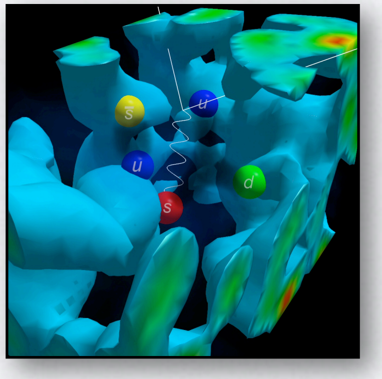
HARD SCATTERINGS AND SOFT GLUONS

Lorenzo Magnea

University of Torino - INFN Torino

QCD@LHC14 - Suzdal - 25/08/14



Outline

- Motivation
- One word about one loop
- A bird's eye over two loops
- Breaking ground at three loops
- Soft gluons to all loops: from theory ...
- ... to phenomenology
- Outlook

MOTIVATION

Where we stand

- The LHC had a wonderful first run, culminating in the Nobel-Prize-winning discovery of the Higgs boson.
  - *We expected it, but it was a great achievement nonetheless!*

Where we stand

- The LHC had a wonderful first run, culminating in the Nobel-Prize-winning discovery of the Higgs boson.
  - *We expected it, but it was a great achievement nonetheless!*
- The Standard Model of particle physics has been verified to an unexpected, even amazing degree of accuracy.
  - *But the Holy Grail - new physics - is not in sight.*

Where we stand

- The LHC had a wonderful first run, culminating in the Nobel-Prize-winning discovery of the Higgs boson.
  - *We expected it, but it was a great achievement nonetheless!*
- The Standard Model of particle physics has been verified to an unexpected, even amazing degree of accuracy.
  - *But the Holy Grail - new physics - is not in sight.*
- The absence of new physics signals is quietly shifting the focus of the theory community away from the `Standard BSM' scenario of `Naturalness + Supersymmetry'.
  - *LHC Run Two may effectively kill many cherished models.*

Where we stand

- The LHC had a wonderful first run, culminating in the Nobel-Prize-winning discovery of the Higgs boson.
  - *We expected it, but it was a great achievement nonetheless!*
- The Standard Model of particle physics has been verified to an unexpected, even amazing degree of accuracy.
  - *But the Holy Grail - new physics - is not in sight.*
- The absence of new physics signals is quietly shifting the focus of the theory community away from the `Standard BSM' scenario of `Naturalness + Supersymmetry'.
  - *LHC Run Two may effectively kill many cherished models.*
- The probability of striking, macroscopic new physics signatures with a moderate increase in energy appears low.
  - *We may have to disentangle small distortions from large SM backgrounds.*

Where we stand

- The LHC had a wonderful first run, culminating in the Nobel-Prize-winning discovery of the Higgs boson.
  - *We expected it, but it was a great achievement nonetheless!*
- The Standard Model of particle physics has been verified to an unexpected, even amazing degree of accuracy.
  - *But the Holy Grail - new physics - is not in sight.*
- The absence of new physics signals is quietly shifting the focus of the theory community away from the `Standard BSM' scenario of `Naturalness + Supersymmetry'.
  - *LHC Run Two may effectively kill many cherished models.*
- The probability of striking, macroscopic new physics signatures with a moderate increase in energy appears low.
  - *We may have to disentangle small distortions from large SM backgrounds.*
- In the coming years the need will be for precise predictions of increasingly complex SM processes.
  - A great time for the QCD community!

Where we stand

- The LHC had a wonderful first run, culminating in the Nobel-Prize-winning discovery of the Higgs boson.
  - *We expected it, but it was a great achievement nonetheless!*
- The Standard Model of particle physics has been verified to an unexpected, even amazing degree of accuracy.
  - *But the Holy Grail - new physics - is not in sight.*
- The absence of new physics signals is quietly shifting the focus of the theory community away from the \`Standard BSM' scenario of \`Naturalness + Supersymmetry'.
  - *LHC Run Two may effectively kill many cherished models.*
- The probability of striking, macroscopic new physics signatures with a moderate increase in energy appears low.
  - *We may have to disentangle small distortions from large SM backgrounds.*
- In the coming years the need will be for precise predictions of increasingly complex SM processes.
  - A great time for the QCD community!
- The run-up to the LHC has seen a vast effort and great progress in precision phenomenology: PDF's, jets, hard cross sections, resummations ...
  - *A continuing effort that will hopeful pay off during Run Two!*

ONE WORD ABOUT ONE LOOP

Done.

Done.

- Multi-leg NLO calculations matched with parton showers are now a **commodity**.


BB R 8051


HUMMER
HUMMER
LIKE NOTHING ELSE.

Examples of processes calculated with GoSam

- GoSam + MadDipole/MadGraph/MadEvent
  - $pp \to W^+W^- + 2\,jets$ [Greiner, GH, Mastrolia, Ossola, Reiter, Tramontano '12]
  - $pp \to \tilde{\chi}^0_1\tilde{\chi}^0_1 + jet$ [Cullen, Greiner, GH '12]
  - $pp \to (G \to \gamma\gamma) + 1\,jet$ [Greiner, GH, Reichel, von Soden-Fraunhofen '13]
  - $pp \to \gamma\gamma + 1,2\,jets$ [Gehrmann, Greiner, GH '13]
  - $pp \to HH + 2\,jets$ [Dolan, Englert, Greiner, Spannowsky '13]
- GoSam + Sherpa
  - $pp \to W^+W^+ + 2\,jets$ [Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano '12]
  - $pp \to H + 2\,jets$ [van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano '13]
  - $pp \to W^+W^- b\bar{b}$ [GH, Maier, Nisius, Schlenk, Winter '13]
  - $pp \to t\bar{t} + 0,1\,jet$ (includes shower) [Höche, Huang, Luisoni, Schönherr, Winter '13]
  - $pp \to H\,t\bar{t} + 0,1\,jet$ [van Deurzen, Luisoni, Mastrolia, Mirabella, Ossola, Peraro '13]
- GoSam + Powheg (includes shower)
  - $pp \to HW/HZ + 0,1\,jet$ [Luisoni, Nason, Oleari, Tramontano '13]
- GoSam + Herwig++/Matchbox (includes shower)
  - $pp \to Z + jet$ [Bellm, Gieseke, Greiner, GH, Plätzer, Reuschle, von Soden-Fraunhofen '13]
- GoSam + MadDipole/MadGraph/MadEvent + Sherpa
  - $pp \to H + 3\,jets$ [Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabel

Heinrich

n-Tuple availability

- The n-Tuple files are available
  - On the grid
  - On castor at CERN
- For a range of processes

Process	Pathname	Energy	Jet cut
W+ + 1,2,3,4 jets	Wp<n>j	7TeV	25GeV
W+ + 1,2,3 jets	Wp<n>j	8TeV	20GeV
W- + 1,2,3,4 jets	Wm<n>j	7TeV	25GeV
W- + 1,2,3 jets	Wm<n>j	8TeV	20GeV
Z/gamma* + 1,2 jets	Zee<n>j	7TeV	25GeV
Z/gamma* + 3,4 jets	Zee<n>j	7TeV	20GeV
Z/gamma* + 1,2,3 jets	Zee<n>j	8TeV	20GeV
2,3,4 jets	PureQCD<n>j	7TeV,8TeV	40GeV

From http://blackhat.hepforge.org/trac/wiki/Availability

Loopfest 2014, Brooklyn, 18th June

Maître

[J. Alwall et al. 1405.0301]

Automatic NLO in the SM (2014)

	Process	Syntax	Cross section (pb)			
Vector-boson pair +jets			LO 13 TeV		NLO 13 TeV	
h.1	$pp \to W^+W^-$ (4f)	p p > w+ w-	$7.355 \pm 0.005 \cdot 10^1$	[illegible]	$1.028 \pm 0.003 \cdot 10^2$	[illegible]
h.2	$pp \to ZZ$	p p > z z	$1.097 \pm 0.002 \cdot 10^1$	[illegible]	$1.415 \pm 0.005 \cdot 10^1$	[illegible]
h.3	$pp \to ZW^\pm$	p p > z wpm	$2.777 \pm 0.003 \cdot 10^1$	[illegible]	$4.487 \pm 0.013 \cdot 10^1$	[illegible]
h.4	$pp \to \gamma\gamma$	p p > a a	$2.510 \pm 0.002 \cdot 10^1$	[illegible]	$6.593 \pm 0.021 \cdot 10^1$	[illegible]
h.5	$pp \to \gamma Z$	p p > a z	$2.523 \pm 0.004 \cdot 10^1$	[illegible]	$3.695 \pm 0.013 \cdot 10^1$	[illegible]
h.6	$pp \to \gamma W^\pm$	p p > a wpm	$2.954 \pm 0.005 \cdot 10^1$	[illegible]	$7.124 \pm 0.026 \cdot 10^1$	[illegible]
h.7	$pp \to W^+W^-j$ (4f)	p p > w+ w- j	$2.865 \pm 0.003 \cdot 10^1$	[illegible]	$3.730 \pm 0.013 \cdot 10^1$	[illegible]
h.8	$pp \to ZZj$	p p > z z j	$3.662 \pm 0.003 \cdot 10^0$	[illegible]	$4.830 \pm 0.016 \cdot 10^0$	[illegible]
h.9	$pp \to ZW^\pm j$	p p > z wpm j	$1.605 \pm 0.005 \cdot 10^1$	[illegible]	$2.086 \pm 0.007 \cdot 10^1$	[illegible]
h.10	$pp \to \gamma\gamma j$	p p > a a j	$1.022 \pm 0.001 \cdot 10^1$	[illegible]	$2.292 \pm 0.010 \cdot 10^1$	[illegible]
h.11*	$pp \to \gamma Zj$	p p > a z j	$8.310 \pm 0.017 \cdot 10^0$	[illegible]	$1.220 \pm 0.005 \cdot 10^1$	[illegible]
h.12*	$pp \to \gamma W^\pm j$	p p > a wpm j	$2.546 \pm 0.010 \cdot 10^1$	[illegible]	$3.713 \pm 0.015 \cdot 10^1$	[illegible]
h.13	$pp \to W^+W^+jj$	p p > w+ w+ j j	$1.484 \pm 0.006 \cdot 10^{-1}$	[illegible]	$2.251 \pm 0.011 \cdot 10^{-1}$	[illegible]
h.14	$pp \to W^-W^-jj$	p p > w- w- j j	$6.752 \pm 0.007 \cdot 10^{-2}$	[illegible]	$1.003 \pm 0.003 \cdot 10^{-1}$	[illegible]
h.15	$pp \to W^+W^-jj$ (4f)	p p > w+ w- j j	$1.144 \pm 0.002 \cdot 10^1$	[illegible]	$1.396 \pm 0.005 \cdot 10^1$	[illegible]
h.16	$pp \to ZZjj$	p p > z z j j	$1.344 \pm 0.002 \cdot 10^0$	[illegible]	$1.706 \pm 0.011 \cdot 10^0$	[illegible]
h.17	$pp \to ZW^\pm jj$	p p > z wpm j j	$8.038 \pm 0.009 \cdot 10^0$	[illegible]	$9.139 \pm 0.031 \cdot 10^0$	[illegible]
h.18	$pp \to \gamma\gamma jj$	p p > a a j j	$5.377 \pm 0.029 \cdot 10^0$	[illegible]	$7.501 \pm 0.032 \cdot 10^0$	[illegible]
h.19*	$pp \to \gamma Zjj$	p p > a z j j	$3.260 \pm 0.009 \cdot 10^0$	[illegible]	4.242 ± 0.016	
h.20*	$pp \to \gamma W^\pm jj$	p p > a wpm j j	$1.233 \pm 0.002 \cdot 10^1$	[illegible]	1.448 ± 0.005	

Hirschi

First OpenLoops Applications (Higgs and Top phenomenology)

- MEPS@NLO for $\ell\ell\nu\nu$+0,1 jets, Cascioli, Höche, Krauss, Maierhöfer, S. P. , Siegert, arXiv:1309.0500
- MC@NLO pp $\to t\bar{t}b\bar{b}$ with $m_b > 0$, Cascioli, Maierhöfer, Moretti, S. P. , Siegert, arXiv:1309.5912
- NLO for pp $\to W^+W^-b\bar{b}$ with $m_b > 0$, Cascioli, Kallweit, Maierhöfer, S. P., arXiv:1312.0546
- NNLO for pp $\to \gamma Z$ production, Grazzini, Kallweit, Rathlev, Torre, arXiv:1309.7000
- NLO merging for pp $\to$ HH+0,1 jets, Maierhöfer, Papaefstathiou, arXiv:1401.0007
- MEPS@NLO for $t\bar{t}$+0,1,2 jets, Höche, Krauss, Maierhöfer, S. P. , Schönherr, Siegert arXiv:1402.6293
- MEPS@NLO for WWW+0,1 jets, Höche, Krauss, S. P. , Schönherr, Thompson arXiv:1403.7516
- NNLO for $q\bar{q} \to t\bar{t}$ production, Abelof, Gehrmann-de Ridder, Maierhöfer, S.P. , arXiv:1404.6493
- NNLO for pp $\to ZZ$ production, Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, S.P. , Rathlev, Tancredi, Weihs, arXiv:1405.2219

Technical Motivation

- technical stress tests: multi-particle and multi-scale problems, loop-induced processes, multiple resonances, ...
- beyond parton-level NLO: MC@NLO, MEPS@NLO and NNL

Pozzorini

From David Kosower's summary talk at LoopFest 2014

Directory of NLO (and related) tools

one-loop providers

GoSam: http://gosam.hepforge.org

NJet: https://bitbucket.org/njet/njet

Helac-NLO and more: http://helac-phegas.web.cern.ch/helac-phegas/

BlackHat NTupleReader: https://blackhat.hepforge.org/trac/wiki/BlackHatSherpaNtuples

FeynCalc: http://feyncalc.org

scalar integrals

QCDLoop: http://qcdloop.fnal.gov

OneLoop: http://helac-phegas.web.cern.ch/helac-phegas/OneLOop.html

tensor integrals

PJFry: https://github.com/Vayu/PJFry/

Golem95C: https://golem.hepforge.org

LanHEP: http://theory.sinp.msu.ru/~semenov/lanhep.html

FeynRules: https://feynrules.irmp.ucl.ac.be

MC tools

ThePeg: https://thepeg.hepforge.org

FastJet: http://fastjet.fr

Rivet: http://rivet.hepforge.org

LHAPDF: https://lhapdf.hepforge.org

IR subtraction codes

MadDipole: https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/MadDipole

AutoDipole: http://www-zeuthen.desy.de/~moch/autodipole/

MC event generators

Sherpa: https://sherpa.hepforge.org/

MCFM: http://mcfm.fnal.gov

Herwig++/Matchbox: http://herwig.hepforge.org

MG5_aMC@NLO (incl. MadLoop+MadFKS): https://launchpad.net/mg5amcnlo

HEJ: http://hej.web.cern.ch/HEJ/

Powheg-Box (V2): http://powhegbox.mib.infn.it

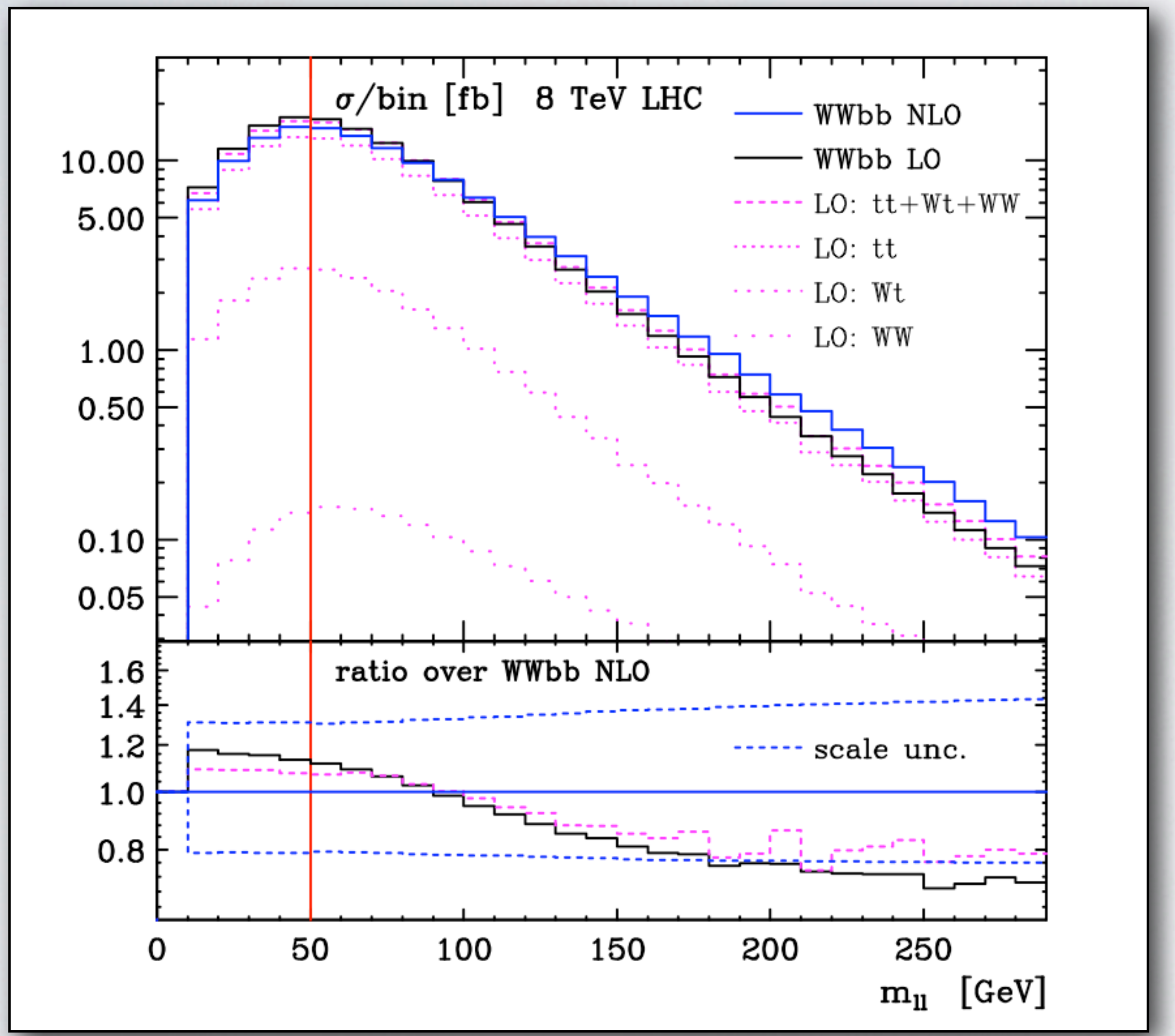
VBFNLO: https://www.itp.kit.edu/~vbfnloweb/

(PowHel: http://grid.kfki.hu/twiki/bin/view/DbTheory/PowHe)l

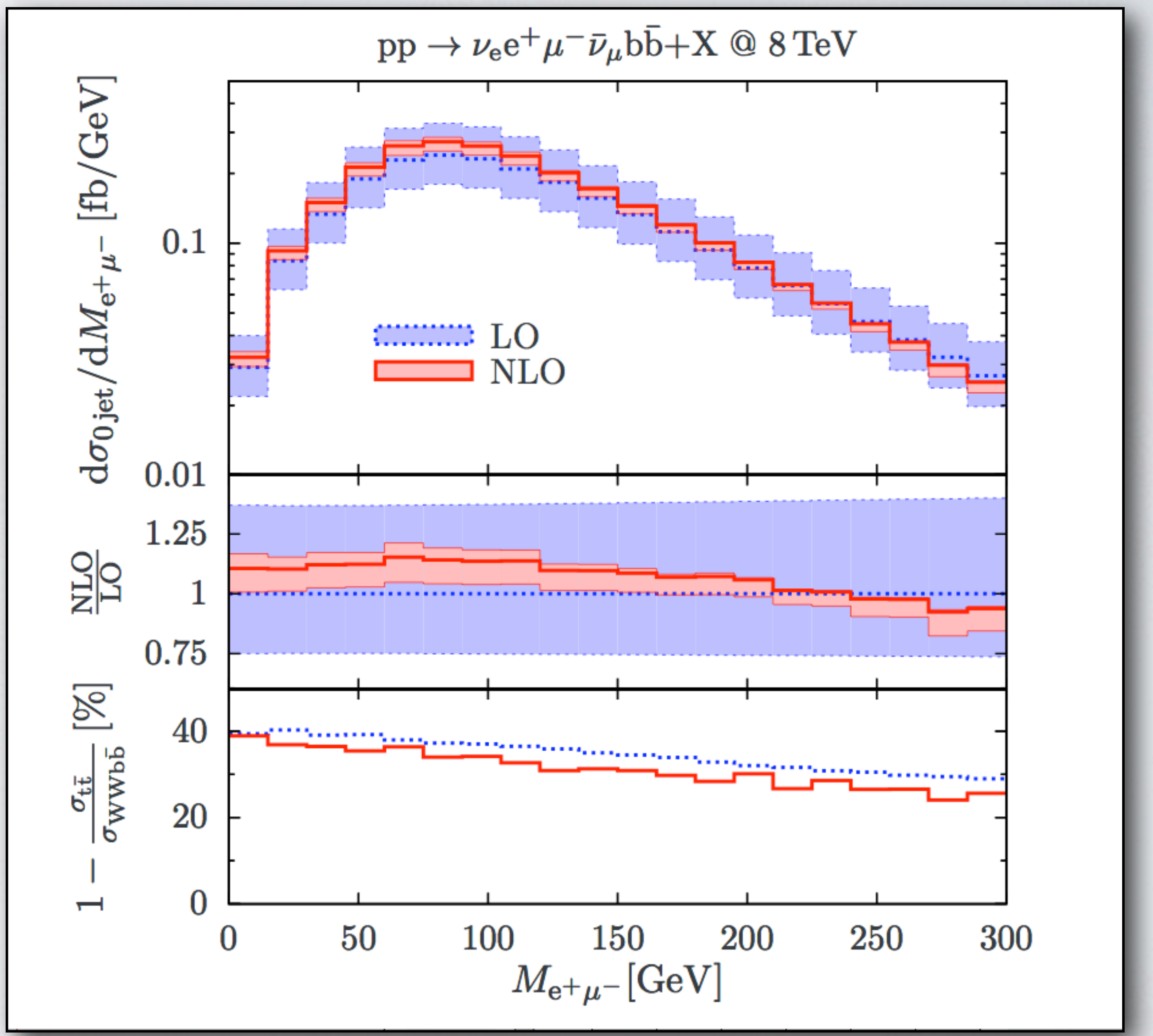
From Simon Badger's talk at ICHEP 2014

One NLO example

- Higgs decays to WW* have a large branching ratio but no mass peak and large backgrounds.
- A precise estimate requires computing p p → l ν l ν b b at NLO with massive b quarks.
- This is now done by two groups including off-shell effects and full interference.



