Parton distributions overview

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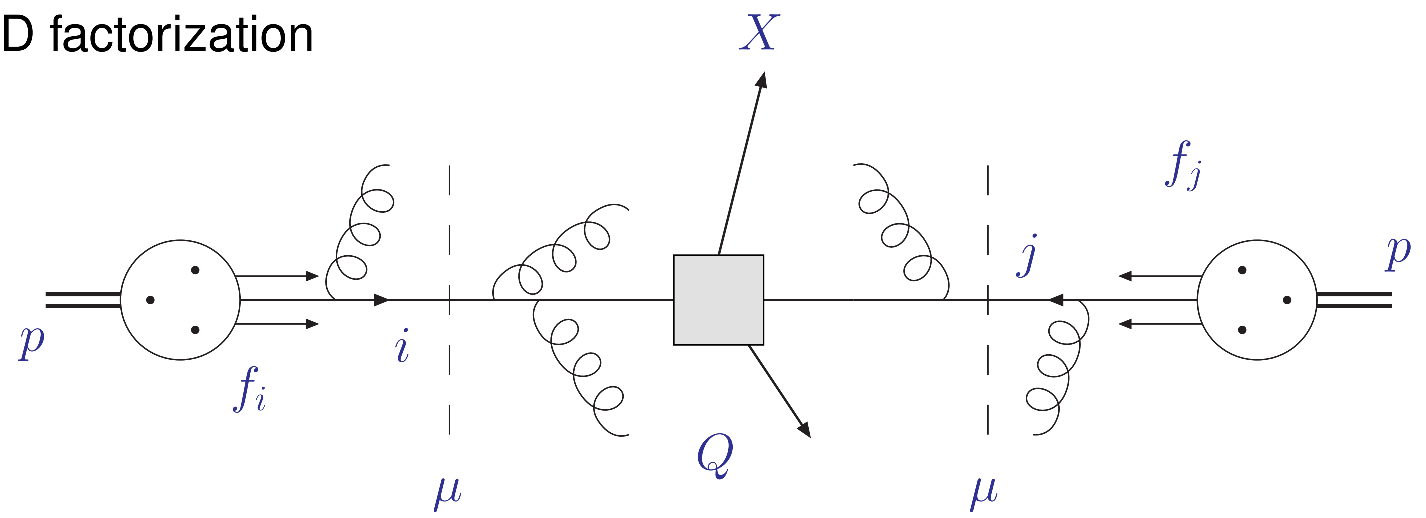
Workshop *QCD@LHC2014*, Suzdal, Aug 25, 2014

Plan

- Cross sections in perturbative QCD
- Non-perturbative input parameters
  - parton distributions
  - strong coupling $\alpha_s(M_Z)$
  - heavy quark masses
- Constraints from new measurements
  - LHC: $W^{\pm}$- and Z-boson production
  - Nomad, Chorus: charm production data in neutrino-nucleon DIS

