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In collaboration with: Rikkert Frederix, Stefano Frixione, Juan Rojo and Mark Sutton

## **Nature of the Problem**

#### Main goal:

- constraining **Parton Distribution Functions** (PDFs) by including as many data as possible from the LHC with the highest accuracy possible.
- Problem:
  - presently, hadronic NLO(+PS) calculations are too time-consuming to be directly employed in a PDF fit.
- The common solution adopted is:
  - **interpolating the PDFs** (and  $\alpha_s$ ) on the (x,Q<sup>2</sup>)-plane with some suitable polynomial basis on a finite number of nodes.
  - **Precomputing the hadronic cross section** by using the basis members as input (rather than PDFs themselves).
    - Time-cosuming step that must be done only once.
  - **Reconstructing the original calculation** by means of the numerical convolution of the precomputed cross sections with an arbitrary PDF set.
    - Very fast  $\Rightarrow$  suitable for PDF fits.

## **Nature of the Problem**

#### The objective of our work is:

- to solve this problem once and for all in a **general manner**.
- This is actually possible thanks to the fact that NLO(+PS) calculations can now be routinely done by means of **automated codes**.

#### The ingredients here are:

- MadGrap5\_aMC@NLO [arXiv:1405.0301]
  - an automated cross section calculator that contains all the ingredients relevant to the computation of LO and NLO cross sections, with or without matching to parton showers.

#### • **APPLgrid** [arXiv:0911.2985]

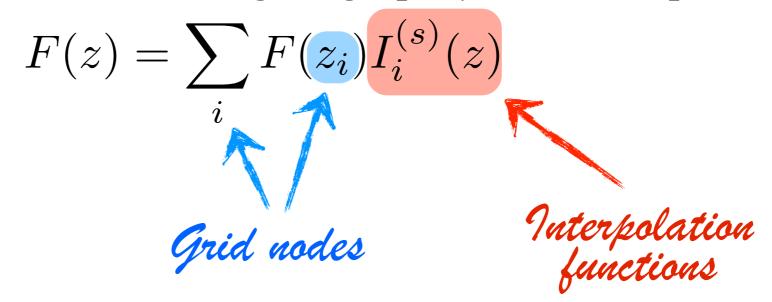
• a framework that implements the strategy for the fast computation of cross sections outlined in the previous slide.

#### The result is:

**aMCfast** [arXiv:1406.7693]:

• an automated interface which bridges MadGraph5\_aMC@NLO with APPLgrid.

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$$F(z) = \sum_{i} F(z_i) I_i^{(s)}(z)$$

Suppose you want to compute numerically the following integral, e.g. by Monte Carlo methods:

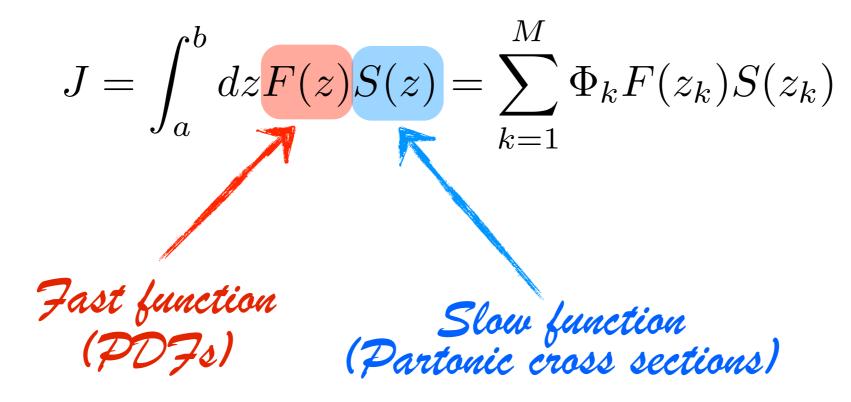
$$J = \int_{a}^{b} dz F(z) S(z) = \sum_{k=1}^{M} \Phi_{k} F(z_{k}) S(z_{k})$$
Prove the interval [a b]

Random points in the interval [a,b]

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(1-dimentional) interpolation grid independent of F(z): precomputed and stored

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• Once  $G_i$  has been precomputed, the *a posteriori* computation of J with any function F(z) will be extremely fast.

• The generalization of this procedure to the realistic case of a hard NLO cross section is straightforward, considering that:

 $d\sigma^{(\text{NLO})} \leftrightarrow \left\{ d\sigma^{(\text{NLO},\alpha)} \right\}_{\alpha=E,S,C,SC}$  Event & Counterevents

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$$d\sigma^{(\text{NLO},\alpha)} = f_1(x_1^{(\alpha)}, \mu_F^{(\alpha)}) f_2(x_2^{(\alpha)}, \mu_F^{(\alpha)}) W^{(\alpha)} d\chi_{Bj} d\chi_{n+1}$$
  
$$\mathcal{PD7s} \qquad \mathcal{Partonic cross section}$$

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$$W^{(\alpha)} = \alpha_s^{b+1}(\mu_R^{(\alpha)}) \left[ W_0^{(\alpha)} + W_F^{(\alpha)} \ln\left(\frac{\mu_F^{(\alpha)}}{Q}\right) + W_R^{(\alpha)} \ln\left(\frac{\mu_R^{(\alpha)}}{Q}\right) \right] + \alpha_s^b(\mu_R^{(\alpha)}) W_B \delta_{\alpha S}$$

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- 4 slow functions  $\Rightarrow$  4 interpolation grids.
- The **fast functions** are functions of 4 independent variables  $(x_1, x_2, \mu_F, \mu_R) \Rightarrow$  4-dimentional interpolation grids needed.

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#### • 4 slow functions $\Rightarrow$ 4 interpolation grids.