Dilepton mass spectrum with MG5_aMC@NLO
from Rikkert Frederix 1311.4893.

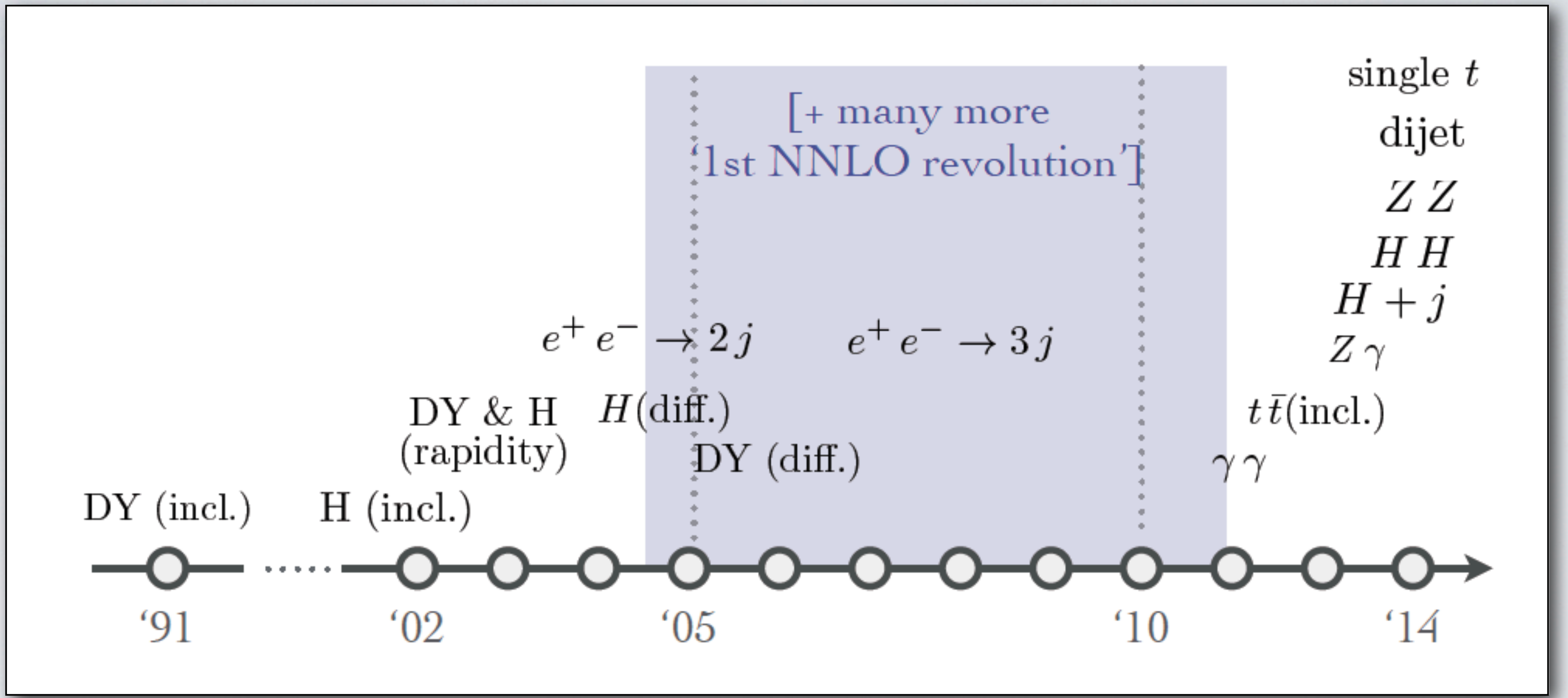


Dilepton mass spectrum with OpenLoops
from Cascioli et al. 1312.0546

A BIRD'S EYE VIEW OVER TWO LOOPS

NNLO revolutions

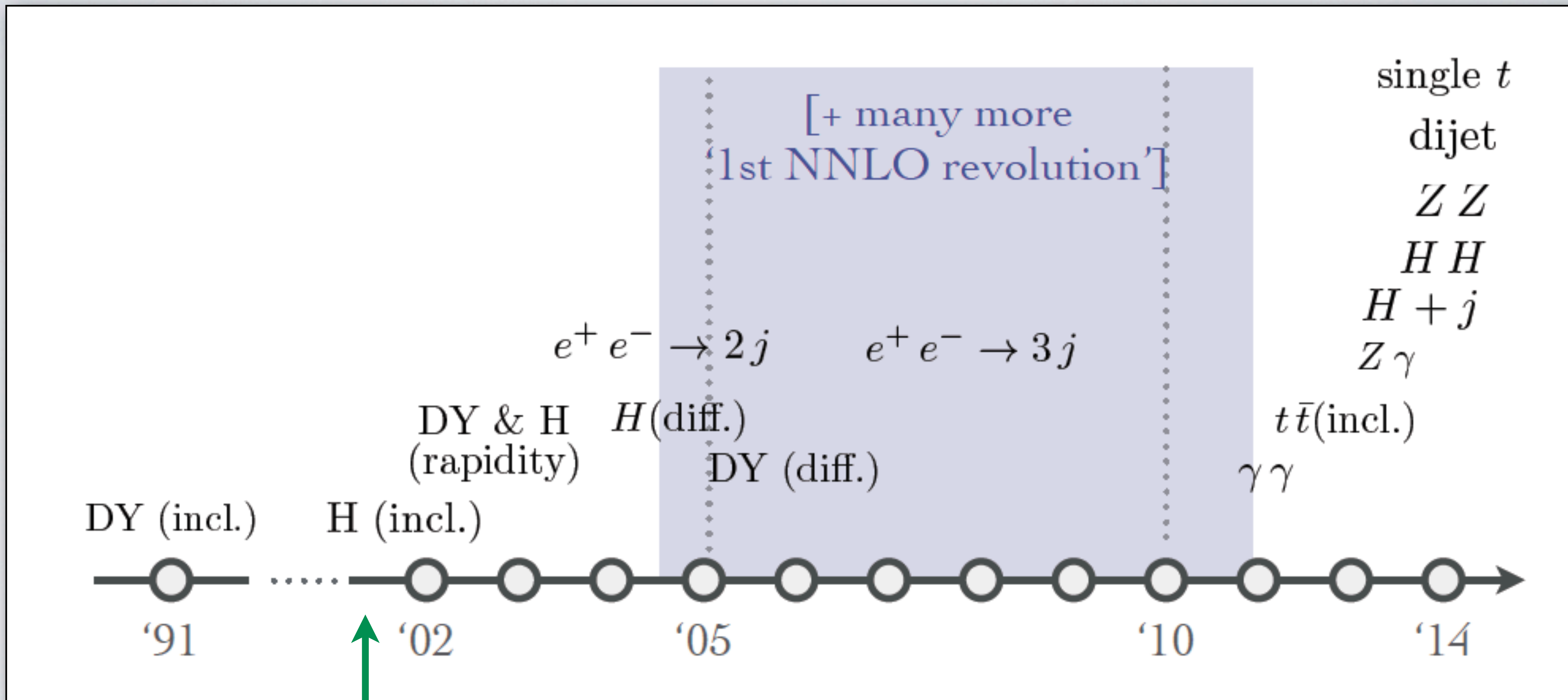
- Two-loop calculations are not yet a commodity: they are largely custom-made and expensive.
- A major stumbling block has been the subtraction of infrared and collinear singularities.
- Progress has been slow but is rapidly speeding up: automation is on the way.



From Claude Duhr's talk at ICHEP 2014

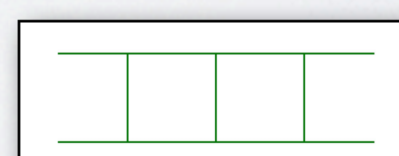
NNLO revolutions

- Two-loop calculations are not yet a commodity: they are largely custom-made and expensive.
- A major stumbling block has been the subtraction of infrared and collinear singularities.
- Progress has been slow but is rapidly speeding up: automation is on the way.



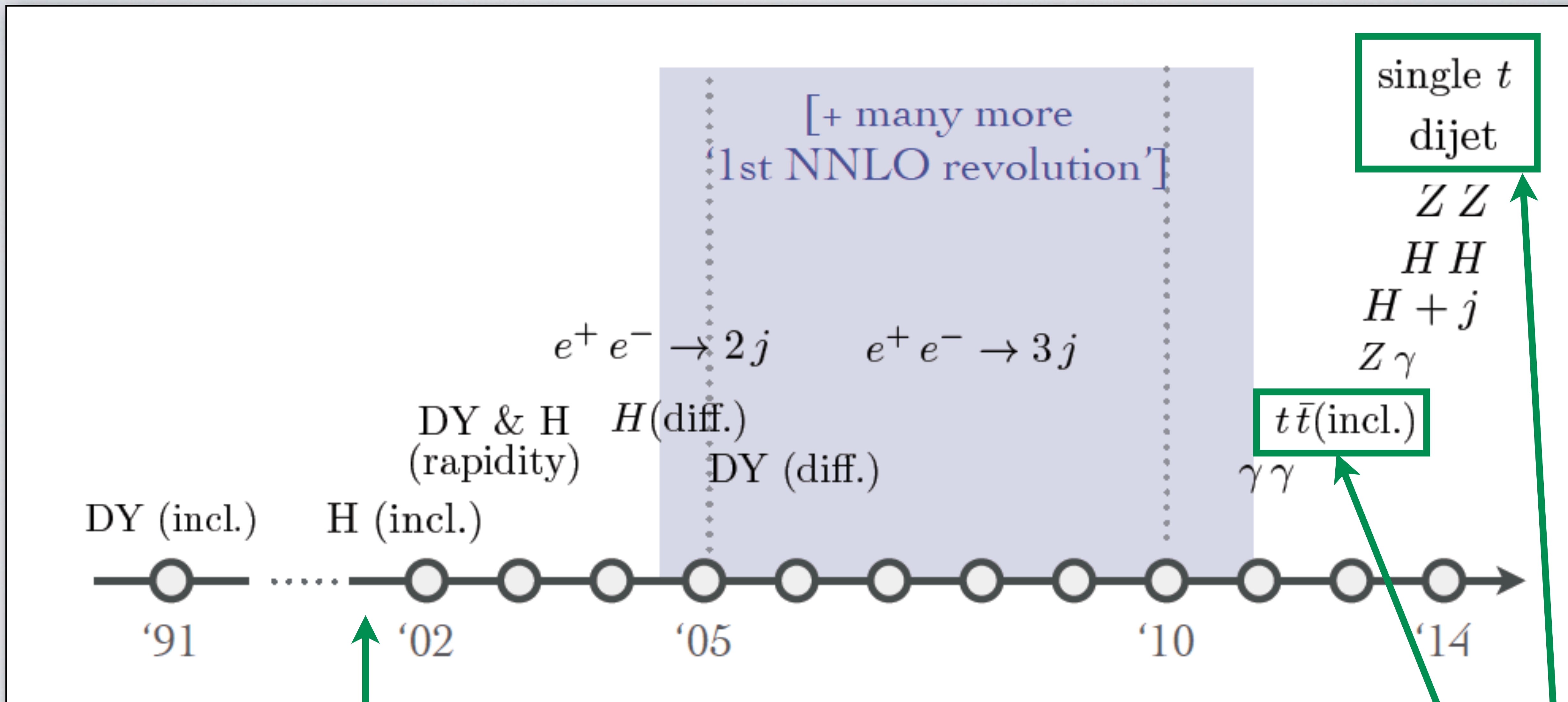
From Claude Duhr's talk at ICHEP 2014

V. Smirnov hep-ph/9905323



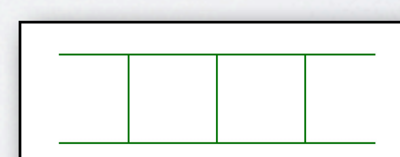
NNLO revolutions

- Two-loop calculations are not yet a commodity: they are largely custom-made and expensive.
- A major stumbling block has been the subtraction of infrared and collinear singularities.
- Progress has been slow but is rapidly speeding up: automation is on the way.



From Claude Duhr's talk at ICHEP 2014

V. Smirnov hep-ph/9905323



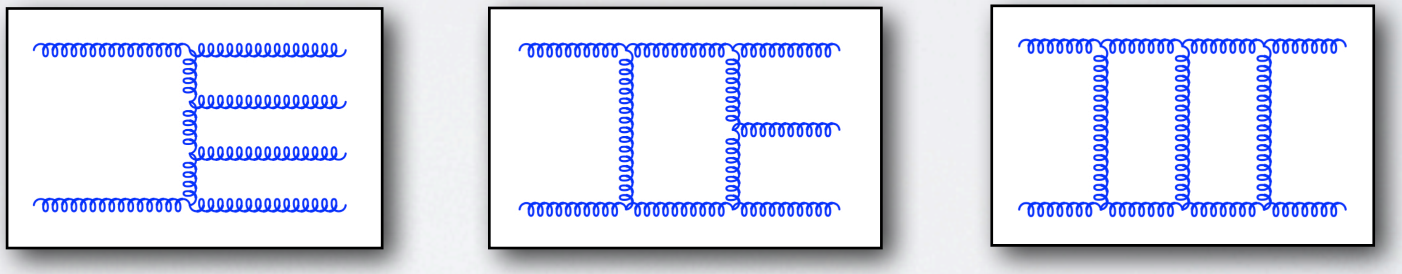
Full color
Masses

The NNLO subtraction problem

- A well-known problem: infrared and collinear divergences cancel between final states with different particle content and different phase spaces.
- The cancellation must be performed locally in phase space to allow for generic observables.
- "Simple" subtraction counterterms must be constructed in each phase space.
- A surprisingly hard problem, on the table for more than a decade.

$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

Different final-state multiplicities conspire to cancel infrared an collinear poles



	analytic	FS colour	IS colour	local
antenna subtraction	✓	✓	✓	✗
STRIPPER	✗	✓	✓	✓
q_T subtraction	✓	✗	✓	✓
reverse unitarity	✓	✗	✓	-
Trócsányi et al	✗	✓	✗	✓

Comparing subtraction algorithms, from James Currie's talk at LoopFest

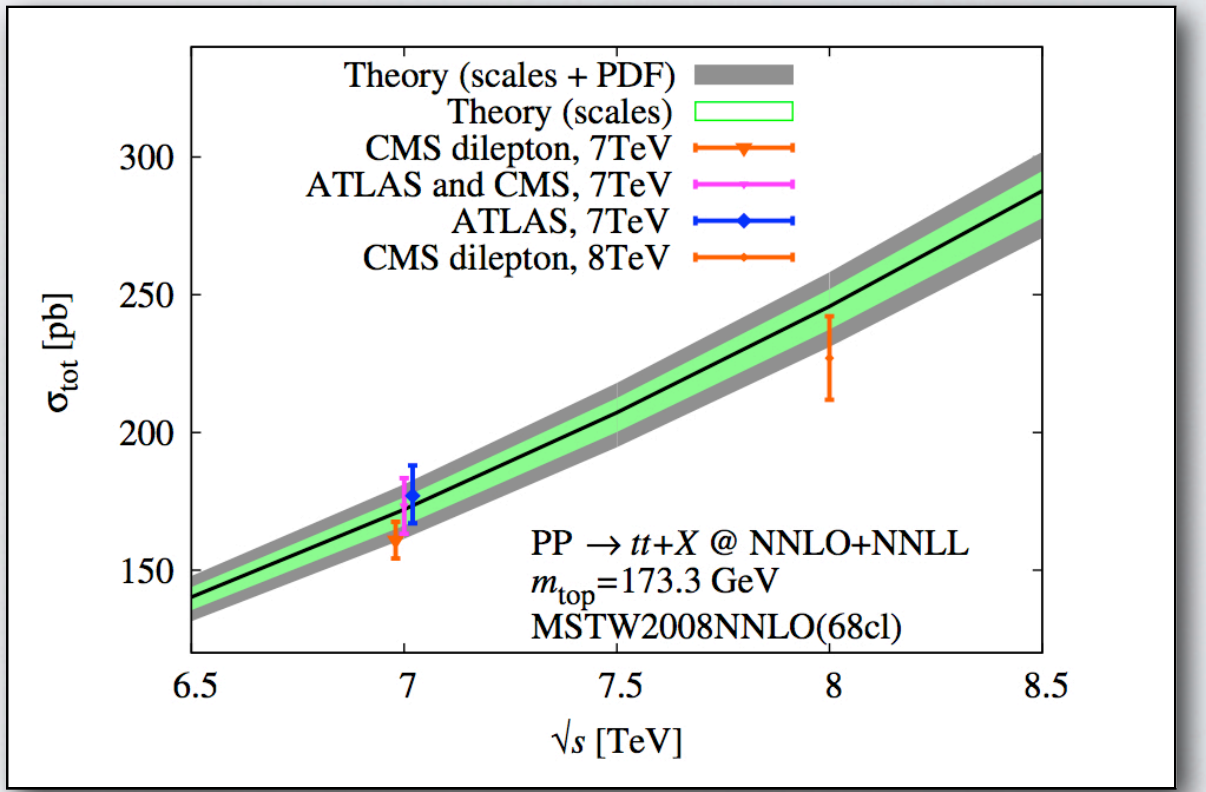
- Several solutions are now available.
- Analytical vs. numerical approaches.
- Dedicated vs. general algorithms.
- Several groups at work.
- No silver bullet yet.