QCD factorization

- QCD factorization

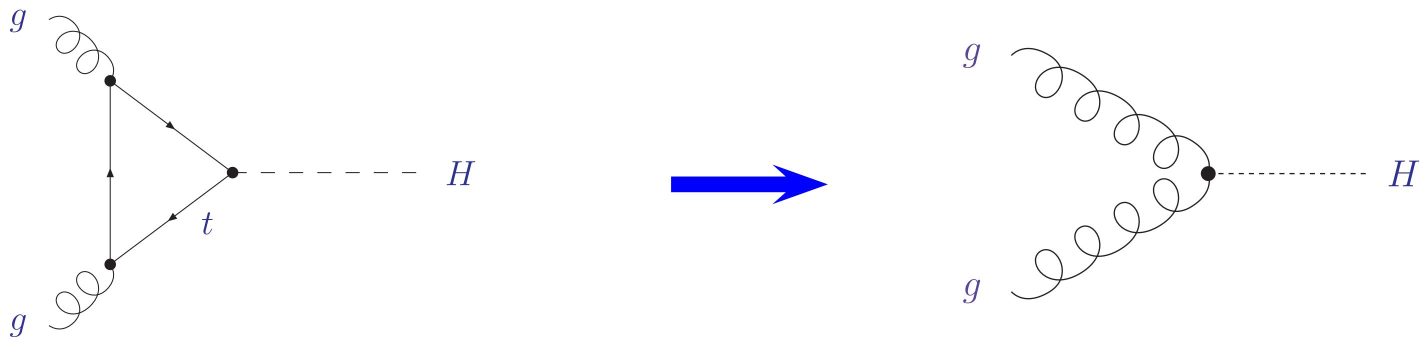


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

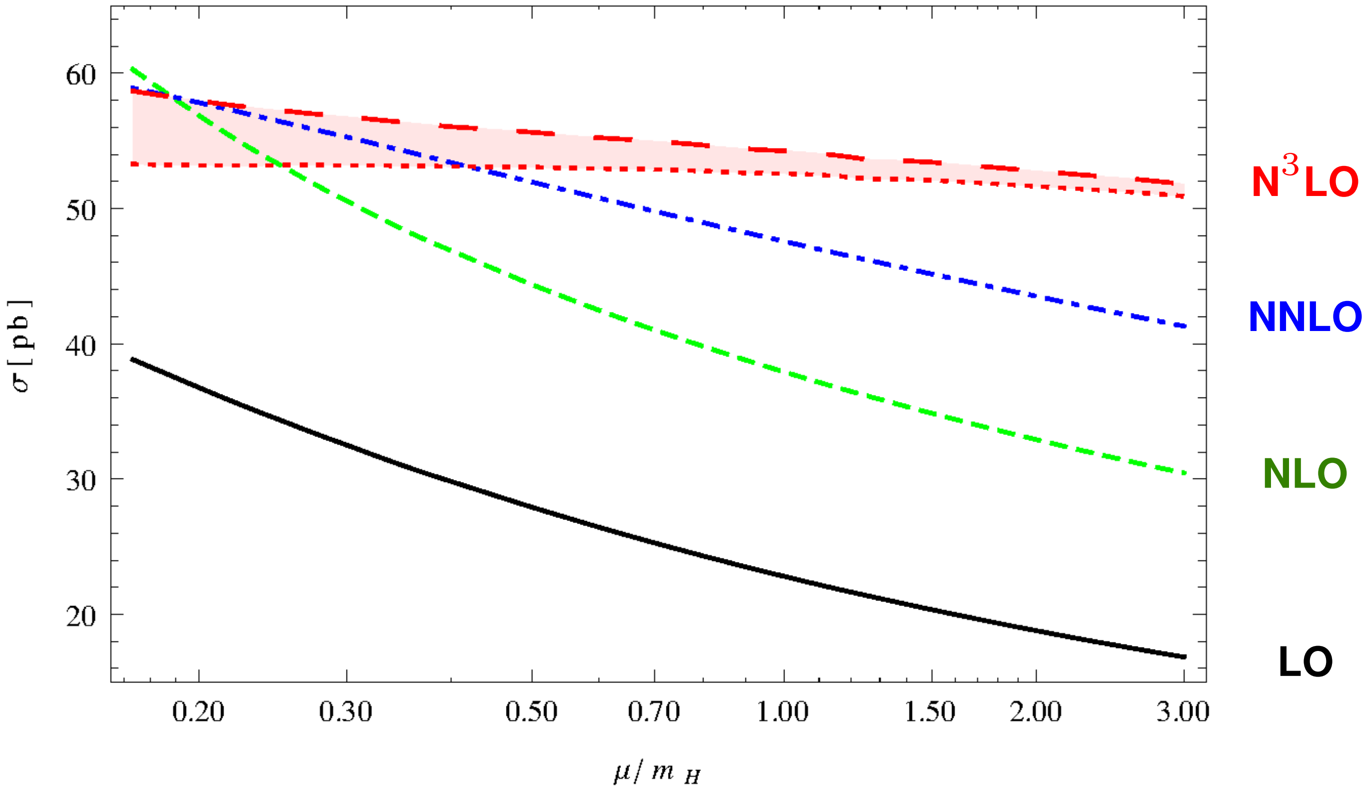
Higgs production in gg-fusion

Effective theory



- Hard scattering cross section $\hat{\sigma}_{ij \to H+X}$ dominated by gluon-gluon fusion
  - typically treated in effective theory in limit $m_t \to \infty$; Lagrangian $\mathcal{L} = -\frac{1}{4} \frac{H}{v} C_H \, G^{\mu\nu\, a} \, G^a_{\mu\nu}$
- QCD corrections significant
  - NNLO corrections still large
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement with soft N^{3}LO corrections S.M., Vogt '05; Laenen, Magnea '05: in x-space '$+$'-distributions $\alpha_s^k \ln^{2k-1}(1-x)/(1-x)$
  - NNLL resummation Catani, de Florian, Grazzini, Nason '03; Ahrens et al. '10; [...]; Ahmed, Mahakhud, Rana, Ravindran '14
  - soft-virtual corrections completed recently: $\delta(1-x)$-term at N^{3}LO
    Anastasiou et al.'14

Perturbation theory at work



- Apparent convergence of perturbative expansion
  - improvement with soft-virtual N^{3}LO corrections (all powers of $\ln^k N$) supplemented by $\ln^k N/N$ corrections from physical kernels
    de Florian, Mazzitelli, S.M., Vogt to appear
- Perturbative stability under renormalization scale variation
  - point of minimal sensitivity around $\mu = m_H/2$
  - overall theoretical uncertainty $\Delta\sigma(H)$ less than 5%

Dependence on parton distributions

- NNLO cross section $\sigma(gg \to H + X)$ at LHC with uncertainties:

$$\sigma(H) + \Delta\sigma(\text{scale}) + \Delta\sigma(\text{PDF} + \alpha_\text{s})$$

	LHC7	LHC8	LHC14
ABM11	$13.23\ ^{+1.35}_{-1.31}\ ^{+0.30}_{-0.30}$	$16.99\ ^{+1.69}_{-1.63}\ ^{+0.37}_{-0.37}$	$44.68\ ^{+4.02}_{-3.78}\ ^{+0.85}_{-0.85}$
ABM12	$13.28\ ^{+1.35}_{-1.32}\ ^{+0.31}_{-0.31}$	$17.05\ ^{+1.68}_{-1.64}\ ^{+0.39}_{-0.39}$	$44.81\ ^{+4.01}_{-3.80}\ ^{+0.94}_{-0.94}$
HiggsXSWG	$15.13\ ^{+1.07}_{-1.18}\ ^{+1.15}_{-1.07}$	$19.27\ ^{+1.39}_{-1.50}\ ^{+1.45}_{-1.33}$	$49.85\ ^{+6.08}_{-4.19}\ ^{+3.69}_{-3.09}$

