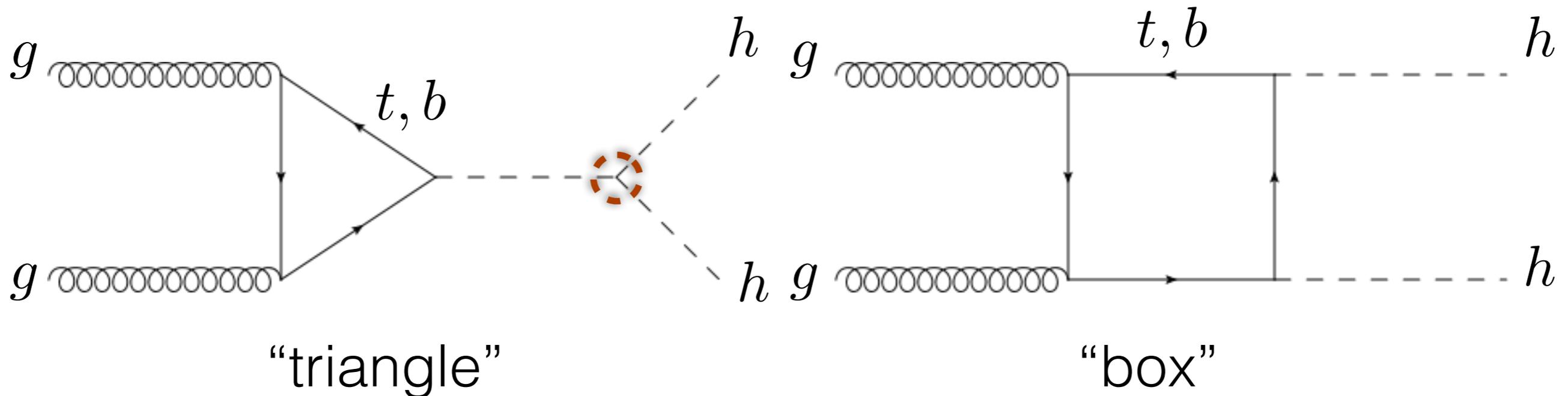
- on the other hand:  $\sigma(pp \rightarrow hh @ 14 \text{ TeV}) \sim 40 \text{ fb}$   
 $\Rightarrow$  hard, but worth investigating!
- at leading order, gluon-fusion-dominated.



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 (i.e. integrate it out)

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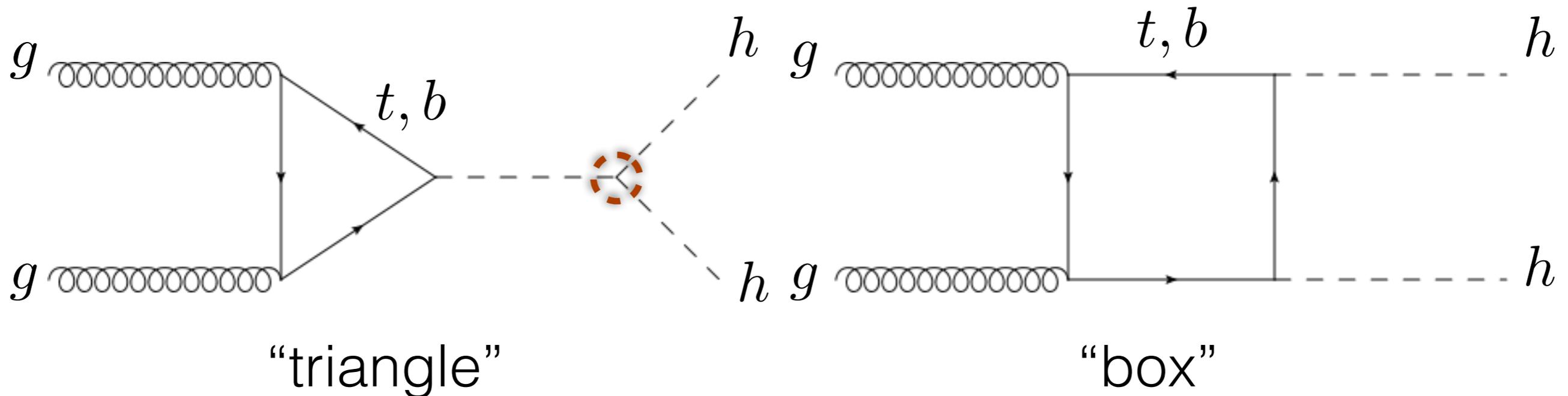
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- use shrink down the top loop to an effective vertex?  
 (i.e. integrate it out)  $\longrightarrow$  **No!**:  $4M_h^2 > M_{\text{top}}^2$

# search strategies @ LHC

- Higgs bosons in **hh** are relatively hard:

$$\longrightarrow (p_{T,\text{peak}} \sim 100\text{-}200 \text{ GeV})$$

- can use boosted-jet techniques (+ jet substructure).

- search for hh final states in:

(+)

(-)

$$hh \rightarrow (b\bar{b})(\tau^+\tau^-)$$

low bkg

**$\tau$** -tagging

$$hh \rightarrow (b\bar{b})(\gamma\gamma)$$

v. low bkg

j-to-photon

$$hh \rightarrow (b\bar{b})(W^+W^-)$$

leptons +  $E_{\text{miss}}$

$t\bar{t}$

$$hh \rightarrow (b\bar{b})(b\bar{b})$$

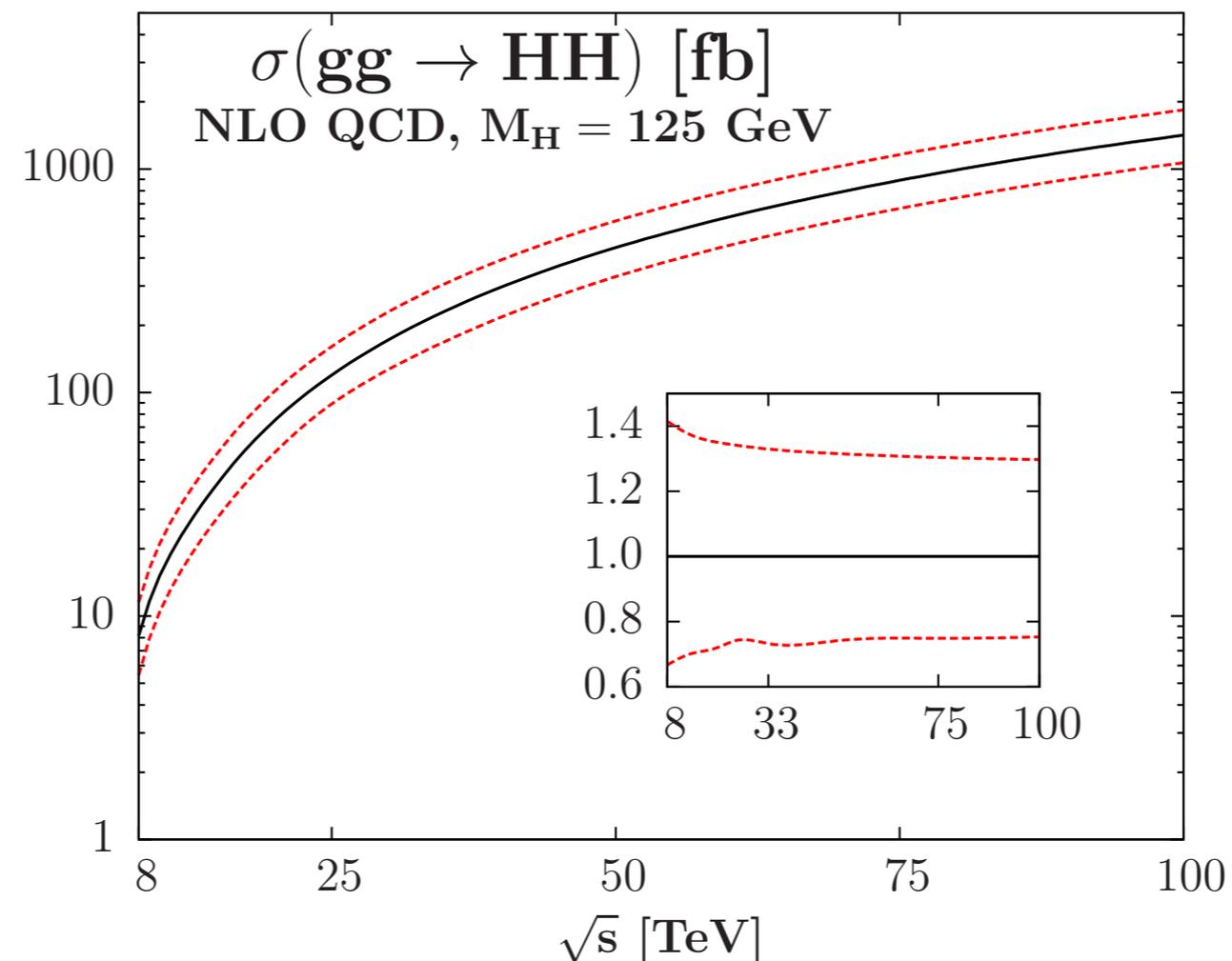
highest BR ( $\sim 1/3$ )

QCD

- possible discovery of SM signal at high-lumi LHC ( $3000 \text{ fb}^{-1}$ ).

# theoretical uncertainty (I)

- unfortunately, theoretical uncertainty on cross section predictions is rather large, e.g.:



[Baglio, Djouadi,  
 Gröber, Mühlleitner,  
 Quevillon, Spira,  
 1212.5581]

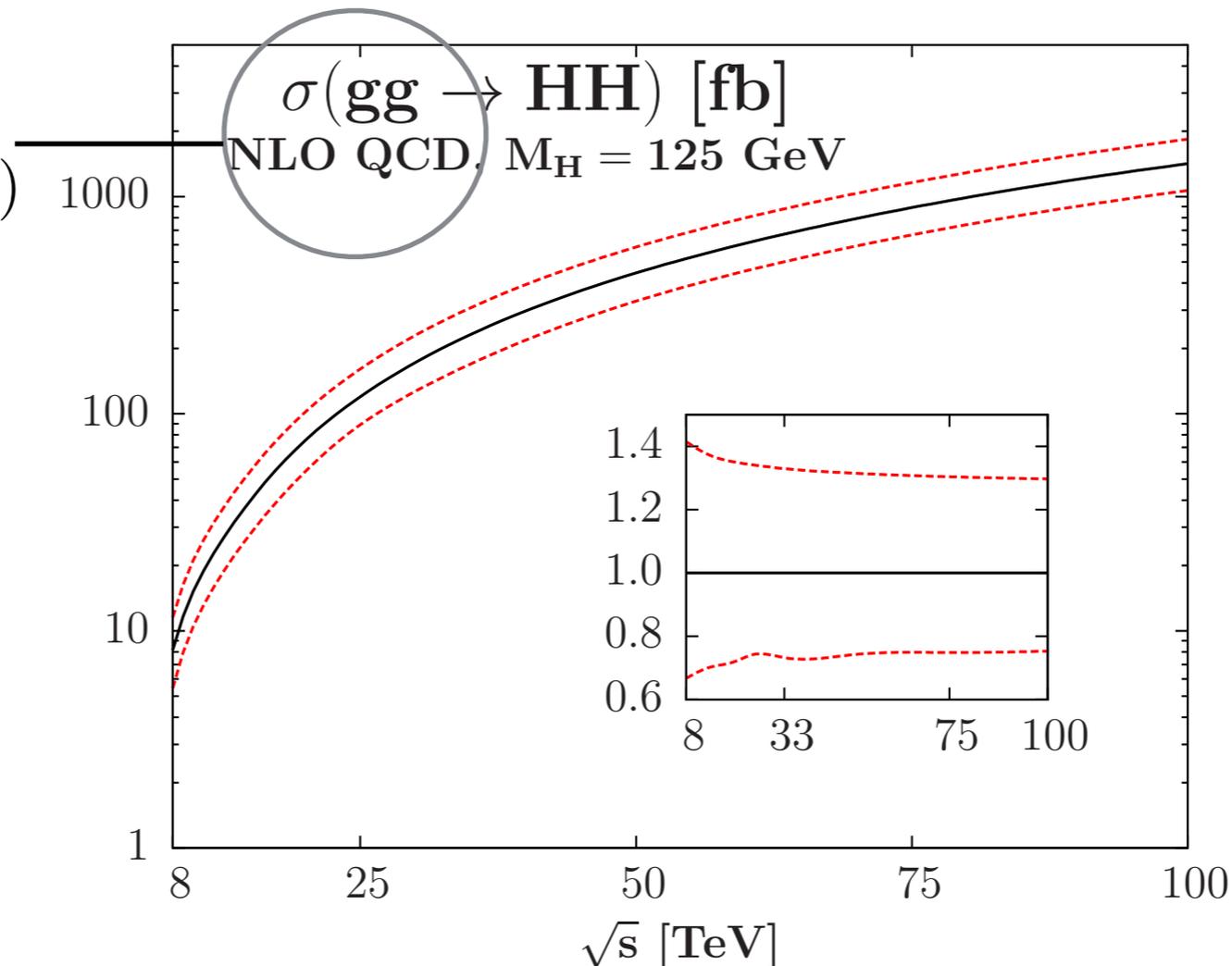
*Figure 10: The total cross section (black/full) of the process  $gg \rightarrow HH + X$  at the LHC for  $M_H = 125$  GeV as a function of  $\sqrt{s}$  including the total theoretical uncertainty (red/dashed). The insert shows the relative deviation from the central cross section.*

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$$\text{NLO} = \text{LO (full } M_t) + \text{NLO } (M_t \rightarrow \infty)$$

[Plehn, Spira, Zerwas, hep-ph/9603205 and Dawson, Dittmaier, Spira hep-ph/9805244]



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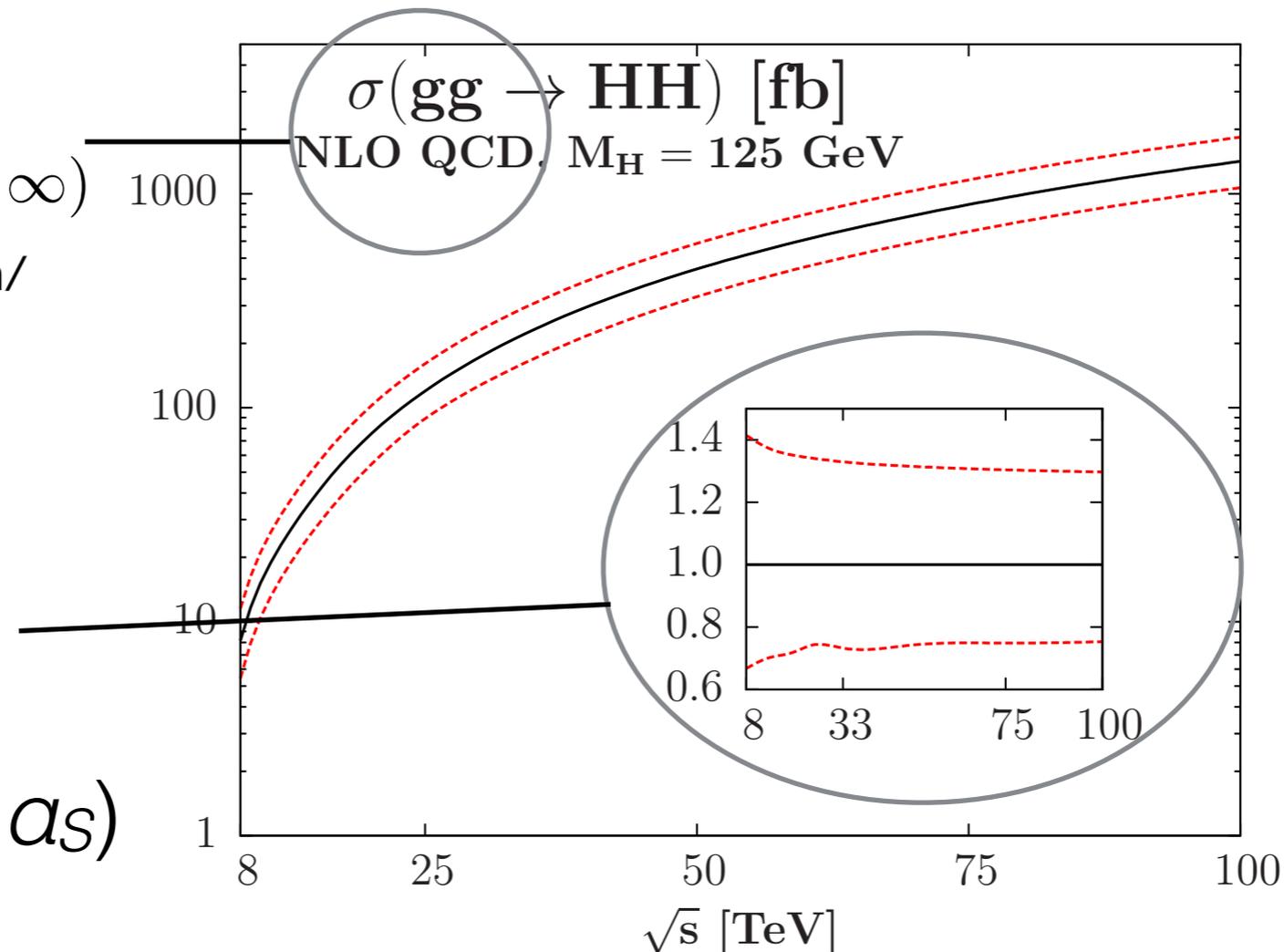
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**30-40%  
uncertainty**  
(scale + PDF +  $\alpha_s$ )



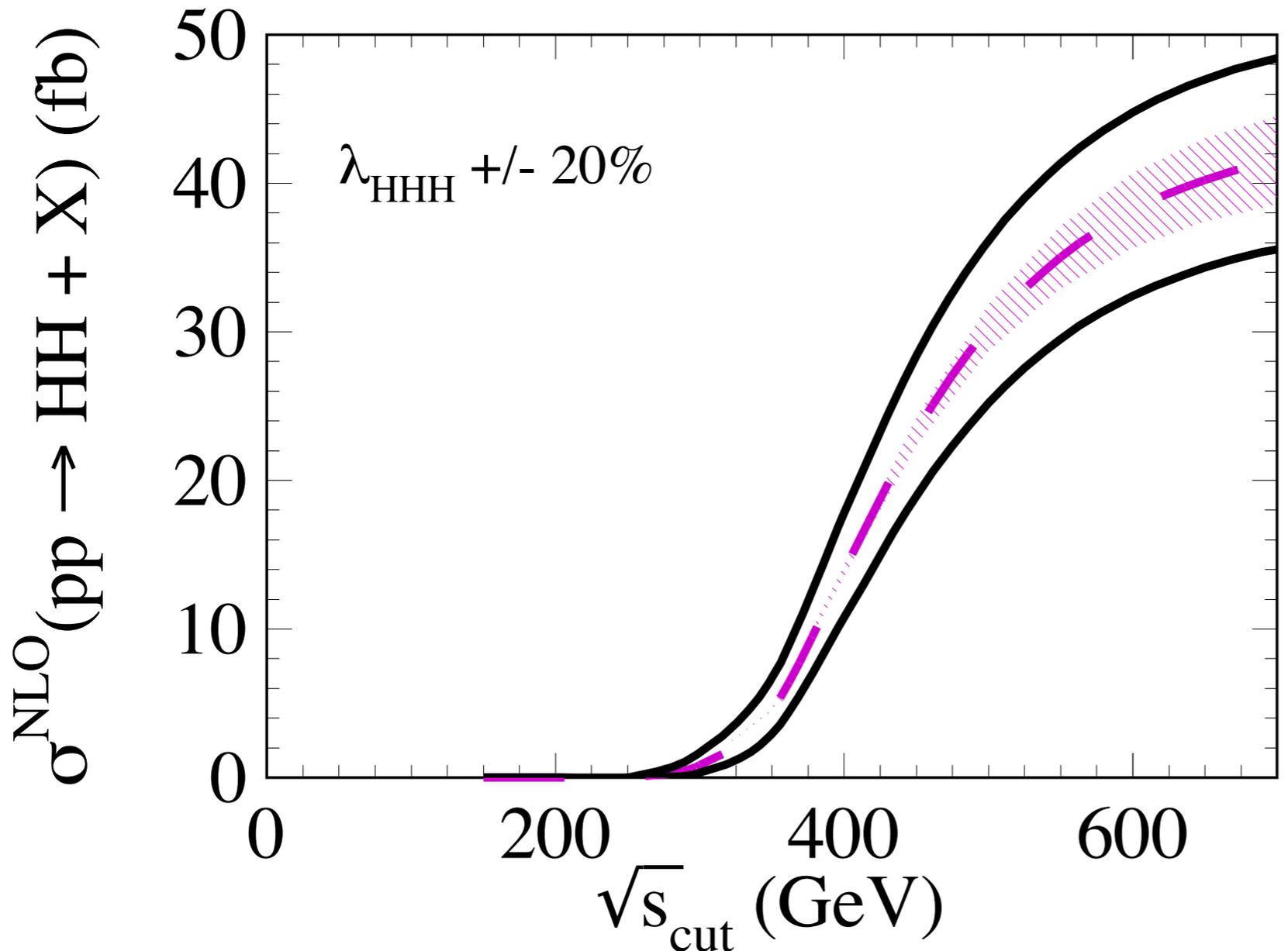
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# theoretical uncertainty (II)

- plus: low-energy theorem,  $M_t \rightarrow \infty$ , introduces further uncertainty.

[Grigo, Hoff, Melnikov, Steinhauser, 1305.7340]  $\longrightarrow$  corrections to NLO  $\sigma$  up to  $\mathcal{O}(1/M_t^8)$



$\sqrt{s_{\text{cut}}}$ : upper cut partonic c.o.m. energy.

**black**: variations of the self-coupling by  $\pm 20\%$ .

**violet**: uncertainty due to un-calculated  $1/M_t$  corrections.  $\longrightarrow \mathcal{O}(10\%)$

# higher-order corrections

- the “ideal” solution: calculate the full NLO **hh** production.
- but this is challenging: **two loops** with **two mass scales** is currently the state-of-the-art + requires time and effort.
- until then:
  - NLO, NNLO in  $M_t \rightarrow \infty$  (low-energy theorem),
  - resummation.