Top pair production

- The first complete QCD calculation of a NNLO cross section involving four colored partons. Full complexity of color exchange comes into play. A major achievement.
- Highly relevant for phenomenology: the heaviest particle lies closest to the new physics.
- The subtraction of IR and collinear poles is performed purely numerically (STRIPPER).
- The structure of singularities is slightly simplified with respect to the massless case.
- Partial analytic results are available (R. Bonciani et al. 1309.4450): a challenging calculation with interesting analytical features.

TABLE II. Pure NNLO theoretical predictions for various colliders and c.m. energies.

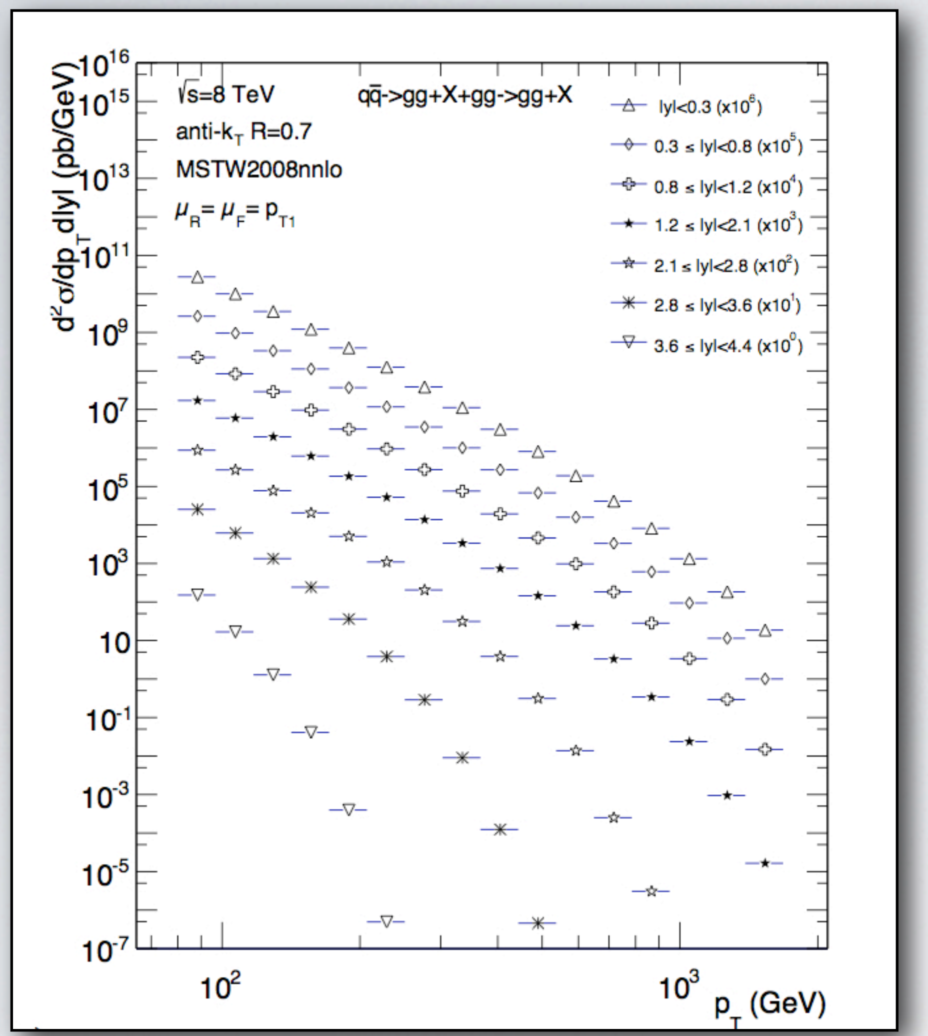
Collider	σ_{tot} [pb]	scales [pb]	PDF [pb]
Tevatron	7.009	$^{+0.259(3.7\%)}_{-0.374(5.3\%)}$	$^{+0.169(2.4\%)}_{-0.121(1.7\%)}$
LHC 7 TeV	167.0	$^{+6.7(4.0\%)}_{-10.7(6.4\%)}$	$^{+4.6(2.8\%)}_{-4.7(2.8\%)}$
LHC 8 TeV	239.1	$^{+9.2(3.9\%)}_{-14.8(6.2\%)}$	$^{+6.1(2.5\%)}_{-6.2(2.6\%)}$
LHC 14 TeV	933.0	$^{+31.8(3.4\%)}_{-51.0(5.5\%)}$	$^{+16.1(1.7\%)}_{-17.6(1.9\%)}$



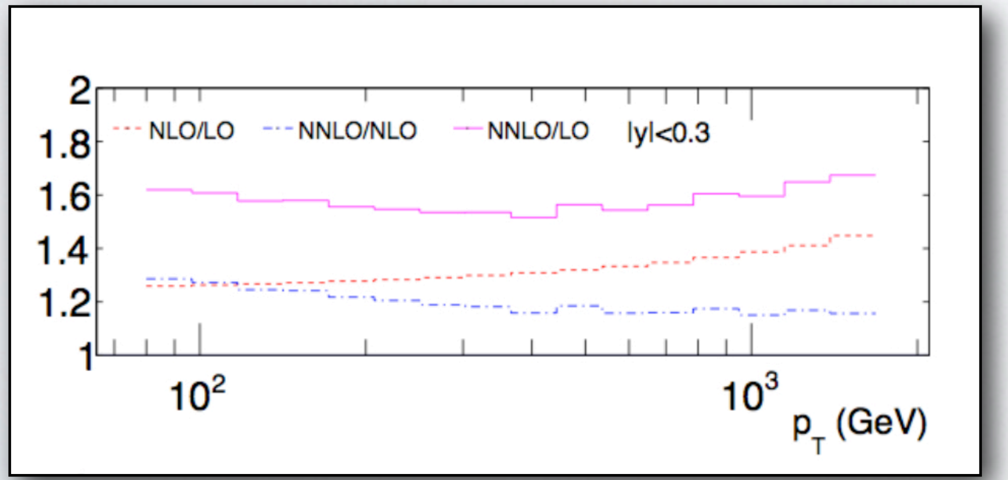
Numerical results and comparison with LHC data for the NNLO top pair production cross section, from M. Czakon, P. Fiendler and A. Mitov, 1303.6254

Dijet cross sections

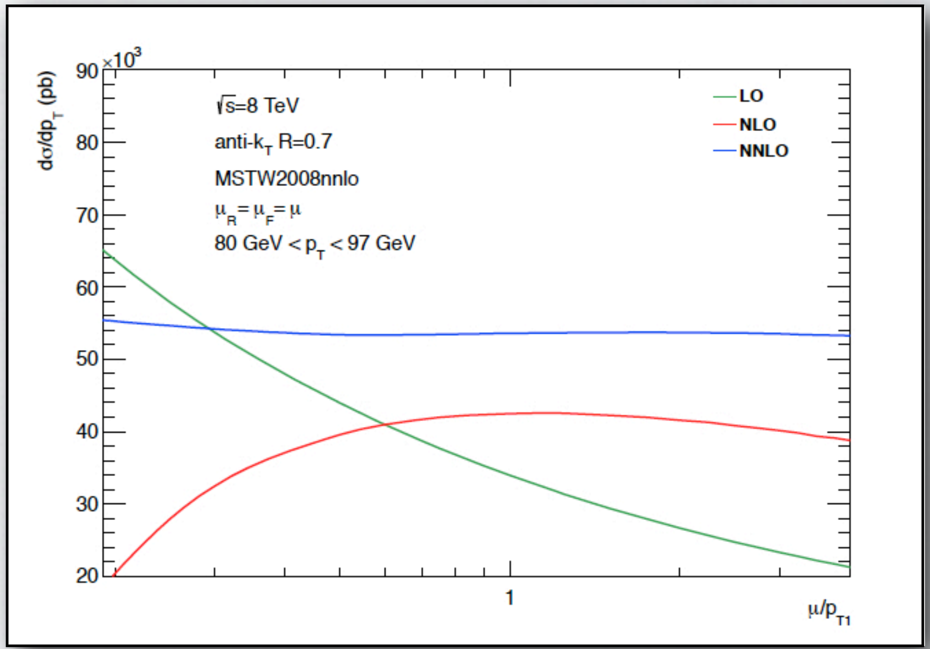
- A multi-million dollar calculation, spanning a decade.
- Some partonic channel completed: gluon-gluon full color, q-qbar leading color.
- Some partonic channels on the way: quark-gluon is phenomenologically important.
- Analytic subtraction of singularities using `antennas' lead to highly complex calculations.
- Important for a range of phenomenological issues: parton distributions, α_s, high-energy probes. Jets enter in essentially all LHC cross sections.



Single inclusive NNLO jet cross section at LHC 8 for fixed p_T and y, anti-k_T algorithm with R = 0.7. Gluon jets only. From J. Currie et al. 1407.5558.



Variations between LO, NLO and NNLO at central rapidities



Scale variations of the LO, NLO and NNLO jet cross sections, gluons only. From J. Currie's talk at LoopFest 2014.

A good NNLO harvest

- Preliminary results for differential distributions in Higgs + one jet production (Boughezal, Caola, Melnikov, Petriello, Schultze; Chen, Gehrmann, Glover, Jaquier).
  - *Note: the two groups use different subtraction techniques.*
- Differential distributions for t-channel single top production in the `structure function' approximation (Brucherseifer, Caola, Melnikov).
  - *Non-factorizable contribution is color-subleading.*
- Differential distributions for ZZ production (using q_T subtraction) (Cascioli et nine al.).
  - *The method is generalizable to all EW di-boson final states.*
- Preliminary results for $\gamma^* \gamma^*$ production presented by Lazopoulos at LoopFest
  - *A stepping stone to a general code for electroweak final states.*
- Very recent! Differential distributions for associated ZH production (Ferrera, Grazzini, Tramontano, 1407.4747).
- Even more recent! Virtual corrections to NNLO HH production (effectively a 2.5 loop calculation) (Grigo, Melnikov, Steinhauser, 1408.2422).
- All master integrals required for pp->VV' two-loop amplitudes now known (Caola, Henn, Melnikov, Smirnov, 1404.5590).
- Progress towards the construction of a general basis for two-loop master integrals (Mastrolia et al.; Badger, Frellesvig, Zhang, 1407.3133)

BREAKING GROUND AT THREE LOOPS

"Drell-Yan" at N^{3}LO

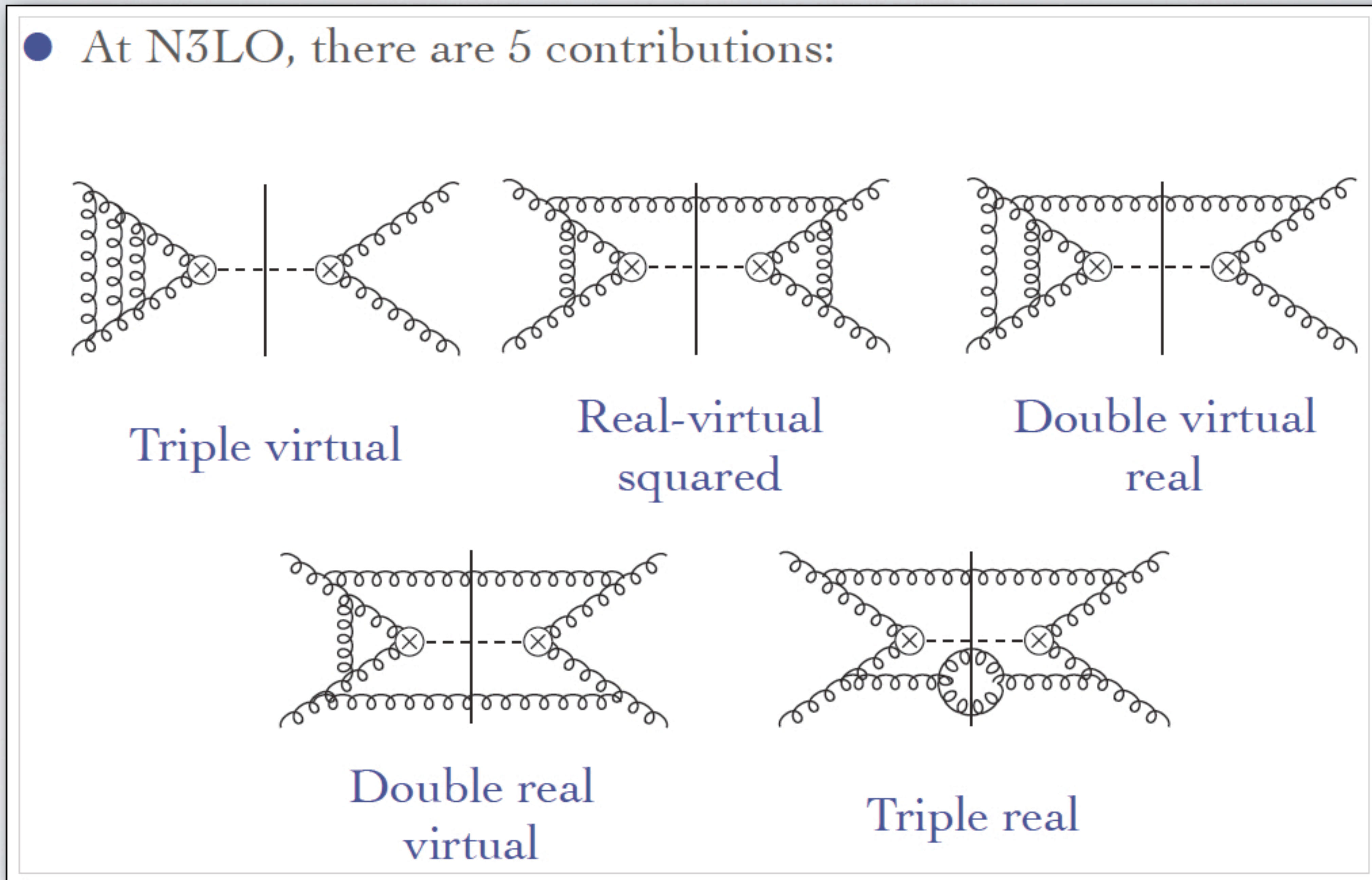
- After the landmark calculation of three-loop DIS structure functions by Moch, Vermaseren and Vogt a decade ago, the next great PQCD challenge is the computation of a cross section without an OPE at three loops. The "Drell-Yan" process is the best candidate.
- At LHC, "Drell-Yan" means vector boson production and Higgs production via gluon fusion. The phenomenological impact is evident, especially given the large corrections to Higgs production at one and two loops.
- Approximate three-loop results using threshold and Regge limits exist (Moch, Vogt, 2005; LM, Laenen, 2005; Ball, Bonvini, Forte, Marzani, Ridolfi, 2013).
- The full calculation is now being tackled step by step (Anastasiou, Duhr, Dulat, Herzog, Mistiberger 1311.1425; Kilgore 1312.1296; Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistiberger 1403.4616).
- The leading term in the threshold expansion is now known

$$\widehat{\sigma} = \widehat{\sigma}(z) \, , \qquad z = \frac{Q^2}{\hat{s}} \, , \qquad \widehat{\sigma}(z) = \widehat{\sigma}_{SV} + \widehat{\sigma}_0 + (1 - z) \, \widehat{\sigma}_1 + \mathcal{O}\big[(1 - z)^2\big]$$

- The three-loop soft-virtual contribution is fully predicted by threshold resummation except for $\delta(1 - z)$ contributions which are the new result.
- The "Drell-Yan" timeline: 1979 - 1991 - 2002 - 2015?

"Drell-Yan" at N^{3}LO

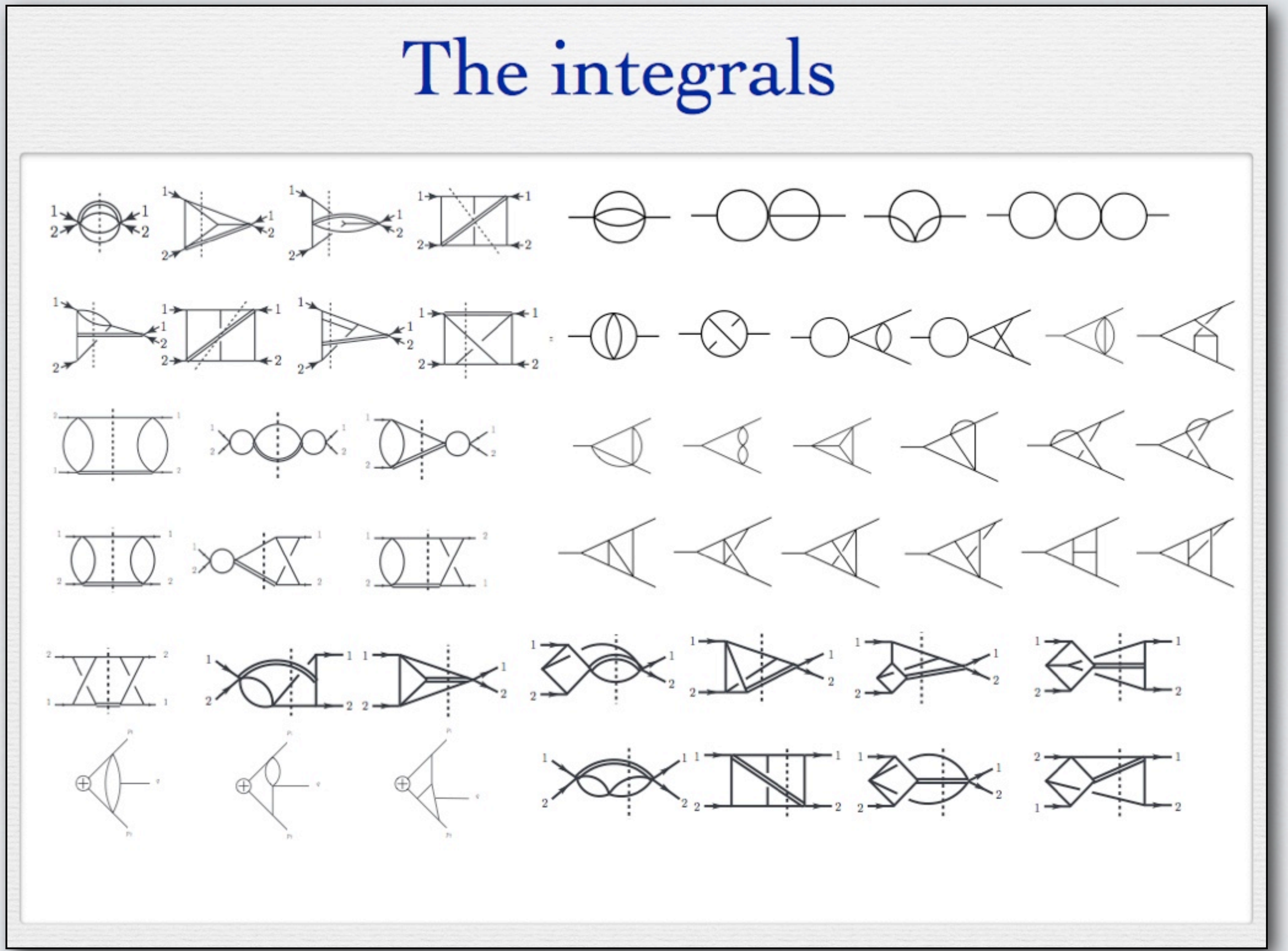
- A massive calculation with many ingredients, and O(10^3) master integrals to evaluate.



From Claude Duhr's talk at ICHEP 2014

"Drell-Yan" at N^3LO

- A massive calculation even in the soft-virtual approximation, with 50 master integrals.