- Comparison for PDF sets at NNLO
  - ABM11, ABM12 Alekhin, Blümlein, S.M. '13 and HiggsXSWG (used by ATLAS and CMS, relying on MSTW08 Martin, Stirling, Thorne, Watt '09 and with soft gluon resummation)
- Largest differences in predictions from PDFs and value of α_s
  - ABM central values significantly lower by some $11 - 14\%$
  - even more dramatic for Tevatron Baglio, Djouadi '10; Baglio, Djouadi, Ferrag, Godbole '11
- PDF and α_s differences significantly larger than residual theory uncertainty due to incomplete N^{3}LO QCD corrections

Non-perturbative parameters

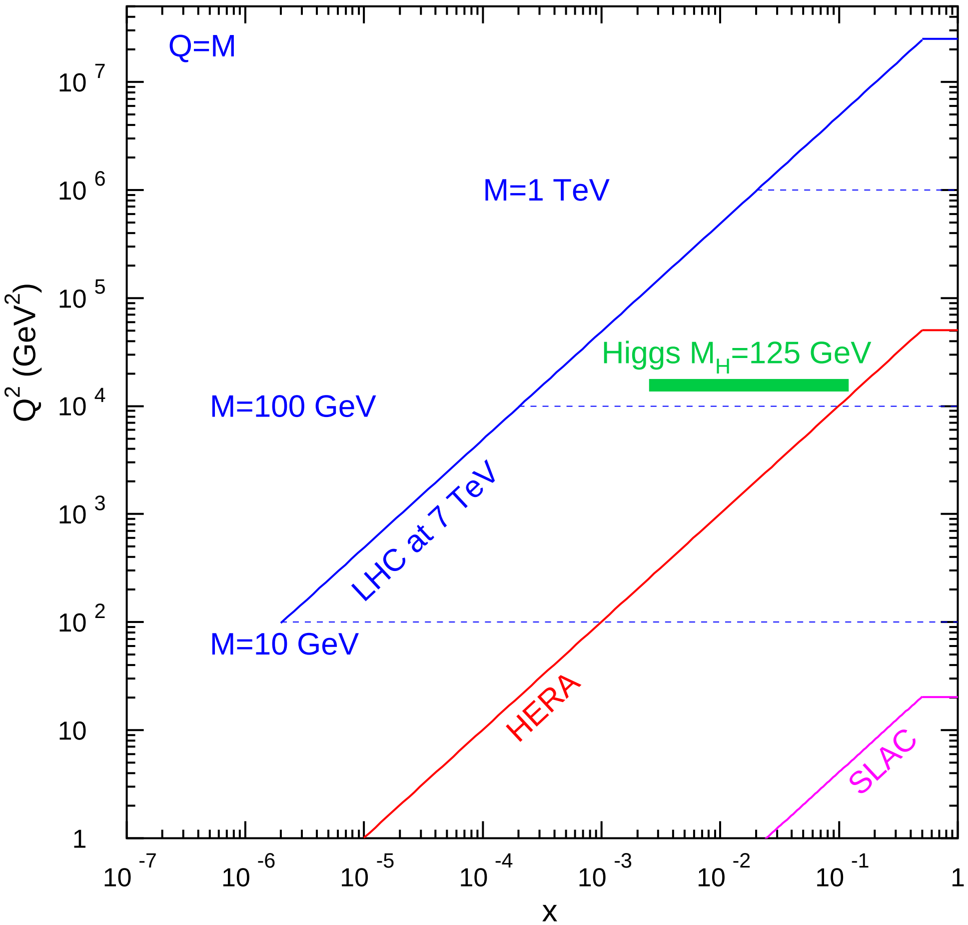
Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparision to experimental data
  - masses of heavy quarks m_c, m_b, m_t
  - parton distribution functions $f_i(x, \mu^2)$
  - strong coupling constant $\alpha_s(M_Z)$

Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - renormalization and factorization scales μ_R, μ_F
  - chosen scheme (e.g. ($\overline{MS}$ scheme)
  - ...

Parton luminosity at LHC



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics at effective $\langle x \rangle = M/\sqrt{S}$
  - 100 GeV physics: small-x, sea partons
  - TeV scales: large-x

Parton distribution fits

Global PDF fits

- PDF sets currently available
  - ABM12 Alekhin, Blümlein, S.M. '13
  - CT10 Gao et al. '13
  - HERAPDF (v1.5) H1 & ZEUS Coll. '11
  - JR09 Jimenez-Delgado, Reya '09
  - MSTW Martin, Stirling, Thorne, Watt '09
  - NNPDF (NN23) Ball et al. '12

Iterative cycle of PDF fits

i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of the non-perturbative parameters

- parton distributions
- strong coupling $\alpha_s(M_Z)$
- heavy quark masses

LHC measurements

General remarks

- QCD corrections important
  - require theory predictions to NNLO accuracy
- PDF fits with 3-flavors for DIS, 5-flavors for LHC data (matching from 3 to 5-flavors)
  - QCD evolution over large range

Benchmark processes

- Complete NNLO QCD corrections available for
  - $W^{\pm}$- and Z-boson production
    Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - top-quark hadro-production Czakon, Fiedler, Mitov '13
- Jet data from Tevatron and LHC
  - QCD corrections only NLO known
  - possible impact of jet definition and algorithm
  - ongoing effort towards NNLO (corrections expected to be as big as $\mathcal{O}(15 - 20\%)$) Gehrmann-De Ridder, Gehrmann, Glover, Pires '13