# NLO hh

[Plehn, Spira, Zerwas, hep-ph/9603205 and Dawson, Dittmaier, Spira hep-ph/9805244]

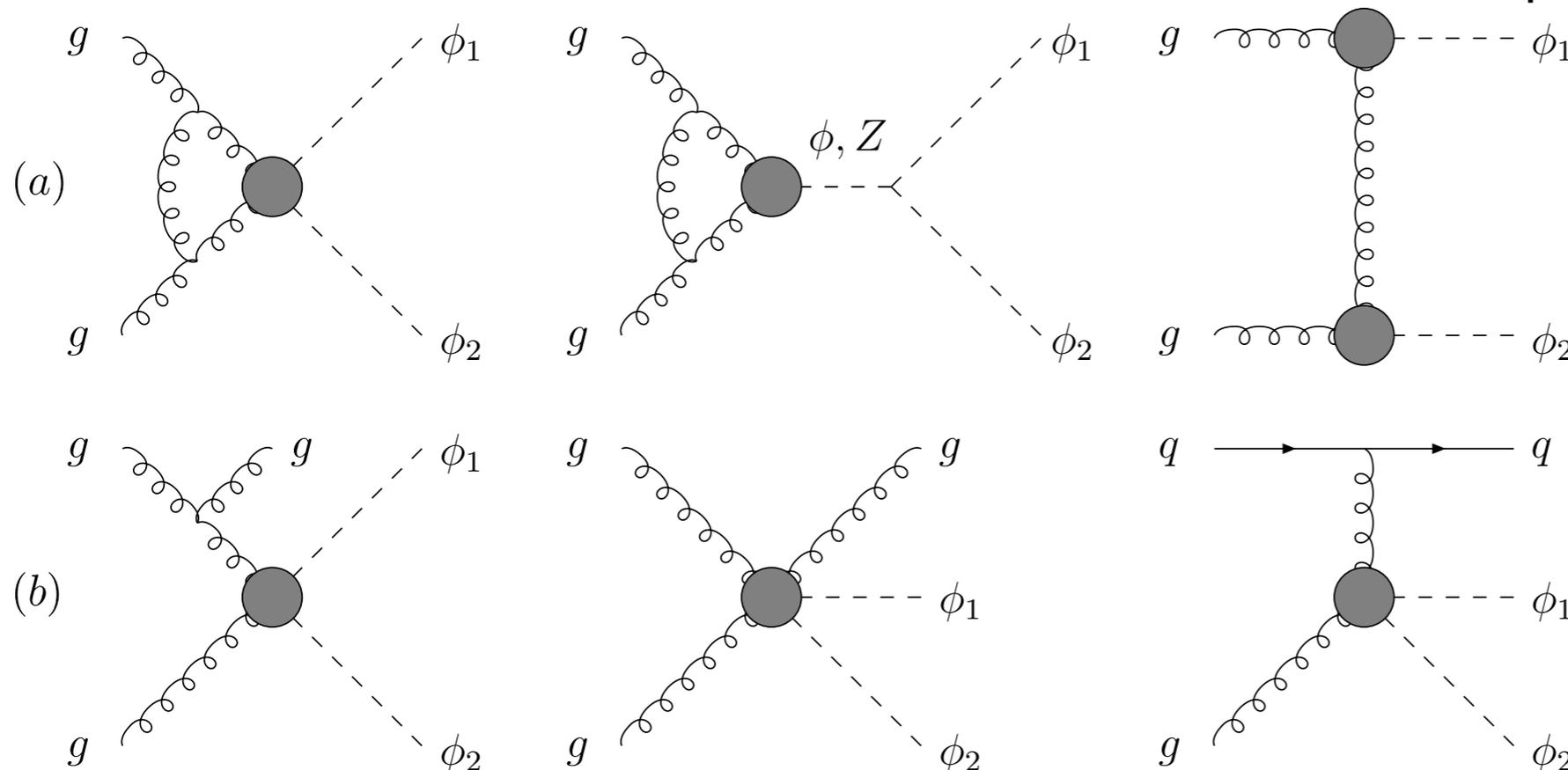
- LO: at one loop, using full top mass dependence.
- NLO: using the low-energy theorem:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^{\mu\nu,a} G_{\mu\nu}^a \left( C_H \frac{h}{v} - C_{HH} \frac{h^2}{2v^2} \right)$$

coefficients:

$$C_X \sim \sum_{n \geq 0} C_X^{(i)} \alpha_S^{i+1}$$

$$X \in (H, HH)$$



“improve validity of results: **insert full expressions for form factors [...]**”

# NNLO hh

- low energy theorem ( $M_t \rightarrow \infty$ ):

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^{\mu\nu,a} G_{\mu\nu}^a \left( C_H \frac{h}{v} - C_{HH} \frac{h^2}{2v^2} \right)$$

known up to:  $\mathcal{O}(\alpha_S^3)$

$\mathcal{O}(\alpha_S^2)$

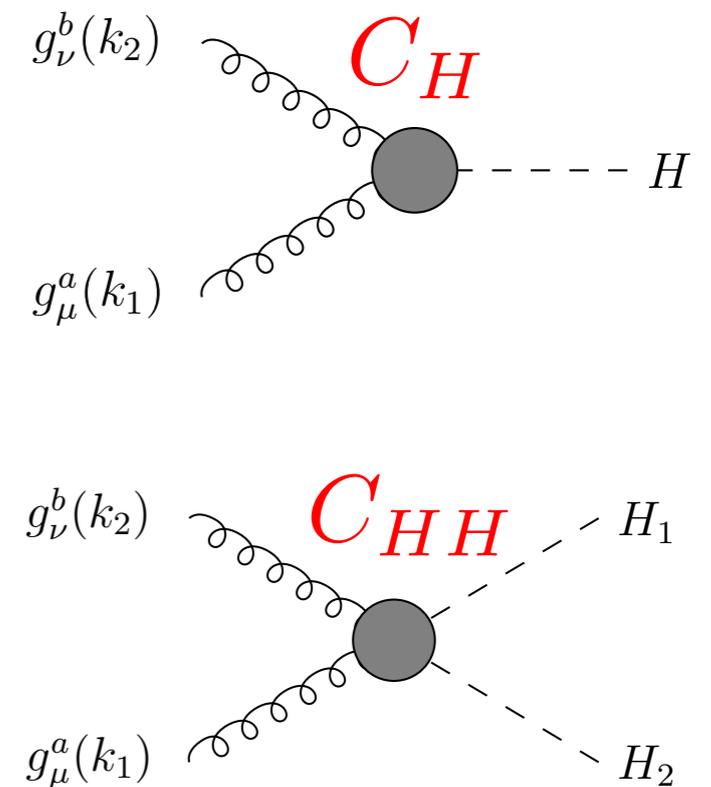
also:  $C_{HH}^{(0)} = C_H^{(0)}$  and  $C_{HH}^{(1)} = C_H^{(1)}$

[de Florian, Mazzitelli  
1305.5206,  
1309.6594]

→

**NNLO** assuming:

$$C_{HH}^{(2)} = C_H^{(2)}$$



# NNLO hh

[de Florian, Mazzitelli, 1309.6594]  $\longrightarrow$  2.5% variation on  $\sigma_{\text{total}}$  in

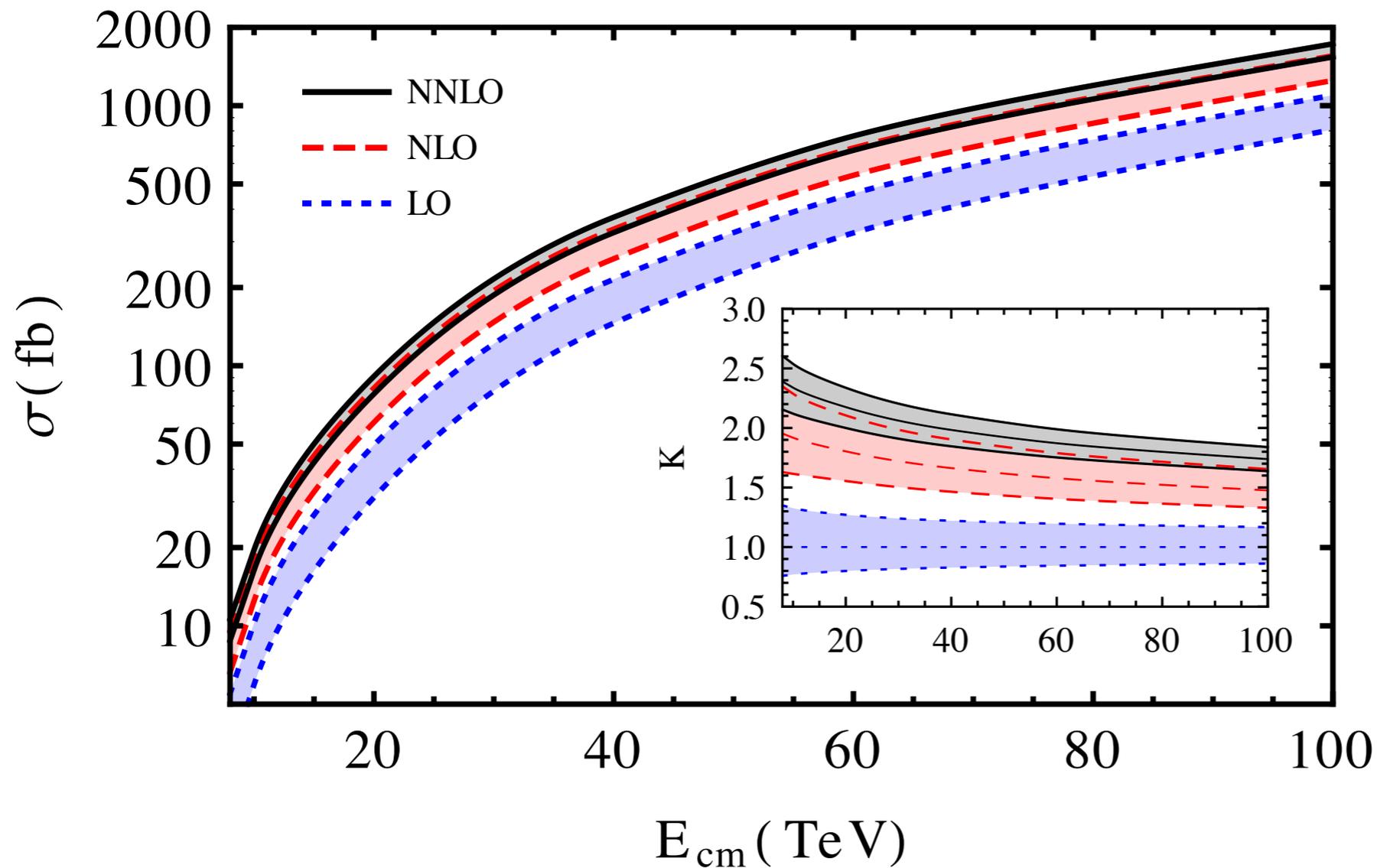
$$0 < C_{HH}^{(2)} < 2C_H^{(2)}$$

- more recently ( $\sim 2$  weeks ago): calculation of the three-loop matching coefficient  $C_{HH}^{(2)}$ .

[Grigo, Melnikov, Steinhauser, 1408.2422]

- they find that  $C_{HH}^{(2)} / C_H^{(2)} \approx 1.8$ .
- causes  $\sim 1\%$  variation on  $\sigma_{\text{total}}$  with respect to previous assumption.
- (but note: interesting change in threshold behaviour.)

# NNLO hh



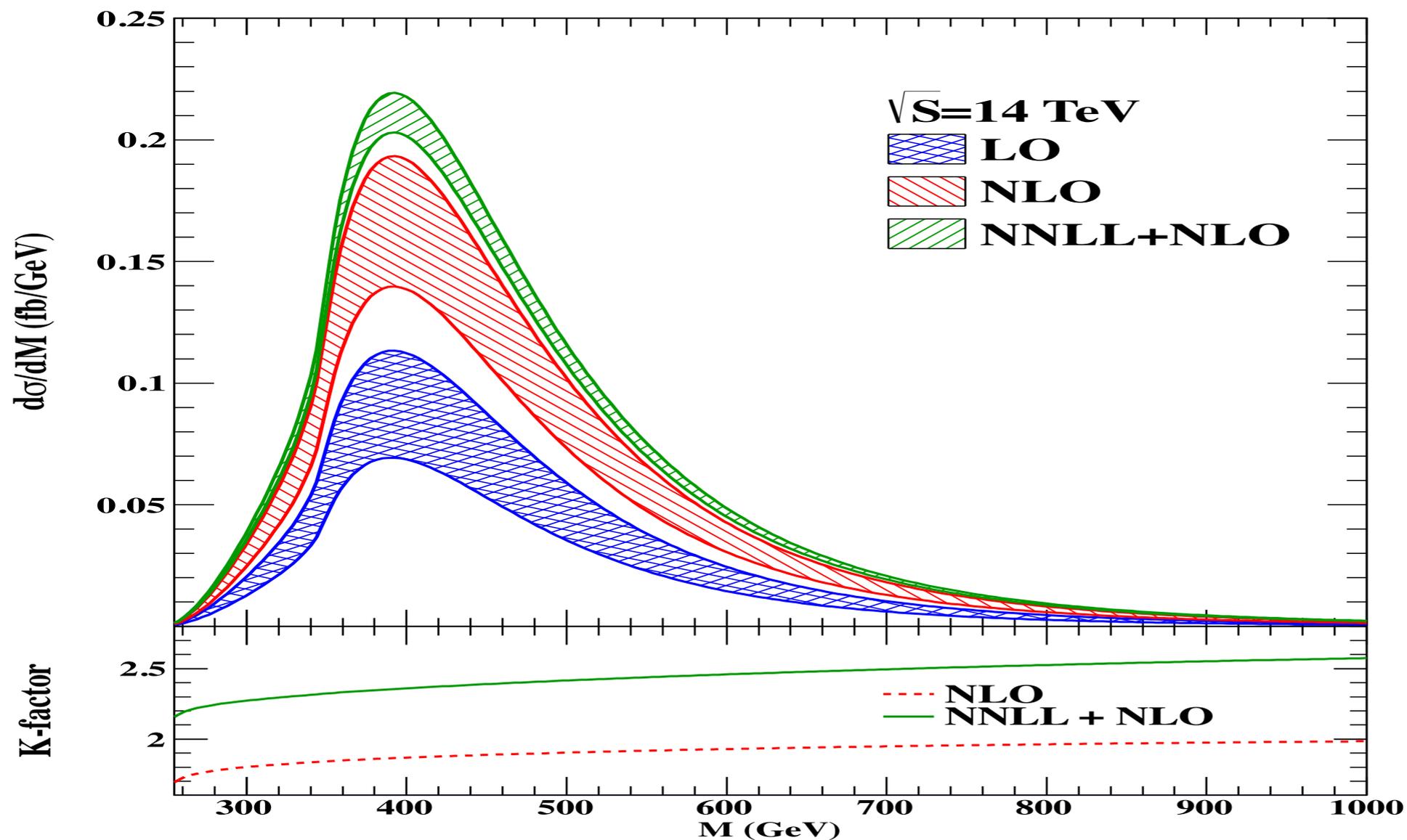
[de Florian, Mazzitelli,  
1309.6594]

- scale uncertainty  $\sim 20\%$  @ NLO VS  $\sim 8\%$  at NNLO
- PDF+ $a_s \sim 10\%$ .
- and convergence of perturbative series is improved!

# resummation

[D. Y. Shao, C. S. Li, H. T. Li, J. Wang, 1301.1245 ]

- threshold resummation in Soft-Collinear Effective Theory (SCET).
- claim: scale uncertainty reduced to  $\sim 8\%$ .



invariant  
mass  
distribution



# Monte Carlo event generation

- so far: discussed theoretical calculations of  $\sigma_{\text{total}}$  and some theoretical (parton-level) distributions.
- for detailed experimental analyses: one needs a Monte Carlo event generator.
- at leading order: simply interface to a parton shower (Herwig, Pythia, Sherpa, etc.).
- recently: go beyond using merging (MLM) or matching (MC@NLO).



# merging via MLM

[Q. Li, Q. Yan, X. Zhao, 1312.3830]

[P. Maierhöfer, **AP**, 1401.0007]

- supplement the parton shower (PS) (soft/collinear QCD radiation) with exact matrix elements (MEs).
- use a merging scheme to put PS and MEs together, avoiding double-counting.
- MLM method “matches” jets to partons according to a “merging” scale and vetoes accordingly.



# merging via MLM

[Q. Li, Q. Yan, X. Zhao, 1312.3830]

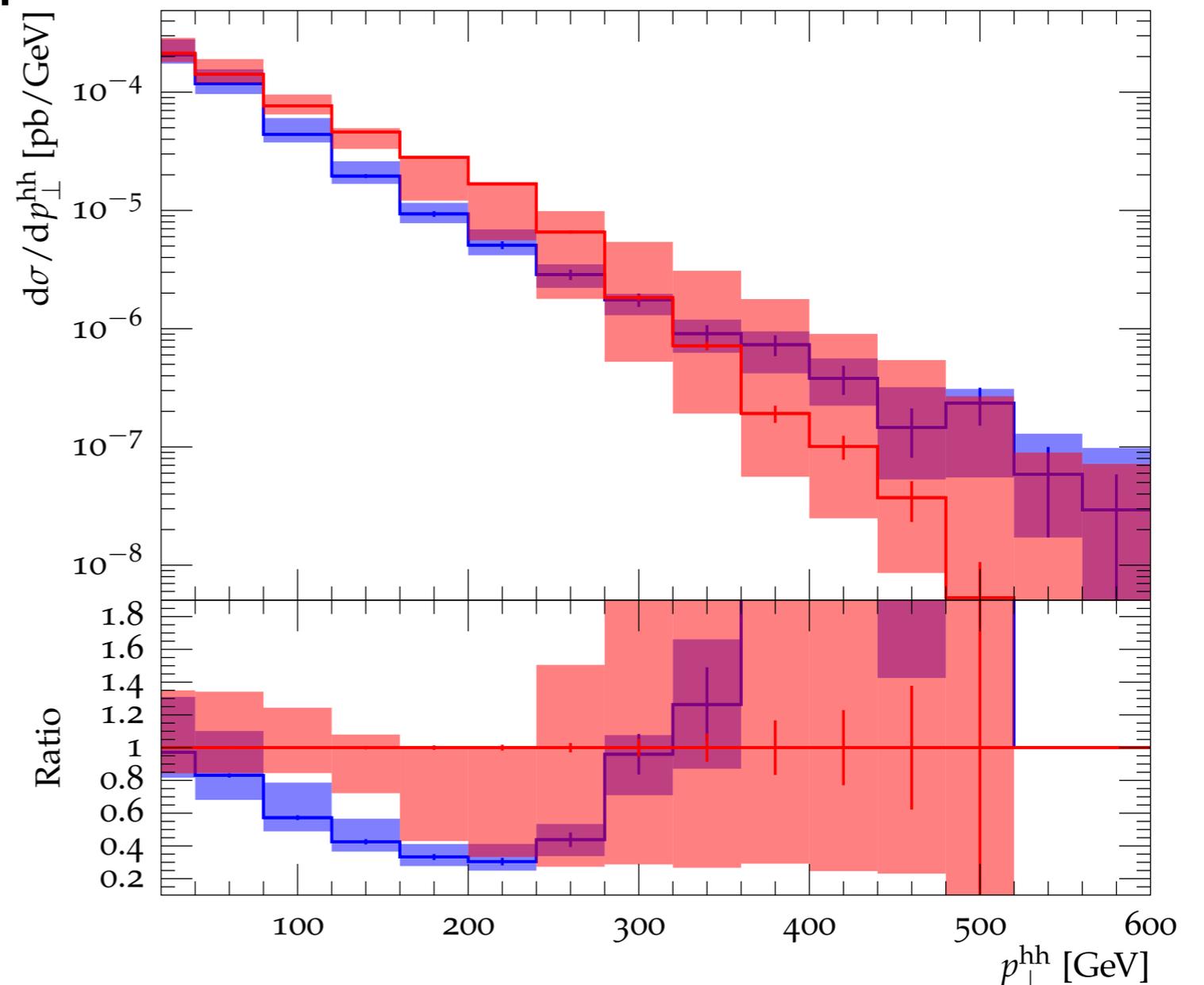
[P. Maierhöfer, **AP**, 1401.0007]

- implementation using MadGraph+Pythia, [Q. Li, Q. Yan, X. Zhao, 1312.3830]
- our implementation: using **OpenLoops** generator: evaluates one-loop MEs efficiently using numerical & tensor integral reduction. [F. Cascioli, P. Maierhöfer, S. Pozzorini, 1111.5206]
- kinematical description of the first jet at high- $p_T$ : via exact ME for  $hh+1$  parton.
- MLM merging performed in Herwig++.

# merging via MLM

[P. Maierhöfer, *AP*, 1401.0007]

- scale uncertainty reduction: from leading-log in PS to LO in ME for the first jet  $p_T$ .
- e.g. transverse momentum of Higgs boson pair.
- **red**: parton shower, **blue**: merged sample.



# matching using MC@NLO



[R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer,  
P. Torrielli, E. Vryonidou, M. Zaro, 1401.7340]

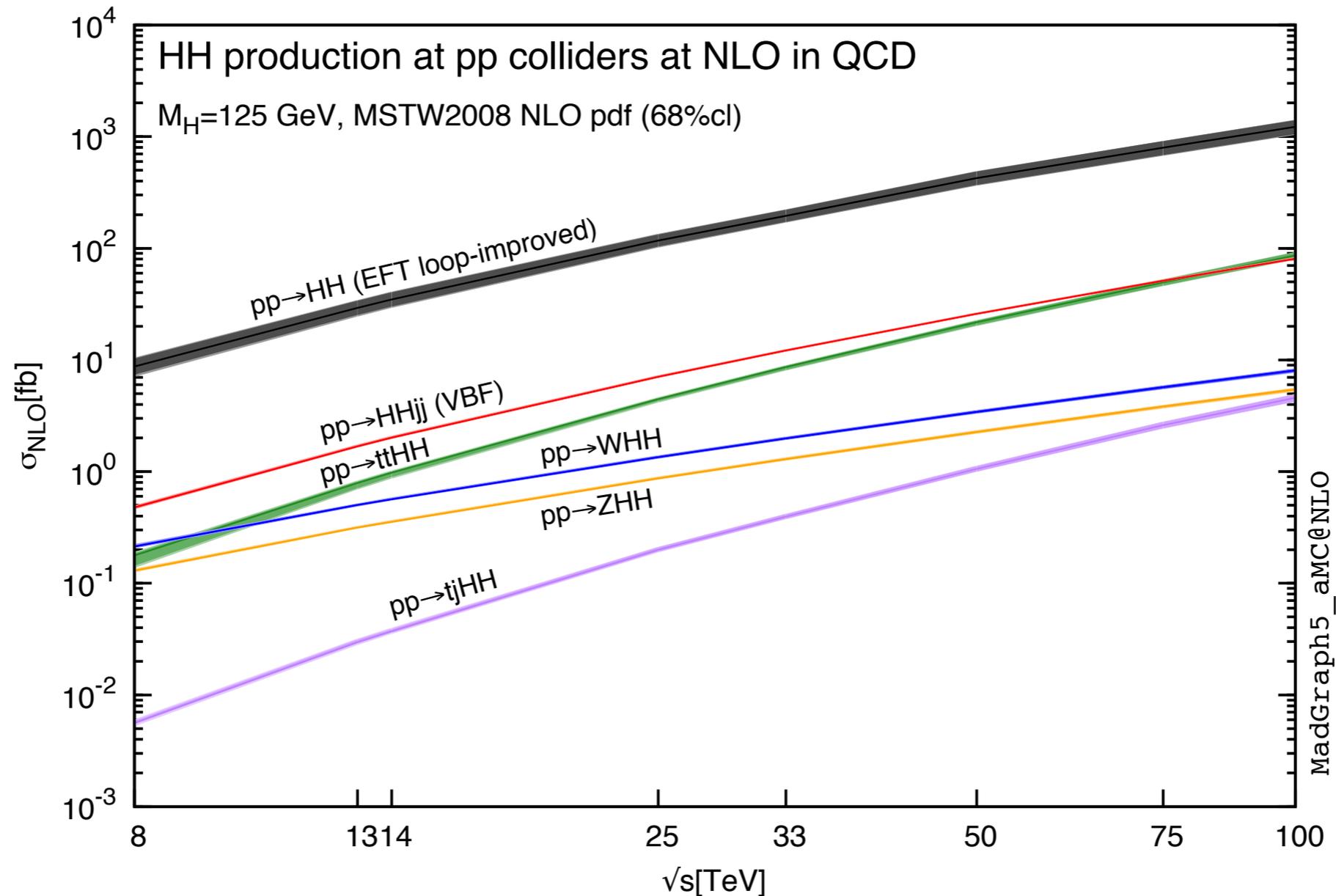
- use exact LO and real emission MEs (hh+1 parton) as was done with merging.
- use the “two-loop” virtual corrections as obtained using the low energy theorem ( $M_t \rightarrow \infty$ ), reweight according to exact LO.
- match via MC@NLO method: removes the double-counting resulting from combination of hh+PS and hh+1 parton ME.

# matching using MC@NLO



[R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, P. Torrielli, E. Vryonidou, M. Zaro, 1401.7340]

- other **hh** production processes also included in the aMC@NLO framework:





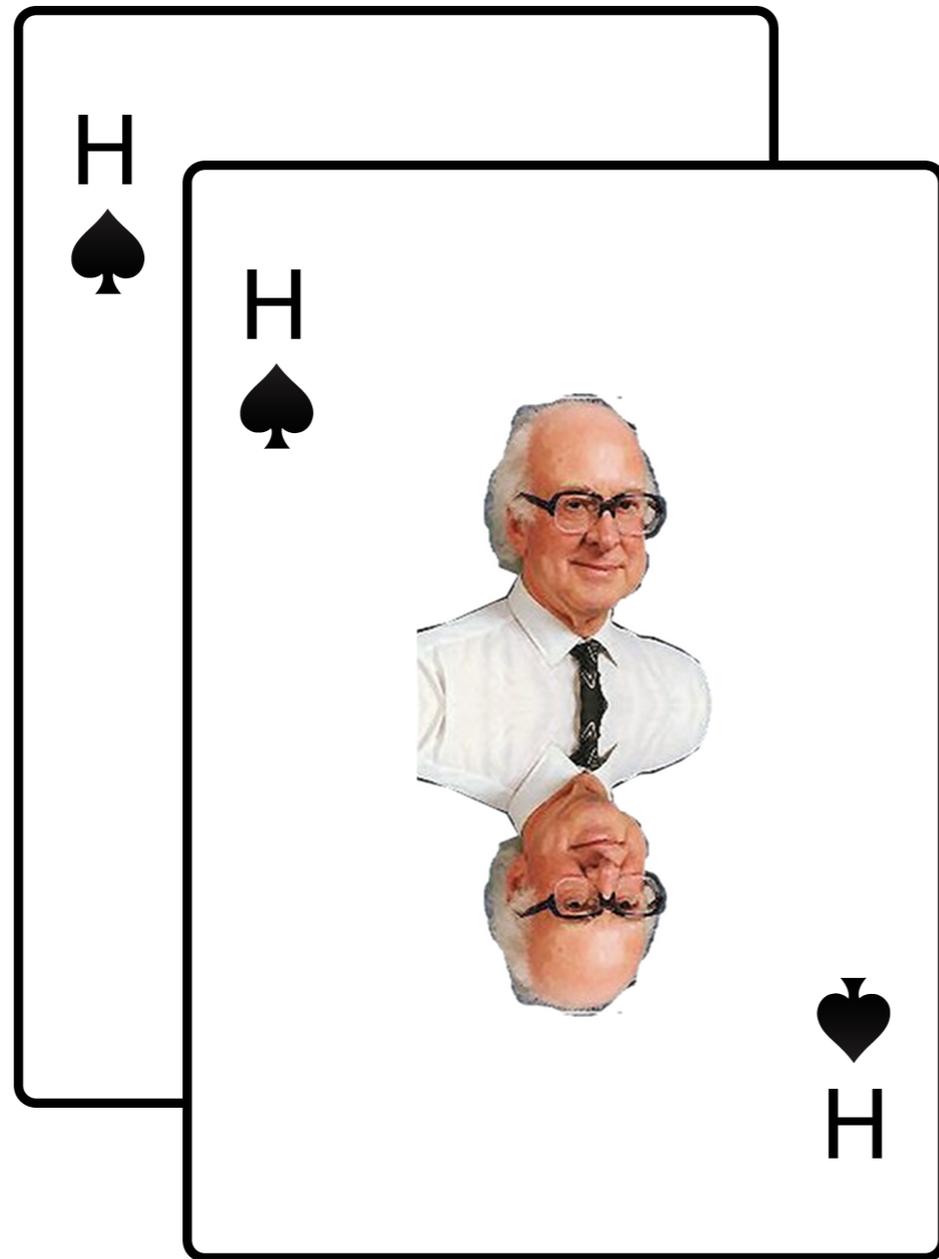
# outlook

- the process is still very much under investigation.
- evidently, there's still a lot to do:
  - full NLO calculation,
  - investigation of search strategies,
  - preparation of ATLAS/CMS experiments (improved triggers?),
  - studies at future colliders ( $e^+e^-$  or high-energy hadron colliders)
  - ...

# conclusions

- **hh** production can provide useful information on the nature of the Higgs boson through its couplings to itself and other SM particles.
- the relation of **hh** to single **h** production could verify whether the **h** is a part of the SM doublet or something more exotic.
- effort is required from both theorists, phenomenologists and experimentalists to turn **hh** into a useful tool at the LHC and future experiments.