• The **fast functions** are functions of 4 independent variables  $(x_1, x_2, \mu_F, \mu_R) \Rightarrow$  4-dimentional interpolation grids needed.

• But assuming  $\mu_F \propto \mu_R \Rightarrow$  **3-dimentional** interpolation grids.

## **The aMCfast Interface** *The NLO Case: a Short Description*

#### • The **aMCfast** interface proceeds through three phases:

**Initialisation phase:** aMCfast provides APPLgrid with:

- the total number of grids needed (equal to the sum over all observables of the number of bins of each observable, times four).
- the grid spacings, the interpolation orders, and the interpolation ranges (this information is under the user's control).

#### Running phase:

- aMCfast gets all the needed information (kinematics and weight functions W) eventby-event from MadGraph5\_aMC@NLO.
- This information is then fed to APPLgrid, whose grid-filling internal routines iteratively construct the interpolation grids.

#### Termination phase:

• The grids are finally written to file in the APPLgrid format.

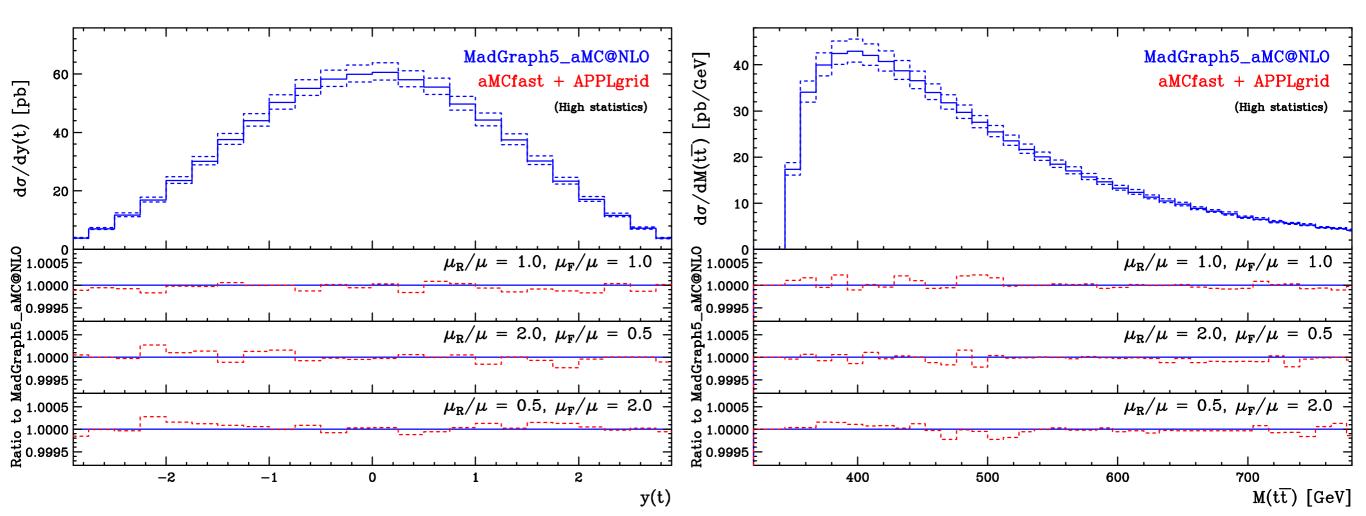
## **The aMCfast Interface** Validation: The Setup

Given a process and an observable, we compute the respective differential distribution in two different ways:

- directly, by means of MadGraph5\_aMC@NLO (Reference),
- a posteriori, convoluting the grids constructed with aMCfast (**Reconstructed**).
- In our approach, the distributions must be in agreement for:
  - **any statistics** (4-grids approach), for testing we choose:
    - low (~10<sup>3</sup> phase space points per integration channel),
    - high (~10<sup>6</sup> phase space points per integration channel),
  - **any scale combination**, for testing we choose:
    - $\bullet \mu_F = \mu \quad \mu_R = \mu,$
    - $\bullet \mu_F = 2\mu \qquad \mu_R = \mu/2,$
    - $\bullet \mu_F = \mu/2 \qquad \mu_R = 2\mu.$
- No PDF variation considered here.