From Claude Duhr's talk at Loops&Legs 2014

“Drell-Yan” at N^{3}LO

- The three-loop **soft-virtual** approximation to Higgs production in gluon fusion.

$$
\begin{aligned}
\hat{\eta}^{(3)}(z) = \delta(1-z)\Big\{ & C_A^3\Big(-\frac{2003}{48}\zeta_6 + \frac{413}{6}\zeta_3^2 - \frac{7579}{144}\zeta_5 + \frac{979}{24}\zeta_2\zeta_3 - \frac{15257}{864}\zeta_4 - \frac{819}{16}\zeta_3 + \frac{16151}{1296}\zeta_2 + \frac{215131}{5184}\Big) \\
& + N_F\Big[C_A^2\Big(\frac{869}{72}\zeta_5 - \frac{125}{12}\zeta_3\zeta_2 + \frac{2629}{432}\zeta_4 + \frac{1231}{216}\zeta_3 - \frac{70}{81}\zeta_2 - \frac{98059}{5184}\Big) \\
& \quad + C_A C_F\Big(\frac{5}{2}\zeta_5 + 3\zeta_3\zeta_2 + \frac{11}{72}\zeta_4 + \frac{13}{2}\zeta_3 - \frac{71}{36}\zeta_2 - \frac{63991}{5184}\Big) + C_F^2\Big(-5\zeta_5 + \frac{37}{12}\zeta_3 + \frac{19}{18}\Big)\Big] \\
& + N_F^2\Big[C_A\Big(-\frac{19}{36}\zeta_4 + \frac{43}{108}\zeta_3 - \frac{133}{324}\zeta_2 + \frac{2515}{1728}\Big) + C_F\Big(-\frac{1}{36}\zeta_4 - \frac{7}{6}\zeta_3 - \frac{23}{72}\zeta_2 + \frac{4481}{2592}\Big)\Big]\Big\} \\
+ \Big[\frac{1}{1-z}\Big]_+ \Big\{ & C_A^3\Big(186\,\zeta_5 - \frac{725}{6}\zeta_3\zeta_2 + \frac{253}{24}\zeta_4 + \frac{8941}{108}\zeta_3 + \frac{8563}{324}\zeta_2 - \frac{297029}{23328}\Big) + N_F^2 C_A\Big(\frac{5}{27}\zeta_3 + \frac{10}{27}\zeta_2 - \frac{58}{729}\Big) \\
& + N_F\Big[C_A^2\Big(-\frac{17}{12}\zeta_4 - \frac{475}{36}\zeta_3 - \frac{2173}{324}\zeta_2 + \frac{31313}{11664}\Big) + C_A C_F\Big(-\frac{1}{2}\zeta_4 - \frac{19}{18}\zeta_3 - \frac{1}{2}\zeta_2 + \frac{1711}{864}\Big)\Big]\Big\} \\
+ \Big[\frac{\log(1-z)}{1-z}\Big]_+ \Big\{ & C_A^3\Big(-77\zeta_4 - \frac{352}{3}\zeta_3 - \frac{152}{3}\zeta_2 + \frac{30569}{648}\Big) + N_F^2 C_A\Big(-\frac{4}{9}\zeta_2 + \frac{25}{81}\Big) \\
& + N_F\Big[C_A^2\Big(\frac{46}{3}\zeta_3 + \frac{94}{9}\zeta_2 - \frac{4211}{324}\Big) + C_A C_F\Big(6\,\zeta_3 - \frac{63}{8}\Big)\Big]\Big\} \\
+ \Big[\frac{\log^2(1-z)}{1-z}\Big]_+ \Big\{ & C_A^3\Big(181\,\zeta_3 + \frac{187}{3}\zeta_2 - \frac{1051}{27}\Big) + N_F\Big[C_A^2\Big(-\frac{34}{3}\zeta_2 + \frac{457}{54}\Big) + \frac{1}{2}C_A C_F\Big] - \frac{10}{27}N_F^2 C_A\Big\} \\
+ \Big[\frac{\log^3(1-z)}{1-z}\Big]_+ \Big\{ & C_A^3\Big(-56\,\zeta_2 + \frac{925}{27}\Big) - \frac{164}{27}N_F C_A^2 + \frac{4}{27}N_F^2 C_A\Big\} \\
+ \Big[\frac{\log^4(1-z)}{1-z}\Big]_+ & \Big(\frac{20}{9}N_F C_A^2 - \frac{110}{9}C_A^3\Big) + \Big[\frac{\log^5(1-z)}{1-z}\Big]_+ 8\,C_A^3 \,.
\end{aligned}
$$

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

From Claude Duhr’s talk at Loops&Legs 2014

"Drell-Yan" at N^3LO

- The three-loop soft-virtual approximation to Higgs production in gluon fusion.

$$
\begin{aligned}
\hat\eta^{(3)}(z) = \delta(1-z)\Bigg\{ & C_A^3\left(-\frac{2003}{48}\zeta_6+\frac{413}{6}\zeta_3^2-\frac{7579}{144}\zeta_5+\frac{979}{24}\zeta_2\zeta_3-\frac{15257}{864}\zeta_4-\frac{819}{16}\zeta_3+\frac{16151}{1296}\zeta_2+\frac{215131}{5184}\right)\\
&+N_F\Bigg[C_A^2\left(\frac{869}{72}\zeta_5-\frac{125}{12}\zeta_3\zeta_2+\frac{2629}{432}\zeta_4+\frac{1231}{216}\zeta_3-\frac{70}{81}\zeta_2-\frac{98059}{5184}\right)\\
&\quad+C_AC_F\left(\frac{5}{2}\zeta_5+3\zeta_3\zeta_2+\frac{11}{72}\zeta_4+\frac{13}{2}\zeta_3-\frac{71}{36}\zeta_2-\frac{63991}{5184}\right)+C_F^2\left(-5\zeta_5+\frac{37}{12}\zeta_3+\frac{19}{18}\right)\Bigg]\\
&+N_F^2\left[C_A\left(-\frac{19}{36}\zeta_4+\frac{43}{108}\zeta_3-\frac{133}{324}\zeta_2+\frac{2515}{1728}\right)+C_F\left(-\frac{1}{36}\zeta_4-\frac{7}{6}\zeta_3-\frac{23}{72}\zeta_2+\frac{4481}{2592}\right)\right]\Bigg]\Bigg\}\\
&+\left[\frac{1}{1-z}\right]_+\Bigg\{C_A^3\left(186\zeta_5-\frac{725}{6}\zeta_3\zeta_2+\frac{253}{24}\zeta_4+\frac{8941}{108}\zeta_3+\frac{8563}{324}\zeta_2-\frac{297029}{23328}\right)+N_F^2C_A\left(\frac{5}{27}\zeta_3+\frac{10}{27}\zeta_2-\frac{58}{729}\right)\\
&\quad+N_F\left[C_A^2\left(-\frac{17}{12}\zeta_4-\frac{475}{36}\zeta_3-\frac{2173}{324}\zeta_2+\frac{31313}{11664}\right)+C_AC_F\left(-\frac{1}{2}\zeta_4-\frac{19}{18}\zeta_3-\frac{1}{2}\zeta_2+\frac{1711}{864}\right)\right]\Bigg]\Bigg\}\\
&+\left[\frac{\log(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(-77\zeta_4-\frac{352}{3}\zeta_3-\frac{152}{3}\zeta_2+\frac{30569}{648}\right)+N_F^2C_A\left(-\frac{4}{9}\zeta_2+\frac{25}{81}\right)\\
&\quad+N_F\left[C_A^2\left(\frac{46}{3}\zeta_3+\frac{94}{9}\zeta_2-\frac{4211}{324}\right)+C_AC_F\left(6\zeta_3-\frac{63}{8}\right)\right]\Bigg]\Bigg\}\\
&+\left[\frac{\log^2(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(181\zeta_3+\frac{187}{3}\zeta_2-\frac{1051}{27}\right)+N_F\left[C_A^2\left(-\frac{34}{3}\zeta_2+\frac{457}{54}\right)+\frac{1}{2}C_AC_F\right]-\frac{10}{27}N_F^2C_A\Bigg\}\\
&+\left[\frac{\log^3(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(-56\zeta_2+\frac{925}{27}\right)-\frac{164}{27}N_FC_A^2+\frac{4}{27}N_F^2C_A\Bigg\}\\
&+\left[\frac{\log^4(1-z)}{1-z}\right]_+\left(\frac{20}{9}N_FC_A^2-\frac{110}{9}C_A^3\right)+\left[\frac{\log^5(1-z)}{1-z}\right]_+8C_A^3\,.
\end{aligned}
$$

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

Not predicted by resummation

From Claude Duhr's talk at Loops&Legs 2014

Iterated integrals

- A large class of integrals arising from Feynman diagrams (but not all!) can be expressed as "iterated integrals", yielding functions in the class of polylogarithms. At one loop

$$\log z = -\int_0^{1-z} \frac{dt}{1-t}\,, \qquad \mathrm{Li}_2(z) = \int_0^z \frac{dt}{t} \int_0^t \frac{du}{1-u}$$

- At higher orders one ancounters more general examples, such as Harmonic Polylogarithms or Goncharov Polylogarithms

$$G_{a_1,\ldots,a_n}(z) \equiv \int_0^z \frac{dt}{t-a_1}\, G_{a_2,\ldots,a_n}(t)\,,$$

- Notice that all these integrals are of a "d log" form: at each step one integrates over the logarithm of a simple (here linear) function of the integration variables.
- The parameters a_n are the locations of singular points and have physical meaning.
- Iterated integrals are organized by a powerful underlying algebraic structure, described by the "Symbol" map or by a Hopf algebra with a notion of "Co-product" (Duhr).
- In particular each such function can be assigned a "weight" w, equal to the number of iterations. For example $\mathrm{Li}_2(z)$ has weight $w = 2$, and $\zeta(n)$ has weight $w = n$.
- These structures were uncovered in the context of studies of N=4 Super Yang-Mills theory amplitudes, where they have played a pivotal role.
- We now see powerful new applications to ordinary QCD (Henn, Smirnov, Von Manteuffel)

A basis of pure functions

- Iterated integrals are the centerpiece of a recent breakthrough (J. Henn, 1304.1806) in the calculation of master integrals for general gauge theory amplitudes.
- Consider the standard method of evaluation for multi-loop scattering amplitudes.
  - Reduce the integrals arising from Feynman diagrams to a set of master integrals.
  - Use IBP and Lorentz invariance identities to derive a system of differential equations coupling all master integrals.
  - Solve for the master integrals using simple configurations for boundary conditions.
- For master integrals $f_i(x_n)$, and with $\varepsilon = 2 - d/2$, the system takes the form

$$\frac{\partial}{\partial x_m} f_i(\epsilon, x_n) = A_{ij}^{(m)}(\epsilon, x_n)\, f_j(\epsilon, x_n)$$

- There is a large, not previously exploited freedom to choose the basis of MI's at will.
- Using iterated integrals, J. Henn suggested that an appropriate choice of basis involving uniform weight functions, can lead to a striking double simplification.

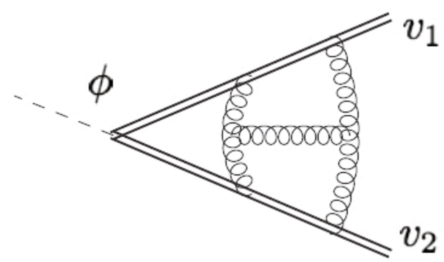
$$\frac{\partial}{\partial x_m} f_i(\epsilon, x_n) = \epsilon\, A_{ij}^{(m)}(x_n)\, f_j(\epsilon, x_n) = \epsilon \sum_k \left[A_k^{(m)}\right]_{ij} \frac{\partial}{\partial x_m}\left[\log \alpha_k(x_n)\right] f_j(\epsilon, x_n)$$

- The system is now easily solved order by order in ε in terms of iterated integrals.
- A key role is played by the alphabet of functions α_k, which encode the kinematic singularities of the amplitude.
- The existence of such a basis of master integrals is not proved in general.
- Whenever this basis of uniform weight functions exists the evaluation of MI's is remarkably simplified.

A wealth of applications

- 8 integral families, e.g.

$$\cos\phi = \frac{v_1 \cdot v_2}{\sqrt{v_1^2}\sqrt{v_2^2}}\,, \quad x = e^{i\phi}$$

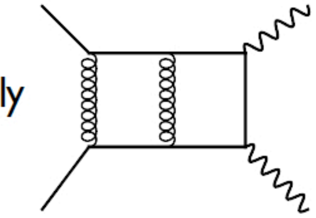


- alphabet $\alpha = \{x, 1+x, 1-x\}$
- 71 master integrals
- application: QCD cusp anomalous dimension

Towards the three-loop QCD cusp anomalous dimension:
Grozin, Henn, Korchemsky, Marquard, 1406.7828.

Vector boson production integrals $pp \to VV'$

- sample integral family



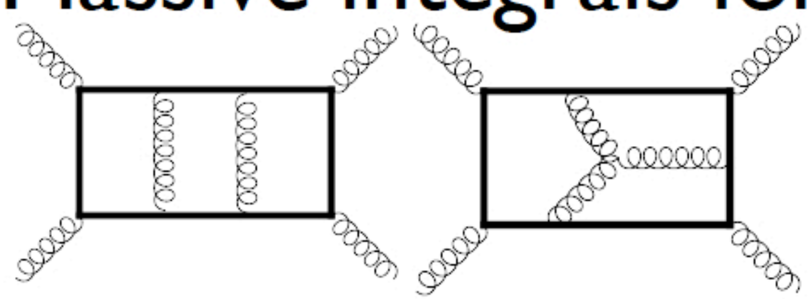
[JMH, Melnikov, V. Smirnov, JHEP 1430 (2014)]
[JMH, Caola, Melnikov, V. Smirnov, 1404.5590]

- variables S, T, M_3^2, M_4^2
- parametrization $\frac{S}{M_3^2} = (1+x)(1+xy)\,, \quad \frac{T}{M_3^2} = -xz\,, \quad \frac{M_4^2}{M_3^2} = x^2 y$
- physical region $0 < x\,, \quad 0 < y < z < 1$
- alphabet

$$\alpha = \{x, y, z, 1+x, 1-y, 1-z, 1+xy, z-y, 1+y(1+x)-z, xy+z, 1+x(1+y-z), 1+xz, 1+y-z, z+x(z-y)+xyz, z-y+yz+xyz\}.$$

All integrals for virtual two-loop double vector boson production:
Caola, Henn, Melnikov, Smirnov, 1404.5590.

Massive integrals for light-by-light scattering



[Caron-Huot, J.M.H., 2014]

- variables m^2, s, t

3 loops and 3 scales!

- full set of 2-loop master integrals (at 3 loops: all finite master integrals in D=4)

All planar two-loop and finite three-loop integrals for light-by-light scattering
Henn, Smirnov, Smirnov 1312.2588 ; Caron-Huot, Henn, 1404.2922.

SOFT GLUONS TO ALL LOOPS

The virtues of large logs

- Multi-scale problems in renormalizable quantum field theories have perturbative corrections of the form $\alpha_s^n \log^k \left(Q_i^2/Q_j^2\right)$, which may spoil the reliability of the perturbative expansion. However, they carry important physical information!
  - Renormalization and factorization logs: $\alpha_s^n \log^n \left(Q^2/\mu^2\right)$
  - High-energy logs: $\alpha_s^n \log^{n-1} \left(s/t\right)$
  - Sudakov logs: $\alpha_s^n \log^{2n-1} \left(1-z\right) , \quad 1 - z = W^2/Q^2, \, 1 - M^2/\hat{s} \, , \, Q_\perp^2/Q^2, \, \ldots$

- Sudakov logs are universal: they originate from infrared and collinear singularities: they exponentiate and can be resummed

$$\underbrace{\frac{1}{\epsilon}}_{\text{virtual}} \; + \underbrace{(Q^2)^\epsilon \int_0^{m^2} \frac{dk^2}{(k^2)^{1+\epsilon}}}_{\text{real}} \qquad \Longrightarrow \qquad \ln(m^2/Q^2)$$

  - For inclusive observables: analytic resummation to high logarithmic accuracy.
  - For exclusive final states: parton shower event generators, (N)LL accuracy.

- Resummation probes the all-order structure of perturbation theory.
  - Power-suppressed corrections to QCD cross sections can be studied.
  - Links to the strong coupling regime can be established for SUSY gauge theories.

The perturbative exponent

A classic way to organize Sudakov logarithms is in terms of the Mellin (Laplace) transform of the momentum space cross section (Catani et al. 93),

$$
\begin{aligned}
d\sigma(\alpha_s, N) &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{2n} c_{nk} \log^k N \, + \, \mathcal{O}(1/N) \\
&= H(\alpha_s) \, \exp\left[\log N \, g_1(\alpha_s \log N) + g_2(\alpha_s \log N) + \alpha_s \, g_3(\alpha_s \log N) + \ldots \right] + \mathcal{O}(1/N)
\end{aligned}
$$

This displays the main features of Sudakov resummation

- Predictive: a k-loop calculation determines g_k and thus a whole tower of logarithms to all orders in perturbation theory.
- Effective:
  - the range of applicability of perturbation theory is extended (finite order: $\alpha_s \log^2 N$ small. NLL resummed: α_s small);
  - the renormalization scale dependence is naturally reduced.
- Theoretically interesting: resummation ambiguities related to the Landau pole give access to non-perturbative power-suppressed corrections.
- Well understood:
  - NLL Sudakov resummations exist for most inclusive observables at hadron colliders, NNLL and approximate N^3LL in simple cases.
  - Different `schools' (USA, Italian, SCET ...) compete, complacency is not an option, active and lively debate.