# Thanks for your attention!

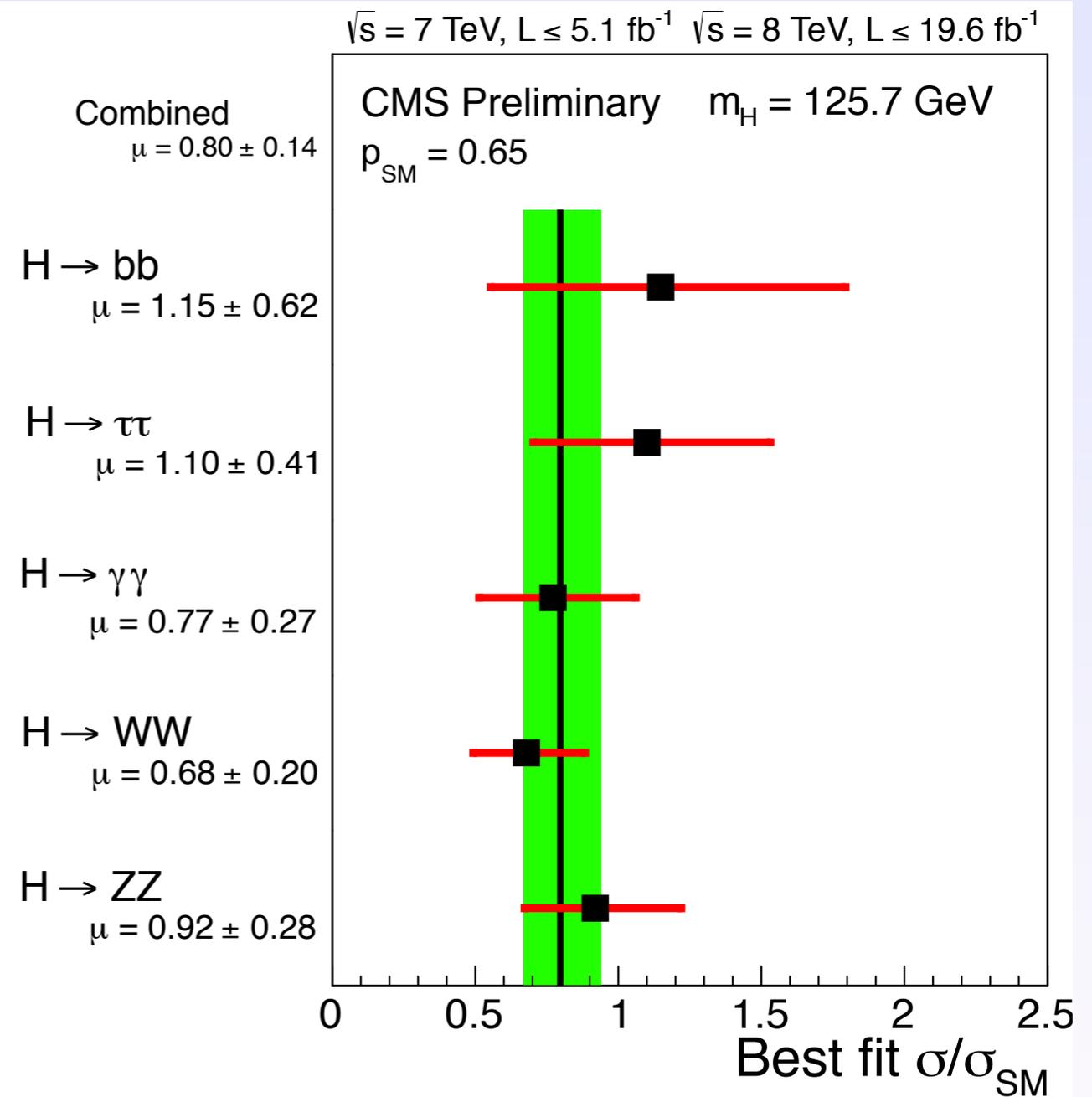
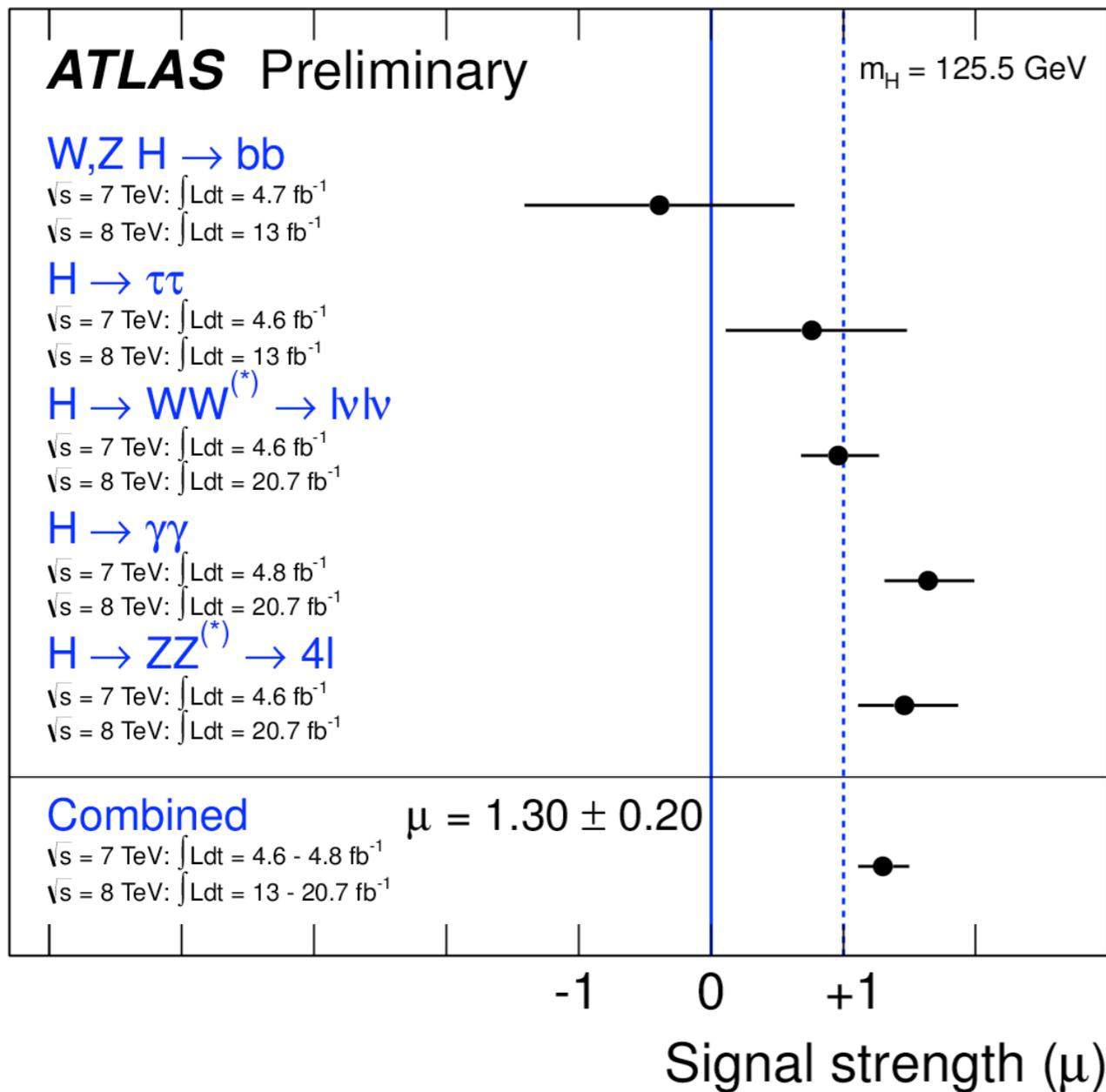


(with apologies  
to Peter Higgs)

appendices

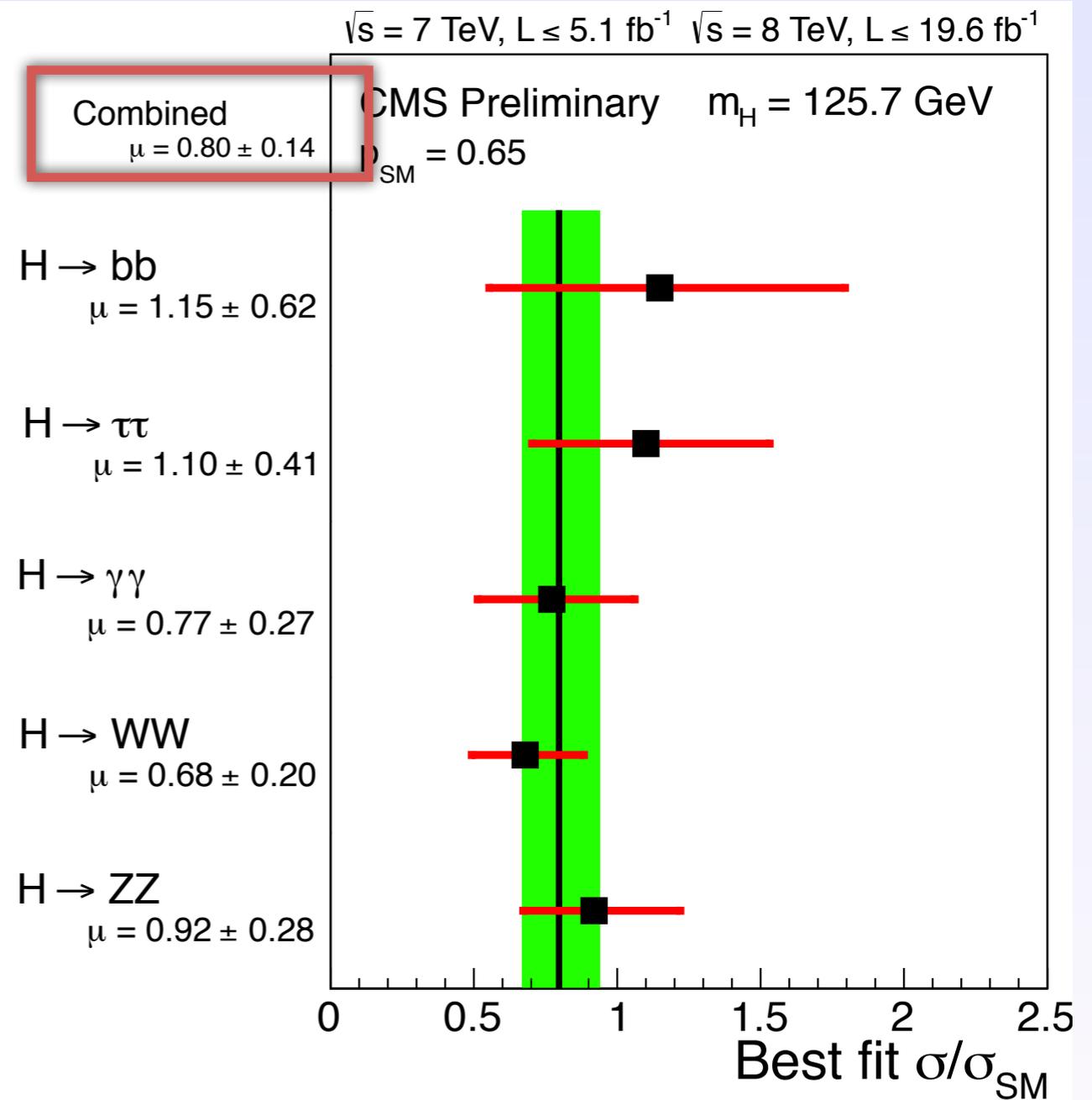
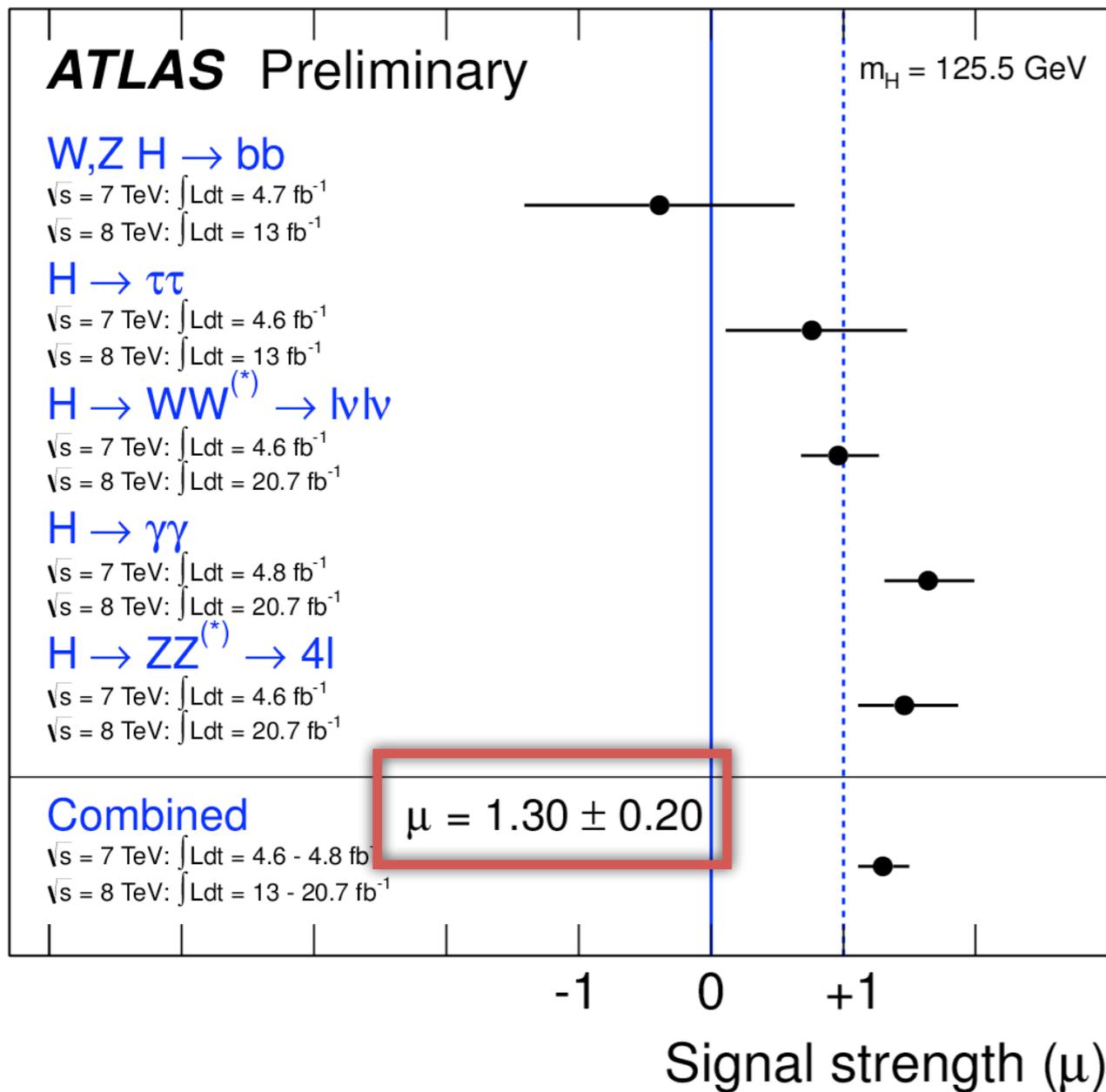
# Higgs Boson signal strengths

$$\mu = \sigma_{\text{obs}} / \sigma_{\text{SM}}$$



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**What about HH, HHH?**



**i. what could we hope to learn  
from multi-Higgs production @  
LHC?**

# electroweak cooking

- ingredients:

$SU(2) \times U(1)$  gauge symmetry

+ complex doublet scalar,  $\phi$

+ potential for  $\phi$  :  $\mathcal{V}(\phi^\dagger \phi)$



# electroweak cooking, steps



- choose a minimum in a particular direction, maintaining U(1) invariance  $\hookrightarrow$  symmetry breaking.

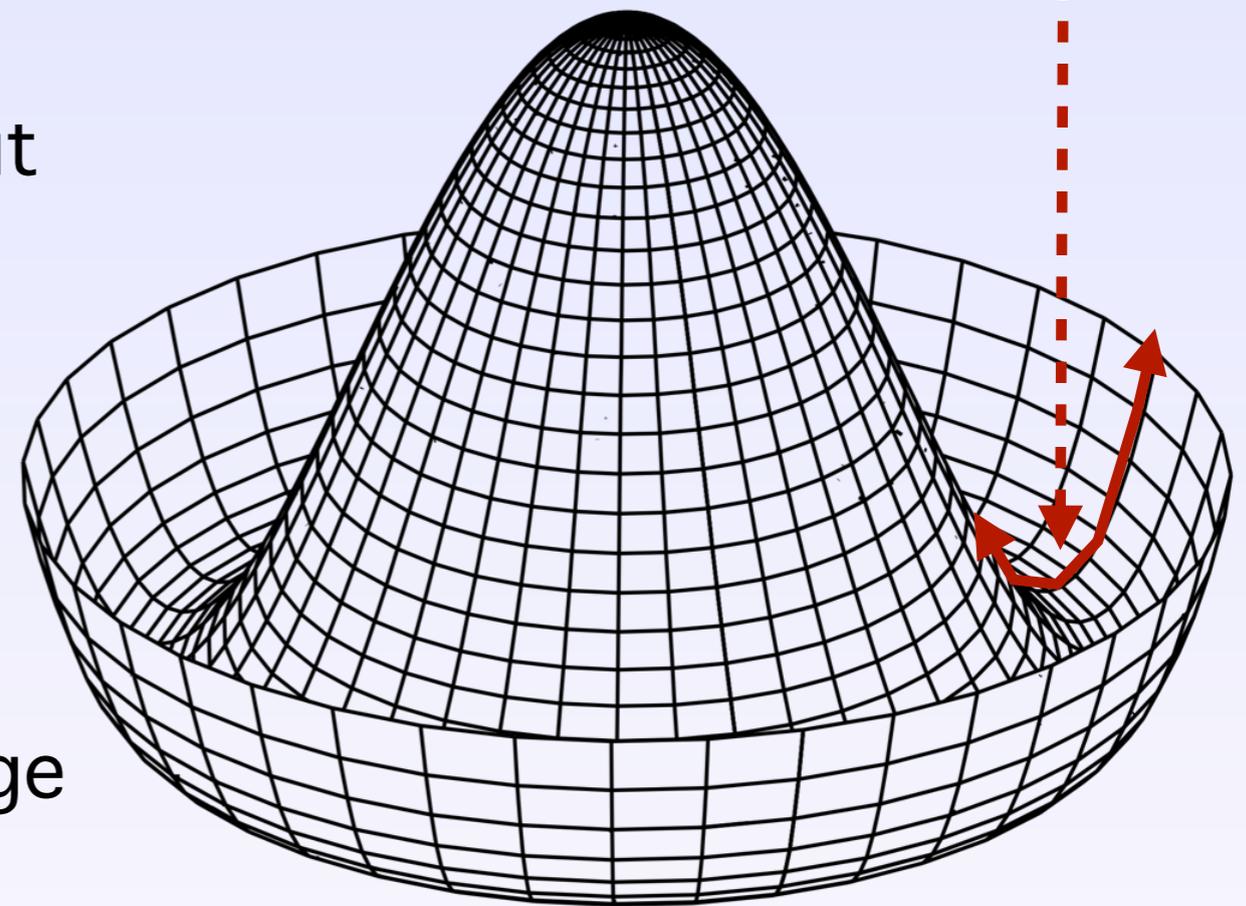
$$\phi_{\min.} \propto (0, v)$$

- fluctuations of scalar field about minimum:

$$\phi \propto (0, v + H)$$

- gauge transformation: absorb Goldstone modes into the gauge bosons.

- recipe makes massive W, Z, massless photons and the Higgs scalar (H). Topped with QCD and served with fermions to complete the SM.



# Higgs potential

- focus on the resulting potential for the scalar field H:

$$\mathcal{V} = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

$$M_H^2 = (2\lambda v^2) \simeq 125 \text{ GeV}$$

- assuming the SM: we already know everything!
- SM prediction:  $\lambda = \frac{M_H^2}{2v^2} \simeq 0.13$ .
- but one wishes to verify the form of the potential in a model-independent way.

# anomalous couplings

$$\mathcal{V} = \frac{1}{2} M_H^2 H^2 + \lambda v H^3 + \frac{\tilde{\lambda}}{4} H^4$$

- we may consider anomalous values for these couplings, i.e. **free** parameters.
- their measurement would be a consistency test for the standard model.
- HH can probe  $\lambda$  and the top Yukawa.
- **(SPOILER ALERT: forget about  $\tilde{\lambda}$  through HHH.)**

# the meaning of anomalous couplings

$$\mathcal{V} = \frac{1}{2} M_H^2 H^2 + \lambda v H^3 + \frac{\tilde{\lambda}}{4} H^4$$

- let's assume we measure  $\lambda = (1 + \delta) \times \lambda_{\text{SM}}$  via HH at the LHC, e.g. through  $\mu(\text{HH})$ :
  1. if  $\delta$  is **small**, we may conclude that the SM is self-consistent.
  2. if  $\delta$  is **large**, there may be some new physics in action.
- (but in reality, this is “only” a consistency test.)
- other options for HH: [e.g. Gupta, Rzehak, Wells, 1305.6397]
  - use concrete models: constraints on param. space.
  - use an effective theory: constraints on coefficients.

# an example: dimension-6 EFT (I)

[see: e.g. T. Plehn, 0910.4182]

- add dimension-6 Higgs operators, e.g.:

$$\mathcal{O}_1 = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi) \quad \text{and} \quad \mathcal{O}_2 = -\frac{1}{3} (\phi^\dagger \phi)^3$$

- parametrised by an unknown mass scale  $\Lambda$ :

$$\mathcal{L}_{\text{D6}} = \sum_{i=1}^2 \frac{f_i}{\Lambda^2} \mathcal{O}_i$$

- go through electroweak “cooking” again...
- ...find new minima, expand  $\Phi$ , generate W/Z masses, massless photon, etc.

# an example: dimension-6 EFT (II)

- the twist is that we have to canonically normalise the Higgs boson kinetic term, i.e.

$$\hookrightarrow \alpha \partial_\mu H' \partial^\mu H' \rightarrow \frac{1}{2} \partial_\mu H \partial^\mu H$$

- one possibility (to avoid momentum-dependent interactions in self-couplings):

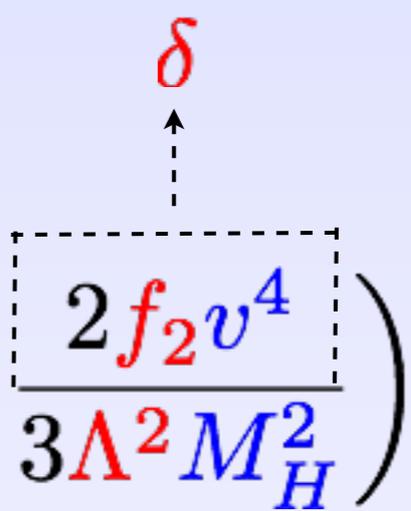
$$H \rightarrow aH + bH^2 + cH^3 + \mathcal{O}(H^4) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

- but: this choice introduces new interactions everywhere in the SM Lagrangian related to  $f_1$ . [again, see T. Plehn, 0910.4182]

# an example: dimension-6 EFT (III)

- let's drop  $f_1$  for the sake of simplicity...
- resulting expressions: ( $f_1=0$ )

$$M_H^2 = 2\lambda v^2 \left( 1 + \frac{f_2 v^2}{2\Lambda^2 \lambda} \right) \quad \text{and} \quad \lambda' = \left( 1 + \frac{2f_2 v^4}{3\Lambda^2 M_H^2} \right) \times \lambda_{\text{SM}}$$

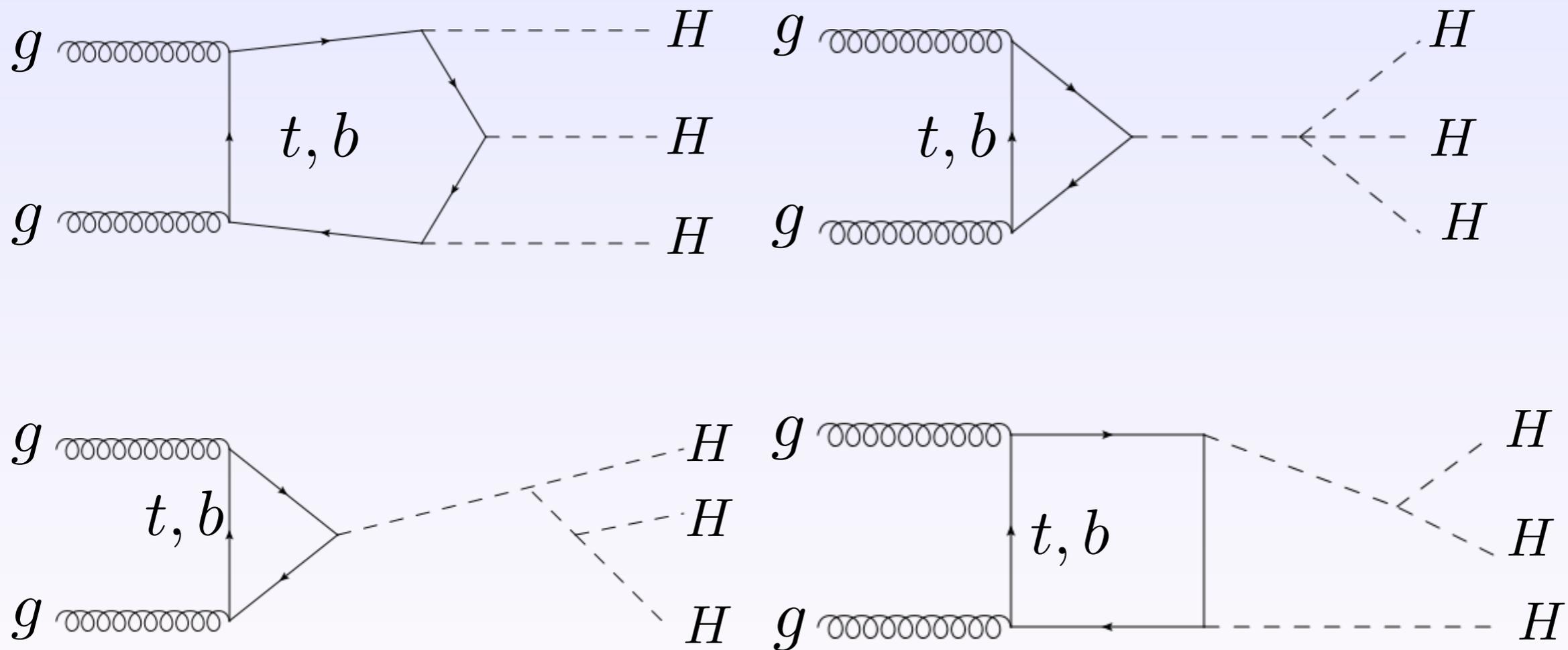


- measuring “effective” self-coupling through HH signal strength would constrain:  $\frac{f_2}{\Lambda^2}$  and  $\lambda$
- had we kept  $f_1$ , simple picture of “effective” self-coupling through HH production no longer holds due to additional interactions.
- for a complete study, add more operators  $f_i$  & use other experimental results.