Example: ABM PDFs

Data considered in the fit

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC
  - semi-inclusive DIS charm production data (HERA)
  - Drell-Yan data (fixed target) E-605, E-866
  - neutrino-nucleon DIS data (di-muon production) CCFR/NuTeV
  - LHC data for $W^{\pm}$- and Z-boson production

Theory considerations

- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Determination of PDFs and strong coupling constant α_s to NNLO QCD
- Consistent scheme for treatment of heavy quarks
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
- Full account of error correlations

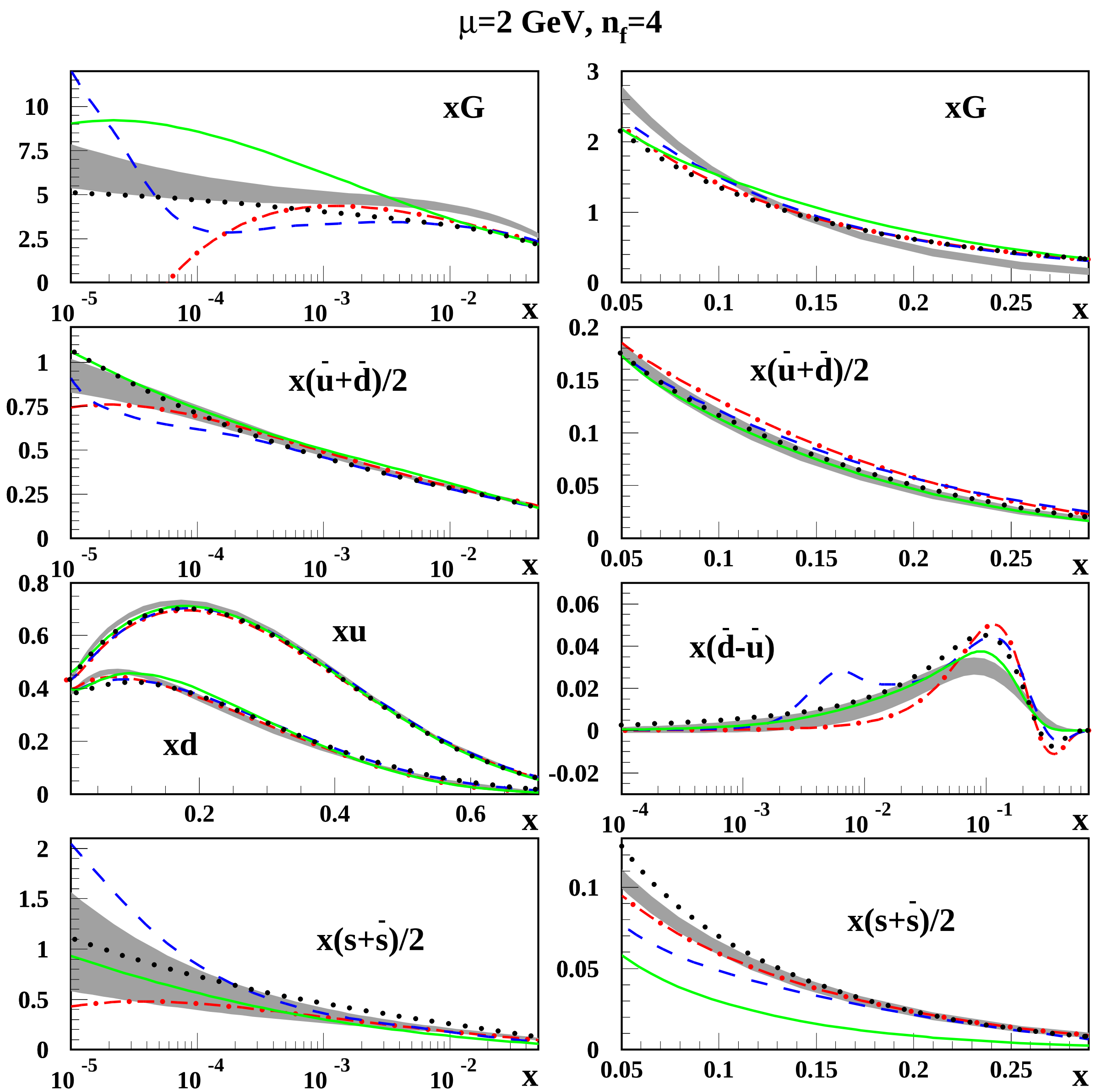
ABM PDF ansatz

- PDFs parameterization at scale $Q_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M. '12
  - ansatz for valence-/sea-quarks, gluon with polynomial $P(x)$
  - strange quark is taken in charge-symmetric form
  - 24 parameters in polynomials $P(x)$
  - 4 additional fit parameters: $\alpha_s^{(n_f=3)}(\mu = 3$ GeV), m_c, m_b and deuteron correction
  - simultaneous fit of higher twist parameters (twist-4)

$$
\begin{aligned}
xq_v(x, Q_0^2) &= \frac{2\delta_{qu} + \delta_{qd}}{N_q^v} x^{a_q} (1-x)^{b_q} x^{P_{qv}(x)} \\
xu_s(x, Q_0^2) = x\bar{u}_s(x, Q_0^2) &= A_{us} x^{a_{us}} (1-x)^{b_{us}} x^{a_{us}\, P_{us}(x)} \\
x\Delta(x, Q_0^2) = xd_s(x, Q_0^2) - xu_s(x, Q_0^2) &= A_\Delta x^{a_\Delta} (1-x)^{b_\Delta} x^{P_\Delta(x)} \\
xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) &= A_s x^{a_s} (1-x)^{b_s} \,, \\
xg(x, Q_0^2) &= A_g x^{a_g} (1-x)^{b_g} x^{a_g\, P_g(x)}
\end{aligned}
$$

- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Parton distributions tuned to LHC data



- 1σ band for ABM12 PDFs (NNLO, 4-flavors) at $\mu = 2$ GeV
  Alekhin, Blümlein, S.M.'13
- comparison with:
  JR09 (solid lines),
  MSTW (dashed dots),
  NN23 (dashes) and
  CT10 (dots)
- Some interesting observations to be made . . .

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (I)

    Alekhin, Blümlein, S.M. ‘12

	a_u	b_u	$\gamma_{1,u}$	$\gamma_{2,u}$	a_d	b_d	A_d	b_Δ	A_u	a_{us}	b_{us}	a_G	b_G
a_u	1.0000	0.9256	0.9638	-0.2527	0.3382	0.2922	0.1143	-0.4267	0.4706	0.3117	0.1422	0.0982	0.1127
b_u		1.0000	0.9574	-0.5608	0.1933	0.1200	0.1058	-0.3666	0.3712	0.2674	0.1537	0.0453	0.1878
$\gamma_{1,u}$			1.0000	-0.4504	0.2328	0.2329	0.0906	-0.3379	0.4106	0.2876	0.0812	0.0491	0.1627
$\gamma_{2,u}$				1.0000	0.3007	0.3119	-0.0242	-0.0118	0.0587	0.0026	-0.0305	0.0949	-0.1876
a_d					1.0000	0.8349	-0.2010	-0.3371	0.3786	0.2592	0.1212	-0.0377	0.1305
b_d						1.0000	-0.2669	-0.0599	0.2768	0.1941	-0.0698	-0.0926	0.2088
A_d							1.0000	-0.2132	0.0549	0.0245	0.2498	-0.0523	0.0614
b_Δ								1.0000	-0.1308	-0.0729	-0.7208	-0.0124	-0.0225
A_u									1.0000	0.9240	-0.0723	0.3649	-0.1674
a_{us}										1.0000	-0.0144	0.2520	-0.1095
b_{us}											1.0000	-0.1274	0.1808
a_G												1.0000	-0.6477
b_G													1.0000

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (II)

    Alekhin, Blümlein, S.M. '12

	$\gamma_{1,G}$	$\alpha_s(3,3\ \mathrm{GeV})$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	A_s	b_s	a_s	a_Δ	m_c	m_b
a_u	-0.0727	-0.0611	0.3383	0.6154	0.2320	-0.0724	-0.0681	-0.0763	-0.0935	0.0026	0.0900	-0.0053
b_u	-0.1130	-0.1725	0.2992	0.4848	0.0849	0.0720	-0.0723	-0.0618	-0.0926	0.0049	0.0349	-0.0118
$\gamma_{1,u}$	-0.1106	-0.1338	0.2753	0.5638	0.1316	-0.0535	-0.0798	-0.0854	-0.1059	-0.0060	0.0817	0.0003
$\gamma_{2,u}$	0.1174	0.2195	-0.0210	0.0822	0.3712	-0.3310	0.0339	0.0143	0.0381	-0.0098	0.0430	-0.0004
a_d	-0.1631	-0.0208	0.0319	0.4974	0.9570	-0.4636	-0.0700	-0.0996	-0.0979	-0.2121	0.1066	-0.0150
b_d	-0.2198	-0.0913	-0.1775	0.4092	0.8985	-0.8498	-0.0533	-0.0669	-0.0806	-0.2252	0.0822	-0.0068
A_d	-0.0825	0.0188	0.8558	-0.0289	-0.2624	0.2852	-0.0075	-0.0189	-0.0180	0.9602	0.0420	0.0120
b_Δ	0.0530	-0.0801	-0.6666	-0.0904	-0.1981	-0.2532	-0.0022	0.0257	0.0048	-0.0260	-0.0166	-0.0056
A_u	0.2502	-0.0157	0.1265	0.7525	0.3047	-0.0668	-0.7064	-0.6670	-0.7267	0.0345	0.2137	0.0358
a_{us}	0.1845	-0.0216	0.0683	0.5714	0.2157	-0.0554	-0.8768	-0.8081	-0.8980	0.0145	0.0430	0.0074
b_{us}	-0.1619	-0.0715	0.5343	-0.3656	0.0293	0.2430	-0.0345	-0.0132	-0.0356	0.1527	-0.0899	-0.0058
a_G	0.8291	0.2306	-0.0260	0.3692	-0.0966	0.1496	0.0087	0.0007	0.0464	-0.0541	-0.0661	0.0417
b_G	-0.9184	-0.6145	0.0538	-0.2770	0.1990	-0.2552	0.0381	0.0616	-0.0468	0.0502	0.1847	0.0861