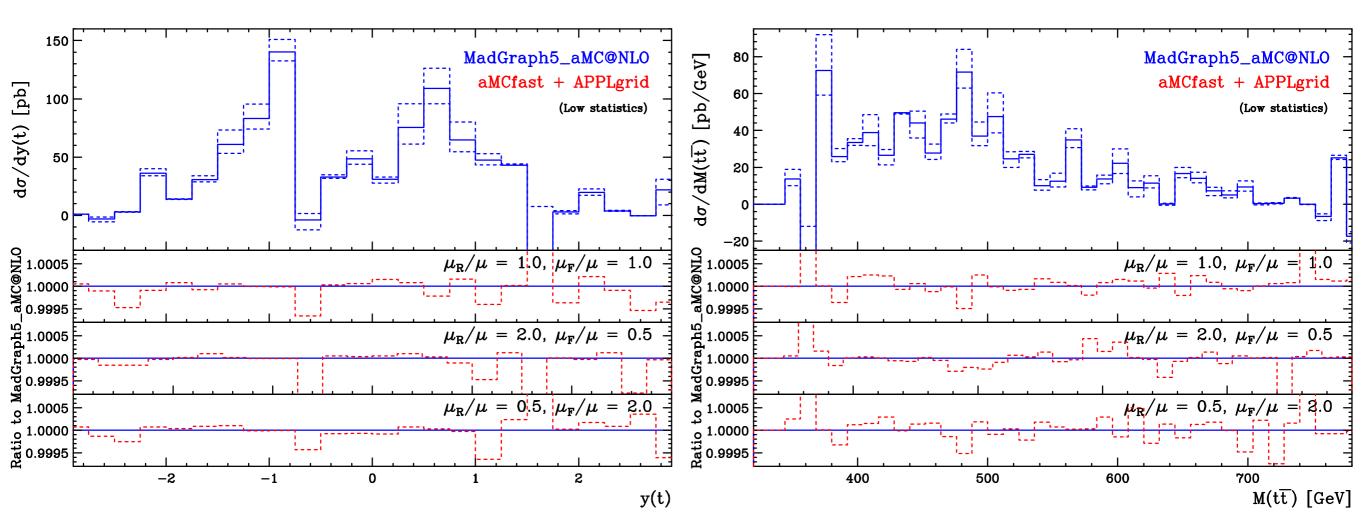
Validation: Top-Quark Pair Production

- Important for constraining the large-x gluon.
- We looked at the following observables:
  - the rapidity distribution of the top quark (left),
  - the invariant mass distribution of the top pair (right).
- High statistics plots:



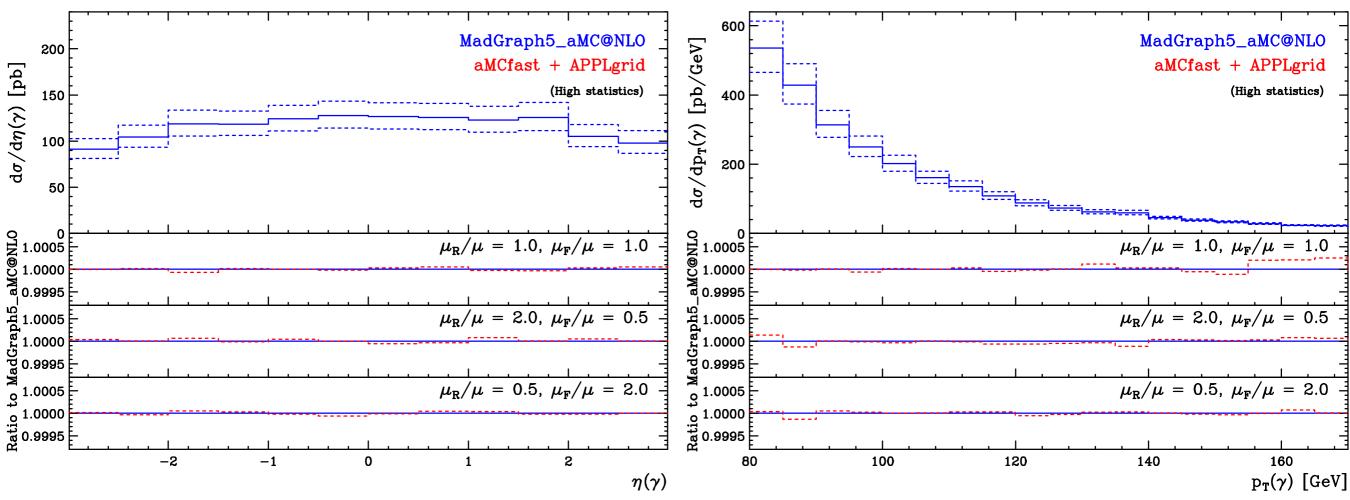
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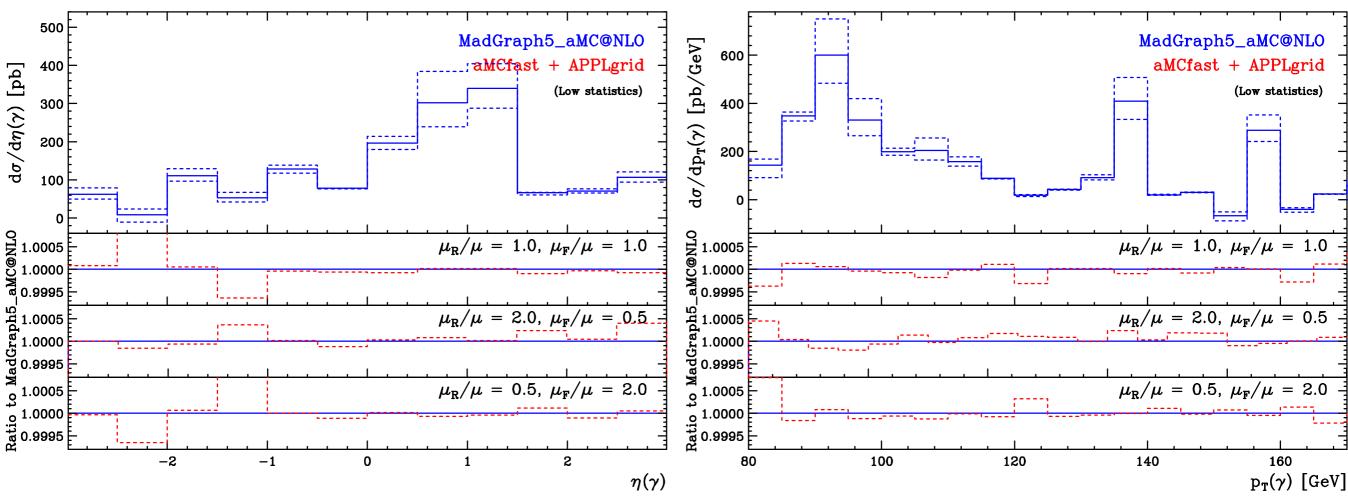
Validation: Photon Production with one Jet

- Important for the gluon in the region relevant for Higgs production in gluon fusion.
- We looked at the following observables:
  - the pseudo-rapidity distribution of the photon (left),
  - the tranverse momentum distribution of the photon (right).
- High statistics plots:



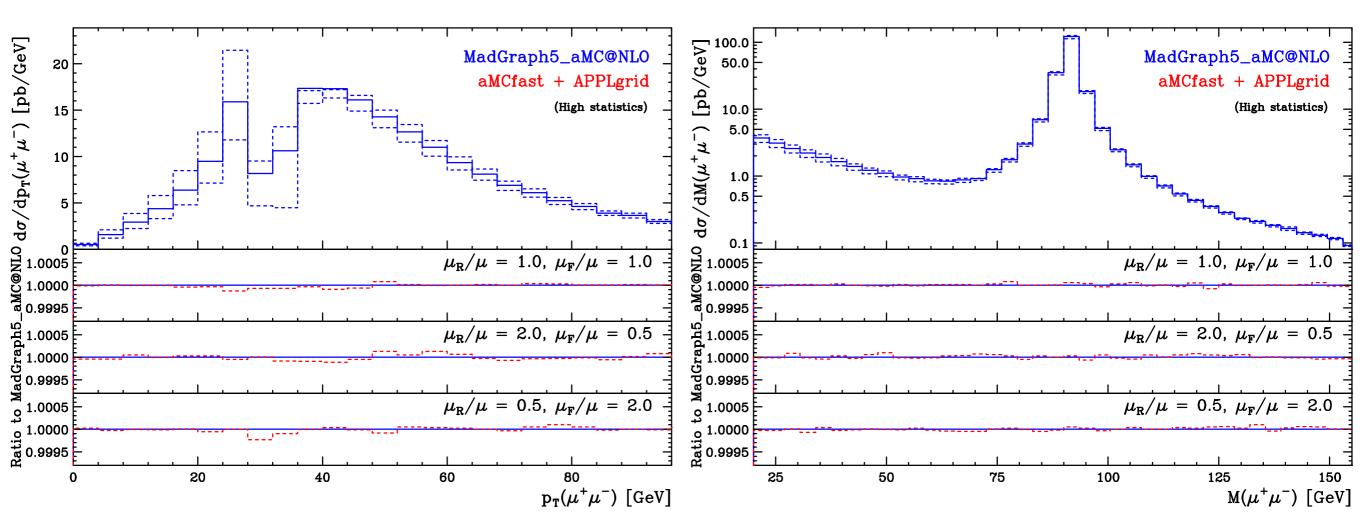
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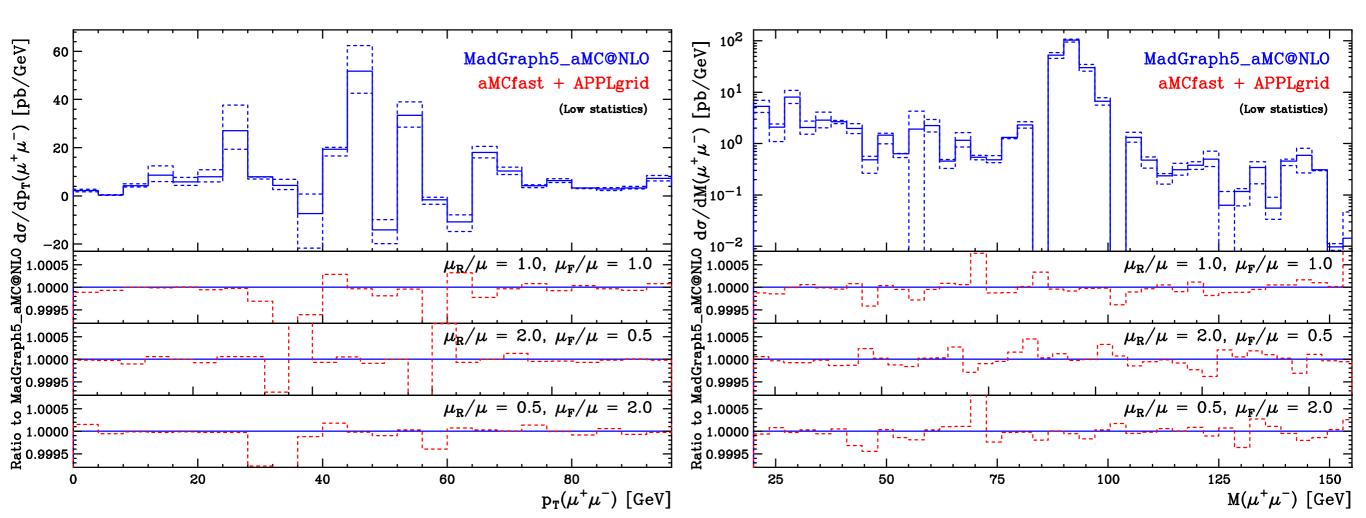
Validation: Dilepton Production with one Jet

- Relevant for quarks and antiquarks in the large-x region.
- We looked at the following observables:
  - the transverse momentum distribution of the lepton pair (left),
  - the invariant mass distribution of the lepton pair (right).
- High statistics plots:



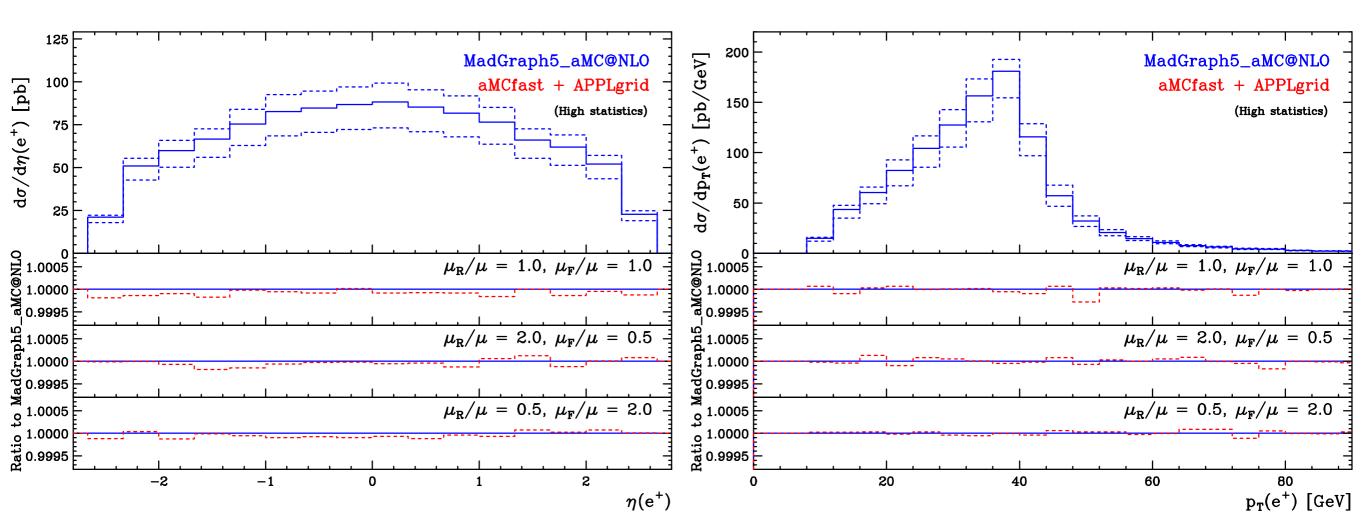
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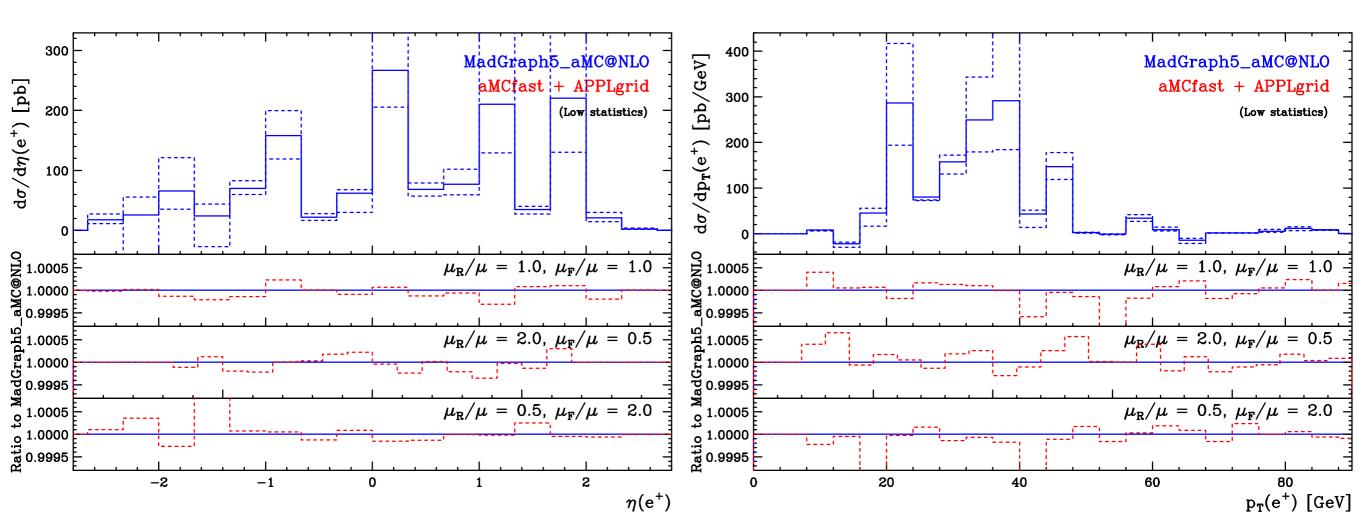
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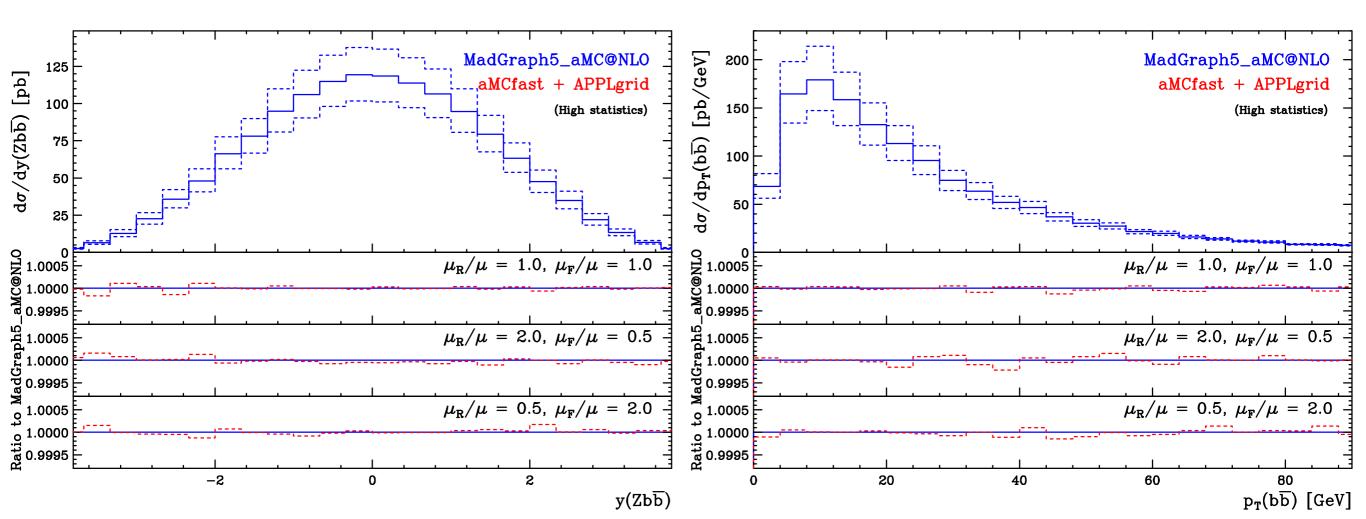
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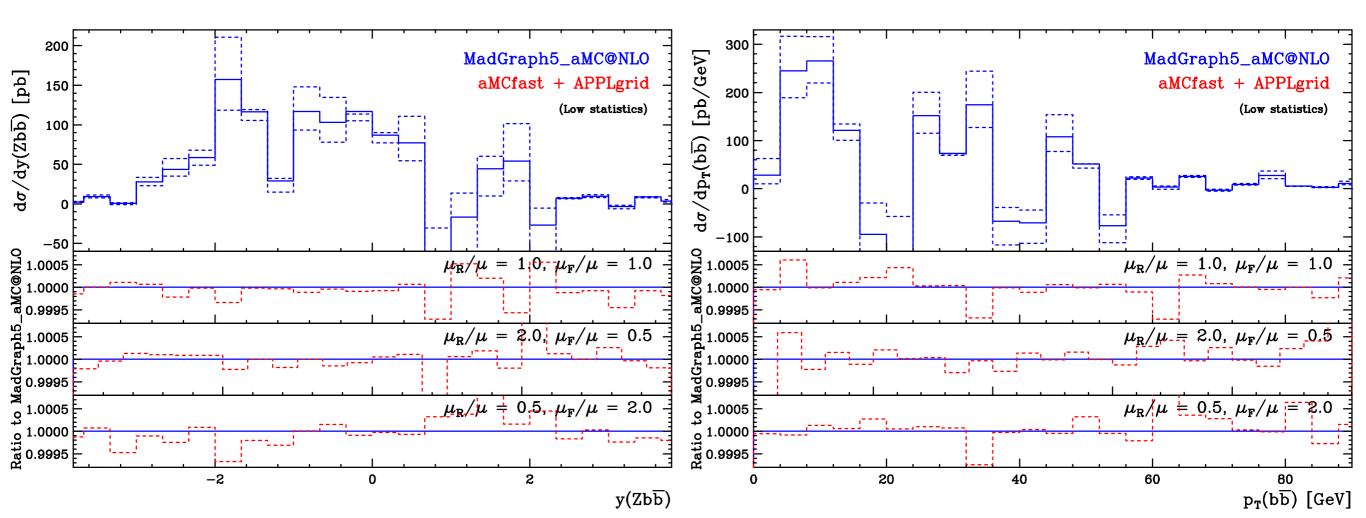
Validation: Z + bb Production