# **ii. multi-Higgs processes @ hadron colliders**

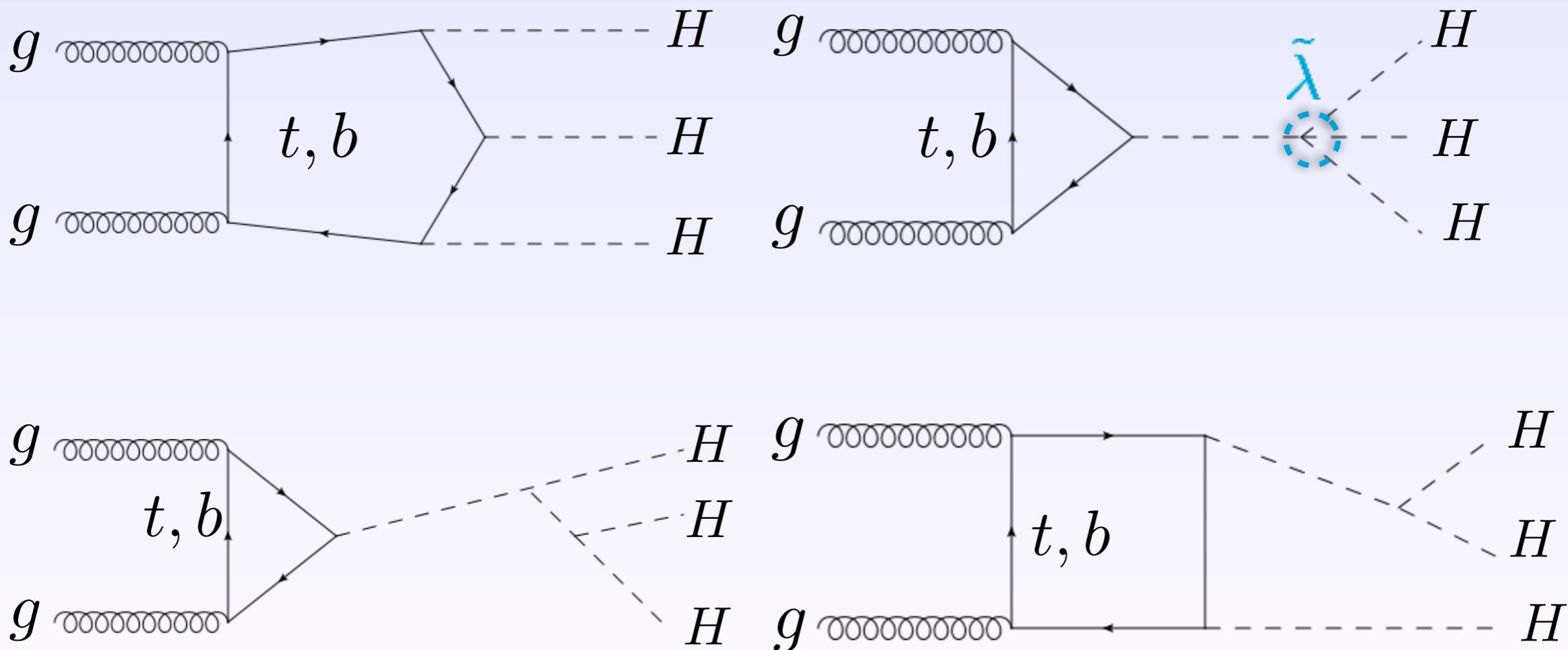
# SM HHH production @ LHC

- triple Higgs boson production at hadron colliders,
- contributing diagrams:  $gg \rightarrow HHH$



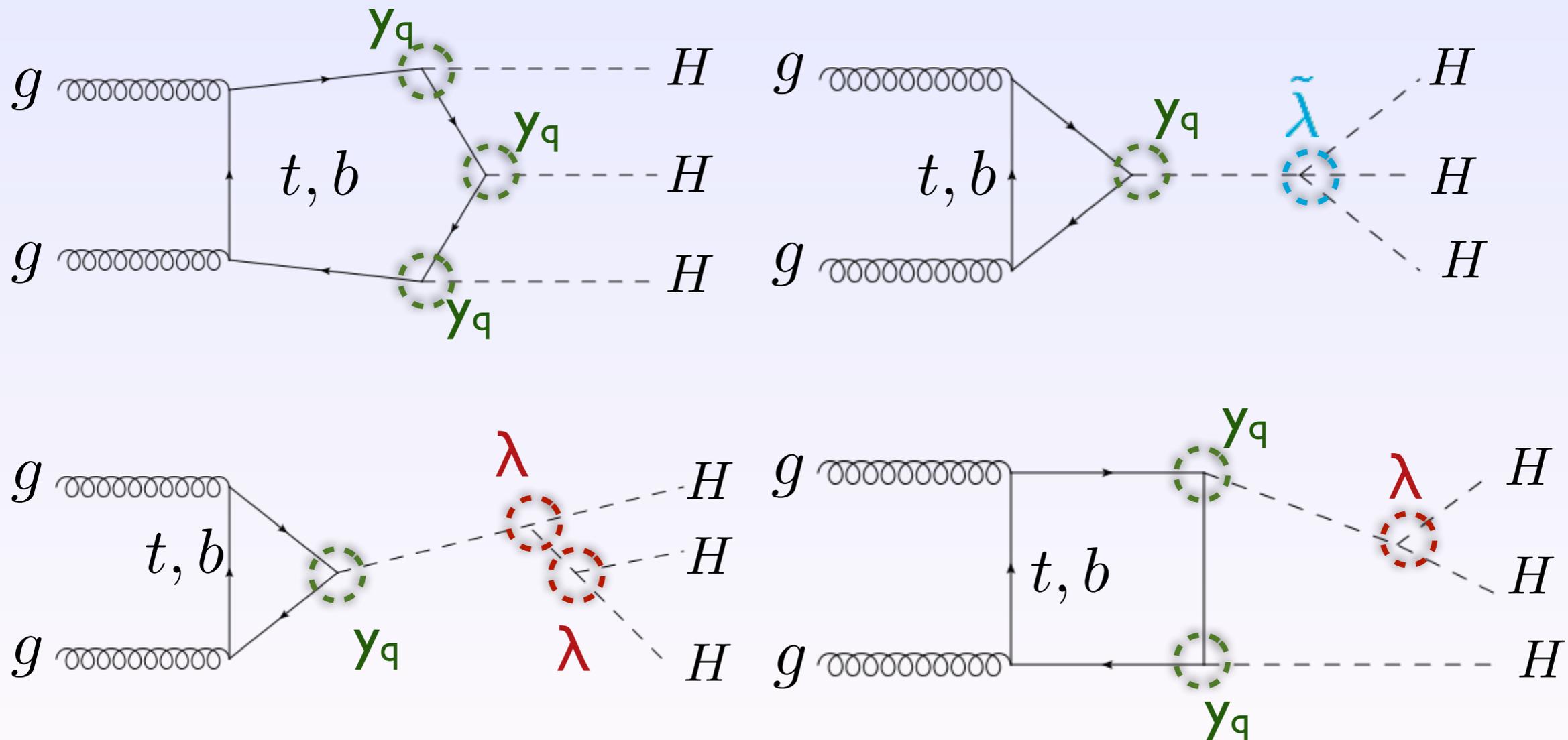
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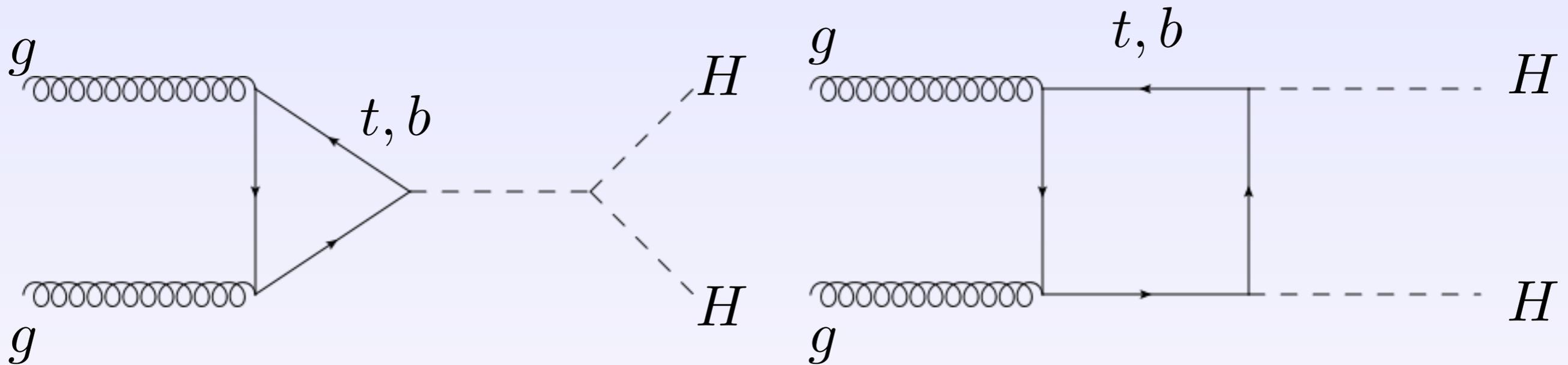
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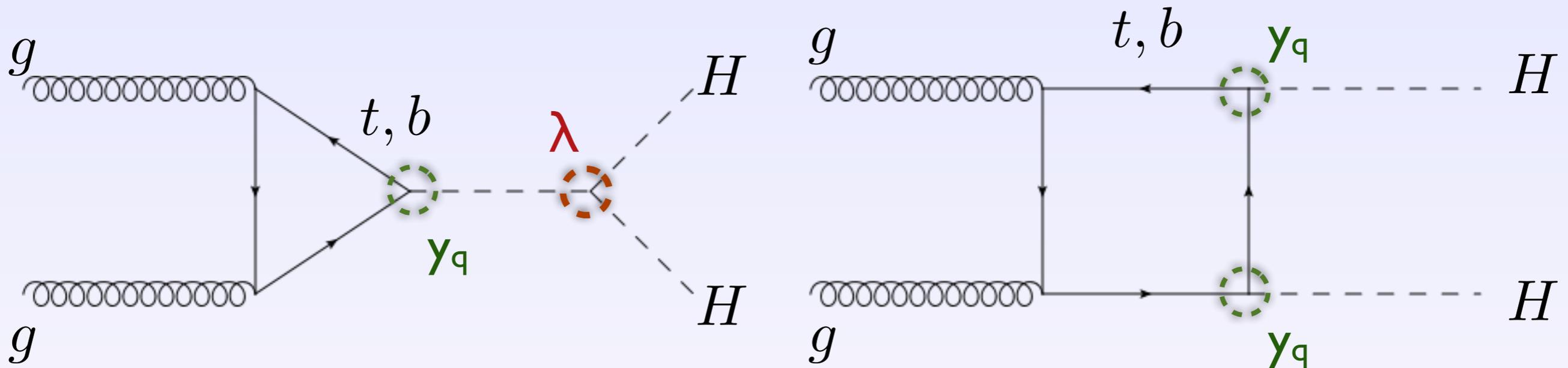
- dominant initial state: gluon-gluon fusion.
- leading order, two diagrams:



- effective theory (infinite top mass) insufficient:  $Q^2 \gtrsim M_{\text{top}}^2$ .
- loop calculation necessary to reproduce kinematical properties.

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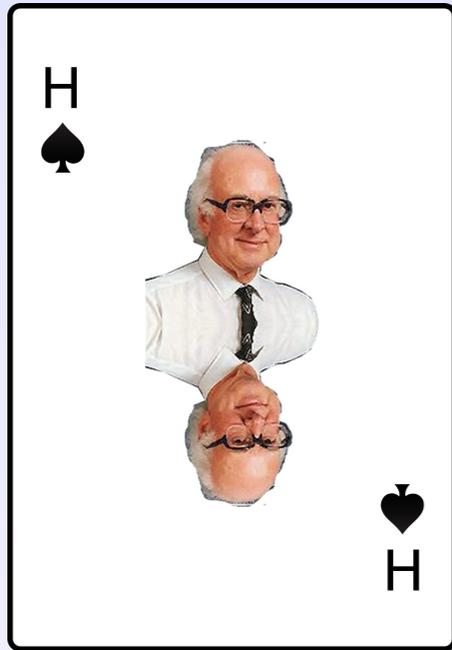
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# multi-Higgs cross sections (14 TeV LHC)



(with apologies to  
Peter Higgs!)

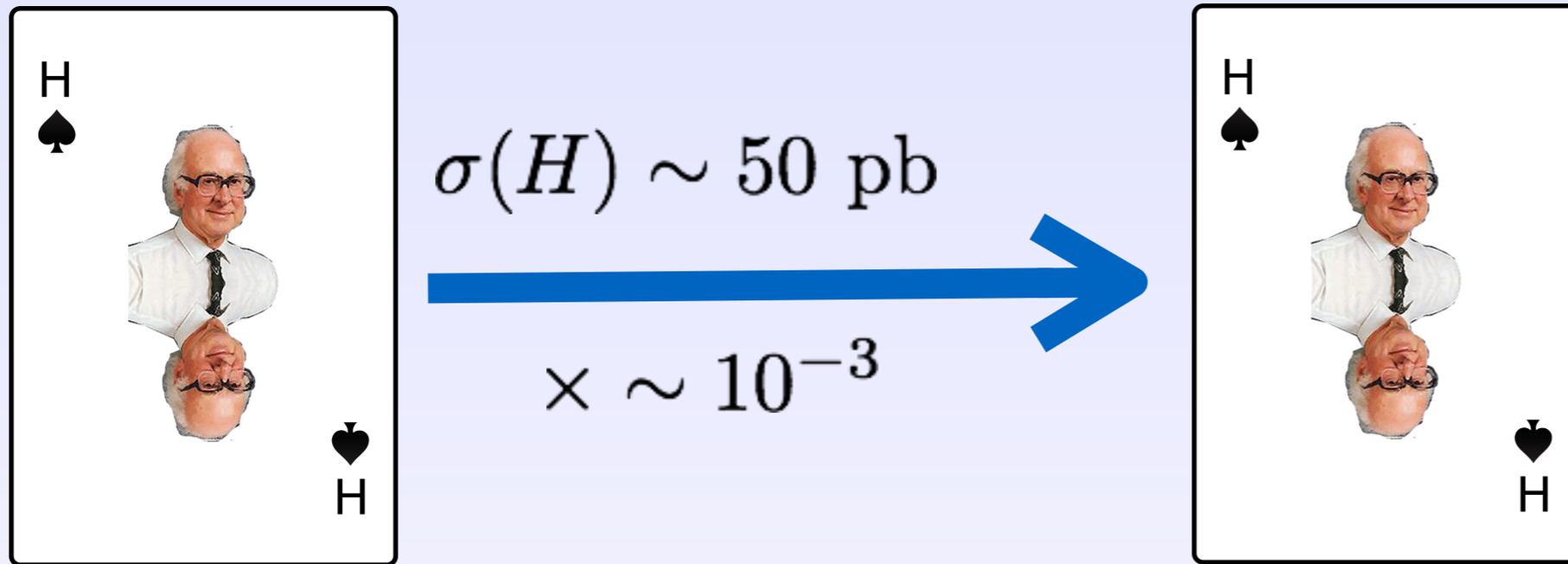
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$$\sigma(H) \sim 50 \text{ pb}$$

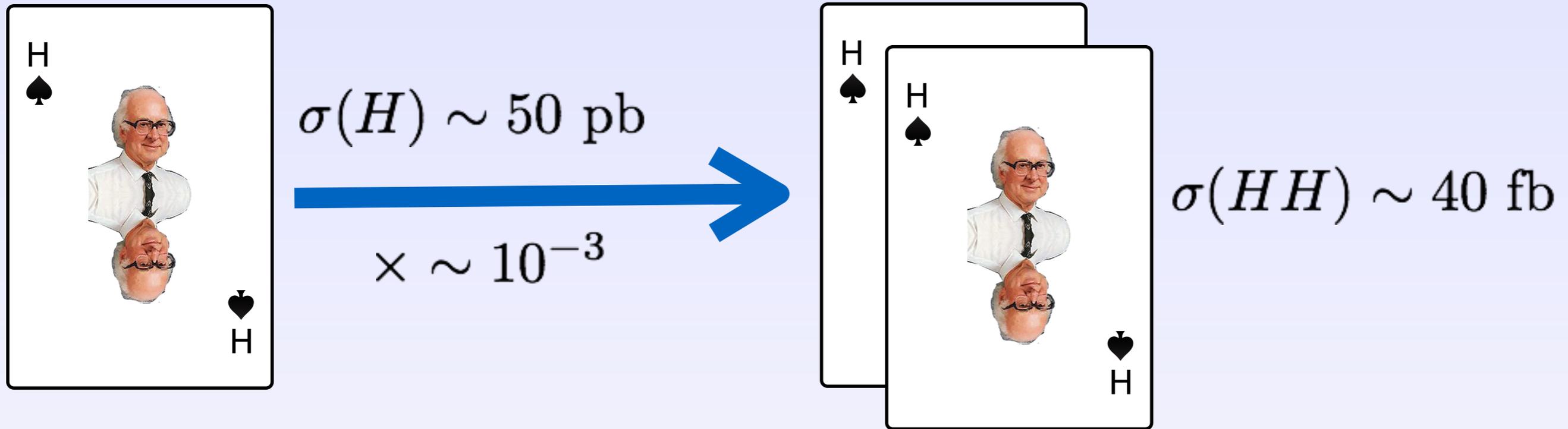
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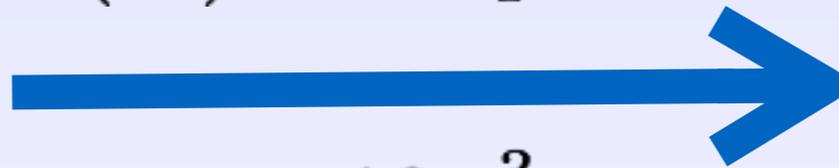


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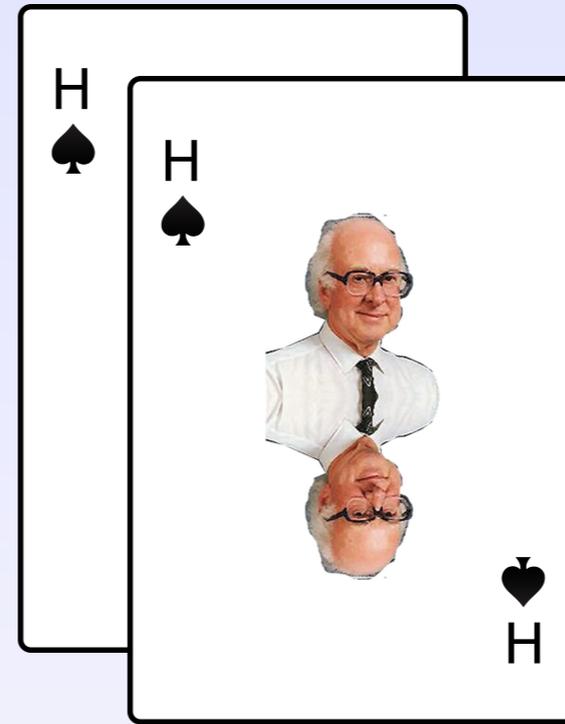
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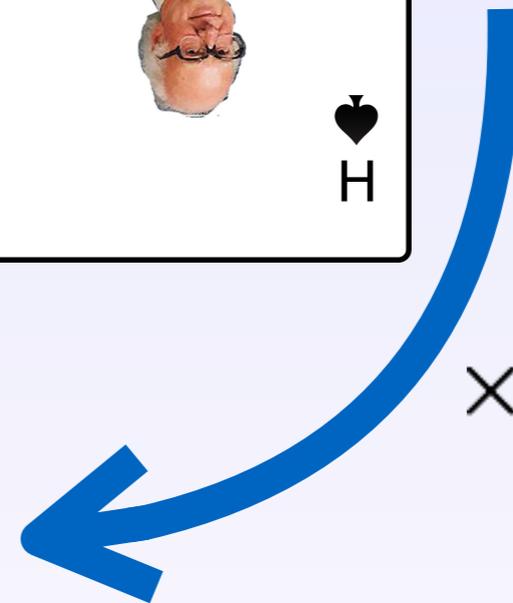
$$\sigma(H) \sim 50 \text{ pb}$$



$$\times \sim 10^{-3}$$



$$\sigma(HH) \sim 40 \text{ fb}$$



$$\times \sim 10^{-3}$$



(with apologies to  
Peter Higgs!)

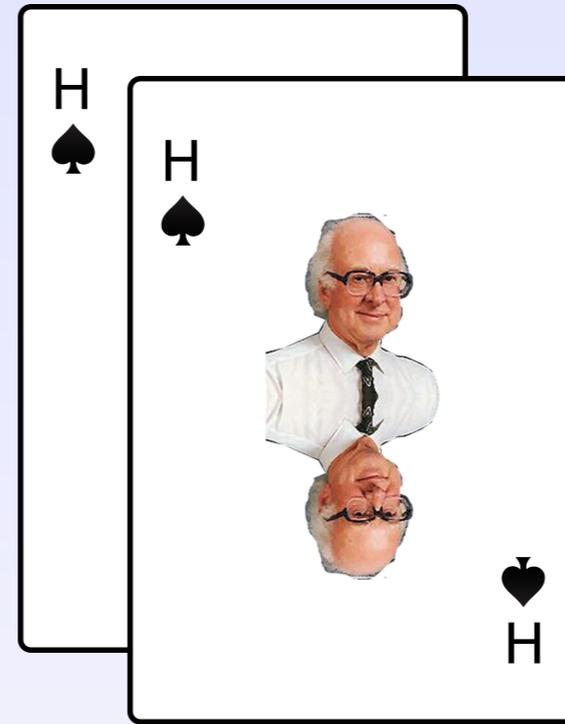
# multi-Higgs cross sections (14 TeV LHC)



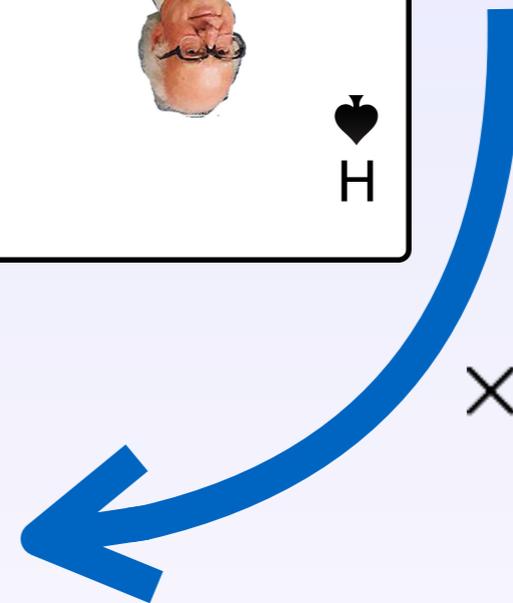
$$\sigma(H) \sim 50 \text{ pb}$$



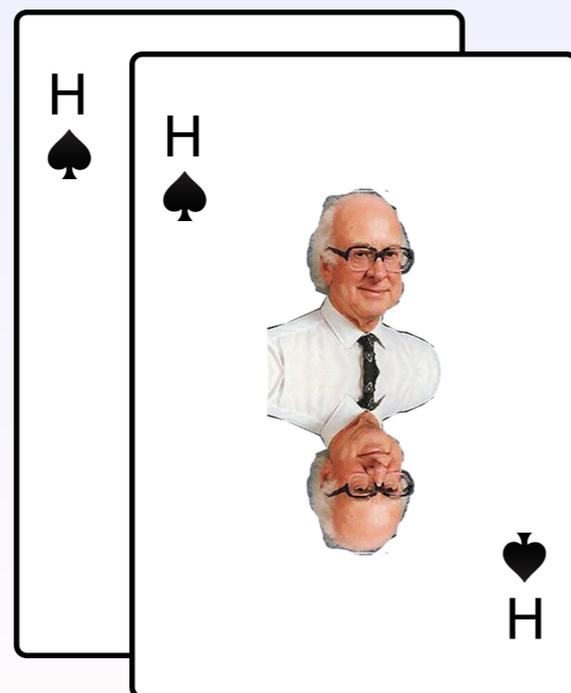
$$\times \sim 10^{-3}$$



$$\sigma(HH) \sim 40 \text{ fb}$$



$$\times \sim 10^{-3}$$



(with apologies to  
Peter Higgs!)