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

Covariance matrix

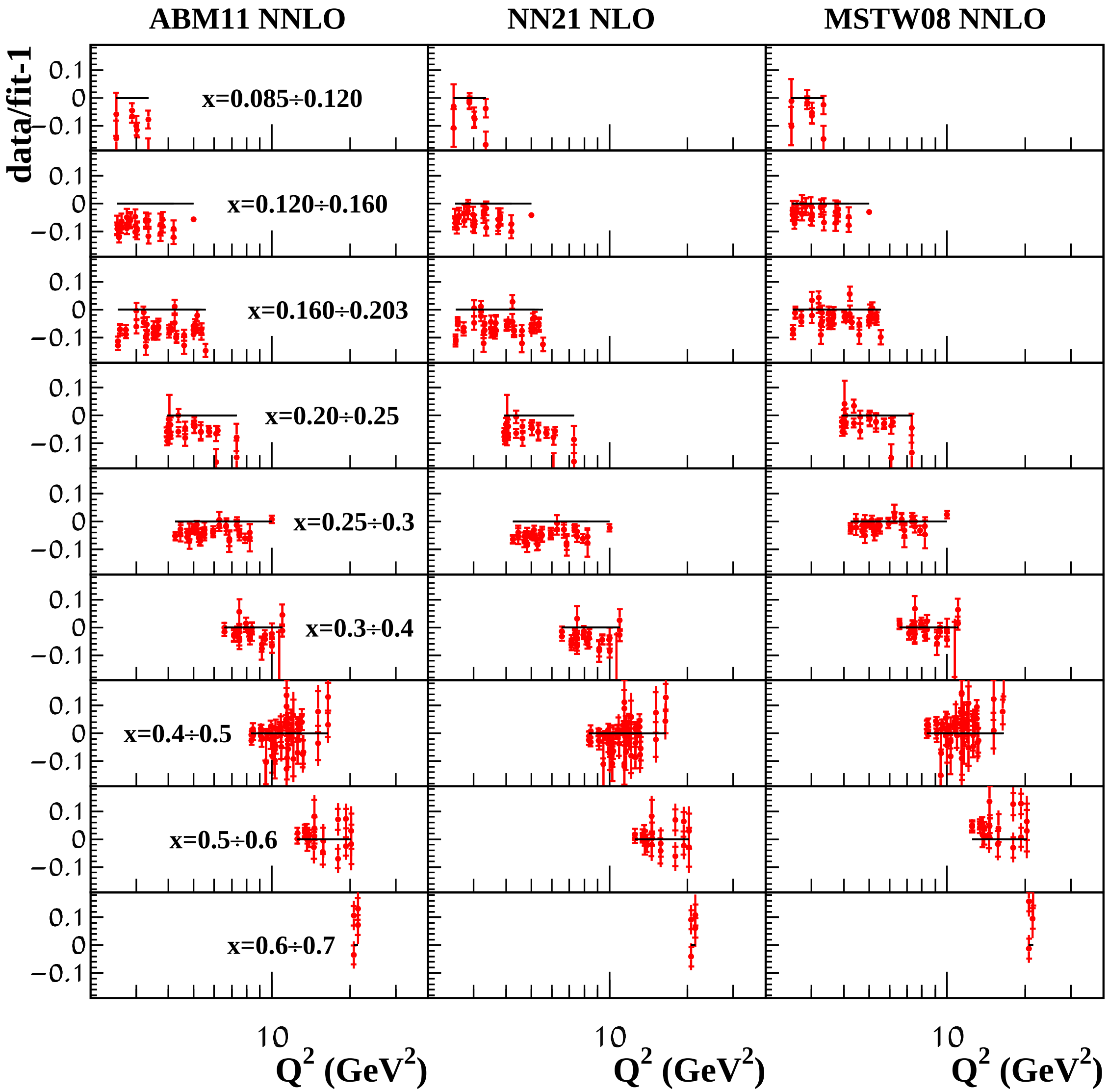
- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (III)

    Alekhin, Blümlein, S.M. '12

	$\gamma_{1,G}$	$\alpha_s(3,3\ \mathrm{GeV})$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	A_s	b_s	a_s	a_Δ	m_c	m_b
$\gamma_{1,G}$	1.0000	0.3546	-0.0876	0.2751	-0.2215	0.2410	-0.0539	-0.0634	0.0122	-0.0658	-0.1149	-0.0474
$\alpha_s(3,3\ \mathrm{GeV})$		1.0000	0.0601	0.1127	-0.0761	0.1534	-0.0176	-0.0121	0.0883	0.0022	-0.5641	-0.0526
$\gamma_{1,\Delta}$			1.0000	0.0699	-0.1081	0.3796	-0.0050	-0.0329	-0.0175	0.7098	0.0418	0.0113
$\gamma_{1,us}$				1.0000	0.4099	-0.1547	-0.2622	-0.3181	-0.2801	-0.0785	0.1870	0.0103
$\gamma_{1,d}$					1.0000	-0.6540	-0.0688	-0.0892	-0.0974	-0.2332	0.0999	-0.0093
$\gamma_{2,d}$						1.0000	0.0212	0.0128	0.0413	0.1876	-0.0396	-0.0049
A_s							1.0000	0.8584	0.9689	-0.0109	0.0596	0.0116
b_s								1.0000	0.8826	-0.0173	-0.0777	0.0003
a_s									1.0000	-0.0204	-0.0845	-0.0145
a_Δ										1.0000	0.0385	0.0085
m_c											1.0000	0.1451
m_b												1.0000

Testing higher twist

- Fit of SLAC data without higher twist contributions (data cut at $W^2 > 12.5\ \mathrm{GeV}^2$, $Q^2 > 2.5\ \mathrm{GeV}^2$)