- This is just an example of complicated process.
- We looked at the following observables:
  - the rapidity distribution of the Zbb system (left),
  - the transverse momentum distribution of the Zbb system (right).
- High statistics plots:



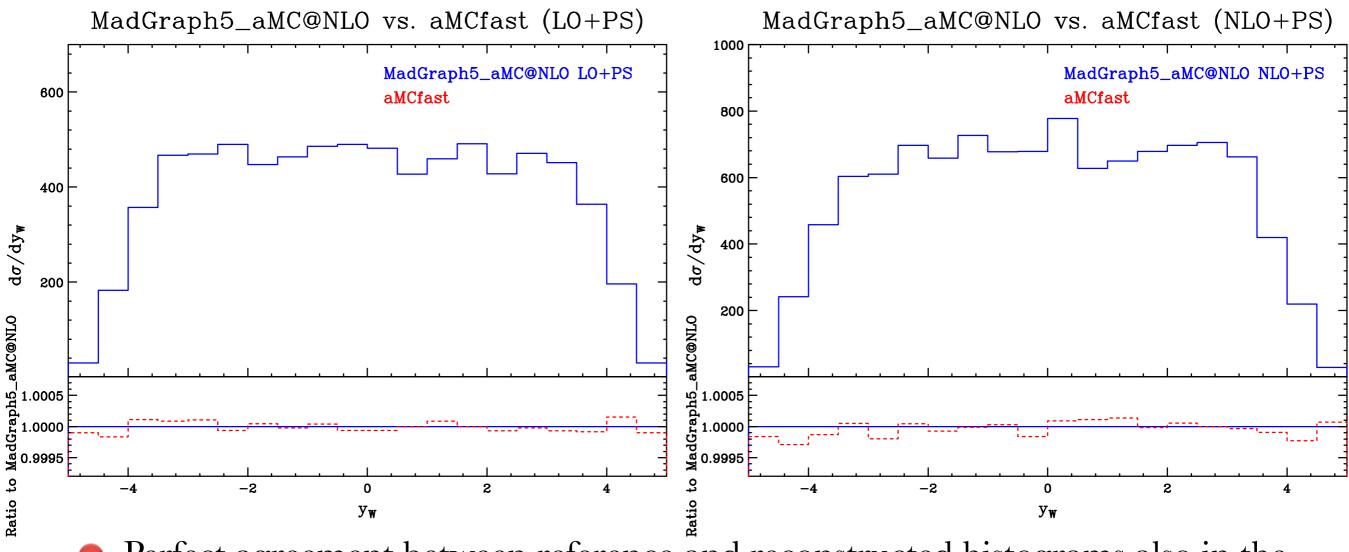
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The aMCfast Interface *The NLO + PS Case: Preliminary Results*We are presently working on extending aMCfast to the (N)LO+PS mode of MadGraph5\_aMC@NLO.