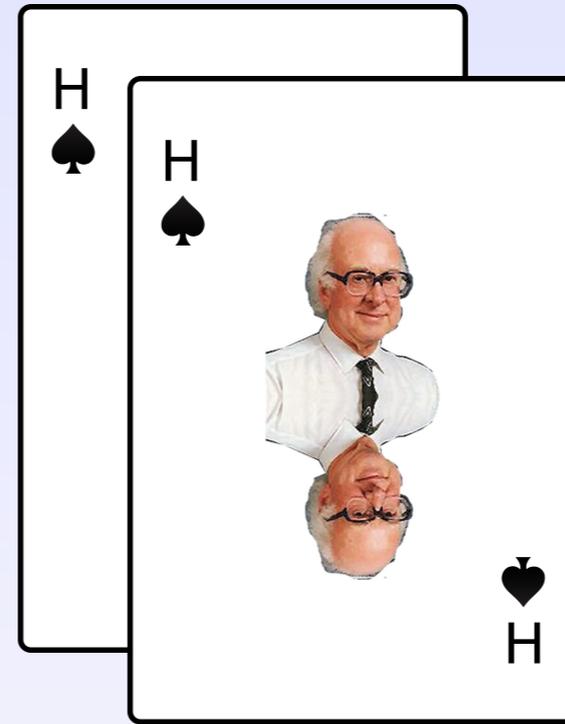
# multi-Higgs cross sections (14 TeV LHC)



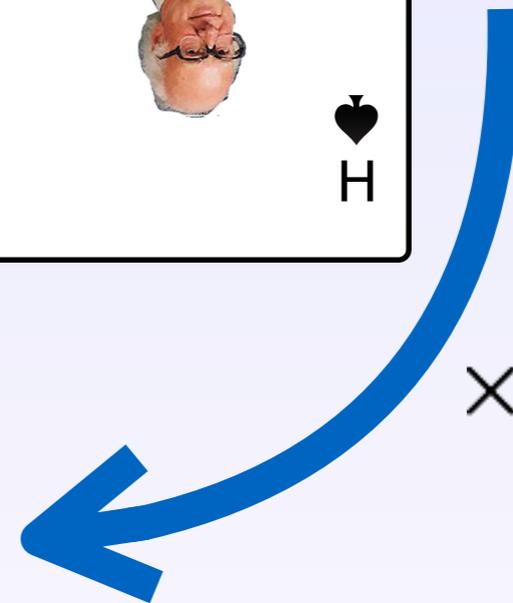
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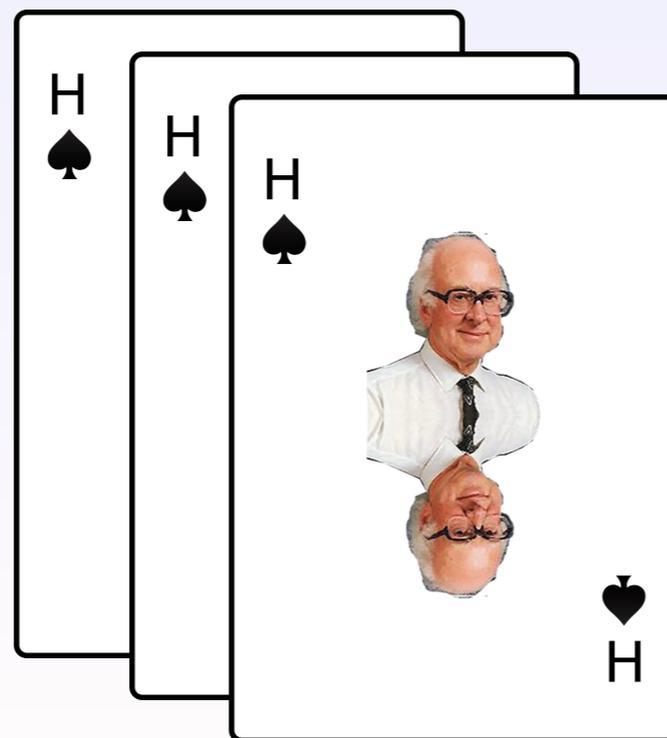
$$\times \sim 10^{-3}$$



$$\sigma(HH) \sim 40 \text{ fb}$$



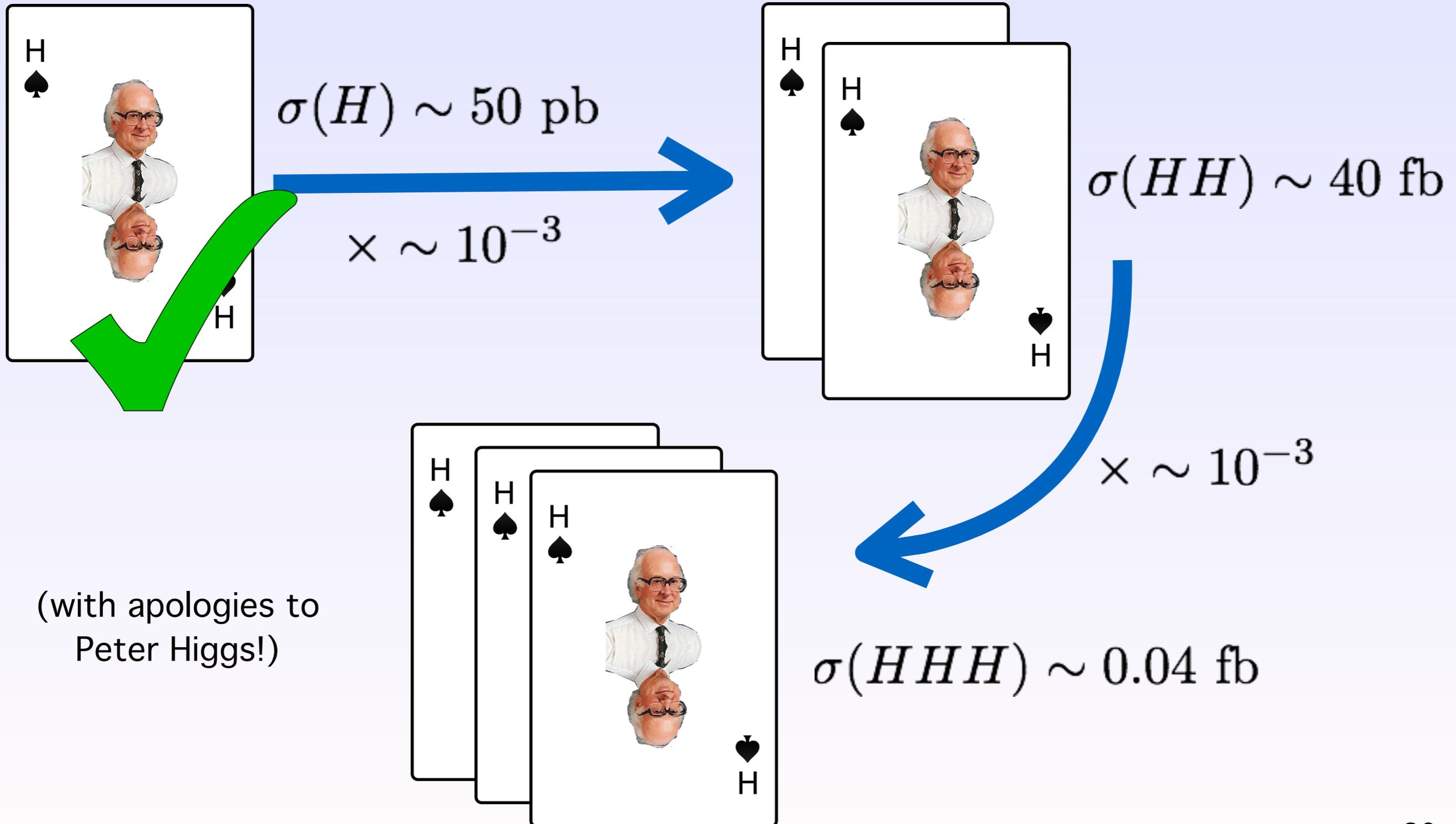
$$\times \sim 10^{-3}$$



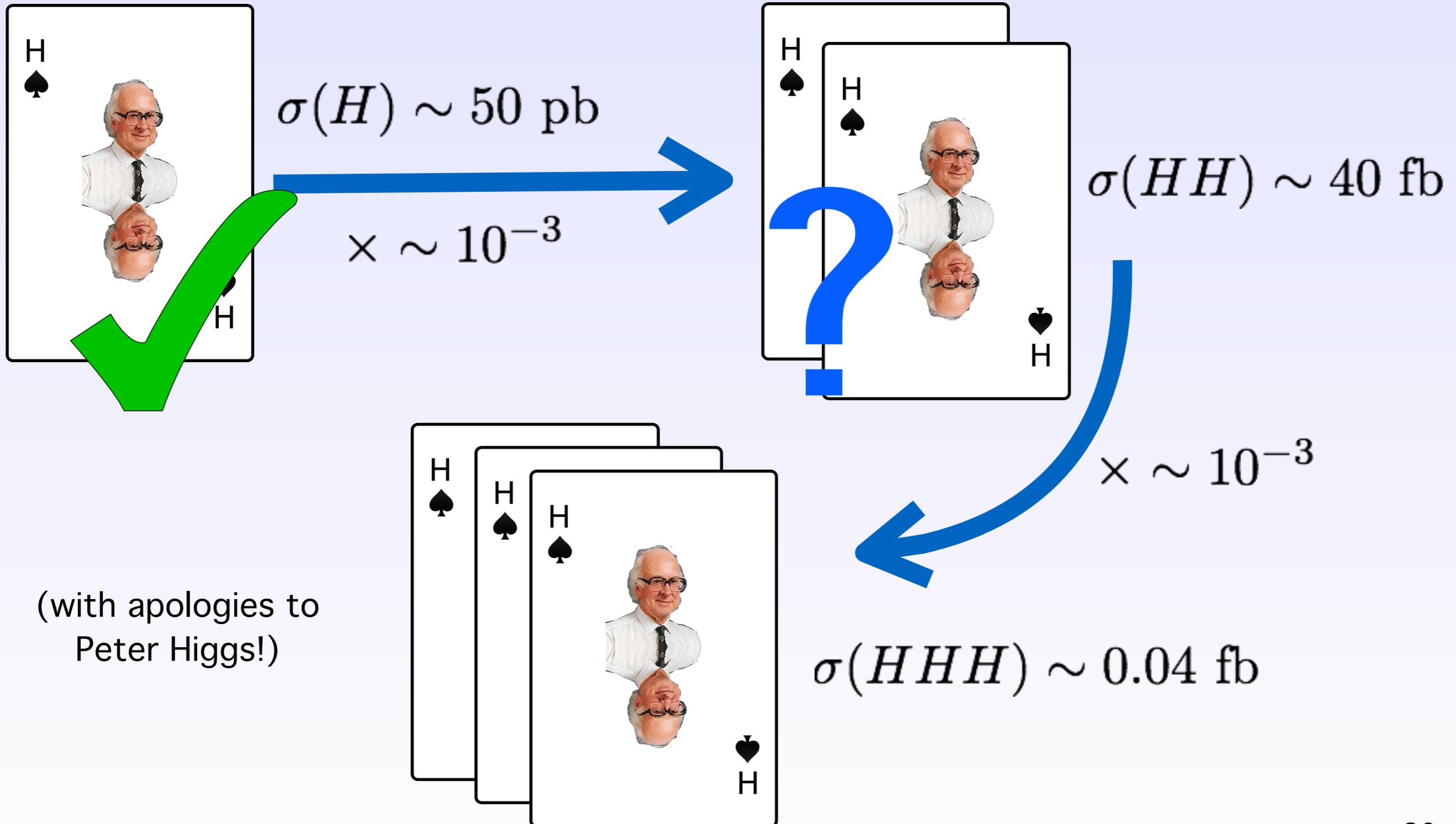
$$\sigma(HHH) \sim 0.04 \text{ fb}$$

(with apologies to  
Peter Higgs!)

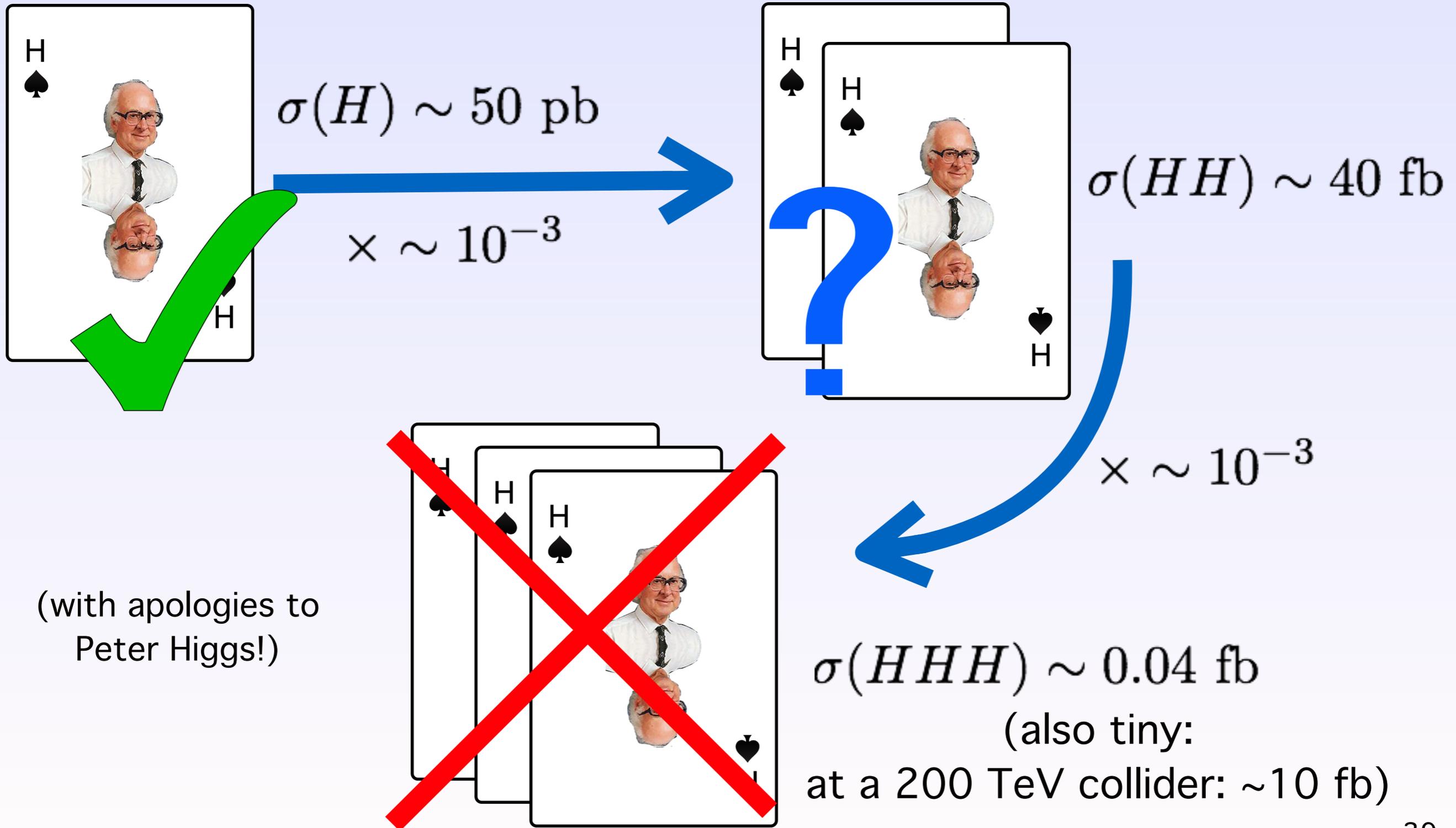
# multi-Higgs cross sections (14 TeV LHC)



# multi-Higgs cross sections (14 TeV LHC)



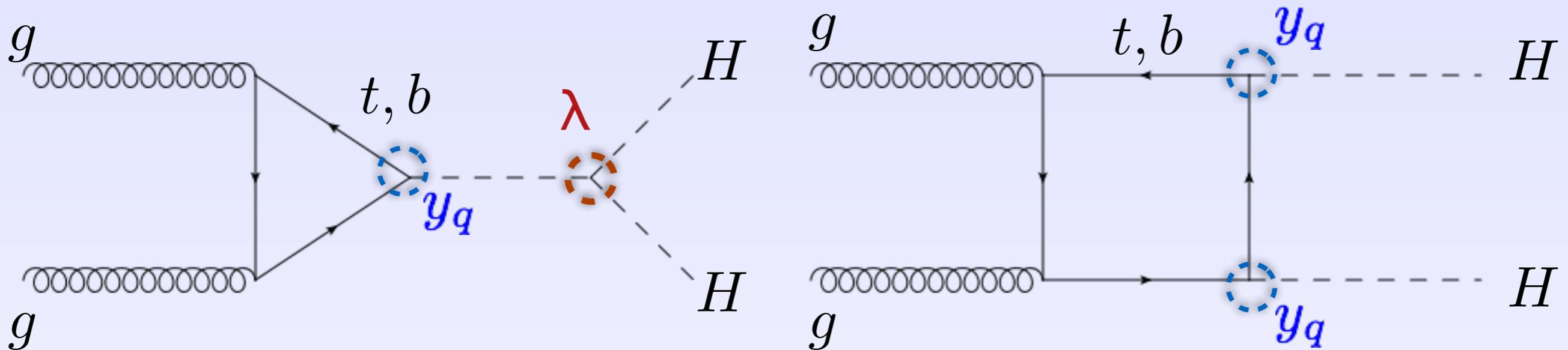
# multi-Higgs cross sections (14 TeV LHC)





# iii. HH production @ LHC, in gory detail

# HH production @ LO



box and triangle topologies,

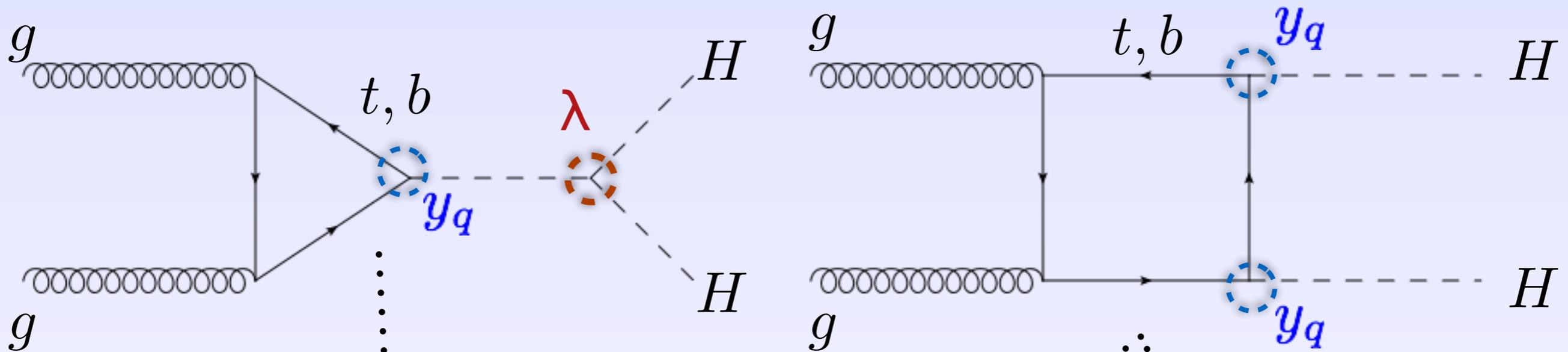
Lorentz structures for spin-0 and spin-2 gg configurations.

$$\sigma_{HH}^{LO} = \left| \sum_q (\lambda y_q C_{q,\text{tri}}^{(\text{spin}-0)} + y_q^2 C_{q,\text{box}}^{(\text{spin}-0)}) \right|^2 + \left| \sum_q y_q^2 C_{q,\text{box}}^{(\text{spin}-2)} \right|^2$$

(sum over quarks  $q = t, b$ )

(couplings normalized to SM:  $\lambda = 1$ ,  $y_q = 1$  is the SM)

# HH production @ LO



box and triangle topologies,

Lorentz structures for spin-0 and spin-2 gg configurations.

$$\sigma_{HH}^{LO} = \left| \sum_q (\lambda y_q C_{q,\text{tri}}^{(\text{spin}-0)} + y_q^2 C_{q,\text{box}}^{(\text{spin}-0)}) \right|^2 + \left| \sum_q y_q^2 C_{q,\text{box}}^{(\text{spin}-2)} \right|^2$$

(sum over quarks  $q = t, b$ )

(couplings normalized to SM:  $\lambda = 1$ ,  $y_q = 1$  is the SM)

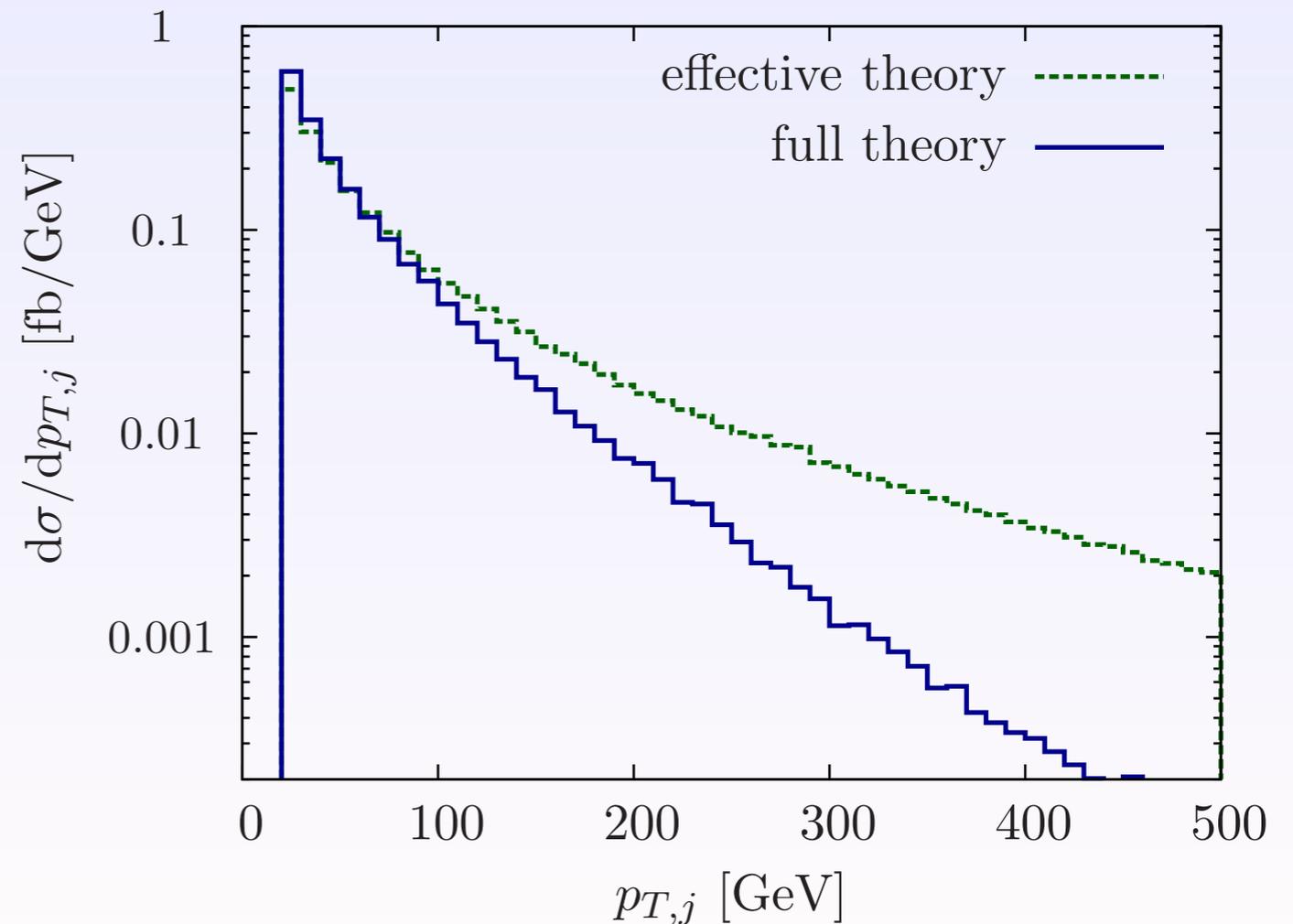
# effective theory gone wild

- for HH: FAILS since  $Q^2 \gtrsim 4M_H^2 > M_t^2$ .
- the K-factor (NLO/LO) at HH threshold is strongly affected by power-suppressed  $1/M_{\text{top}}$  terms. [Grigo, Hoff, Melnikov, Steinhauser, 1305.7340]
- does not describe the kinematics of the process properly:

e.g., spectrum of the hardest jet in

$$pp \rightarrow HH + j + X$$

[Dolan, Englert, Spannowsky 1206.5001]



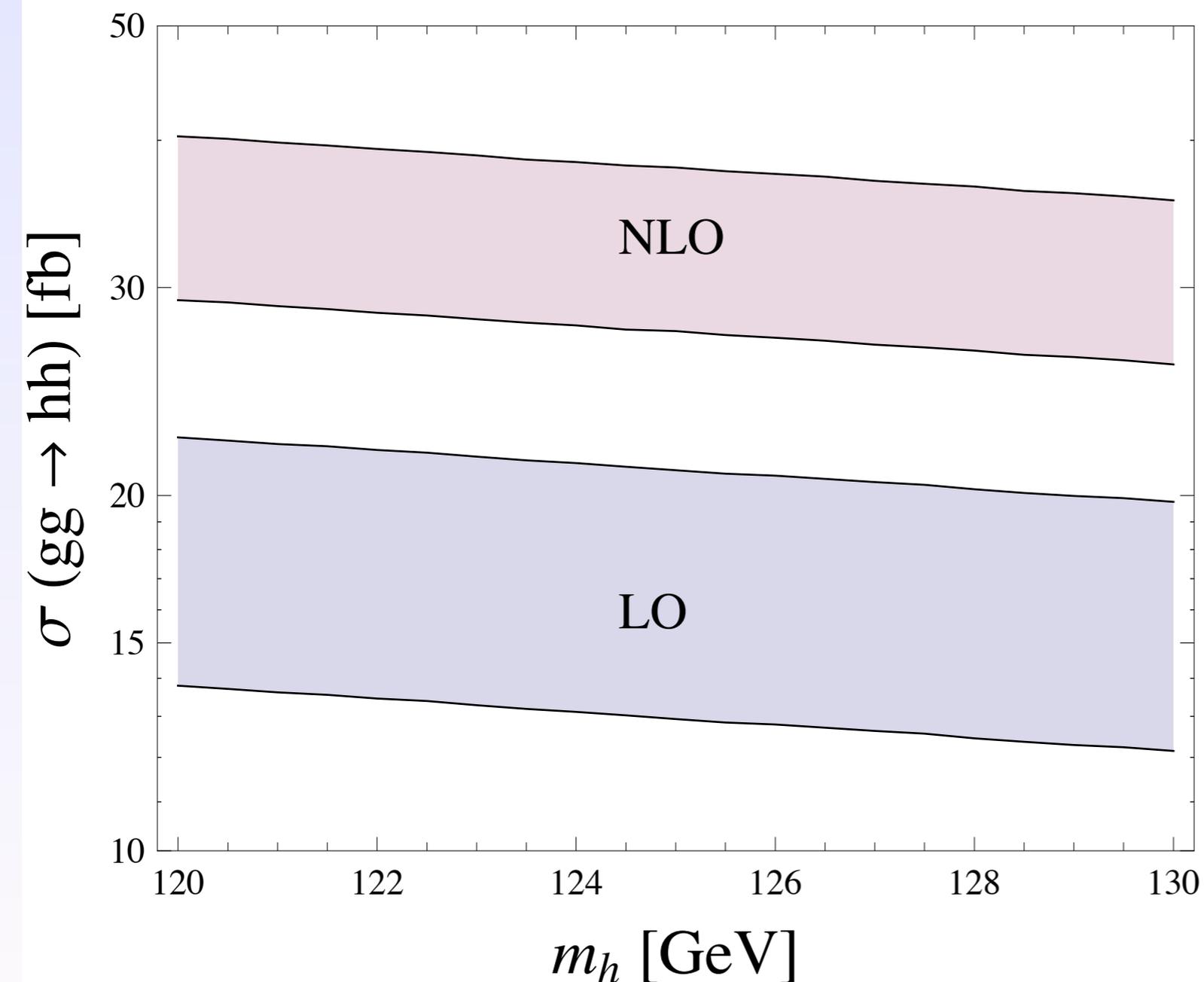
# HH production @ (N)NLO

- (N)NLO calculations only available in the infinite top mass limit. [Dawson, Dittmaier, Spira, [hep-ph/9805244]], [de Florian, Mazzitelli, 1309.6594]
- K-factor (w.r.t. LO) in this limit  $\sim 2$ .
- $\sigma_{NNLO}/\sigma_{NLO} \sim 1.2$



# HH cross section @ 14 TeV

$$\sqrt{s} = 14 \text{ TeV}, m_{hh}/2 < \mu_F = \mu_R < 2 m_{hh}$$



$$\sigma^{NLO}(M_H = 125 \text{ GeV}) = 32.3^{+5.6}_{-4.7} \text{ fb}$$

(using HPAIR by M. Spira)

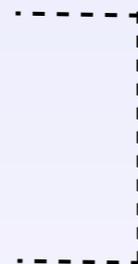
[AP, Li Lin Yang, and José Zurita, 1209.1489]

# improving the Monte Carlo (I)

- go beyond LO + parton shower
- merging/matching (e.g. MLM or CKKW/MC@NLO or POWHEG)
- HH production, no full NLO calculation: use the effective theory NLO or merge to higher-multiplicities.