Strong coupling constant

Essential facts

- $\alpha_s(M_Z)$ from e^+e^- data high
- $\alpha_s(M_Z)$ from DIS data low
- World average 1992
  $\alpha_s(M_Z) = 0.117 \pm 0.004$

Bethke, Catani CERN TH-6484/92

	Process	Ref.	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	order of perturb.
1	R_τ [LEP]	[7-10]	1.78	$0.318^{+0.048}_{-0.039}$	$0.117^{+0.006}_{-0.005}$	$^{+0.003}_{-0.004}$	$^{+0.005}_{-0.004}$	NNLO
2	R_τ [world]	[2]	1.78	0.32 ± 0.04	$0.118^{+0.004}_{-0.006}$	–	–	NNLO
3	DIS [ν]	[3]	5.0	$0.193^{+0.019}_{-0.018}$	$0.111^{+0.006}_{-0.007}$	$^{+0.004}_{-0.006}$	0.004	NLO
4	DIS [μ]	[12]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
5	$J/\Psi, \Upsilon$ decay	[4]	10.0	$0.167^{+0.015}_{-0.011}$	$0.113^{+0.007}_{-0.005}$	–	–	NLO
6	e^+e^- [σ_{had}]	[14]	34.0	0.163 ± 0.022	0.135 ± 0.015	–	–	NNLO
7	e^+e^- [shapes]	[15]	35.0	0.14 ± 0.02	0.119 ± 0.014	–	–	NLO
8	$p\bar{p} \to b\bar{b}X$	[11]	20.0	$0.136^{+0.025}_{-0.024}$	$0.108^{+0.015}_{-0.014}$	0.006	$^{+0.014}_{-0.013}$	NLO
9	$p\bar{p} \to W$ *jets*	[13]	80.6	0.123 ± 0.027	0.121 ± 0.026	0.018	0.020	NLO
10	$\Gamma(Z^0 \to \text{had.})$	[5]	91.2	0.133 ± 0.012	0.133 ± 0.012	0.012	$^{+0.003}_{-0.001}$	NNLO
11	Z^0 ev. shapes							
	ALEPH	[7]	91.2	$0.119^{+0.008}_{-0.010}$		–	–	NLO
	DELPHI	[8]	91.2	0.113 ± 0.007		0.002	0.007	NLO
	L3	[9]	91.2	0.118 ± 0.010		–	–	NLO
	OPAL	[10]	91.2	$0.122^{+0.006}_{-0.005}$		0.001	$^{+0.006}_{-0.005}$	NLO
	SLD	[6]	91.2	$0.120^{+0.015}_{-0.013}$		0.009	$^{+0.012}_{-0.009}$	NLO
	Average	[6-10]	91.2		0.119 ± 0.006	0.001	0.006	NLO
12	Z^0 ev. shapes							
	ALEPH	[7]	91.2	0.125 ± 0.005		0.002	0.004	resum.
	DELPHI	[8]	91.2	0.122 ± 0.006		0.002	0.006	resum.
	L3	[9]	91.2	0.126 ± 0.009		0.003	0.008	resum.
	OPAL	[10]	91.2	$0.122^{+0.003}_{-0.006}$		0.001	$^{+0.003}_{-0.006}$	resum.
	Average	[7-10]	91.2		0.123 ± 0.005	0.001	0.005	resum.

Table 1: Summary of measurements of α_s. For details see text.

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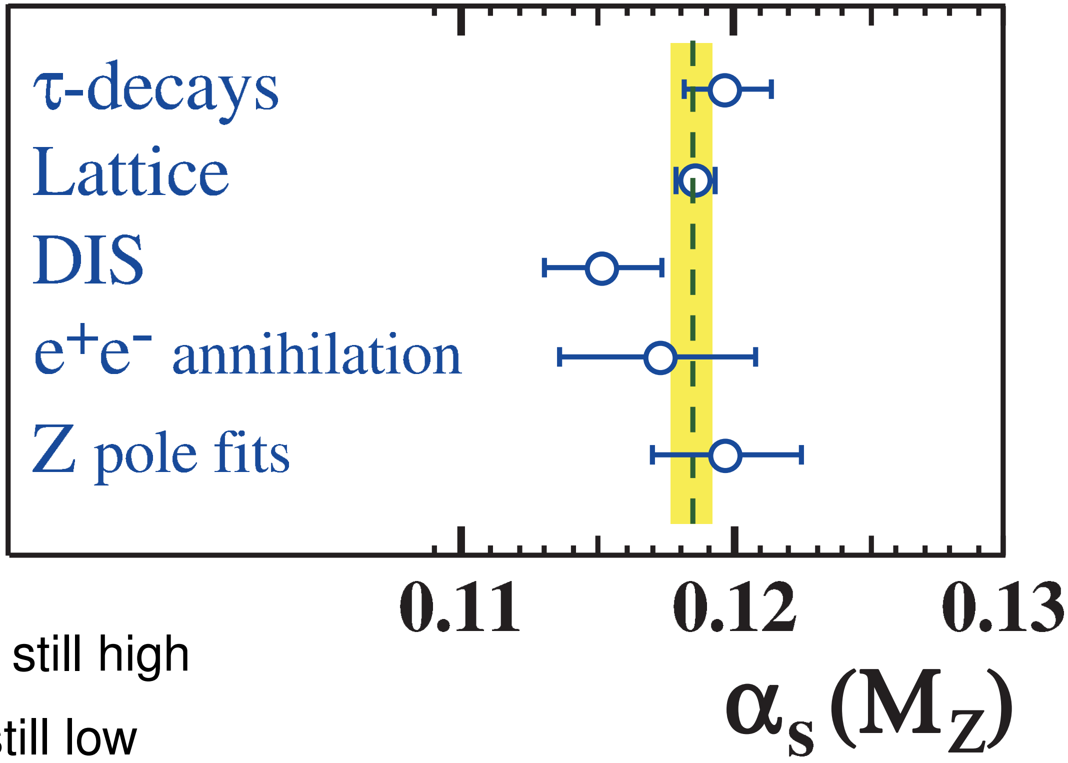
α_s 2014