• Preliminary results are already available  $(e^+v + \text{Herwig6})$ :

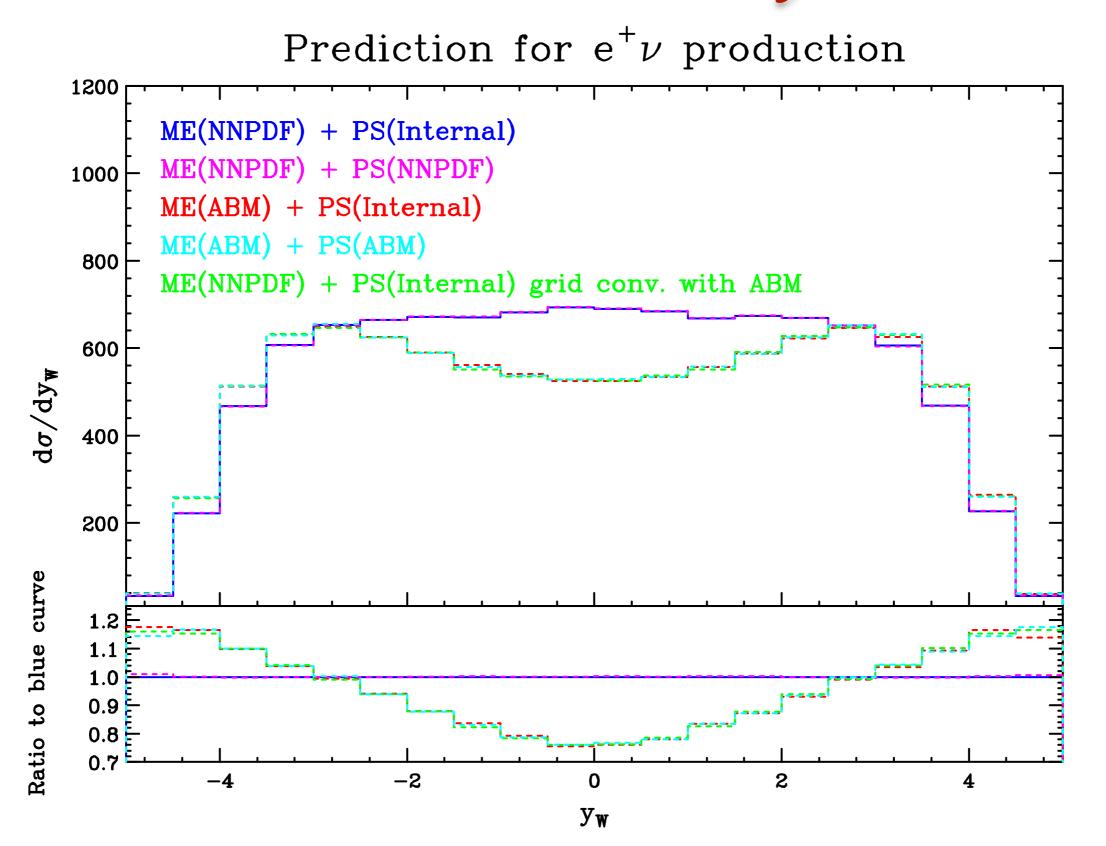


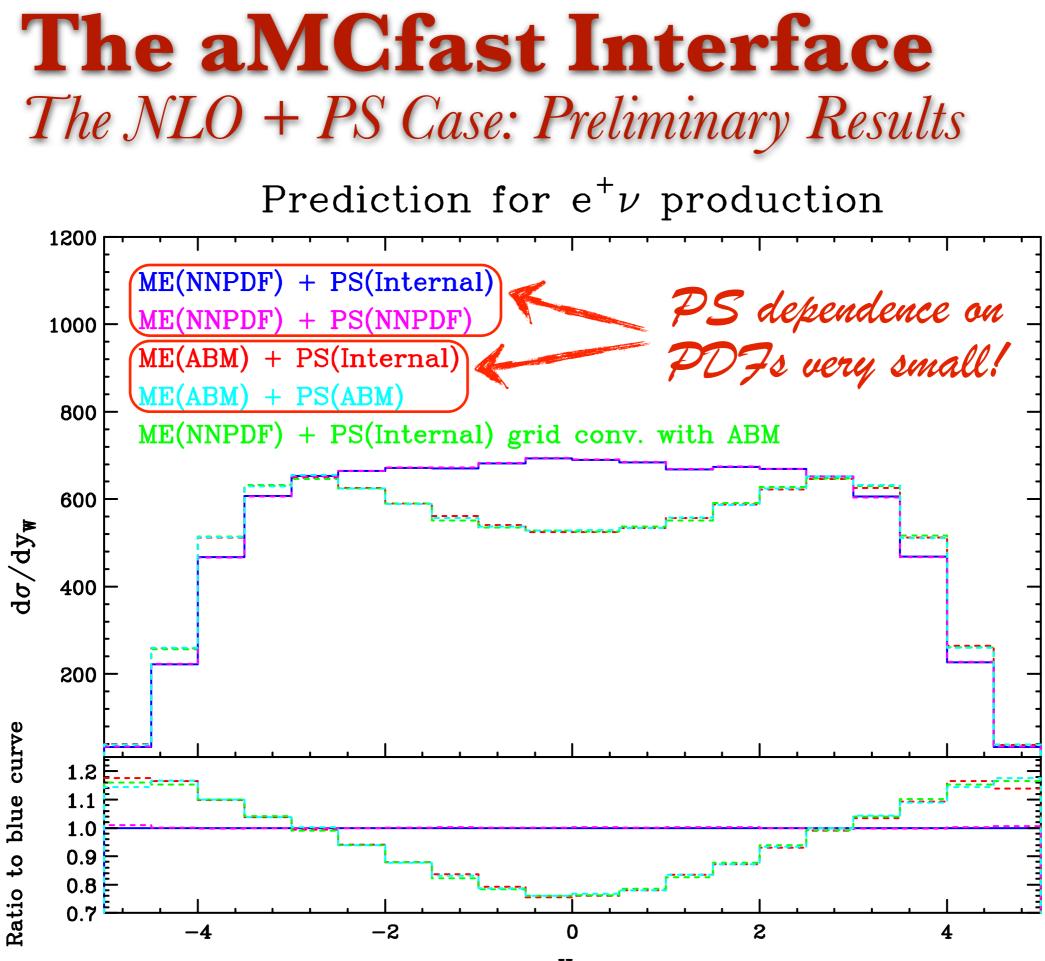
 Perfect agreement between reference and reconstructed histograms also in the low statistics regime, as in the fixed-order case.