P. Maierhöfer, *AP*, 1401.0007

Q. Li, Q. Yan, X. Zhao, 1312.3830



MLM merging up to 1 extra parton.

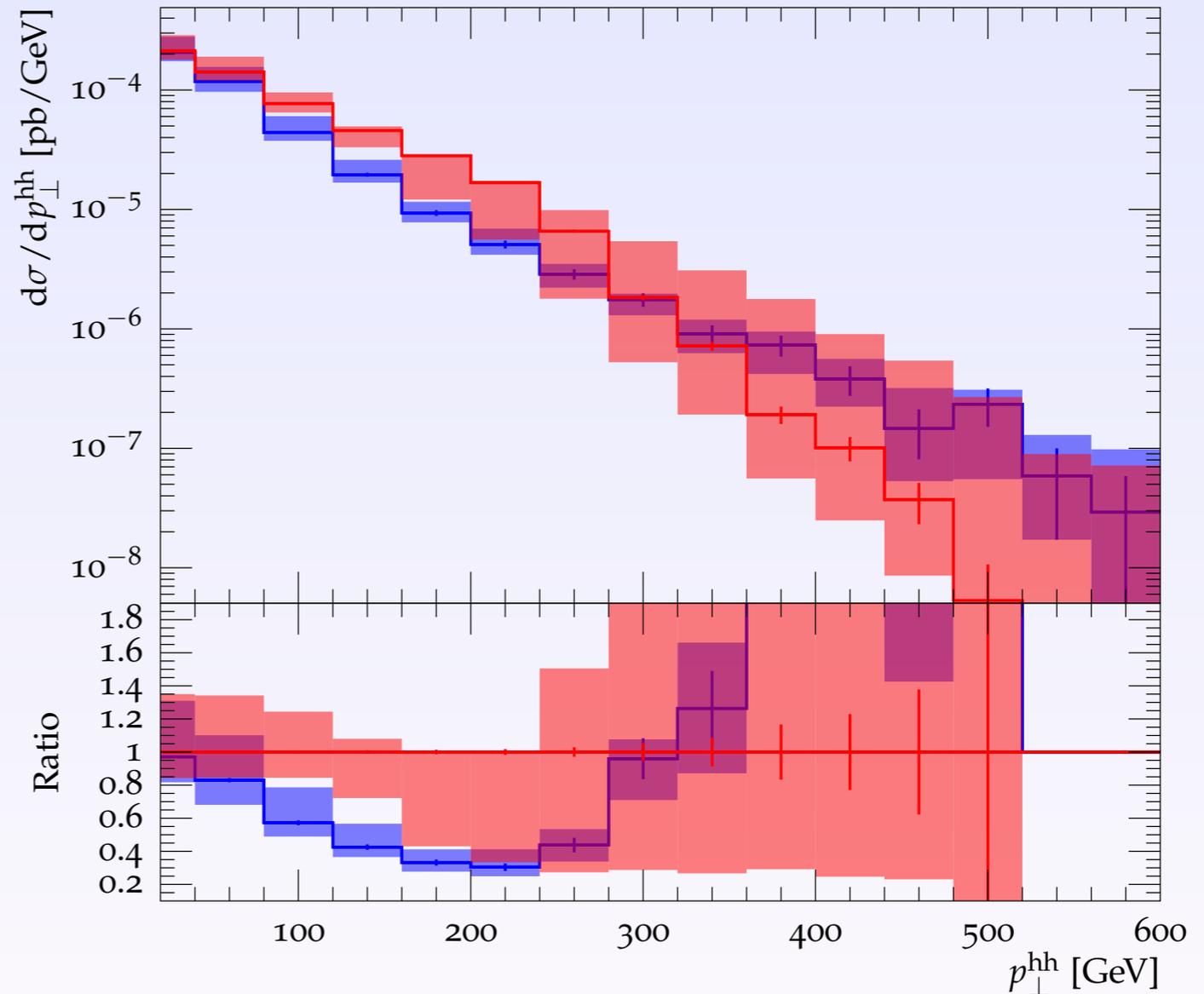
R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, P. Torrielli, E. Vryonidou, M. Zaro, 1401.7340 --> MC@NLO with NLO EFT.

- using these improved samples, systematic uncertainties can be reduced.

# improving the Monte Carlo (II)

- (leading log to LO in first jet  $p_{T}$ : similar to improvement in scale uncertainty from LO to NLO.)

- e.g., transverse momentum of Higgs pair (red: parton shower, blue: merged sample)



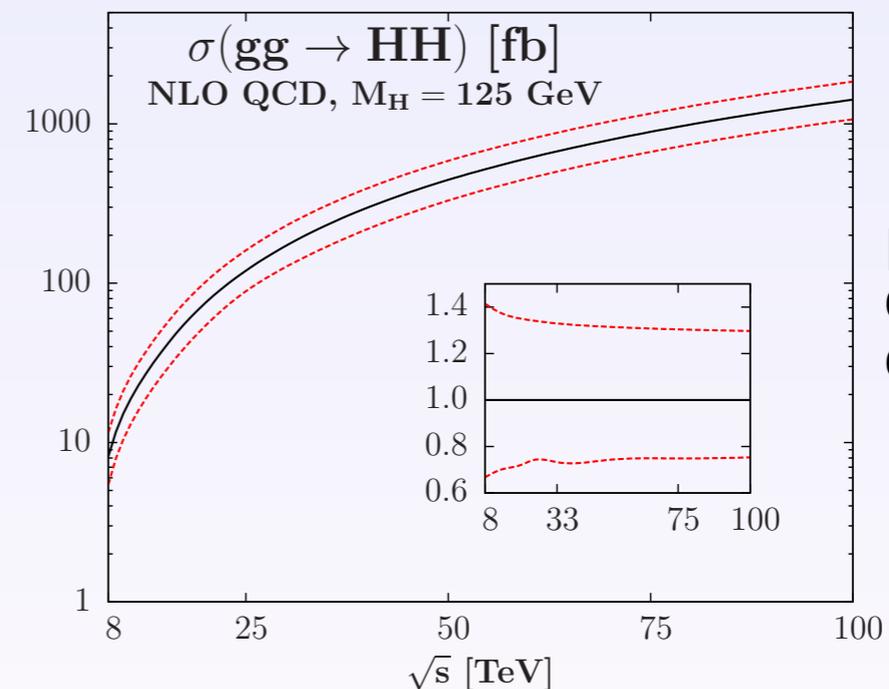
[P. Maierhöfer, AP, 1401.0007]



# **iv. searching for HH @ LHC14**

# challenges

- small cross section, implying high luminosity (600/fb or 3000/fb: end-of-lifetime or HL-LHC).
- + large theoretical uncertainties on this cross section.
- generating sufficiently large Monte Carlo background samples:
  - $N_{\text{events}} = O(1000/\text{fb}) \times O(100 \text{ pb}) = O(10^8)$
- simulating experimental efficiencies,
  - jet-to- $\gamma$  mis-tagging,
  - $\tau$ -tagging, b-tagging.



[Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira, 1212.5581]

Figure 10: The total cross section (black/full) of the process  $gg \rightarrow \text{HH} + X$  at the LHC for  $M_H = 125 \text{ GeV}$  as a function of  $\sqrt{s}$  including the total theoretical uncertainty (red/dashed). The insert shows the relative deviation from the central cross section.



# branching ratios ( $M_H = 125 \text{ GeV}$ )

$$BR[b\bar{b}b\bar{b}] = 33.3\%$$

$$BR[b\bar{b}WW] = 24.8\%$$

$$BR[b\bar{b}\tau\tau] = 7.29\%$$

$$BR[WWWW] = 4.62\%$$

$$BR[WW\tau\tau] = 2.71\%$$

$$BR[\tau\tau\tau\tau] = 0.399\%$$

$$BR[b\bar{b}ZZ] = 0.305\%$$

$$BR[b\bar{b}\gamma\gamma] = 0.263\%$$

$$BR[b\bar{b}Z\gamma] = 0.178\%$$

$$BR[b\bar{b}\mu\mu] = 0.025\%$$

note: each 1% corresponds to  
 $\sim 100$  events per  $300 \text{ fb}^{-1}$  of  
luminosity @ LHC14.

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$$BR[b\bar{b}b\bar{b}] = 33.3\%$$



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may provide  
constraints

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may provide  
constraints

# branching ratios ( $M_H = 125 \text{ GeV}$ )

$$BR[b\bar{b}b\bar{b}] = 33.3\% \quad \checkmark$$

$$BR[b\bar{b}WW] = 24.8\% \quad \checkmark$$

$$BR[b\bar{b}\tau\tau] = 7.29\% \quad \checkmark$$

$$BR[WWWW] = 4.62\% \quad ?$$

$$BR[WW\tau\tau] = 2.71\% \quad ?$$

$$BR[\tau\tau\tau\tau] = 0.399\% \quad ?$$

$$BR[b\bar{b}ZZ] = 0.305\% \quad \times$$

$$BR[b\bar{b}\gamma\gamma] = 0.263\% \quad \checkmark$$

$$BR[b\bar{b}Z\gamma] = 0.178\% \quad \times$$

$$BR[b\bar{b}\mu\mu] = 0.025\% \quad \times$$

note: each 1% corresponds to  
 $\sim 100$  events per  $300 \text{ fb}^{-1}$  of  
luminosity @ LHC14.

may provide  
constraints

$$HH \rightarrow b\bar{b}\tau\tau$$

Dolan, Englert, Spannowsky, [1206.5001], Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira [1212.5581].

- BR = 7.29%, cross section  $\sim 2.4\text{fb}$  ( $\sim 700$  events @  $300\text{fb}^{-1}$ ).
- reconstruction of  $\tau$  leptons experimentally delicate.
- backgrounds relatively low: electroweak and top decays with taus in the final states.
- Higgses naturally boosted: use a fat jet: sub-structure of the two b-quark system: like in Higgs+vector boson.  
[Butterworth, Davison, Rubin, Salam, 0802.2470]  $\rightarrow$  “BDRS”
- results promising given a high  $\tau$ -tagging efficiency (80%), b-tagging assumed 70%, low fake rates.
- $S \sim 50$  versus  $B = 100$  at  $600\text{fb}^{-1}$  ( $\sim 5\sigma$ ).

$$HH \rightarrow b\bar{b}\gamma\gamma$$

Baur, Plehn, Rainwater, [hep-ph/031005], Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira [1212.5581].

- BR = 0.263%, cross section = 0.09 fb, ( $\sim 27$  events @  $300 \text{ fb}^{-1}$ ).
- low rate but ‘clean’. backgrounds generally low and mostly coming from reducible backgrounds due to mis-identification of b-jets or photons (jet-to- $\gamma$ ).
- $S \sim 30$  versus  $B \sim 60$  at  $3000 \text{ fb}^{-1}$  ( $\sim 4\sigma$ ).



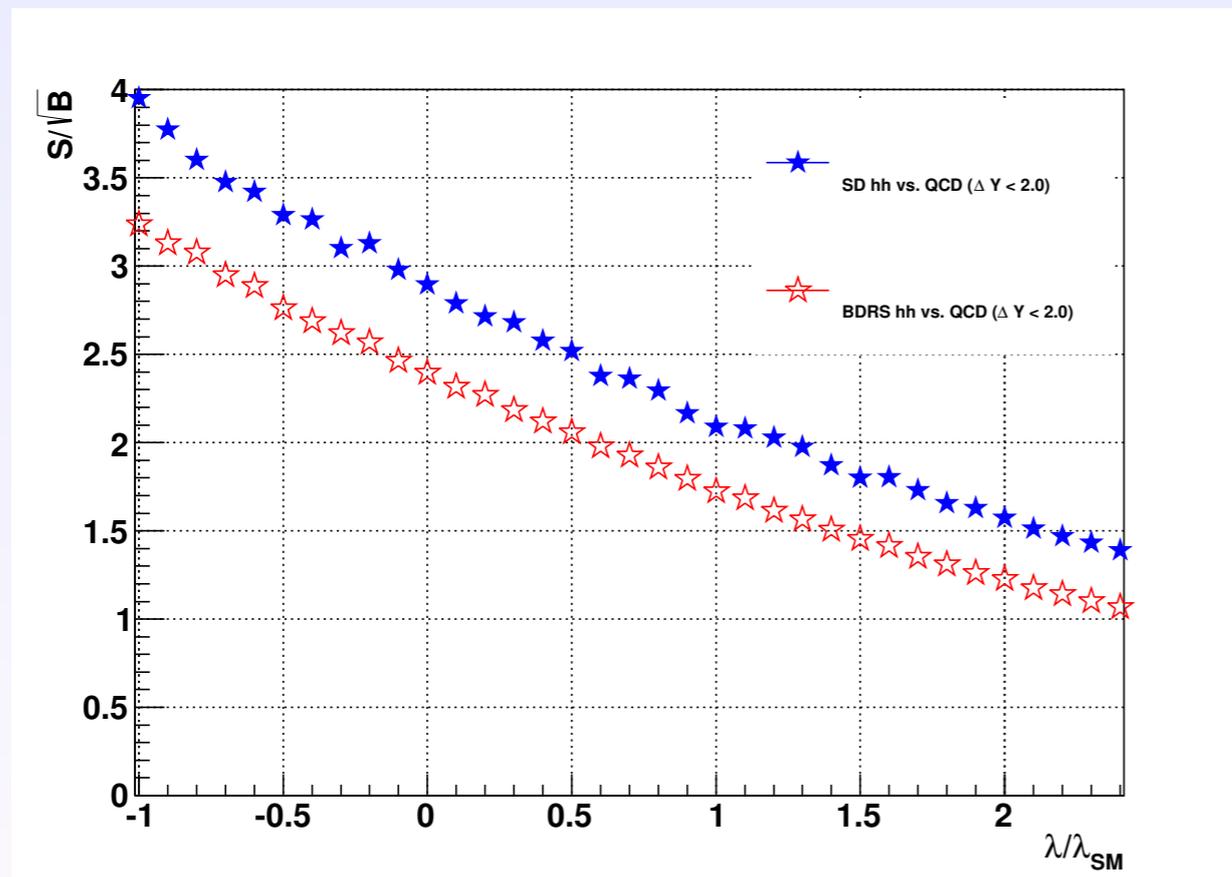
$$HH \rightarrow b\bar{b}WW$$

Dolan, Englert, Spannowsky, [1206.5001], Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira [1212.5581], AP, Li Lin Yang, and José Zurita [arXiv:1209.1489]

- BR = 24.8%, cross section = 8.0 fb, ( $\sim 2400$  events @  $300 \text{ fb}^{-1}$ ).
- high rate, can have leptons + missing energy in the final state.
- **but:** huge backgrounds from top-anti-top production.
- with one leptonic W and one hadronic W was shown to be viable using jet sub-structure techniques. [AP, L. L. Yang, and J. Zurita, 1209.1489]
- S = 11 versus B = 7 at  $600 \text{ fb}^{-1}$  ( $\sim 4\sigma$ ).

# more HH channels? (I)

- $b\bar{b}b\bar{b}$  : highest BR ( $\sigma \sim 10.8$  fb), but fully hadronic (triggering an issue) and huge QCD backgrounds.
- one may use boosted jet techniques to dig out this mode from the QCD background.



- improved triggering strategies necessary!

[Danilo E. Ferreira de Lima, AP,  
Michael Spannowsky, 1404.7139]

**Figure 8:** The best expected significance of the different Higgs tagger methods for different values of  $\lambda$  at  $3000 \text{ fb}^{-1}$  for a 14 TeV LHC.

# more HH channels? (II)

- $b\bar{b}\mu\bar{\mu}$  : small initial cross section, essentially found to be impossible ( $\sigma \sim 0.008$  fb). [Baur, Plehn, Rainwater [hep-ph/0304015]].
- $WWWW$  : good for high-mass Higgs. for low mass seems to be hard due to BR of Ws ( $\sigma \sim 1.5$  fb).
- $\tau\tau\tau\tau$  : low rate and  $\tau$ -tagging ( $\sigma \sim 0.13$  fb).
- $WW\tau\tau$  :  $\tau$ -tagging, W BRs ( $\sigma \sim 0.86$  fb)
- $b\bar{b}Z\gamma$  ,  $b\bar{b}ZZ$  : low rates and BR for Zs ( $\sigma < 0.1$  fb).



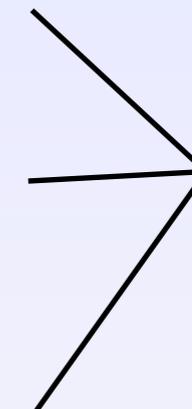
**v. how can we use HH to  
constrain the self-couplings?  
(focus on anomalous coupling picture)**

# how can we measure $\lambda$ ?

- older studies considered analysis of shapes of distributions. [e.g. Baur, Plehn, Rainwater [hep-ph/0310056]].
- shapes may not be so well predicted at the moment.
- moreover, low number of events: must exploit all differences in shapes of distributions to dig signal VS background.
- to start with: use measured rates instead. [F. Goertz, AP, L.L. Yang, J. Zurita, arXiv:1301.3492].

# how can we measure $\lambda$ ?

- e.g. using the three channels shown to be potentially viable, at  $3000 \text{ fb}^{-1}$ , LHC@14 TeV:

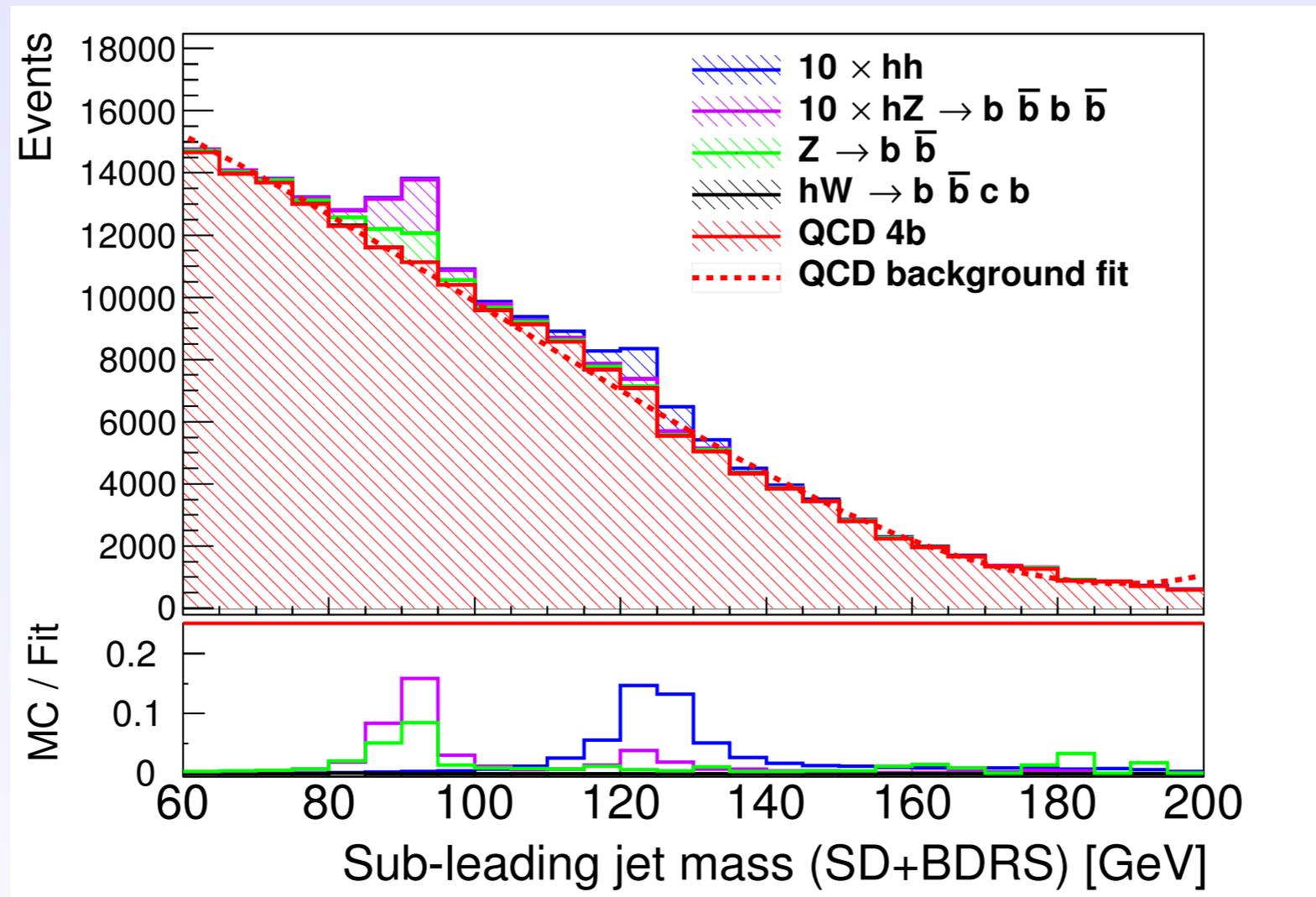
$HH \rightarrow b\bar{b}\tau\tau$	$\Rightarrow$	$\lambda = 1.00^{+0.40}_{-0.31}$	 <p>times the SM value</p>
$HH \rightarrow b\bar{b}\gamma\gamma$	$\Rightarrow$	$\lambda = 1.00^{+0.87}_{-0.52}$	
$HH \rightarrow b\bar{b}WW$	$\Rightarrow$	$\lambda = 1.00^{+0.46}_{-0.35}$	

[F. Goertz, AP, L. L. Yang, J. Zurita, 1301.3492]

- “naively” combining:  $\sim +30\%$ ,  $\sim -20\%$  error.

# how can we measure $\lambda$ ?

- using the ratio with hZ/ZZ peak in the 4b mode.