Color singlet hard scattering

A well-established formalism exists for distributions in processes that are electroweak at tree level (Gardi, Grunberg 07). For an observable r vanishing in the two-jet limit

$$\frac{d\sigma}{dr} = \delta(r)\big[1+\mathcal{O}(\alpha_s)\big] + C_R \frac{\alpha_s}{\pi}\left\{\left[-\frac{\log r}{r} + \frac{b_1 - d_1}{r}\right]_+ + \mathcal{O}(r^0)\right\} + \mathcal{O}(\alpha_s^2)$$

The Mellin (Laplace) transform, $\sigma(N) = \int_0^1 dr\,(1-r)^{N-1}\,\frac{d\sigma}{dr}$

exhibits log N singularities that can be organized in exponential form

$$\sigma\left(\alpha_s, N, Q^2\right) = H(\alpha_s)\,\mathcal{S}\left(\alpha_s, N, Q^2\right) + \mathcal{O}\left(1/N\right)$$

where the exponent of the \`Sudakov factor' is in turn a Mellin transform

$$\mathcal{S}\left(\alpha_s, N, Q^2\right) = \exp\left\{\int_0^1 \frac{dr}{r}\left[(1-r)^{N-1} - 1\right]\mathcal{E}\left(\alpha_s, r, Q^2\right)\right\}$$

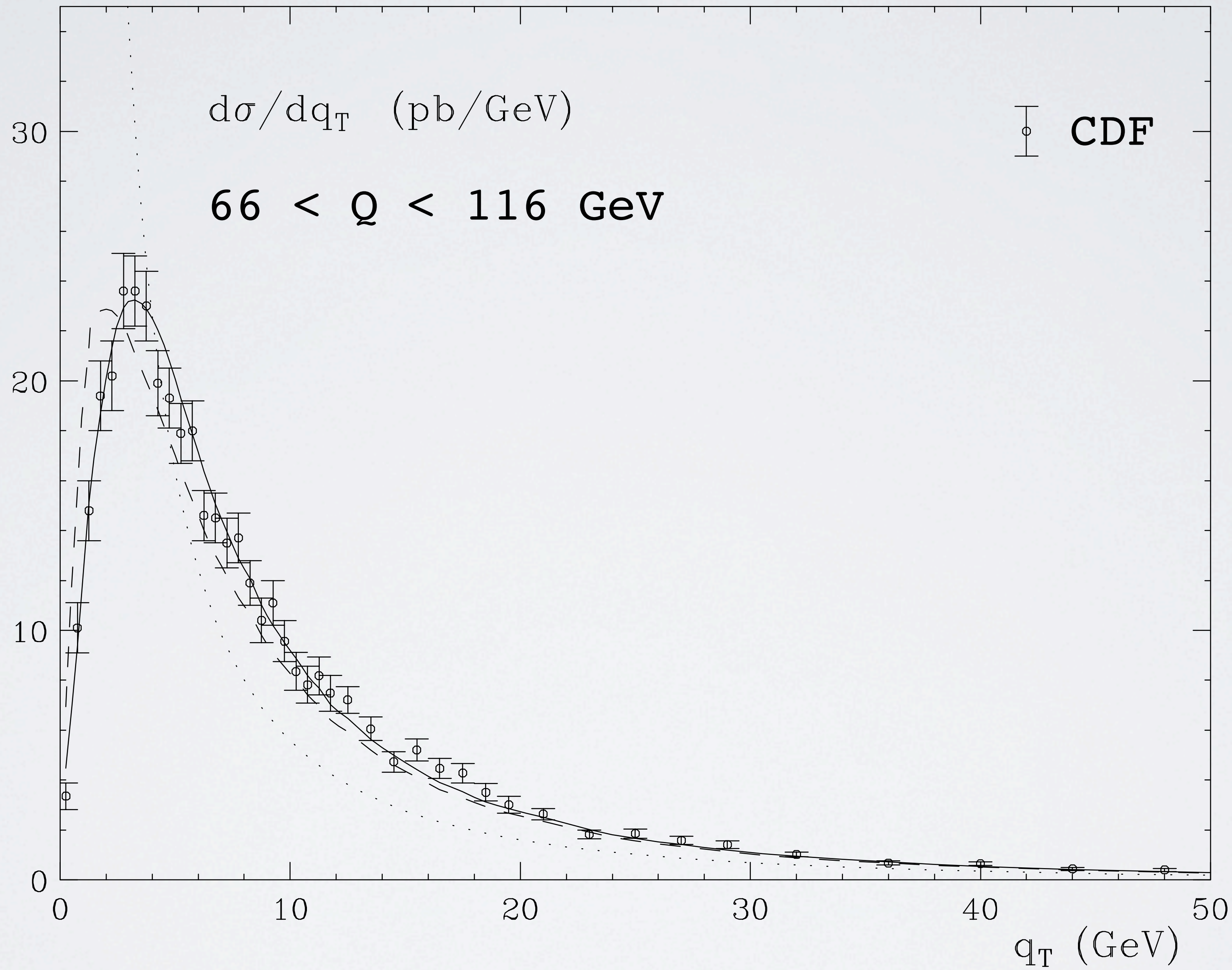
and the general form of the kernel is

$$\mathcal{E}\left(\alpha_s, r, Q^2\right) = \int_{r^2Q^2}^{rQ^2} \frac{d\xi^2}{\xi^2}\,A\left(\alpha_s(\xi^2)\right) + B\left(\alpha_s(rQ^2)\right) + D\left(\alpha_s(r^2Q^2)\right)$$

where A is the cusp anomalous dimension, and B and D have distinct physical characters.

Impact of resummation

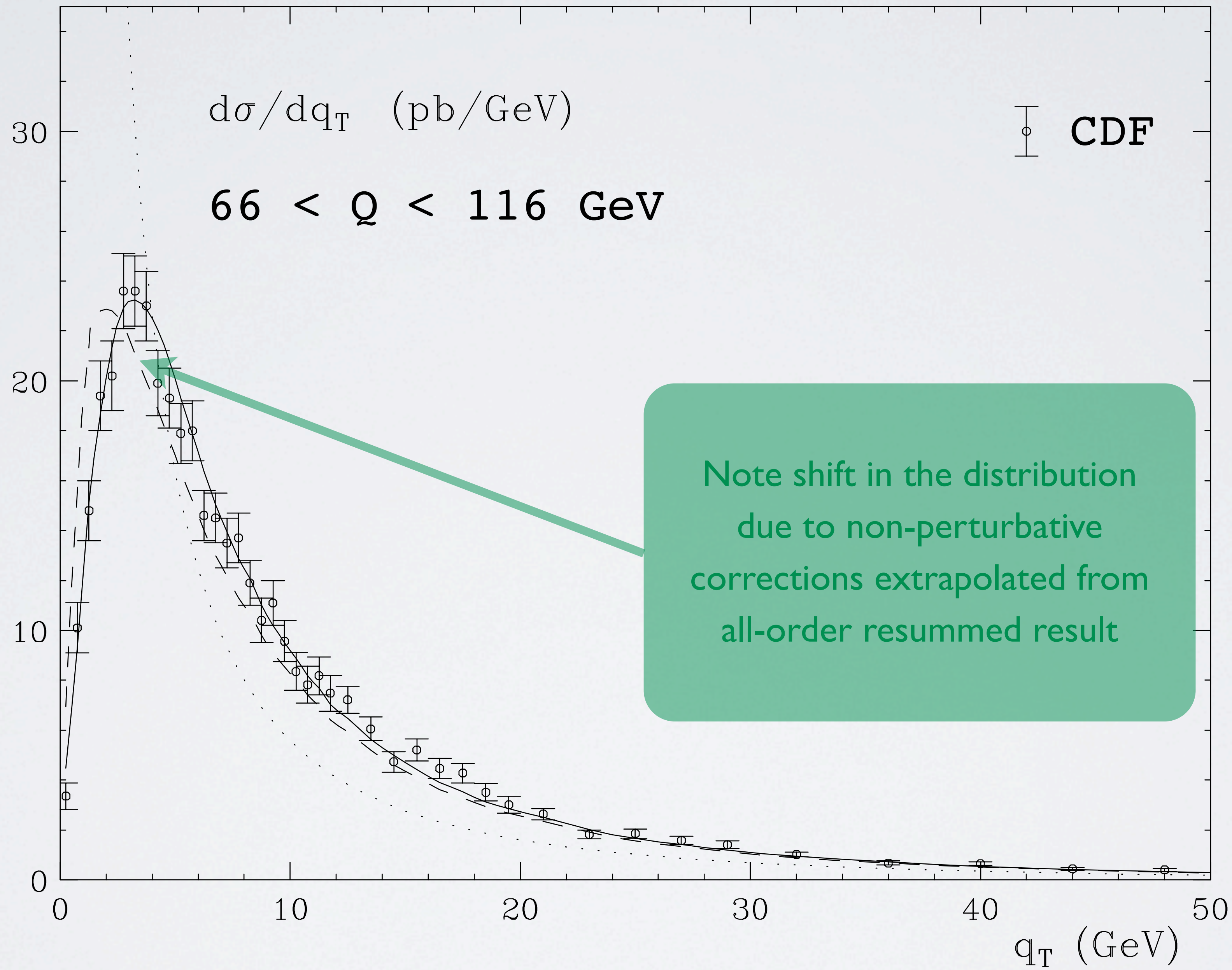
Z-boson q_T spectrum at Tevatron (Kulesza et al. 03)



CDF data on Z production compared with QCD predictions at fixed order (dotted), with joint resummation (dashed), and with the inclusion of power corrections (solid).

Impact of resummation

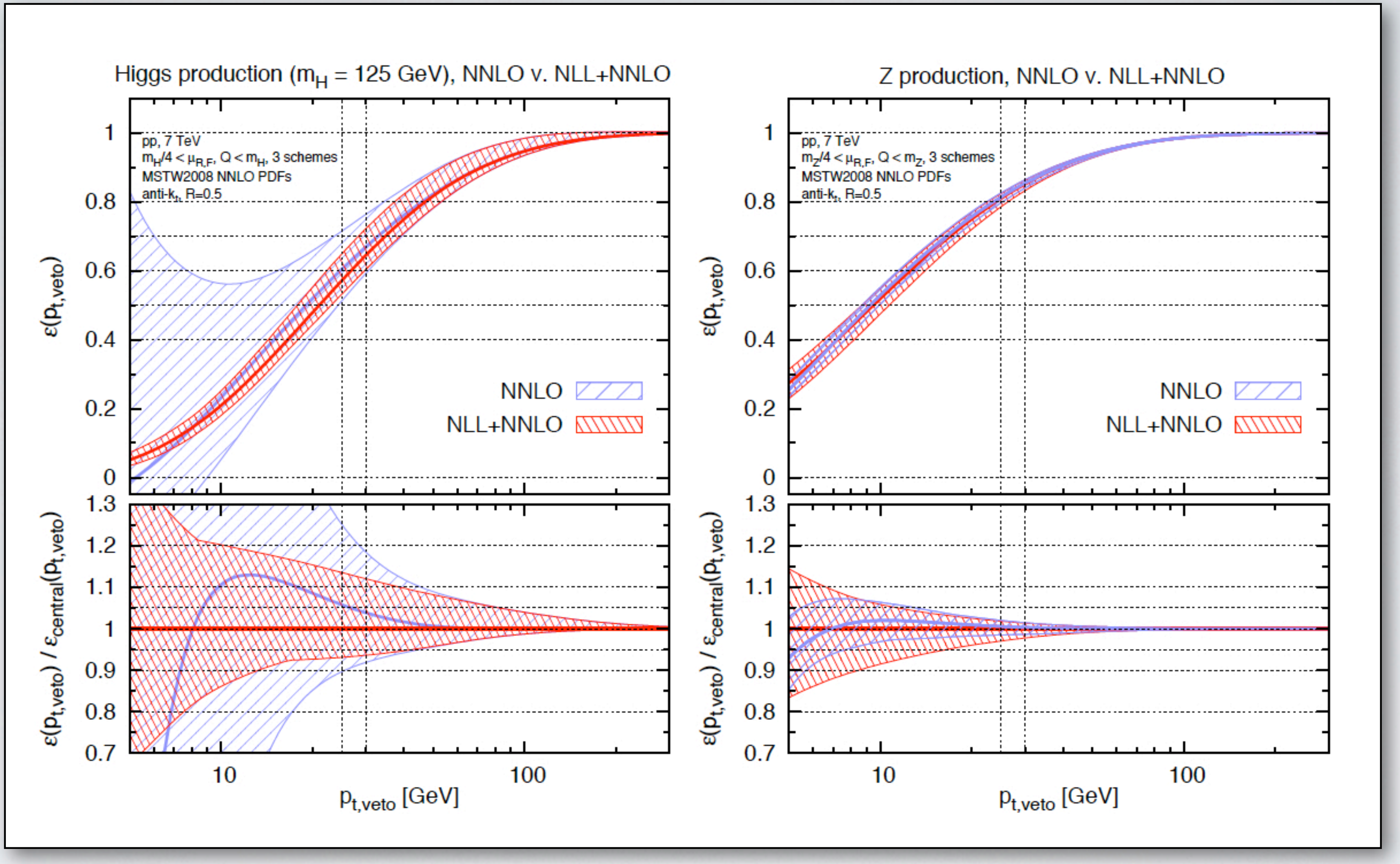
Z-boson q_T spectrum at Tevatron (Kulesza et al. 03)



CDF data on Z production compared with QCD predictions at fixed order (dotted), with (joint) resummation (dashed), and with the inclusion of power corrections (solid).

Complex observables

Jet veto efficiency in Higgs and Z production (Banfi et al., 03/12)

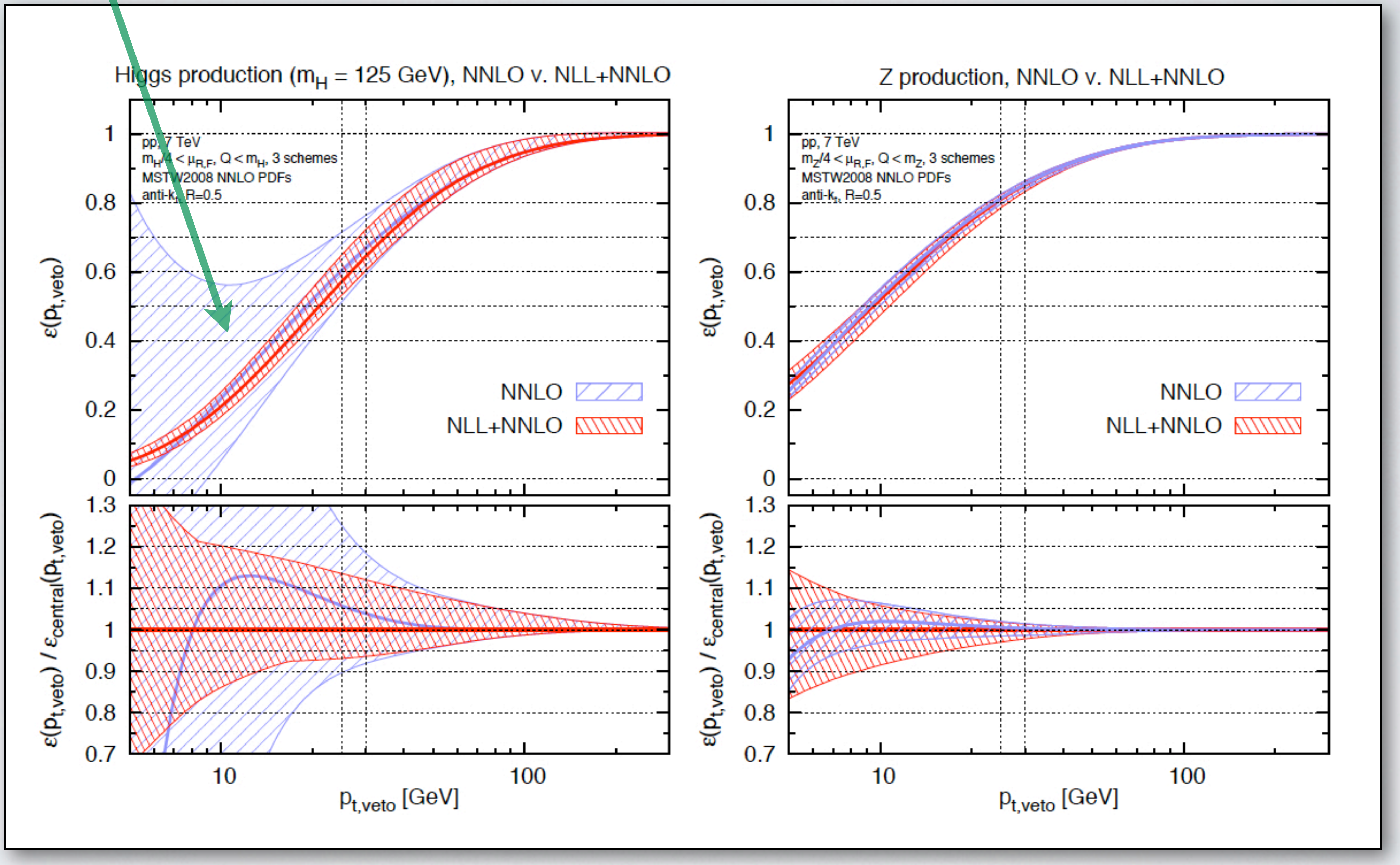


Comparison of NNLO fixed order results and matched resummed NLL-NNLO results for Higgs production with a jet veto (left) and Z production with a jet veto (right). Subsequent improvements include NNLL accuracy (also in SCET, by Becher, Neubert, Rothen, 1307.0025), and exact treatment of quark masses (1308.4634).

Note the sharp reduction of the theoretical uncertaitnty upon resummation

Complex observables

Jet veto efficiency in Higgs and Z production (Banfi et al., 03/12)



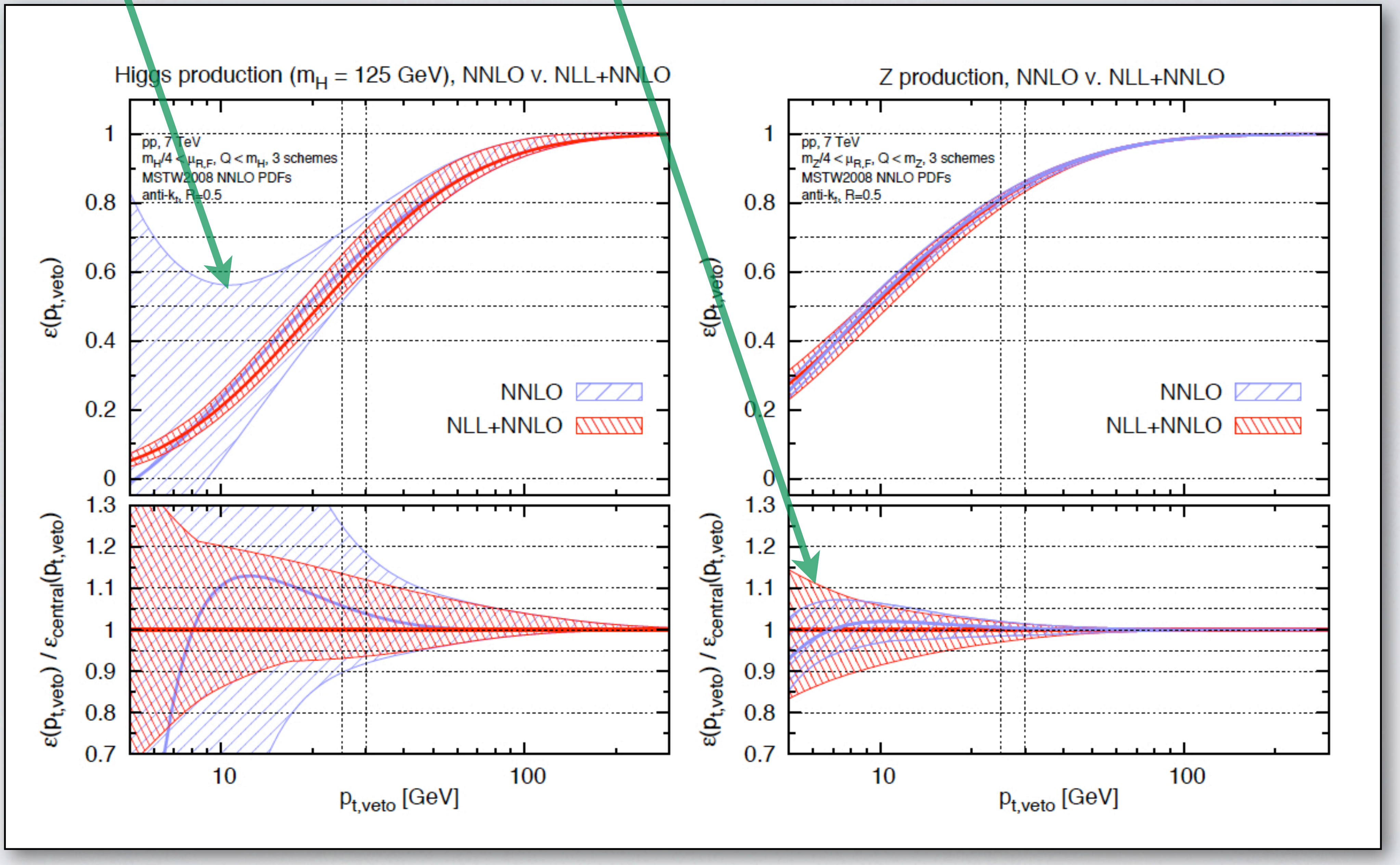
Comparison of NNLO fixed order results and matched resummed NLL-NNLO results for Higgs production with a jet veto (left) and Z production with a jet veto (right). For inclusion of NNLL, see also Becher and Neubert, 05/12.

Note the sharp reduction of the theoretical uncertaitnty upon resummation ...

... which does not always take place!