## **The aMCfast Interface** *The NLO* + *PS Case: Preliminary Results*

- The production of interpolation grids in the presence of PS poses more conceptual questions as compared to the fixed-order case.
- **•** There are **two main issues**:
  - 1) Dependence on PDFs of the **backward PS evolution** cannot be disentangled:
    - expeted to be small as it appears as a ratio of PDFs at the same x but different  $Q^2$ .
  - 2) Dependence on PDFs of the **PS evolution** as a results of different kinematic configurations at the **matrix element** (ME) level when the latter is computed with different PDF sets cannot be removed.
- Need to explicitly check that interpolation grids including PS do not have a (strong) dependence on the PDFs used for the production.

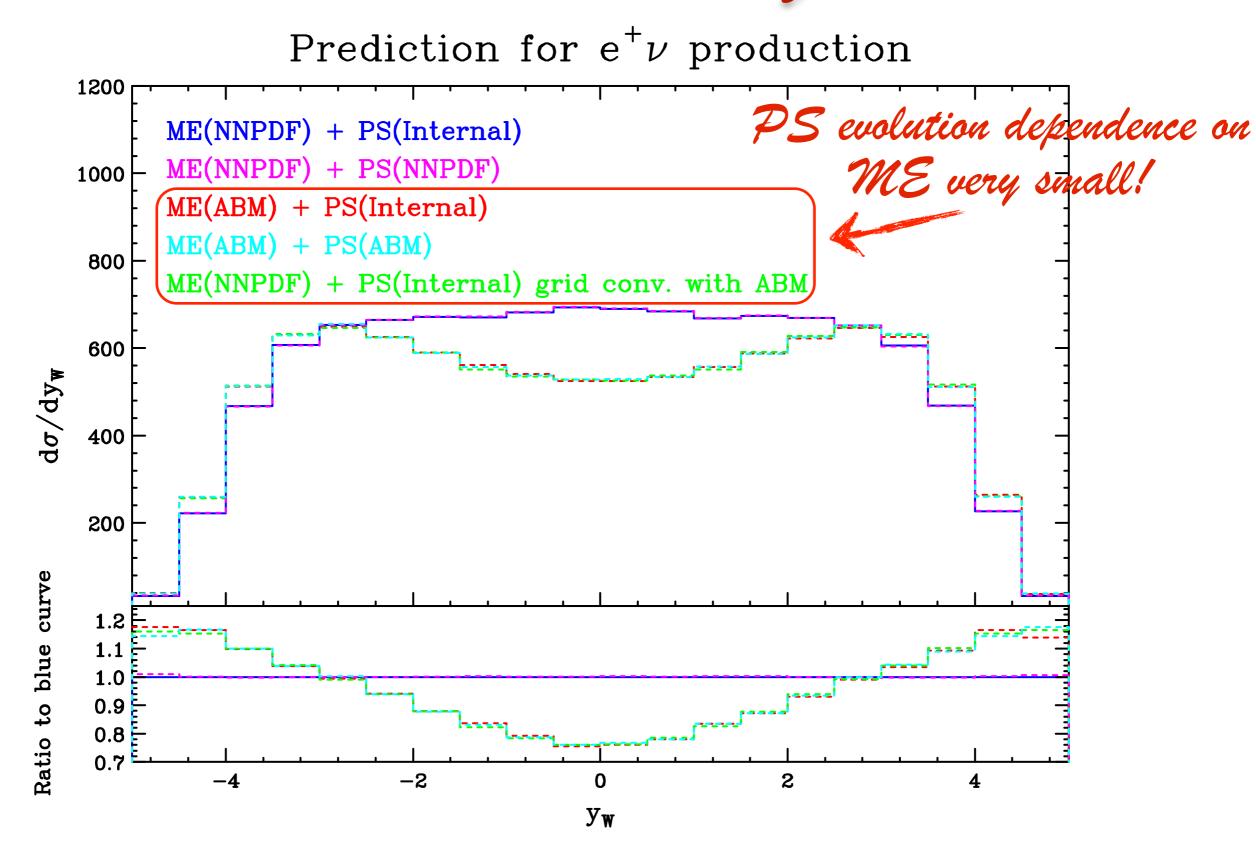
### **The aMCfast Interface** *The NLO* + *PS Case: Preliminary Results*





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### **The aMC fast Interface** *The NLO* + *PS Case: Preliminary Results*



# **Summary and Outlook**

- Summary:
  - **aMCfast** is an automated interface which bridges **APPLgrid** and **MadGraph5\_aMC@NLO**.
  - It allows the user to produce fast interpolation grids for any possible hadronic process up to NLO (in the SM for the time being).
  - It ensures a very high accuracy for any statistics and **any scale** choice.
  - **aMCfast** will make extremly simple the inclusion of new data coming from the LHC in any future PDF fit.
- Outlook:
  - We are presently working on aMCfast in order to interface APPLgrid with Madgraph5\_aMC@NLO when running in the (N)LO+PS mode.
  - Encouraging **preliminary** results.
- For more details on how to install and use aMCfast, you can visit our web page:

### http://amcfast.hepforge.org/