[Danilo E. Ferreira de Lima, AP, Michael Spannowsky, 1404.7139]

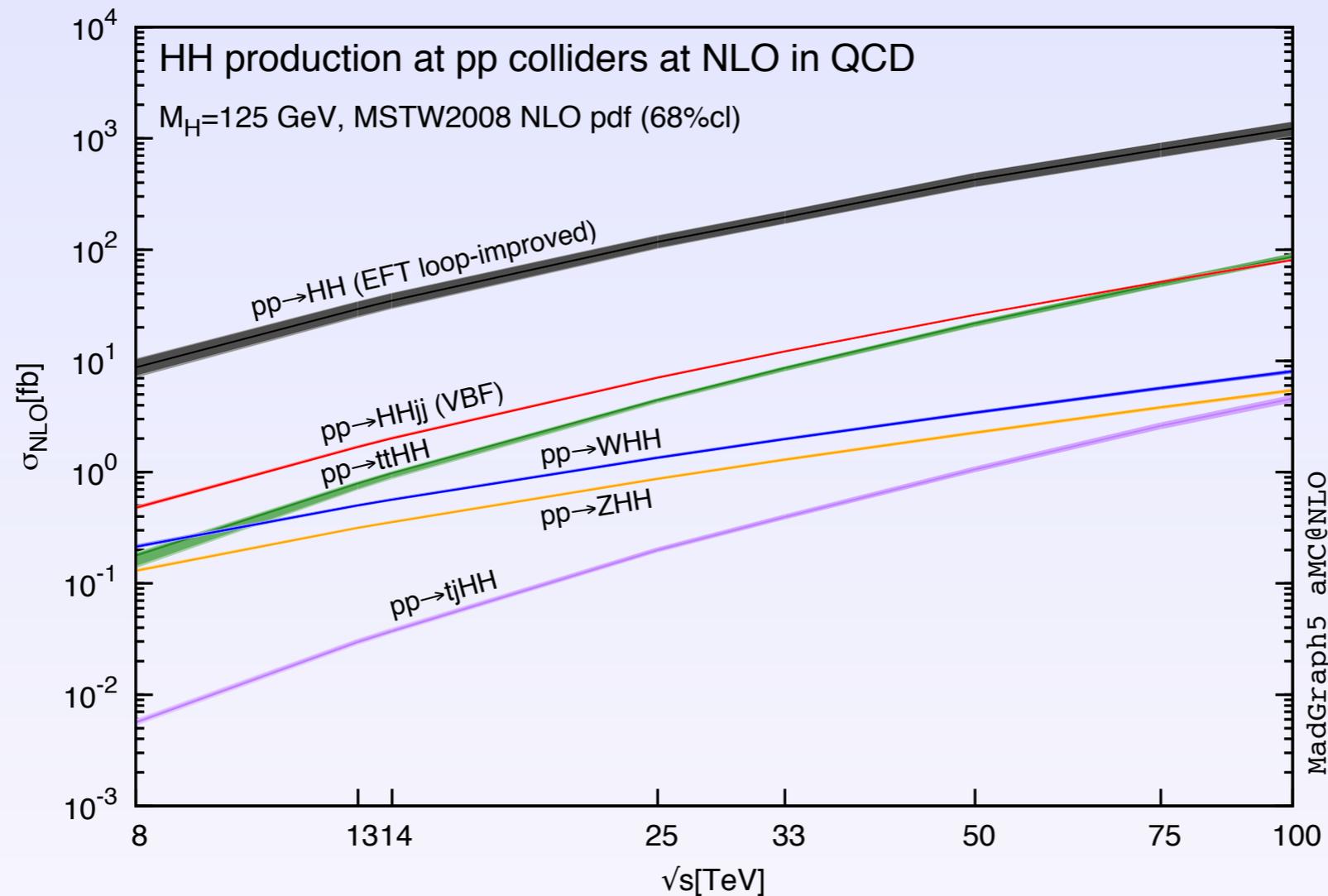
**Figure 9:** A fit of a side band region using a 5<sup>th</sup>-order polynomial, performed with looser selection requirements, using Shower Deconstruction for the leading- $p_T$  Higgs boson identification and BDRS for the sub-leading Higgs mass reconstruction.



**vi. (... and beyond)**

# other production modes?

- several associated production modes exist:



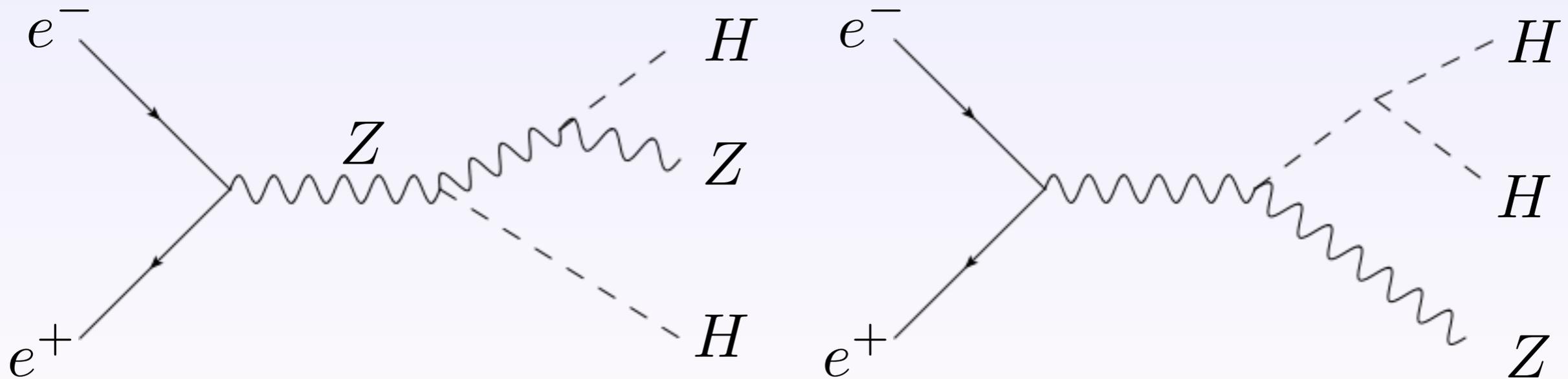
[R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, P. Torrielli, E. Vryonidou, M. Zaro, 1401.7340]

- (note: behaviour w.r.t.  $\lambda$  is different for each channel.)
- with decays  $HH \rightarrow b\bar{b}b\bar{b}$ , could be looked into with sub-structure techniques, but initial cross section low.

# triple coupl. @ lin. colliders (I)

- at a linear collider, a few studies exist,
- based on processes such as:

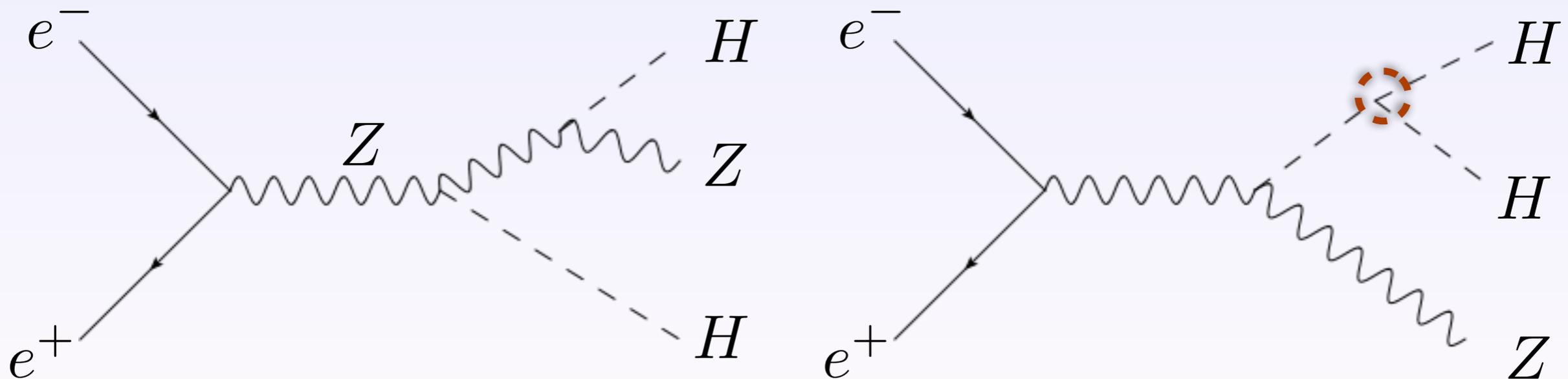
$$e^+e^- \rightarrow ZHH$$



# triple coupl. @ lin. colliders (I)

- at a linear collider, a few studies exist,
- based on processes such as:

$$e^+e^- \rightarrow ZHH$$



# triple coupl. @ lin. colliders (II)

- e.g. ILC [1306.6352] or TESLA TDR [hep-ph/0106315]:

$$e^+e^- \rightarrow ZHH \quad (\text{and both } H \rightarrow b\bar{b})$$

with:

$$\sigma(\sqrt{S} = 500 \text{ GeV}) \simeq 0.15 \text{ fb for: } M_H \simeq 125 \text{ GeV}$$

TESLA TDR (2001): cross section with  $\sim 20\%$  error,

and  $\lambda$  with accuracy  $\sim 20\%$ : at  $1000 \text{ fb}^{-1}$ .

ILC TDR (2013): cross section with  $\sim 27\%$  error,

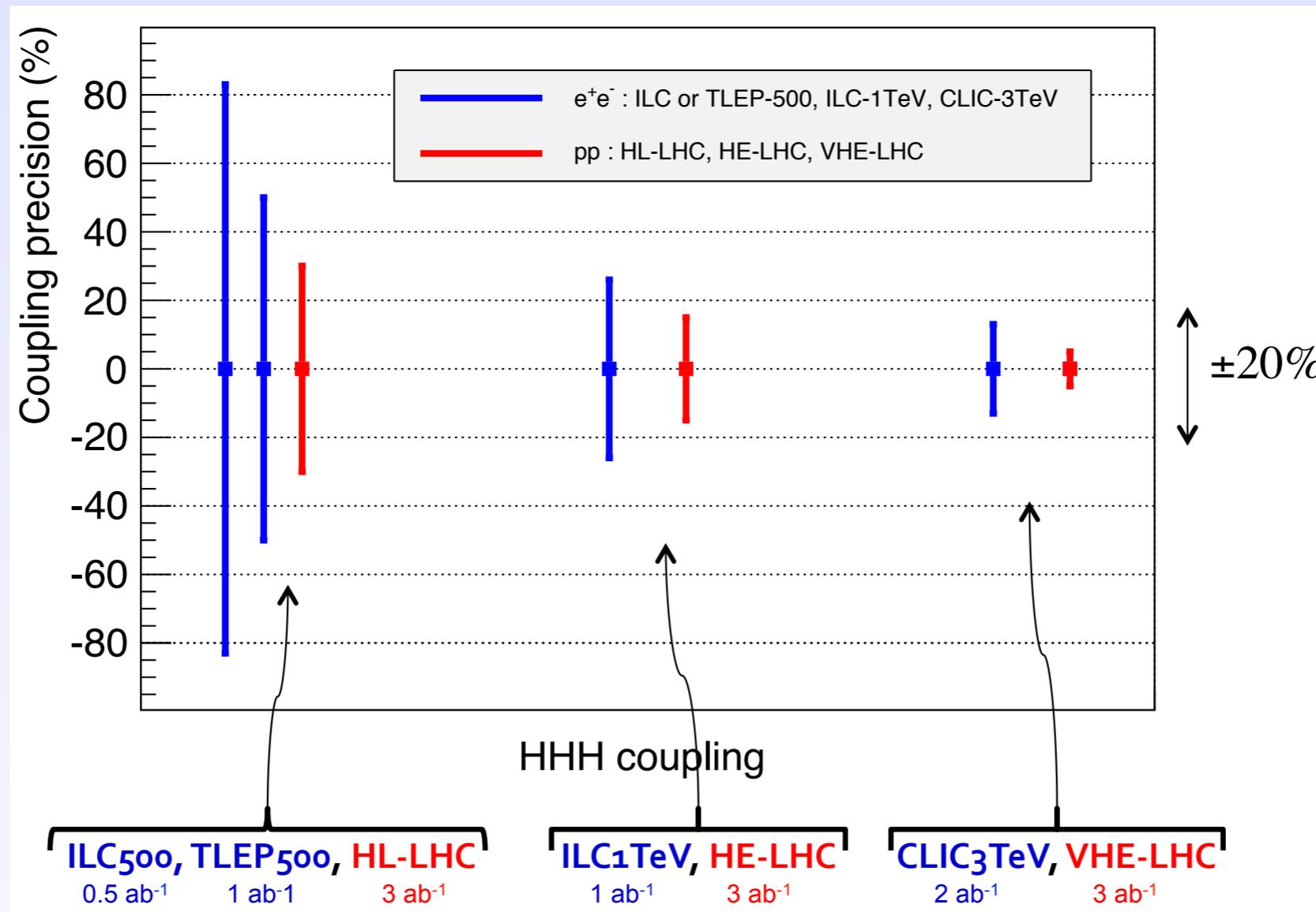
and  $\lambda$  with accuracy  $\sim 44\%$ : at  $2000 \text{ fb}^{-1}$ .

ILC discrepancy:  
‘mis-clustering of  
color-singlet groups’



‘A new jet clustering  
algorithm is now  
being developed.’

# triple coupl. @ future colliders



HE-LHC: 33 TeV

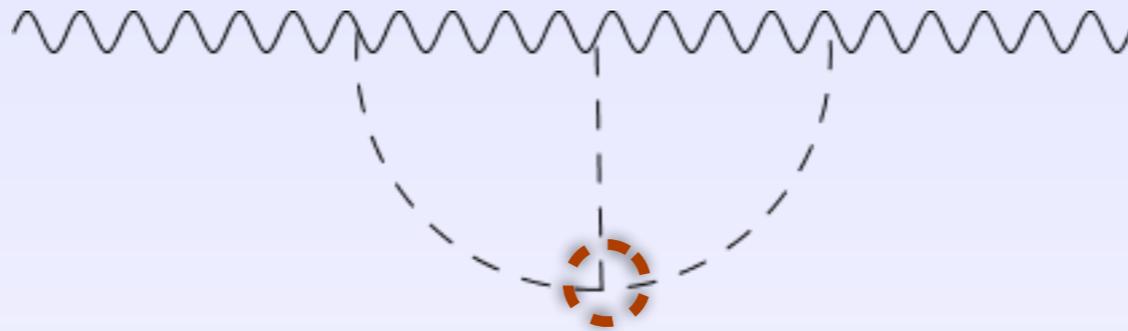
VHE-LHC: 100 TeV

[TLEP Design WG,  
1308.6176]

**Fig. 18:** Expected relative statistical accuracy in % on the trilinear Higgs self-coupling for  $e^+e^-$  (blue) and  $pp$  (red) colliders at the high-energy frontier. The accuracy estimates are given, from left to right, for ILC500, TLEP500, HL-LHC, ILC1000, HE-LHC, CLIC and VHE-LHC, for integrated luminosities of 0.5, 1, 3, 1, 3, 2, and 3  $ab^{-1}$ , respectively.

# indirect constraints? (I)

- e.g. contributions to observables such as the W mass @ two loops via:



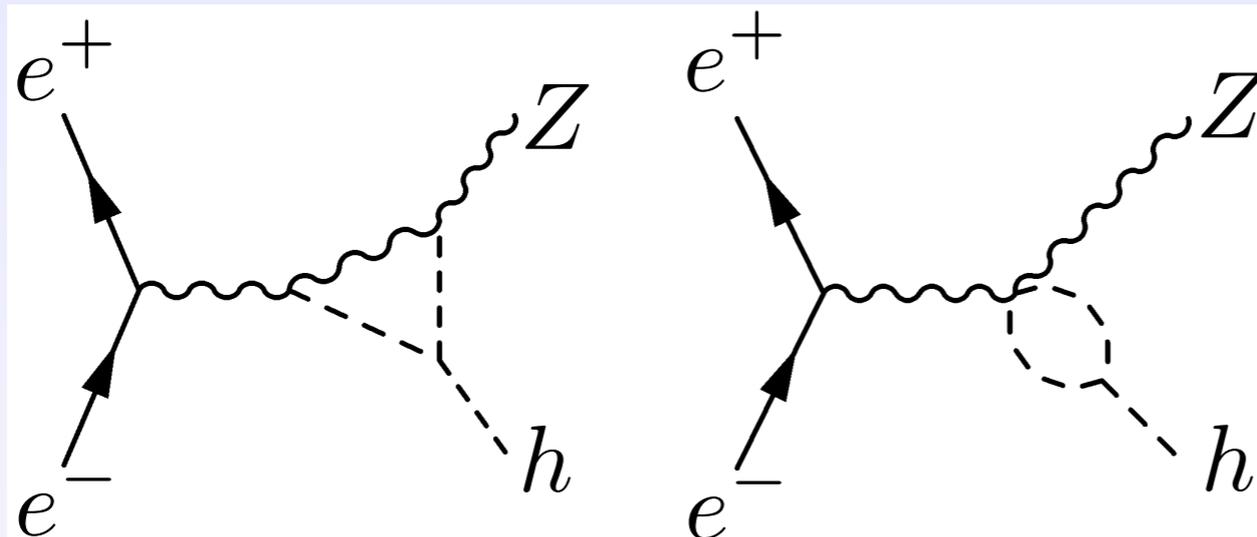
- but SUM of **all** the bosonic contributions only has (in the SM): [e.g. Awramik, Czakon, Freitas, Weiglein, hep-ph/0311148]

$$(\Delta M_W)_{\text{bos.}}^{2\text{-loop}} = \mathcal{O}(0.1 \text{ MeV})$$

- compare to  $\sim 15 \text{ MeV}$ , current experimental uncert. (or factor of 2-3 better in future experiments).
- can never provide constraints (?).

# indirect constraints? (II)

- e.g. contributions to **single Higgs observables** through higher-order corrections.
- e.g.  $e^+e^-$  @ 240 GeV:



[M. McCullough, 1312.3322]

FIG. 1: NLO vertex corrections to the associated production cross section which depend on the Higgs self-coupling. These terms lead to a linear dependence on modifications of the self-coupling  $\delta_h$ .

- may determine triple coupling within  $\sim 30\%$  at 10/ab.

# summary/conclusions

- I have discussed...
  - i. multi-Higgs processes at the LHC,
  - ii. and what we would hope to learn.
  - iii. specifically: HH production,
  - iv. how to go about searching for it, and what possible constraints we could expect.
  - v. prospects for going beyond gluon fusion HH@LHC.

- HH is an interesting channel for HL-LHC and future colliders!
- further work:
  - **theoretically**: improving description of the kinematics and the total cross section (full NLO?), investigate effective theory description,
  - in **phenomenology**: re-examine channels, search new, or use indirect constraints,
  - **experimentally**: assess the viability of the promising channels/methods, improve triggering for this channel!

# special thanks

- special thanks to my collaborators:  
Florian, José, Li Lin, Philipp,  
Michael, Danilo.

# special thanks

- special thanks to my collaborators:  
Florian, José, Li Lin, Philipp,  
Michael, Danilo.
- ...and thanks for your attention!

auxiliary slides



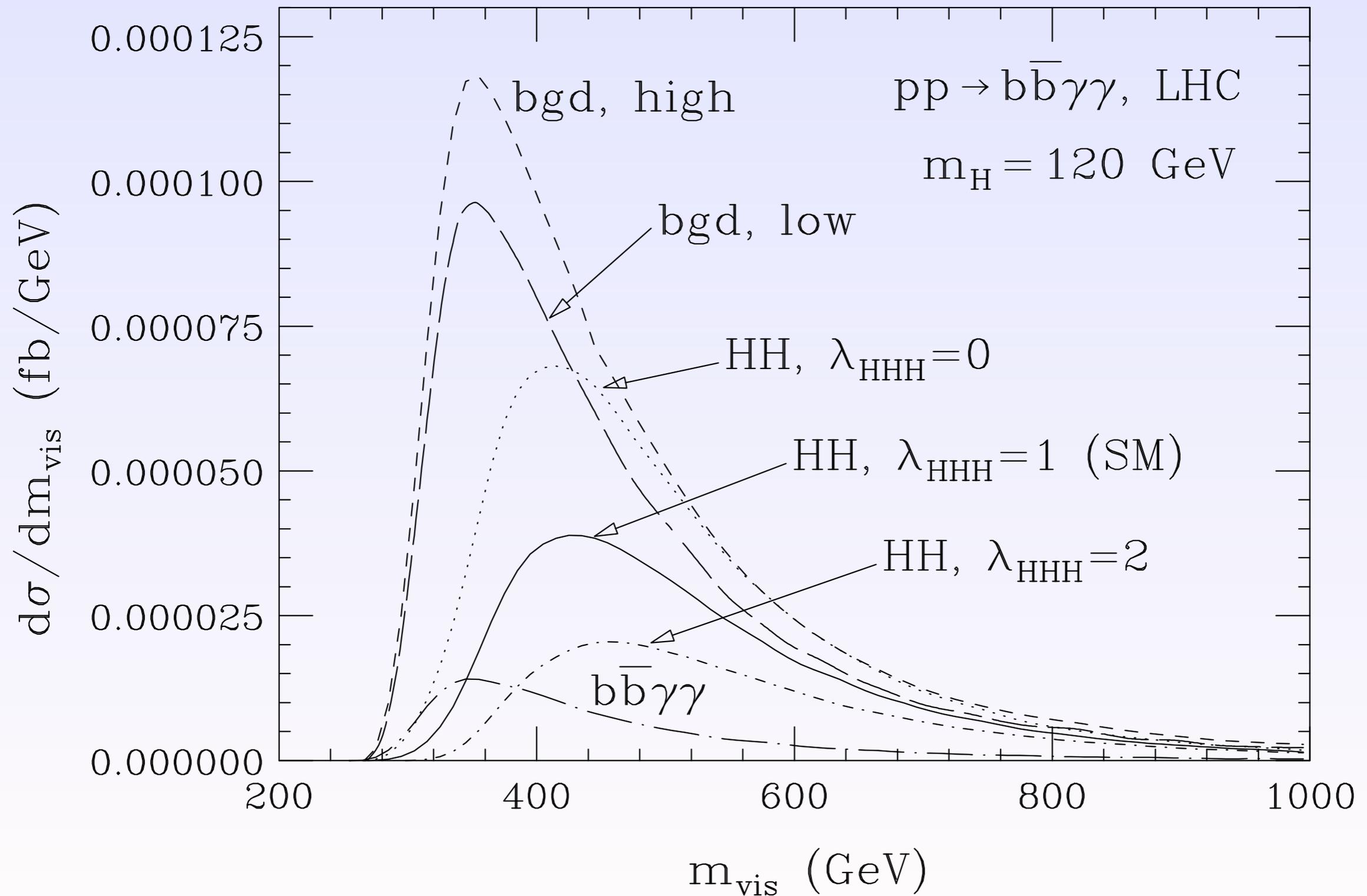
**how do we (actually) measure the  
triple coupling  $\lambda$ ?**

# using differential distributions



- (as seen in: Baur, Plehn, Rainwater [hep-ph/0310056])
- perform the analysis, e.g. for  $b\bar{b}\gamma\gamma$  .
- construct a differential distribution for signal and background using Monte Carlo.
- compare to Monte Carlo events to get expected bounds on the self-coupling.

# using differential distributions (an example from Baur, Plehn, Rainwater):



# using rates (i.e. cross sections)

- differential distributions for both signal and background may not be very well modeled.
- we can use the **total rate** predictions for signal and background instead.
- BUT: these can be dominated by large systematic uncertainties, originating either from:
  - unknown higher-order corrections,
  - parton density function uncertainties,
  - experimental errors,
  - + more.

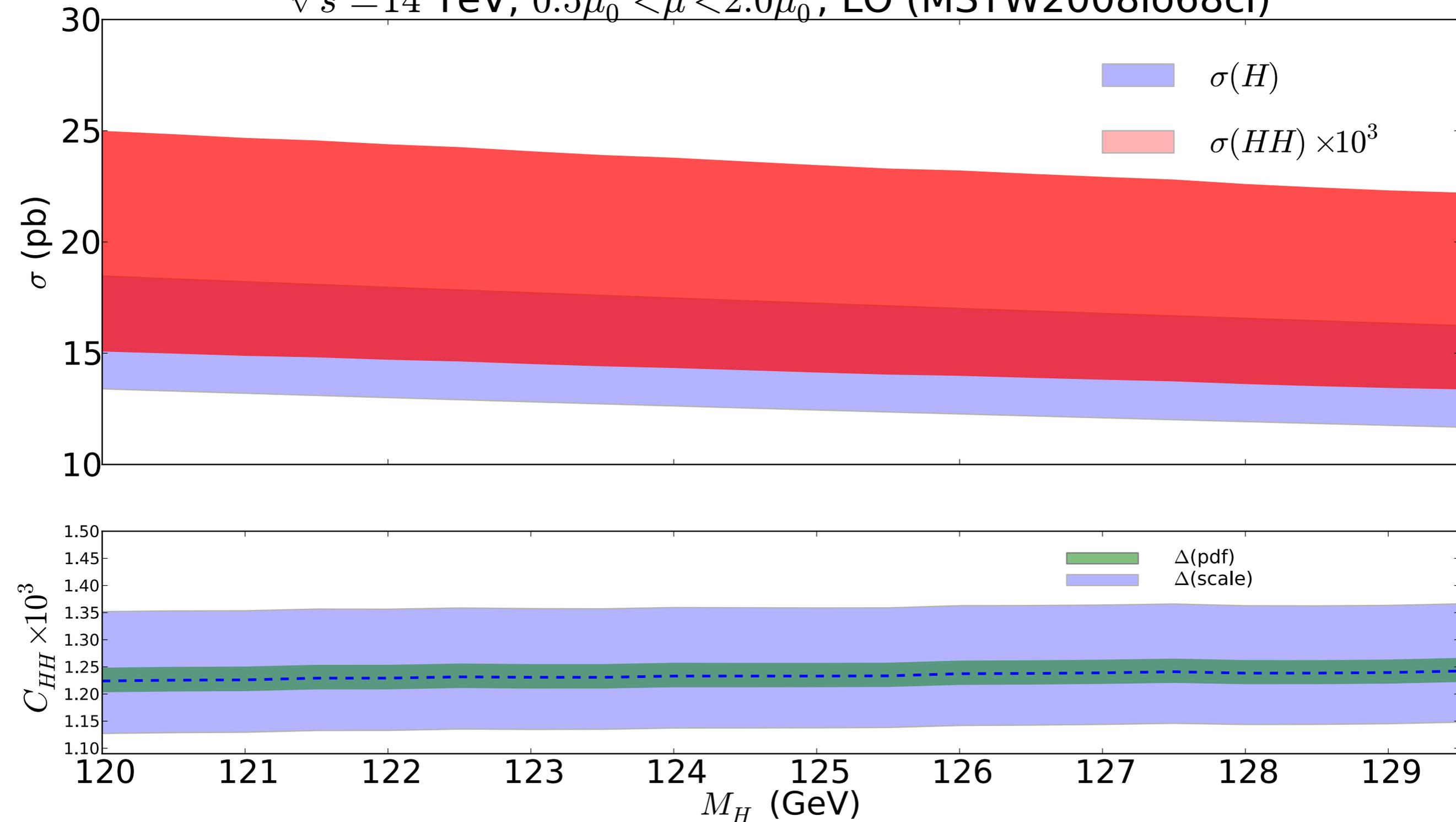
# using ratios of cross sections

- consider: 
$$C_{HH} = \frac{\sigma(gg \rightarrow HH)}{\sigma(gg \rightarrow H)},$$
- **single** Higgs production may possess similar higher-order QCD corrections to Higgs pair production.
- these may cancel out in the ratio, leading to a more stable prediction.
- moreover, experimental systematic uncertainties may cancel out, e.g. the luminosity uncertainty.
- we can check the degree to which extent the scale and pdf uncertainties cancel out.