Complex observables

Jet veto efficiency in Higgs and Z production (Banfi et al., 03/12)

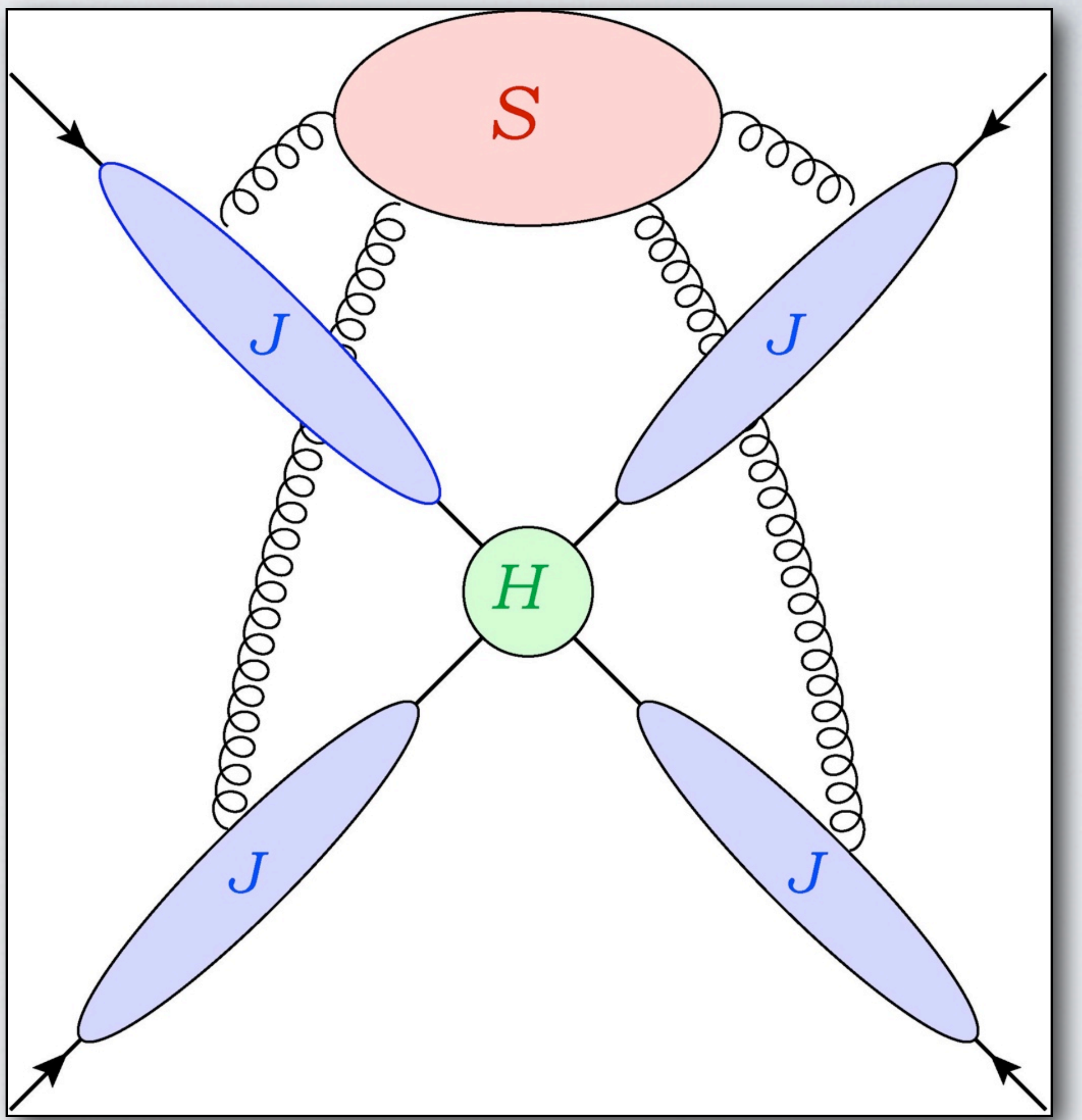


Comparison of NNLO fixed order results and matched resummed NLL-NNLO results for Higgs production with a jet veto (left) and Z production with a jet veto (right). Subsequent improvements include NNLL accuracy (also in SCET, by Becher, Neubert, Rothen, 1307.0025), and exact treatment of quark masses (1308.4634).

TOOLS FOR LARGE LOGS

Soft-collinear factorization

- Sudakov logarithms are remainders of infrared and collinear divergences.
- Divergences arise in scattering amplitudes from leading regions in loop momentum space.
- Power-counting arguments show that soft gluons decouple from the hard subgraph.
- Ward identities decouple soft gluons from jets and restrict color transfer to the hard part.
- Jet functions J represent color singlet evolution of external hard partons.
- The soft function S is a matrix mixing the available color representations.
- In the planar limit soft exchanges are confined to wedges: S is proportional to the identity.
- Beyond the planar limit S is determined by an anomalous dimension matrix Γ_S.
- The matrix Γ_S correlates color exchange with kinematic dependence.

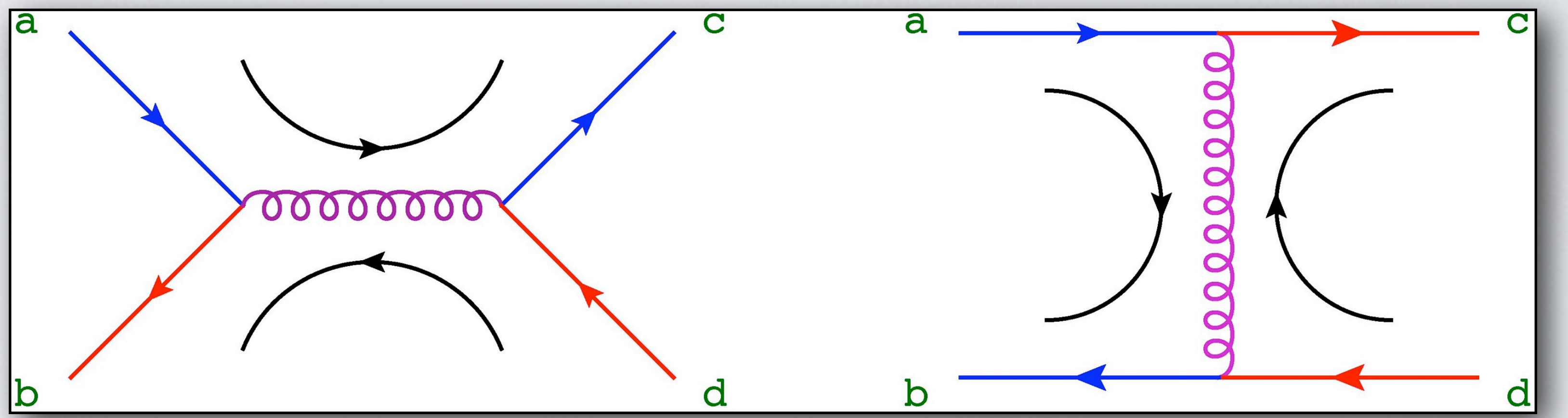


Leading integration regions in loop momentum space for soft-collinear factorization

Color flow

In order to understand the matrix structure of the soft function it is sufficient to consider the simple case of quark-antiquark scattering.

At tree level



Tree-level diagrams and color flows for quark-antiquark scattering

For this process only two color structures are possible. A basis in the space of available color tensors is

$$c^{(1)}_{abcd} = \delta_{ab}\delta_{cd}\,, \qquad c^{(2)}_{abcd} = \delta_{ac}\delta_{bd}$$

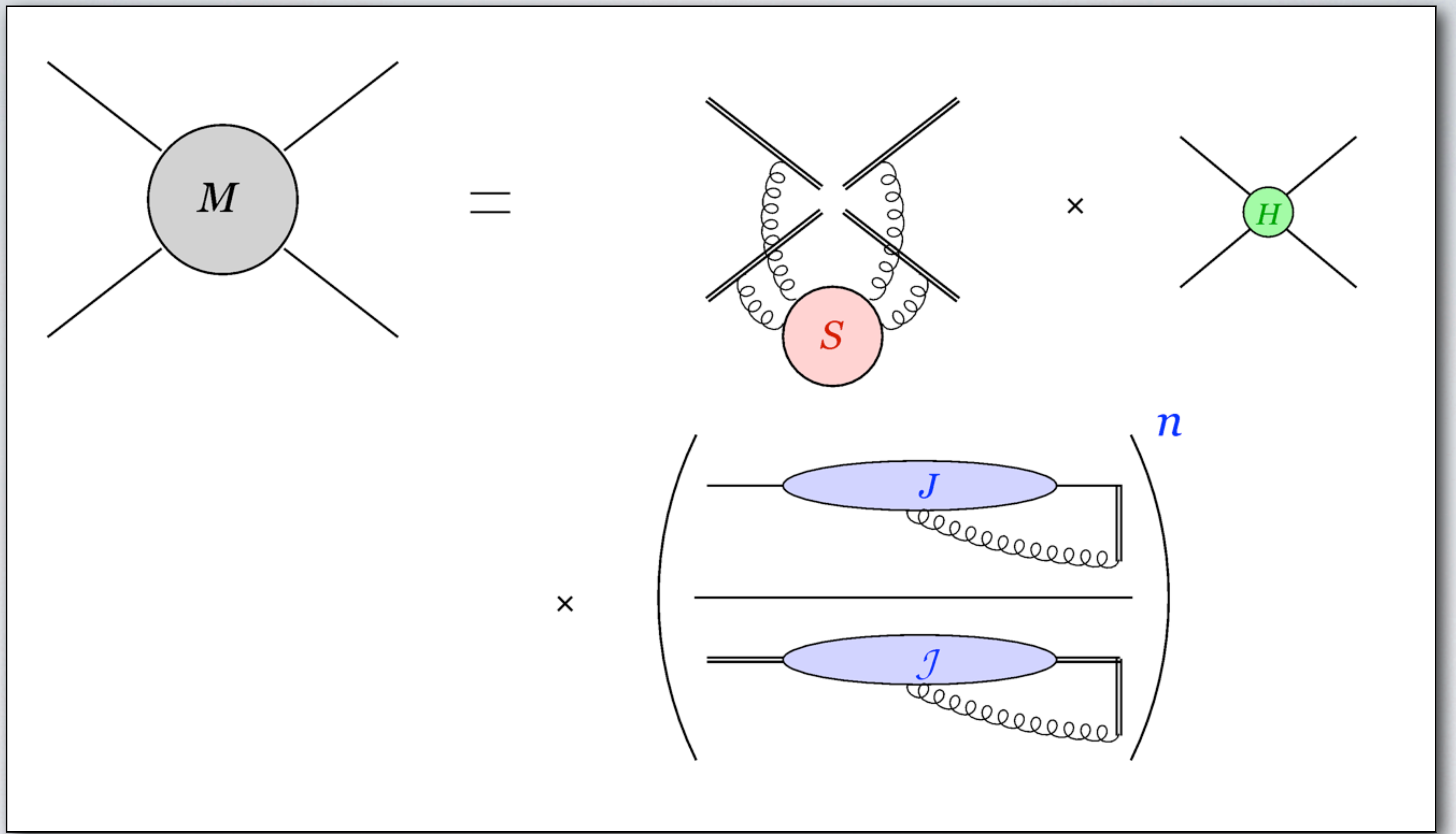
The matrix element is a vector in this space, and the Born cross section is

$$\mathcal{M}_{abcd} = \mathcal{M}_1\, c^{(1)}_{abcd} + \mathcal{M}_2\, c^{(2)}_{abcd} \longrightarrow \sum_{\text{color}} |\mathcal{M}|^2 = \sum_{J,L} \mathcal{M}_J\, \mathcal{M}^*_L\ \mathrm{tr}\left[c^{(J)}_{abcd}\left(c^{(L)}_{abcd}\right)^\dagger\right] \equiv \mathrm{Tr}\Big[HS\Big]_0$$

A virtual soft gluon will reshuffle color and mix the components of this vector

$$\mathrm{QED}: \quad \mathcal{M}_{\mathrm{div}} = S_{\mathrm{div}} \mathcal{M}_{\mathrm{Born}}\,; \qquad \mathrm{QCD}: \quad [\mathcal{M}_{\mathrm{div}}]_J = [S_{\mathrm{div}}]_{JL}\, [\mathcal{M}_{\mathrm{Born}}]_L$$

Sudakov factorization: pictorial



A pictorial representation of Sudakov factorization for fixed-angle scattering amplitudes

Soft Matrices

The soft function S is a matrix, mixing the available color tensors. It is defined by a correlator of Wilson lines.

$$(c_L)_{\{\alpha_k\}} \mathcal{S}_{LK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = \sum_{\{\eta_k\}} \langle 0| \prod_{i=1}^{n} \left[\Phi_{\beta_i}(\infty, 0)_{\alpha_k, \eta_k}\right] |0\rangle \, (c_K)_{\{\eta_k\}} \, ,$$

The soft function S obeys a matrix RG evolution equation

$$\mu \frac{d}{d\mu} \mathcal{S}_{IK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = - \mathcal{S}_{IJ}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) \Gamma^{\mathcal{S}}_{JK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right)$$

- Γ^S is singular due to overlapping UV and collinear poles.

S is a pure counterterm. In dimensional regularization, using $\alpha_s(\mu^2 = 0, \varepsilon < 0) = 0$,

$$\mathcal{S}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = P \exp\left[-\frac{1}{2} \int_0^{\mu^2} \frac{d\xi^2}{\xi^2} \Gamma^{\mathcal{S}}\left(\beta_i \cdot \beta_j, \alpha_s(\xi^2, \epsilon), \epsilon\right)\right] .$$

The determination of the soft anomalous dimension matrix Γ^S is the keystone of the resummation program for multiparton amplitudes and cross sections.

- It governs the interplay of color exchange with kinematics in multiparton processes.
- It is the only source of multiparton correlations for singular contributions.
- Collinear effects are `color singlet' and can be extracted from two-parton scatterings.

The Dipole Formula

For massless partons, the soft anomalous dimension matrix obeys an exact equation based on a `conformal anomaly', which correlates color exchange with kinematics.

The simplest solution to this equation is a sum over color dipoles (Becher, Neubert; Gardi, LM, 09). It gives an ansatz for the all-order singularity structure of all multiparton fixed-angle massless scattering amplitudes: the **dipole formula**.

- All soft and collinear singularities can be collected in a multiplicative operator Z

$$\mathcal{M}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = Z\left(\frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon\right) \, \mathcal{H}\left(\frac{p_i}{\mu}, \frac{\mu_f}{\mu}, \alpha_s(\mu^2), \epsilon\right) ,$$

- Z contains both soft singularities from S, and collinear ones from the jet functions. It must satisfy its own matrix RG equation

$$\frac{d}{d\ln\mu} Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = - Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) \Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) .$$

The matrix Γ has a surprisingly simple dipole structure. It reads

$$\Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = -\frac{1}{4}\, \widehat{\gamma}_K\left(\alpha_s(\mu^2)\right) \sum_{j\neq i} \ln\left(\frac{-2\, p_i \cdot p_j}{\mu^2}\right) \mathbf{T}_i \cdot \mathbf{T}_j + \sum_{i=1}^{n} \gamma_{J_i}\left(\alpha_s(\mu^2)\right) .$$

Note that all singularities are generated by integration over the scale of the coupling.

The Dipole Formula

For massless partons, the soft anomalous dimension matrix obeys an exact equation based on a `conformal anomaly', which correlates color exchange with kinematics.

The simplest solution to this equation is a sum over color dipoles (Becher, Neubert; Gardi, LM, 09). It gives an ansatz for the all-order singularity structure of all multiparton fixed-angle massless scattering amplitudes: the **dipole formula**.

- All soft and collinear singularities can be collected in a multiplicative operator Z

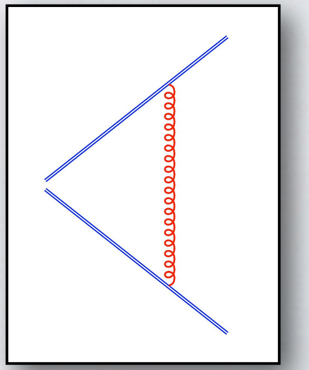
$$\mathcal{M}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = Z\left(\frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon\right) \, \mathcal{H}\left(\frac{p_i}{\mu}, \frac{\mu_f}{\mu}, \alpha_s(\mu^2), \epsilon\right) \, ,$$

- Z contains both soft singularities from S, and collinear ones from the jet functions. It must satisfy its own matrix RG equation

$$\frac{d}{d\ln\mu} Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = - Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) \Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) \, .$$

The matrix Γ has a surprisingly simple dipole structure. It reads

$$\Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = -\frac{1}{4}\,\widehat{\gamma}_K\left(\alpha_s(\mu^2)\right) \sum_{j\neq i} \ln\left(\frac{-2\,p_i\cdot p_j}{\mu^2}\right) \mathbf{T}_i\cdot\mathbf{T}_j + \sum_{i=1}^{n} \gamma_{J_i}\left(\alpha_s(\mu^2)\right) \, .$$



Note that all singularities are generated by integration over the scale of the coupling.

Features of the dipole formula

- All known results for IR divergences of massless gauge theory amplitudes are recovered.
- The absence of multiparton correlations implies remarkable diagrammatic cancellations.
- The color matrix structure is fixed at one loop: path-ordering is not needed.
- All divergences are determined by a handful of anomalous dimensions.
- The cusp anomalous dimension plays a very special role: a universal IR coupling.

Can this be the definitive answer for IR divergences in massless non-abelian gauge theories?

- There are precisely two sources of possible corrections.
  - Quadrupole correlations may enter starting at three loops: they must be tightly constrained functions of conformal cross ratios of parton momenta.

$$\Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = \Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) + \Delta\left(\rho_{ijkl}, \alpha_s(\mu^2)\right) , \qquad \rho_{ijkl} = \frac{p_i \cdot p_j \, p_k \cdot p_l}{p_i \cdot p_k \, p_j \cdot p_l}$$

  - The cusp anomalous dimension may violate Casimir scaling beyond three loops.

$$\gamma_K^{(i)}(\alpha_s) = C_i \, \widehat{\gamma}_K(\alpha_s) + \widetilde{\gamma}_K^{(i)}(\alpha_s)$$

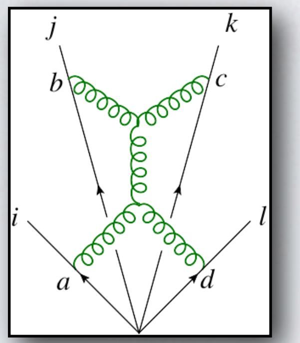
  - The functional form of Δ is further constrained by: collinear limits, Bose symmetry, bounds on weights, high-energy constraints. (Becher, Neubert; Dixon, Gardi, LM, 09).
  - Recent evidence points to a non-vanishing Δ at four loops (Caron-Huot 13).

Features of the dipole formula

- All known results for IR divergences of massless gauge theory amplitudes are recovered.
- The absence of multiparton correlations implies remarkable diagrammatic cancellations.
- The color matrix structure is fixed at one loop: path-ordering is not needed.
- All divergences are determined by a handful of anomalous dimensions.
- The cusp anomalous dimension plays a very special role: a universal IR coupling.

Can this be the definitive answer for IR divergences in massless non-abelian gauge theories?

- There are precisely two sources of possible corrections.



- Quadrupole correlations may enter starting at three loops: they must be tightly constrained functions of conformal cross ratios of parton momenta.

$$\Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = \Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) + \Delta\left(\rho_{ijkl}, \alpha_s(\mu^2)\right) , \qquad \rho_{ijkl} = \frac{p_i \cdot p_j \, p_k \cdot p_l}{p_i \cdot p_k \, p_j \cdot p_l}$$

- The cusp anomalous dimension may violate Casimir scaling beyond three loops.

$$\gamma_K^{(i)}(\alpha_s) = C_i \, \widehat{\gamma}_K(\alpha_s) + \widetilde{\gamma}_K^{(i)}(\alpha_s)$$

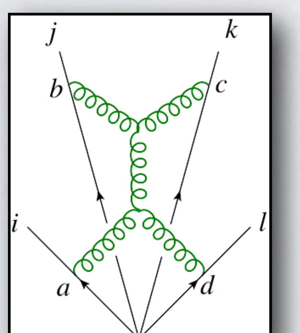
- The functional form of Δ is further constrained by: collinear limits, Bose symmetry, bounds on weights, high-energy constraints. (Becher, Neubert; Dixon, Gardi, LM, 09).
- Recent evidence points to a non-vanishing Δ at four loops (Caron-Huot 13).

Features of the dipole formula

- All known results for IR divergences of massless gauge theory amplitudes are recovered.
- The absence of multiparton correlations implies remarkable diagrammatic cancellations.
- The color matrix structure is fixed at one loop: path-ordering is not needed.
- All divergences are determined by a handful of anomalous dimensions.
- The cusp anomalous dimension plays a very special role: a universal IR coupling.

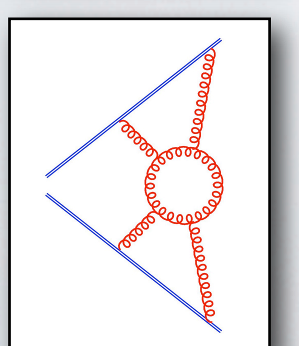
Can this be the definitive answer for IR divergences in massless non-abelian gauge theories?

- There are precisely two sources of possible corrections.



- Quadrupole correlations may enter starting at three loops: they must be tightly constrained functions of conformal cross ratios of parton momenta.

$$\Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = \Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) + \Delta\left(\rho_{ijkl}, \alpha_s(\mu^2)\right) , \qquad \rho_{ijkl} = \frac{p_i \cdot p_j \, p_k \cdot p_l}{p_i \cdot p_k \, p_j \cdot p_l}$$



- The cusp anomalous dimension may violate Casimir scaling beyond three loops.

$$\gamma_K^{(i)}(\alpha_s) = C_i \, \widehat{\gamma}_K(\alpha_s) + \widetilde{\gamma}_K^{(i)}(\alpha_s)$$

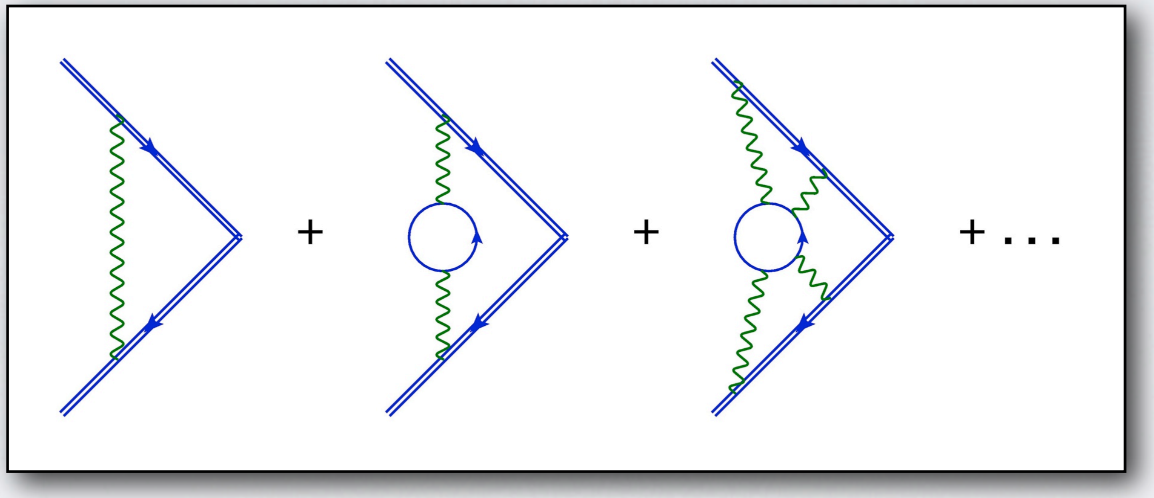
- The functional form of Δ is further constrained by: collinear limits, Bose symmetry, bounds on weights, high-energy constraints. (Becher, Neubert; Dixon, Gardi, LM, 09).
- Recent evidence points to a non-vanishing Δ at four loops (Caron-Huot 13).

Infrared exponentiation

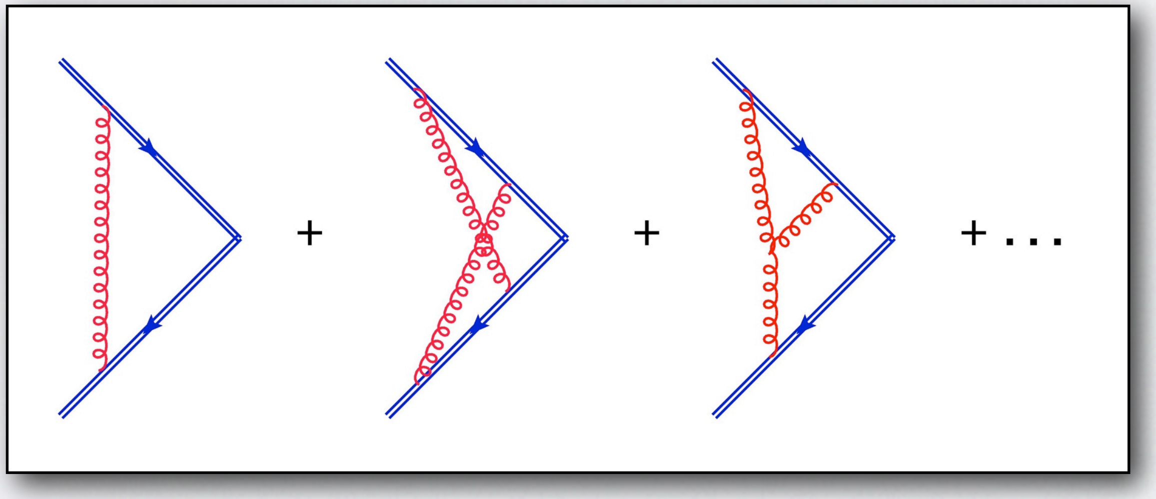
All correlators of Wilson lines, regardless of shape, resum in exponential form.

$$S_n \equiv \langle 0 | \, \Phi_1 \otimes \ldots \otimes \Phi_n \, | 0 \rangle = \exp\left(\omega_n\right)$$

Diagrammatic rules exist to compute directly the logarithm of the correlators.

$\omega_{2,\mathrm{QED}} =$ 

Only connected photon subdiagrams contribute to the logarithm.

$\omega_{2,\mathrm{QCD}} =$ 

Only gluon subdiagrams which are two-eikonal irreducible contribute to the logarithm. They have modified color factors.

For eikonal form factors, these diagrams are called **webs** (Gatheral; Frenkel, Taylor; Sterman).

Multiparticle webs

The concept of web generalizes non-trivially to the case of multiple Wilson lines. (Gardi, Smillie, White, *et al*).

A **web** is a set of diagrams which differ only by the order of the gluon attachments on each Wilson line. They are weighted by modified color factors.

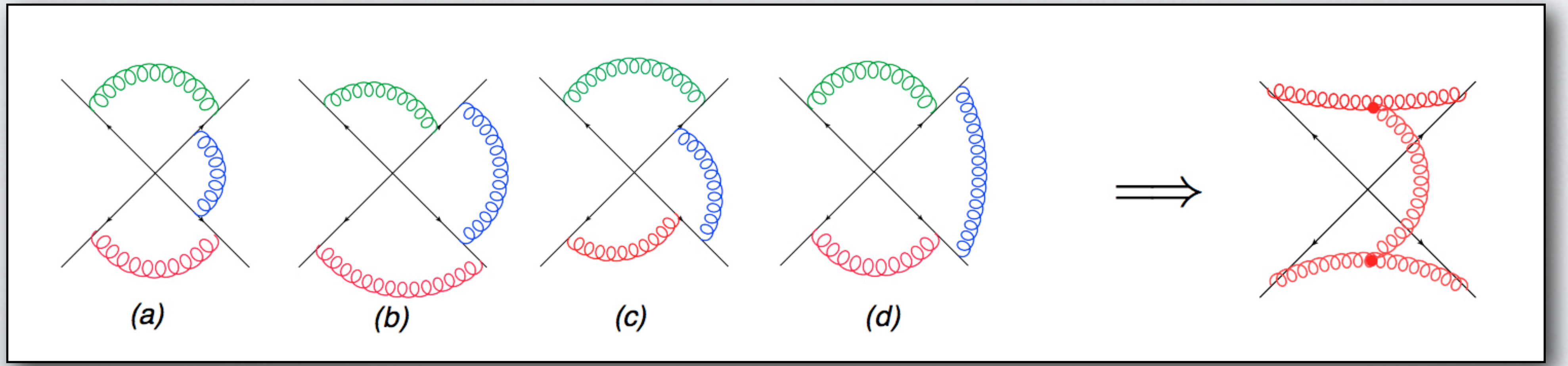
Writing each diagram as the product of its natural color factor and a kinematic factor

$$D = C(D)\mathcal{F}(D)$$

a web W can be expressed as a sum of diagrams in terms of a web mixing matrix R

$$W = \sum_{D} \widetilde{C}(D)\mathcal{F}(D) = \sum_{D,D'} C(D')R(D',D)\mathcal{F}(D)$$

The non-abelian exponentiation theorem holds: each web has the color factor of a fully connected gluon subdiagram (Gardi, Smillie, White).



Computing webs

Bare Wilson-line correlators vanish beyond tree level in dimensional regularization: they are given by scale-less integrals. We require renormalized correlators, which depend on the Minkowsky angles between the Wilson lines.

$$S_{\rm ren}\left(\gamma_{ij},\alpha_s,\epsilon\right) \, = \, S_{\rm bare}\left(\gamma_{ij},\alpha_s,\epsilon\right) Z\left(\gamma_{ij},\alpha_s,\epsilon\right) \, = \, Z\left(\gamma_{ij},\alpha_s,\epsilon\right) \, , \qquad \gamma_{ij} \, = \, \frac{2\beta_i\cdot\beta_j}{\sqrt{\beta_i^2\beta_j^2}}$$

To compute the counterterm Z we make use of an auxiliary, IR-regularized correlator

$$\begin{aligned} \widehat{S}_{\rm ren}\left(\gamma_{ij},\alpha_s,\epsilon,m\right) & = \widehat{S}_{\rm bare}\left(\gamma_{ij},\alpha_s,\epsilon,m\right) Z\left(\gamma_{ij},\alpha_s,\epsilon\right) \\ & \equiv \exp\left(\omega\right)\,\exp\left(\zeta\right) \, = \, \exp\left\{\omega+\zeta+\frac{1}{2}\left[\omega,\zeta\right]+\ldots\right\} \end{aligned}$$

The expression of Z in terms of the anomalous dimension Γ follows from RG arguments

$$Z \, = \, \exp\left[\frac{\alpha_s}{\pi}\frac{1}{2\epsilon}\Gamma^{(1)}+\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{1}{4\epsilon}\Gamma^{(2)}-\frac{b_0}{4\epsilon^2}\Gamma^{(1)}\right)+\left(\frac{\alpha_s}{\pi}\right)^3\left(\frac{1}{6\epsilon}\Gamma^{(3)}+\frac{1}{48\epsilon^2}\left[\Gamma^{(1)},\Gamma^{(2)}\right]+\ldots\right)\right]$$

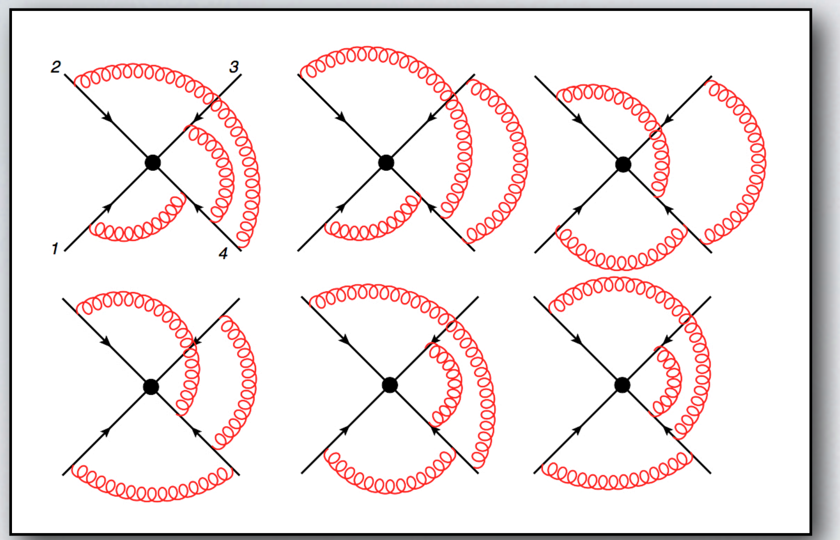
Combining informations one can get Γ directly from the logarithm of the regularized S

$$\begin{aligned} \Gamma^{(1)} & = -2\,\omega^{(1,-1)} \\ \Gamma^{(2)} & = -4\,\omega^{(2,-1)}-2\left[\omega^{(1,-1)},\omega^{(1,0)}\right] \end{aligned} \qquad \omega=\sum_{n=1}^{\infty}\sum_{k=-n}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n\epsilon^k\,\omega^{(n,k)}$$

Computing regularized webs is a game of combinatorics and renormalization theory.

Three-loop progress

The computation of the three-loop multi-particle soft anomalous dimension is under way.

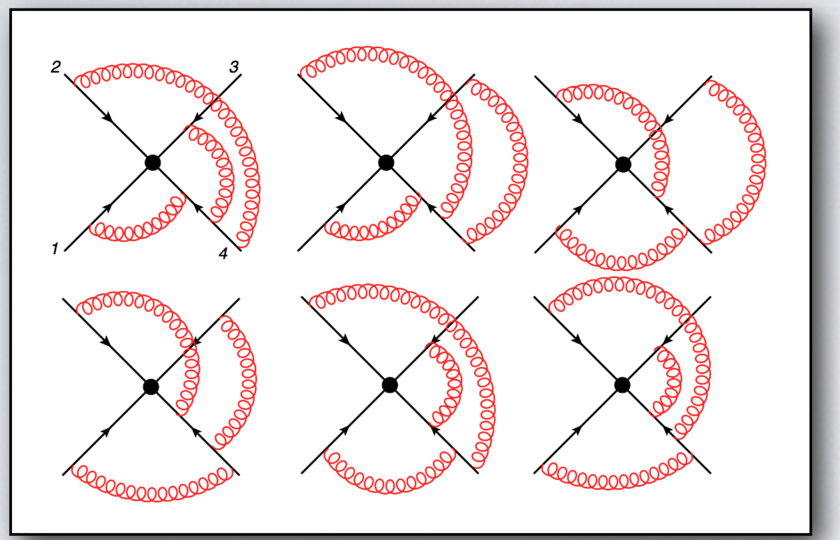


Gardi 1310.5268
Falcioni et al. 1407.3477.

A Multiple Gluon Exchange Web

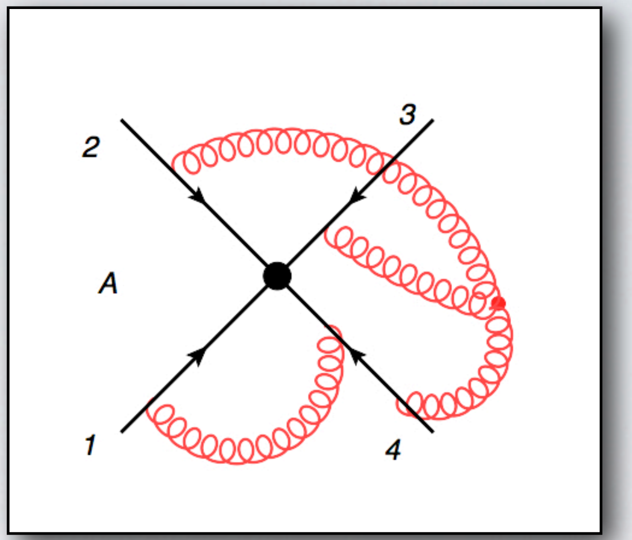
Three-loop progress

The computation of the three-loop multi-particle soft anomalous dimension is under way.



A Multiple Gluon Exchange Web

Gardi 1310.5268
Falcioni et al. 1407.3477.

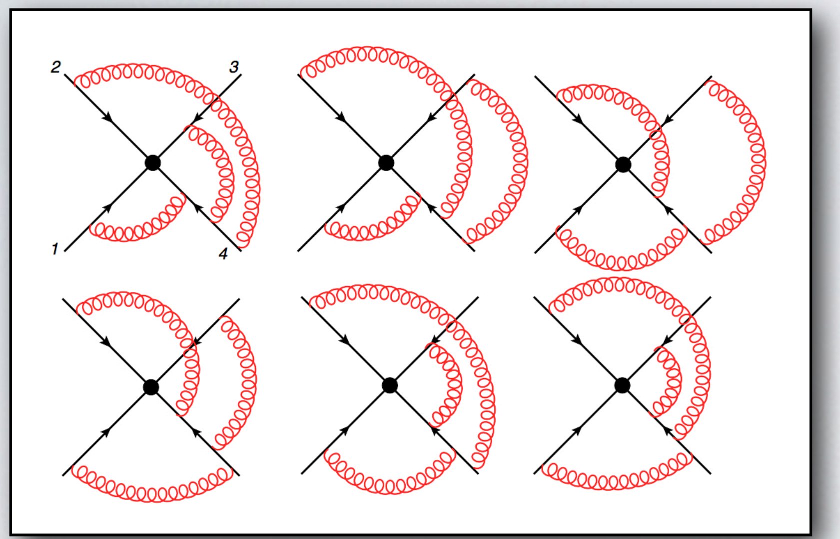


A Tripole Web

Falcioni et al. , in progress.

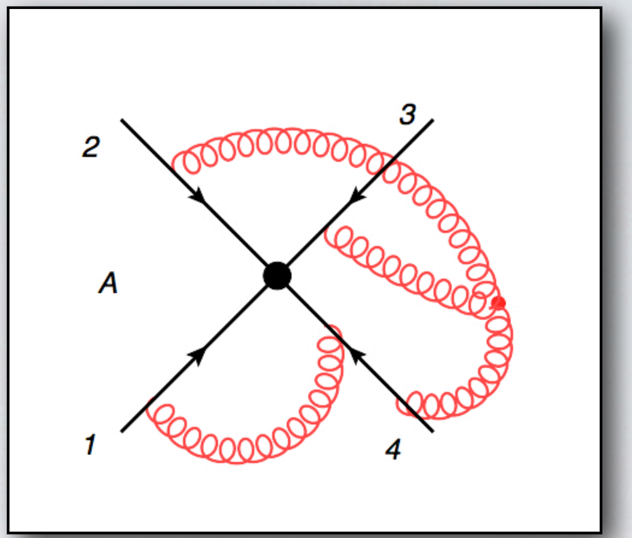
Three-loop progress

The computation of the three-loop multi-particle soft anomalous dimension is under way.



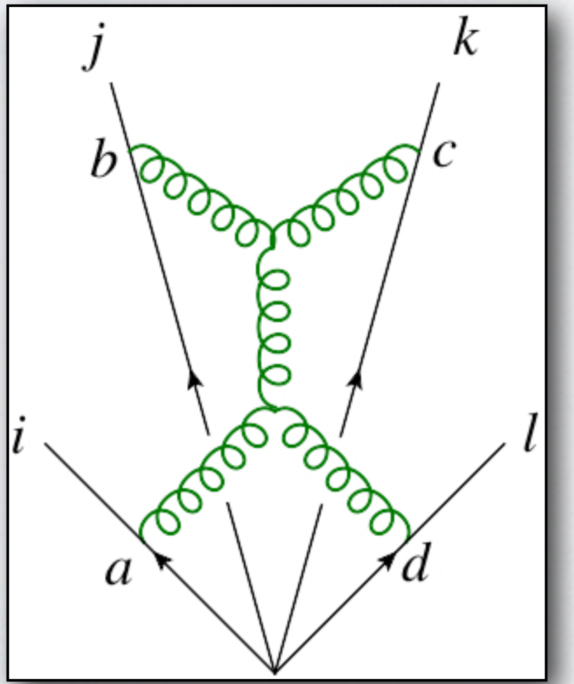
A Multiple Gluon Exchange Web

Gardi 1310.5268
Falcioni et al. 1407.3477.



A Tripole Web

Falcioni et al. ,
in progress.