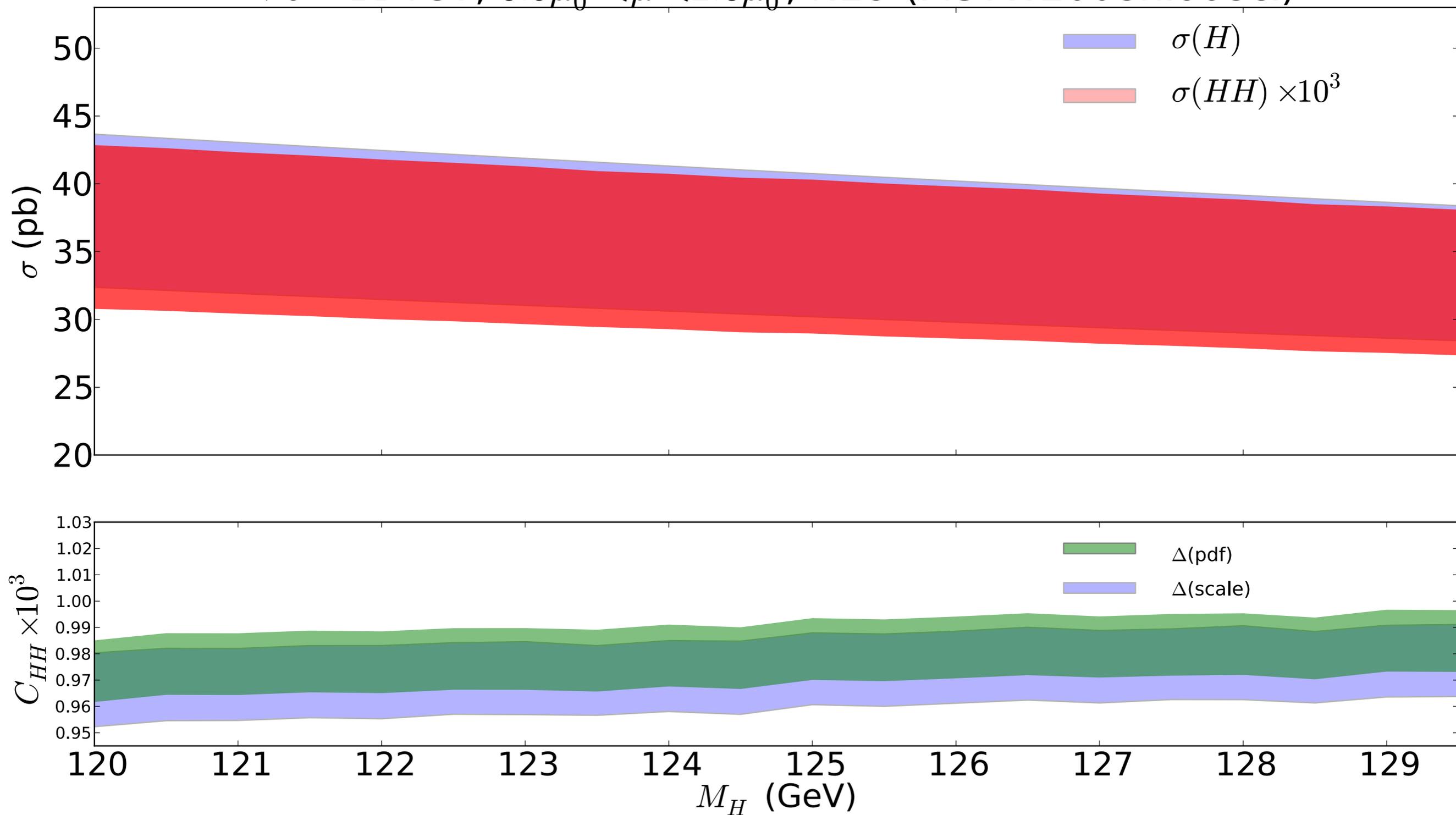
# leading order

$\sqrt{s} = 14 \text{ TeV}, 0.5\mu_0 < \mu < 2.0\mu_0, \text{ LO (MSTW2008lo68cl)}$



# next-to-leading order

$\sqrt{s} = 14 \text{ TeV}, 0.5\mu_0 < \mu < 2.0\mu_0, \text{ NLO (MSTW2008nlo68cl)}$



# comments on ratio

- assuming that the scale uncertainties are correlated is a reasonable assumption.
- ratio goes from  $\sim 1.25$  to  $\sim 1.0$  from LO to NLO even though the K-factor is  $\sim 2$ .
- a total theoretical uncertainty of  $\sim 5\%$  is not unreasonable for the ratio, as opposed to  $\sim 20\%$  for the cross section itself.
- we used the ratio, along with **conservative** expected experimental uncertainties to construct expected exclusion regions.



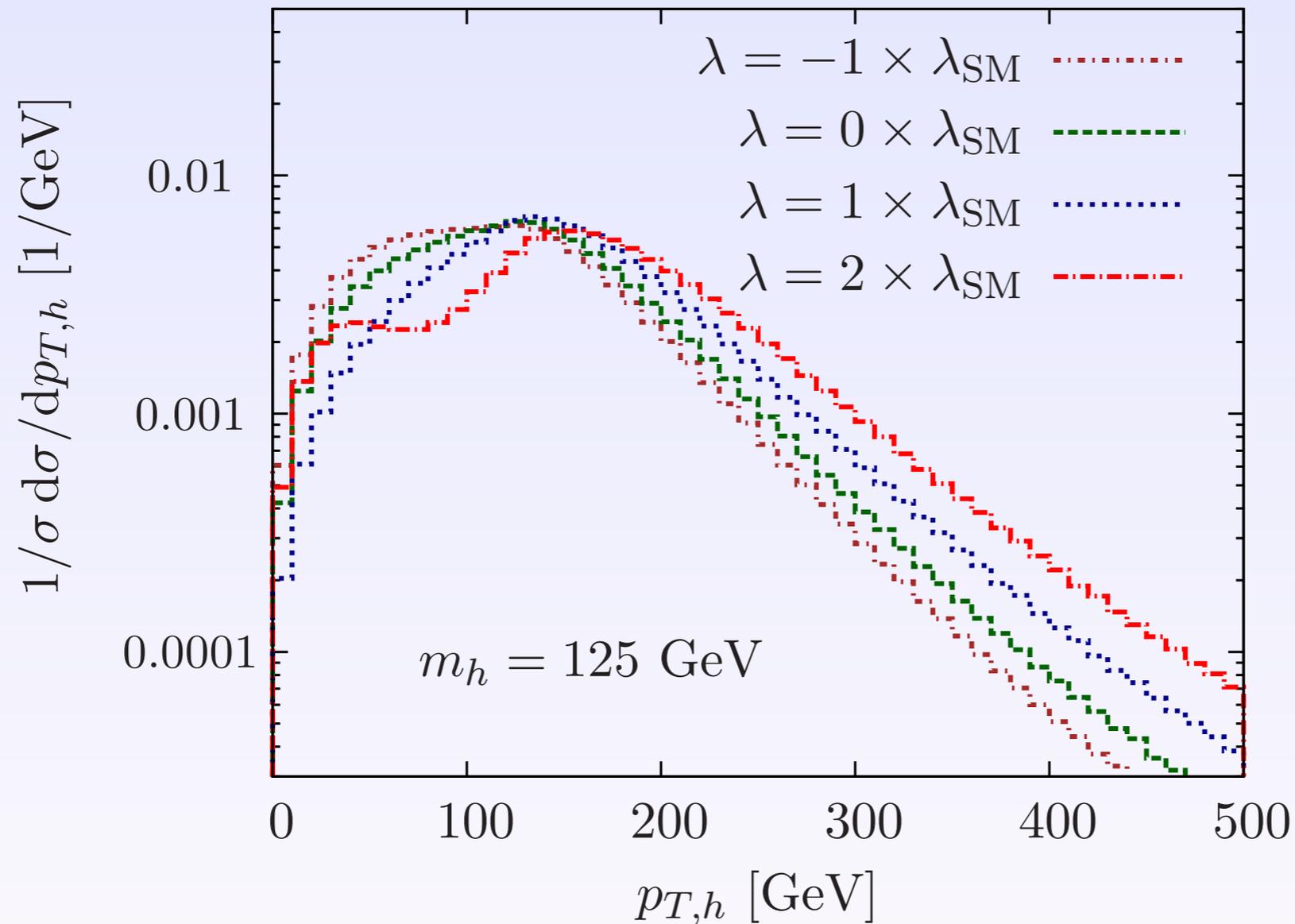
# H+V, BDRS Analysis

[Butterworth, Davison, Rubin, Salam, 0802.2470]

- “BDRS” analysis:
  - Higgs decays to two b-quarks.
  - Cambridge/Aachen jet algorithm,  $R=1.2$ , get “fat jets”.
  - apply a “mass-drop” condition on a hard jet:
    - picks up the decay of a massive particle, e.g.  $H \rightarrow b\bar{b}$
  - “filter” the jet: re-apply the jet algorithm with a smaller  $R$ , on the “fat” jet constituents, take **three** hardest “sub-jets”.
  - ask for the two hardest “sub-jets” to contain **b-tags**.
  - “**filtering**” reduces the effective area of the “Higgs”-jet,
  - hence reduces pollution from **Underlying Event**.

# BDRS analysis on H+H

- the Higgs bosons in HH are **naturally boosted**:



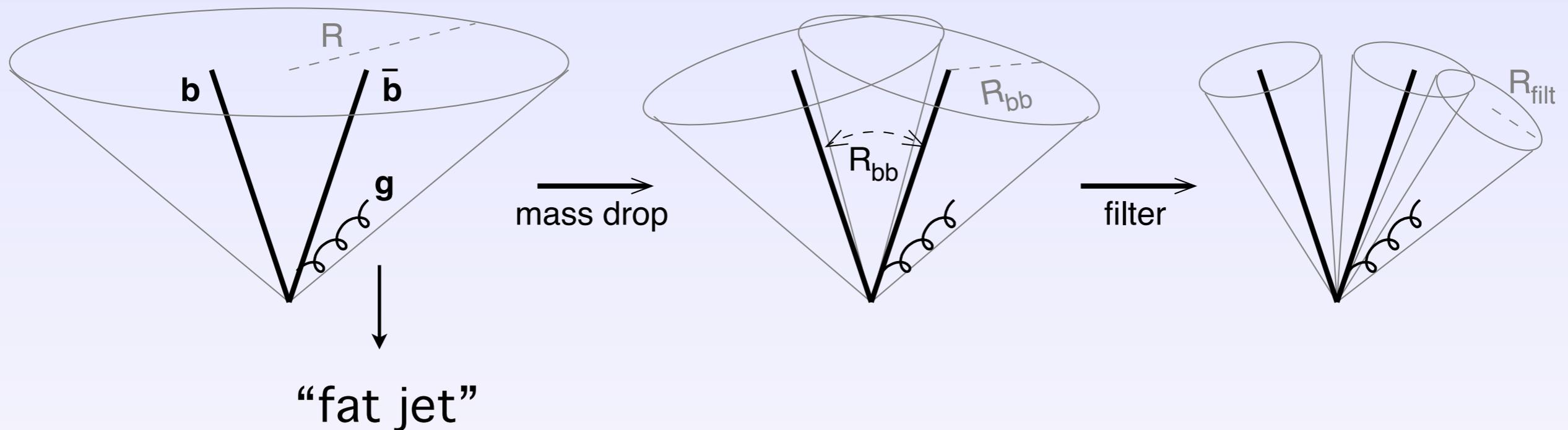
[ Dolan, Englert, Spannowsky, 1206.5001 ]

- + other arguments of BDRS technique apply.

# H+V

- “BDRS” analysis, pictorially:

[Butterworth, Davison, Rubin, Salam, 0802.2470]



- HV: yields good sensitivity ( $4.5\sigma$ ) @ 14 TeV @  $30 \text{ fb}^{-1}$ .
- perhaps an improvement of previous HH results can be also achieved!

# electroweak Lagrangian (I)

- ingredients of the 'recipe':



+ (...)



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an  $SU(2) \times U(1)$  gauge symmetry

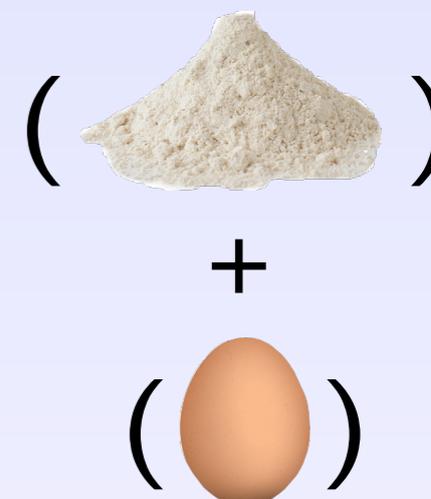
+ a complex doublet scalar,  $\phi$ .

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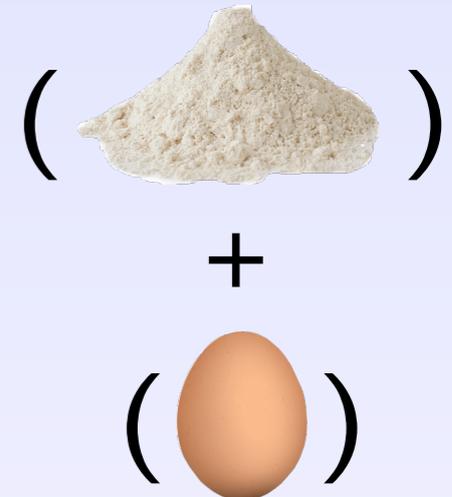


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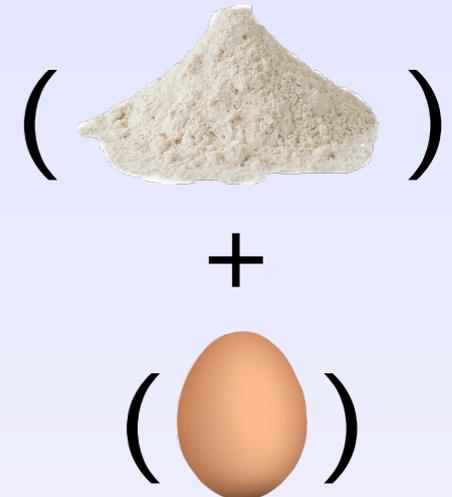
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the covariant derivative:

$$D^\mu = \partial^\mu + ig_2 (T \cdot W^\mu) + iY g_1 B^\mu$$

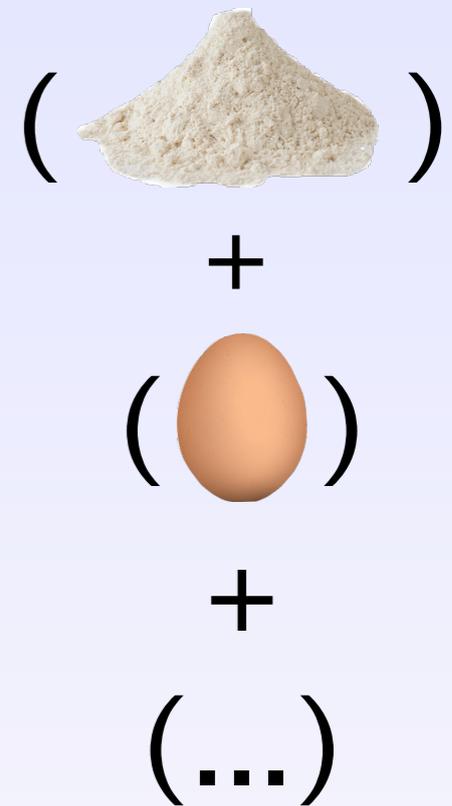
SU(2) coupl.
SU(2) gens.
U(1) coupl.

# electroweak Lagrangian (II)

- with potential:

$$\mathcal{V}(\phi^\dagger \phi) = \lambda(\phi^\dagger \phi)^2 + \mu^2 \phi^\dagger \phi,$$

$$(\lambda > 0, \mu^2 < 0)$$

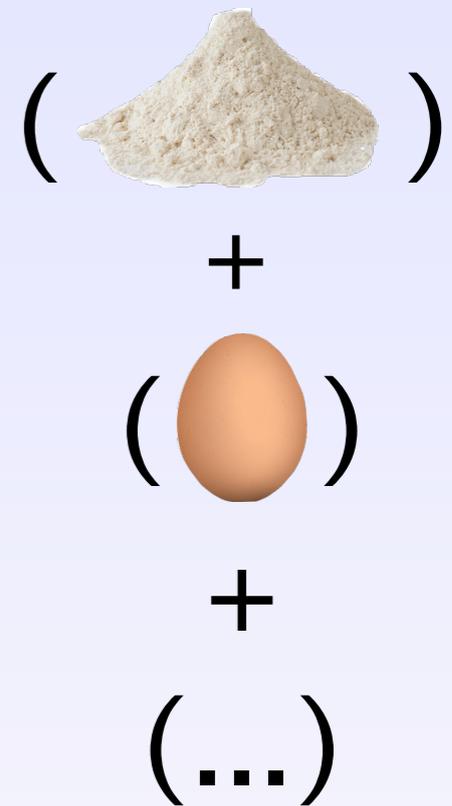


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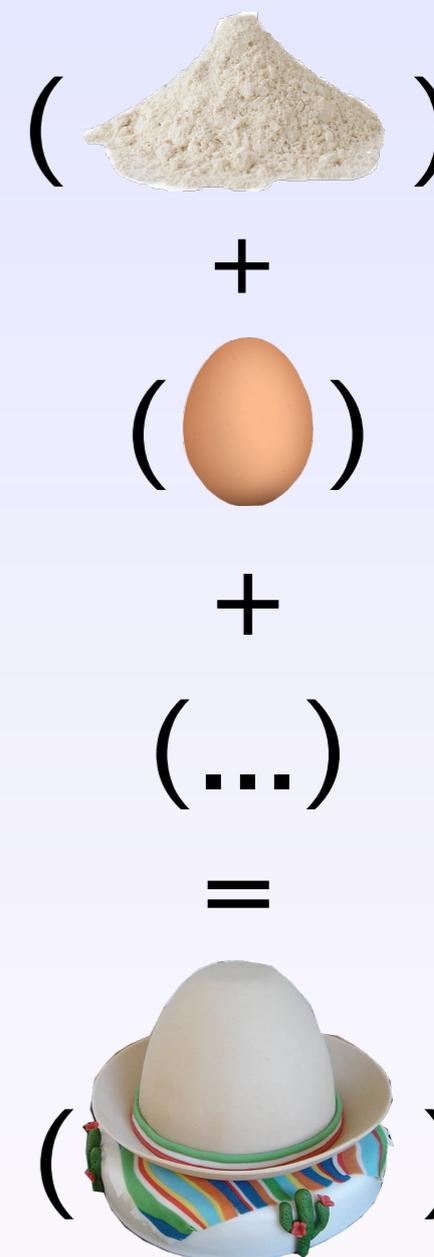
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# electroweak Lagrangian

- further steps:
- choose minimum in particular direction:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (\text{implies: residual } U(1) \text{ invariance})$$

- consider fluctuations of scalar field about that minimum,
- and make a gauge transformation to absorb the Goldstone modes into the gauge bosons.

# electroweak Lagrangian

- hence, after symmetry breaking, the Higgs + SU(2)xU(1) Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2} \partial_\mu H \partial^\mu H - \mathcal{V}(H; \lambda, v) + \frac{(v + H)^2}{8} \begin{pmatrix} 0 & 1 \end{pmatrix} (2g_2 T \cdot W_\mu + g_1 B_\mu) \times (2g_2 T \cdot W^\mu + g_1 B^\mu) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

fluct. about min.  
 $\phi \propto (0, v + H)$

(recall:  $\mu$ ,  $\lambda$  and  $v$  are related and hence only 2/3 are independent.)

↪ 'Free' parameters:  $v, g_1, g_2, \lambda$

# 'fixing' free params. (I)

- diagonalize the quadratic terms in vector boson fields,
- and deduce the masses of Z and W bosons:

$$M_W = \frac{1}{2} v g_2 \qquad M_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}$$

Measured!

- 4-fermion interaction at low energies can fix the Fermi constant:

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$$

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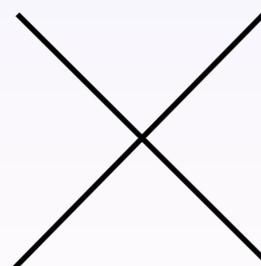
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**WARNING:** Leading Order!

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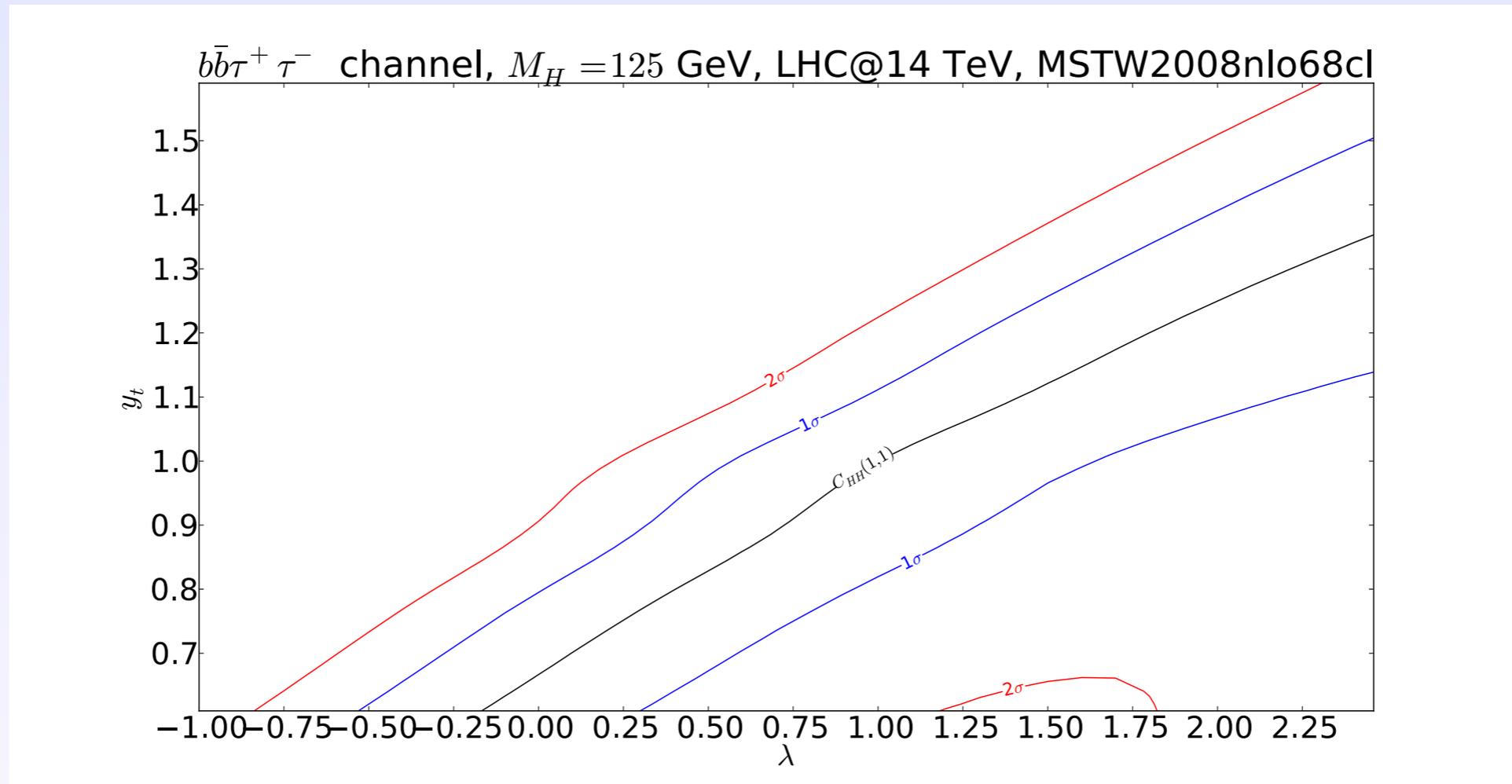
- until very recently, only had 3 out of 4 constraining equations...
- ...in July 2012, we obtained the fourth:

$$M_H = \sqrt{2\lambda}v$$

Measured!

$\hookrightarrow \sim 125 \text{ GeV}$

# HH SM consistency via anomalous couplings



**Figure 9:** The  $1\sigma$  and  $2\sigma$  confidence regions in the  $y_t - \lambda$  plane at  $600 \text{ fb}^{-1}$  for the  $b\bar{b}\tau^+\tau^-$  decay mode, derived using  $C_{HH}$ , within the SM ( $\lambda_{\text{true}} = 1$  and  $y_{t,\text{true}} = 1$ ).

# HH production @ LHC: numerically

- using HPAIR (M. Spira), fits: Florian Goertz, AP, Li Lin Yang, and José Zurita [1301.3492]

$$\sigma_{HH}^{\text{LO}} [\text{fb}] = 5.22\lambda^2 y_t^2 - 25.1\lambda y_t^3 + 37.3y_t^4$$

$$\sigma_{HH}^{\text{NLO}} [\text{fb}] = 9.66\lambda^2 y_t^2 - 46.9\lambda y_t^3 + 70.1y_t^4$$

(couplings  
normalized to  
SM)

neglecting bottom quark contributions:  
O(1%) at total cross section

- negative interference term between triangle and box.
- [interesting: a symmetry point exists at  $\lambda \sim 2.5 y_t$  (NLO)].

# dim-6 EFT with both operators

$$\lambda' = \lambda_{\text{SM}} \left( 1 - \frac{f_1 v^2}{2\Lambda^2} + \frac{2f_2 v^4}{3\Lambda^2 M_H^2} \right)$$

$$\mathcal{L}_{m_f} = -\frac{m_f}{v} \bar{f} f (v + H) \rightarrow$$

$$\mathcal{L}'_{m_f} = -\frac{m_f}{v} \bar{f} f \left[ v + \left( 1 + \frac{f_1 v^2}{2\Lambda^2} \right) H + \frac{f_1 v}{2\Lambda^2} H^2 + \frac{f_1}{6\Lambda^2} H^3 + \mathcal{O}(H^4) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) \right].$$

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