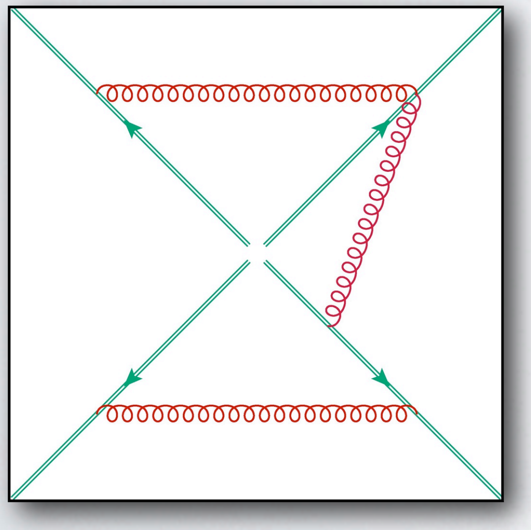
A Quadrupole Web

Almelid, Duhr and Gardi,
in progress.

Beyond the eikonal

- Hadronic cross sections near partonic threshold receive non-singular logarithmic corrections $\alpha_s^p \log^k(1 - z)$, or $\alpha_s^p \log^k N/N$, which may be relevant for phenomenology. Can they also be organized and resummed? (Kraemer et al.; Vogt et al.; Grunberg, ...)
  - For two-parton processes, $O(N^0)$ contributions exponentiate (Laenen, LM, 03).
  - Phenomenological evidence indicates that also `sub-eikonal' logs partly exponentiate.
  - An ansatz summarizes the resummable for Drell-Yan (and DIS) (Laenen et al., 06).

$$\ln\left[\widehat{\omega}(N)\right] = \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z}\, D\left[\alpha_s\left(\frac{(1-z)^2 Q^2}{z}\right)\right]\right.$$
$$\left. + 2\int_{Q^2}^{(1-z)^2Q^2/z}\frac{dq^2}{q^2}\, P_s\left[z, \alpha_s(q^2)\right]\right\}_+$$



Is THIS a web?

- A systematic study of soft-gluon dynamics beyond the eikonal approximation is under way (Laenen et al. 08, 10; Bonocore et al, in prep.).
  - A class of factorizable contributions exponentiate via NE webs

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\sum_{D_{\rm eik}} \tilde{C}(D_{\rm eik})\,\mathcal{F}(D_{\rm eik}) + \sum_{D_{\rm NE}} \tilde{C}(D_{\rm NE})\,\mathcal{F}(D_{\rm NE})\right].$$

  - "Feynman rules" for the NE exponent, including "seagull" vertices.
  - Non-factorizable contribution can be studied using Low's theorem.

Beyond the eikonal

- Hadronic cross sections near partonic threshold receive non-singular logarithmic corrections $\alpha_s^p \log^k(1 - z)$, or $\alpha_s^p \log^k N/N$, which may be relevant for phenomenology. Can they also be organized and resummed? (Kraemer et al.; Vogt et al.; Grunberg, ...)
  - For two-parton processes, $O(N^0)$ contributions exponentiate (Laenen, LM, 03).
  - Phenomenological evidence indicates that also `sub-eikonal' logs partly exponentiate.
  - An ansatz summarizes the resummable for Drell-Yan (and DIS) (Laenen et al., 06).

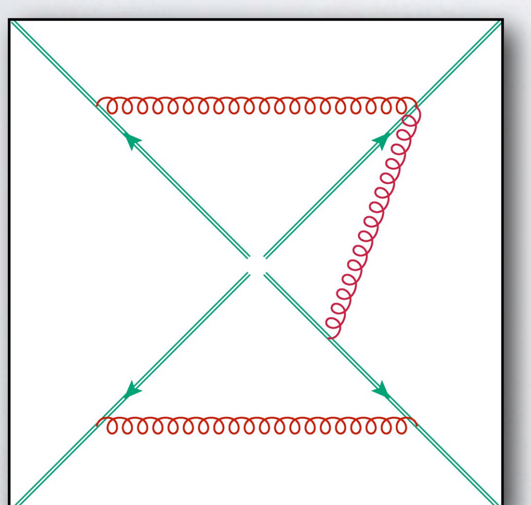
$$\ln\left[\widehat{\omega}(N)\right] = \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z}\, D\left[\alpha_s\left(\frac{(1-z)^2 Q^2}{z}\right)\right] + 2\int_{Q^2}^{(1-z)^2 Q^2/z} \frac{dq^2}{q^2}\, P_s\left[z, \alpha_s(q^2)\right]\right\}_+$$

$$\frac{2}{1-z} \longrightarrow \frac{2\,z}{1-z}$$

- A systematic study of soft-gluon dynamics beyond the eikonal approximation is under way (Laenen et al. 08, 10; Bonocore et al, in prep.).
  - A class of factorizable contributions exponentiate via NE webs

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\sum_{D_{\rm eik}} \tilde{C}(D_{\rm eik})\,\mathcal{F}(D_{\rm eik}) + \sum_{D_{\rm NE}} \tilde{C}(D_{\rm NE})\,\mathcal{F}(D_{\rm NE})\right].$$

  - "Feynman rules" for the NE exponent, including "seagull" vertices.
  - Non-factorizable contribution can be studied using Low's theorem.

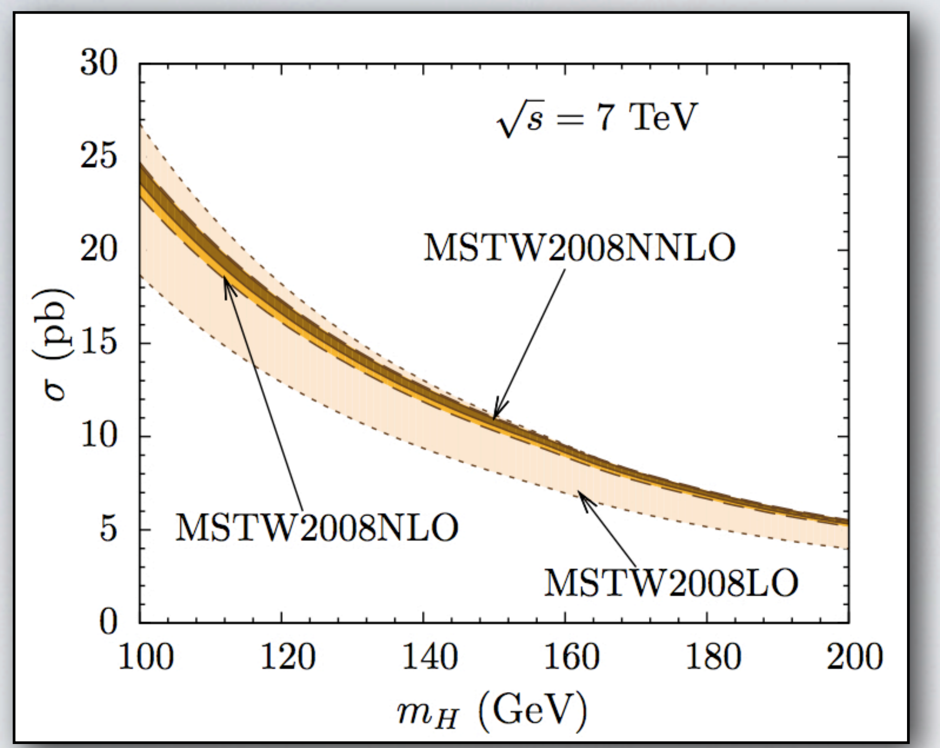


Is THIS a web?

USING LARGE LOGS

No. 28—"Three Log" Load of Sugar Pine at the Mill Pond.

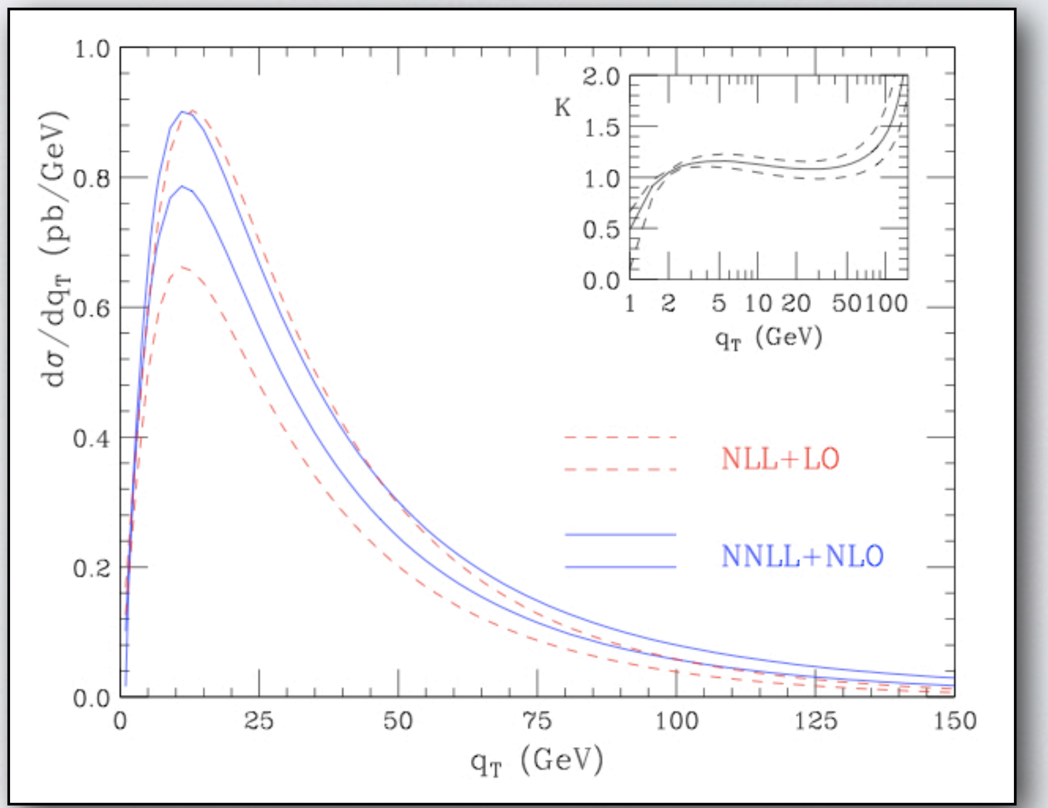
Higgs production



N^{3}LL resummed cross section for Higgs production via gluon fusion at LHC

- The **total cross section** for gg->H is known to N^{3}LL and NNLO+, with NLO EW corrections.
  - One of the best-known observables in the SM.
  - A combined analysis (Ahrens et al. 11) gives a 3% (th) + 8% (pdf) + 1% (mq) uncertainty.
  - Debate on theoretical and pdf uncertainty, initiated in Baglio et al. 11. For consensus see https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CrossSections

- The p_T distribution for gg->H is known to NNLL and NNLO (M. Grazzini et al. 07, 10 Ahrens et al. 11, Boughezal et al. 13).
  - Resummation reduces scale uncertainty.
  - Subtle polarization effects (Catani, Grazzini, 10).
  - `Collinear anomaly' in SCET (Becher, Neubert).
  - Impact of revised three-loop coefficient likely very small.
  - Threshold corrections at large p_T recently computed (Becher et al. 14).



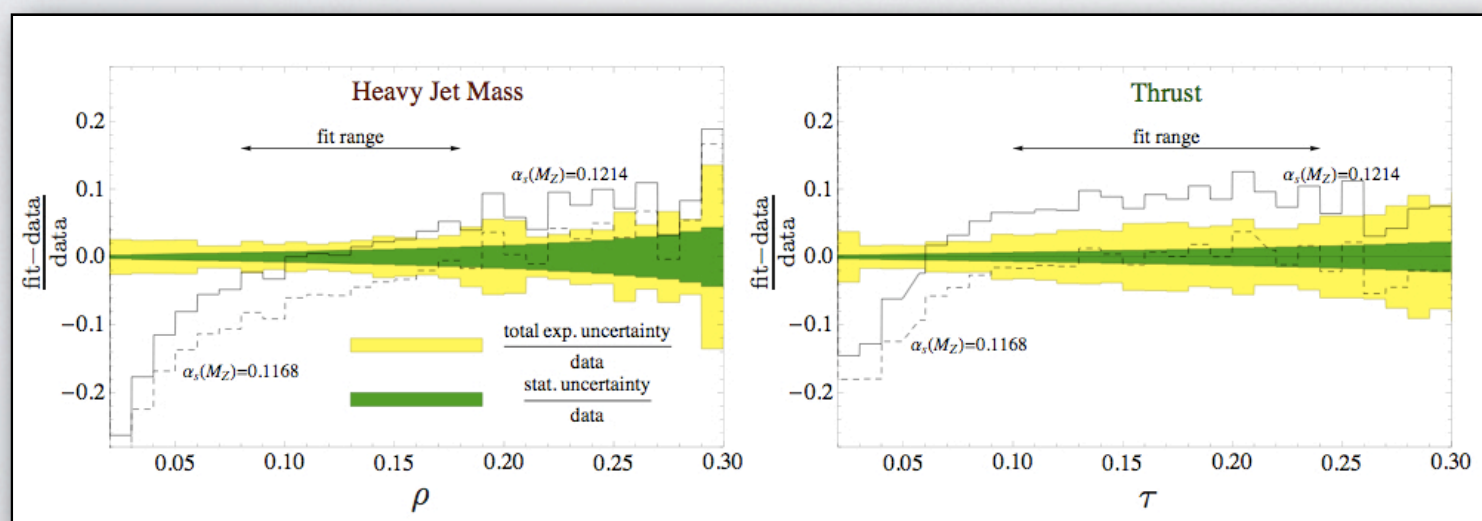
NNLL resummed pT distribution for Higgs production via gluon fusion at LHC

Event shapes

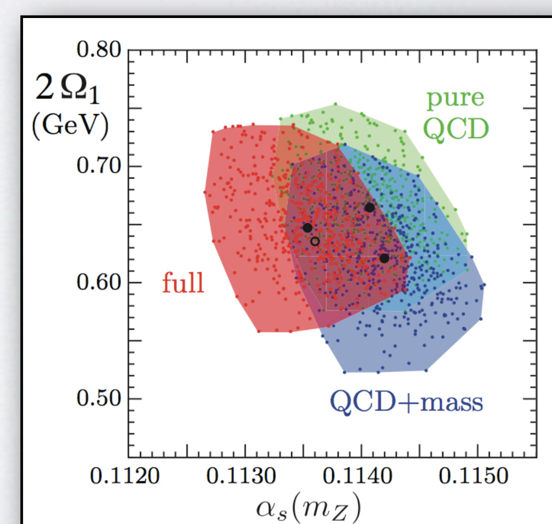
- First studies of event shapes with exact NNLO information and (well) approximated N^{3}LL resummation have appeared (Becher, Schwartz, 08; Schwartz, Cien; Abbate et al. 10).
- The studies deploy neat tricks (Padé approximants, numerical determination of 2-loop soft coefficients) and great care (hadronization, b-mass, QED corrections).
- Perturbative agreement between SCET and standard resummation (Gehrmann et al., 11).
- Significant differences remain in the final results for the strong coupling.

$$
\begin{aligned}
\alpha_s(M_Z^2) &= 0.1172 \pm 0.0022 \qquad \text{thrust (BS)}\\
\alpha_s(M_Z^2) &= 0.1220 \pm 0.0031 \qquad \text{jet mass (SC)}\\
\alpha_s(M_Z^2) &= 0.1135 \pm 0.0010 \qquad \text{thrust (AFHMS)}
\end{aligned}
$$

- Many possible sources of discrepancy, the main suspect remains hadronization/MC.
- The problem is still not fully understood: do we really know α_s to percent accuracy?

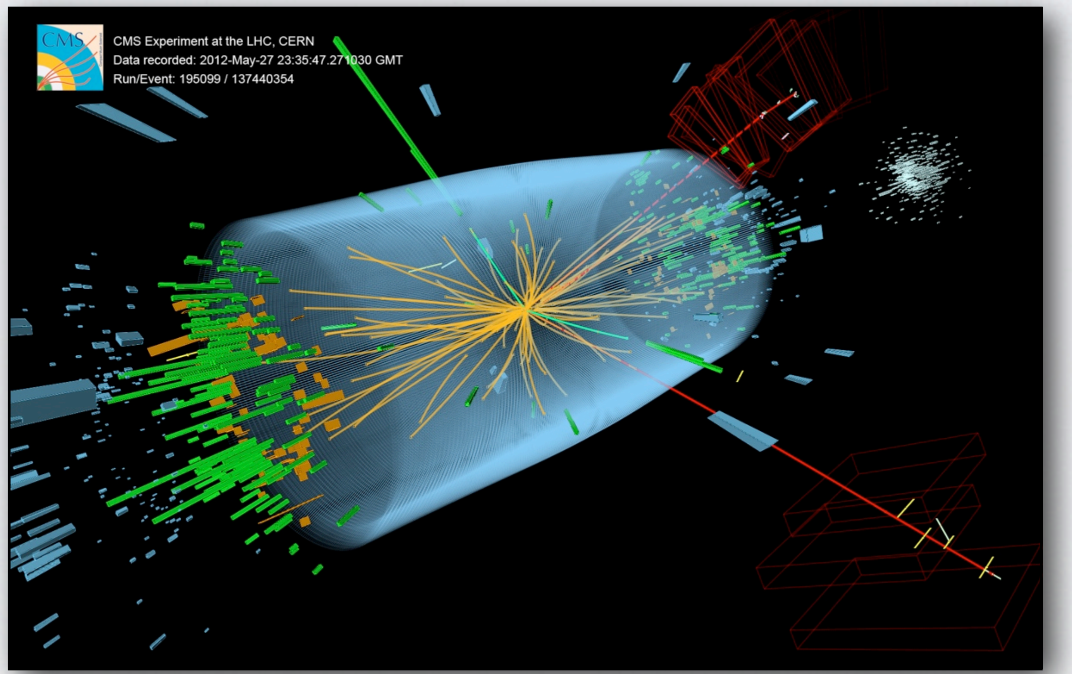


Comparing the α_s fit quality for thrust and heavy jet mass at N^{3}LL (SC)



Joint fit of α_s and hadronization parameter Ω_1 from N^{3}LL thrust (AFHMS)

OUTLOOK



CMS Experiment at the LHC, CERN
Data recorded: 2012-May-27 23:35:47.271030 GMT
Run/Event: 195099 / 137440354

Outlook

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.
- N^3LO is the new NNLO.

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.
- N^3LO is the new NNLO.
- At some point we will have to stop ...

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.
- N^3LO is the new NNLO.
- At some point we will have to stop ...
- ... and just resum ...

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.
- N^3LO is the new NNLO.
- At some point we will have to stop ...
- ... and just resum ...
- The infrared structure of multiparton amplitudes is being clarified to very high and possibly to all orders. In the exponent!

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.
- N^3LO is the new NNLO.
- At some point we will have to stop ...
- ... and just resum ...
- The infrared structure of multiparton amplitudes is being clarified to very high and possibly to all orders. In the exponent!
- Resummation technology is being refined to include new classes of logarithms.

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.
- N^3LO is the new NNLO.
- At some point we will have to stop ...
- ... and just resum ...
- The infrared structure of multiparton amplitudes is being clarified to very high and possibly to all orders. In the exponent!
- Resummation technology is being refined to include new classes of logarithms.
- Resummations will continue to play a pivotal role in precision LHC phenomenology.

Outlook

- Through the vast and remarkable effort of an entire community, ubiquitous and consistent NLO phenomenology at LHC is a reality.
- An ongoing NNLO (r)evolution makes it possible to imagine that in a few years we will be able to state the same about NNLO phenomenology.
- N^3LO is the new NNLO.
- At some point we will have to stop ...
- ... and just resum ...
- The infrared structure of multiparton amplitudes is being clarified to very high and possibly to all orders. In the exponent!
- Resummation technology is being refined to include new classes of logarithms.
- Resummations will continue to play a pivotal role in precision LHC phenomenology.

We are ready for the challenges of LHC Run Two.

THANK YOU!