Bethke in PDG 2012



- $\alpha_s(M_Z)$ from e^+e^- data still high
- $\alpha_s(M_Z)$ from DIS data still low
- World average for $\alpha_s(M_Z)$ based on arithmetic average of (pre-averaged) $\alpha_s(M_Z)$ values from different methods/processes

Measurements of α_s

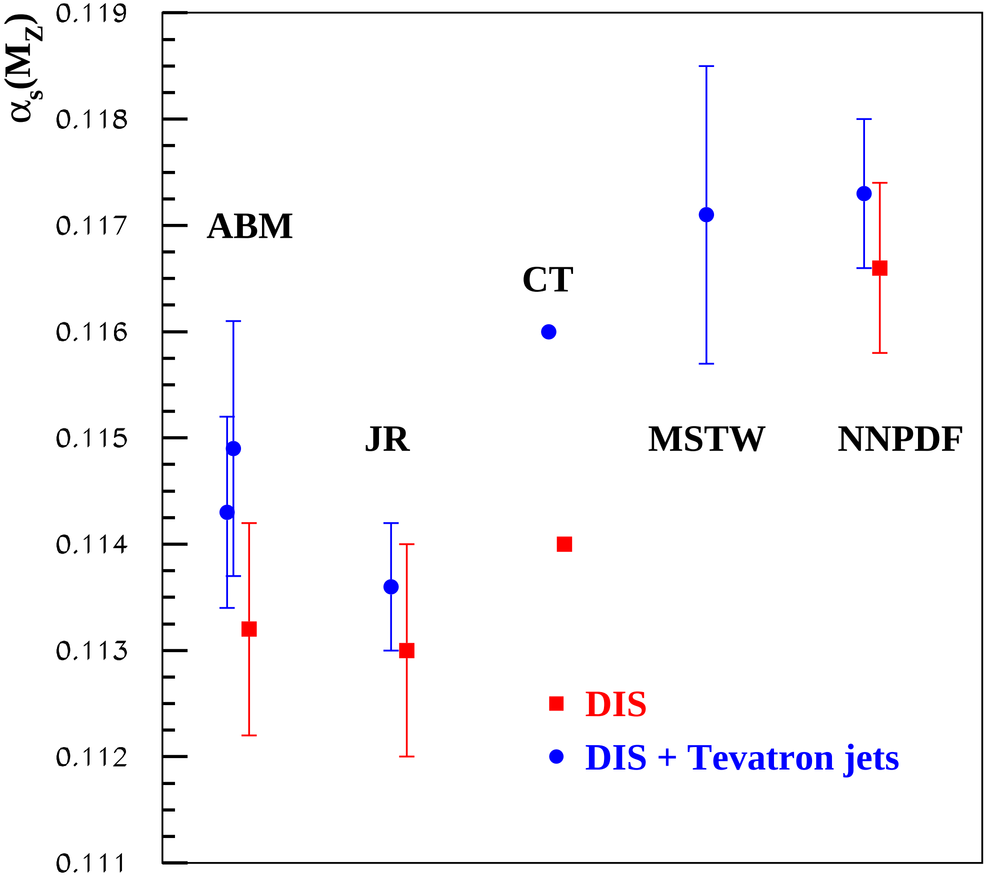
- Values of $\alpha_s(M_Z)$ at NNLO from PDF fits
  (compilation from proceedings of MITP workshop arXiv:1405.4781)

Alekhin [2001]	0.1143 ± 0.013	DIS	Alekhin '01
BBG [2004]	$0.1134\ ^{+\ 0.0019}_{-\ 0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti '06
GRS	0.112	valence analysis, NNLO	Glück, Reya, Schuck '06
ABKM	0.1135 ± 0.0014	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. '09
JR14	0.1136 ± 0.0004	dynamical approach	Jimenez-Delgado, Reya '14
JR14	0.1162 ± 0.0006	including NLO jets	Jimenez-Delgado, Reya '14
MSTW	0.1171 ± 0.0014		Martin, Stirling, Thorne, Watt '09
Thorne	0.1136	[DIS+DY, HT*] (2014)	Thorne '14
ABM11$_J$	$0.1134 - 0.1149$ ± 0.0012	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. '11
ABM13	0.1133 ± 0.0011		Alekhin, Blümlein, S.M. '13
ABM13	0.1132 ± 0.0011	(without jets)	Alekhin, Blümlein, S.M. '13
CTEQ	$0.1159 ... 0.1162$		Gao et al. '13
CTEQ	0.1140	(without jets)	Gao et al. '13
NN21	0.1174 ± 0.0007		NNPDF '11

- Values of $\alpha_s(M_Z)$ at NNLO from related measurements and lattice

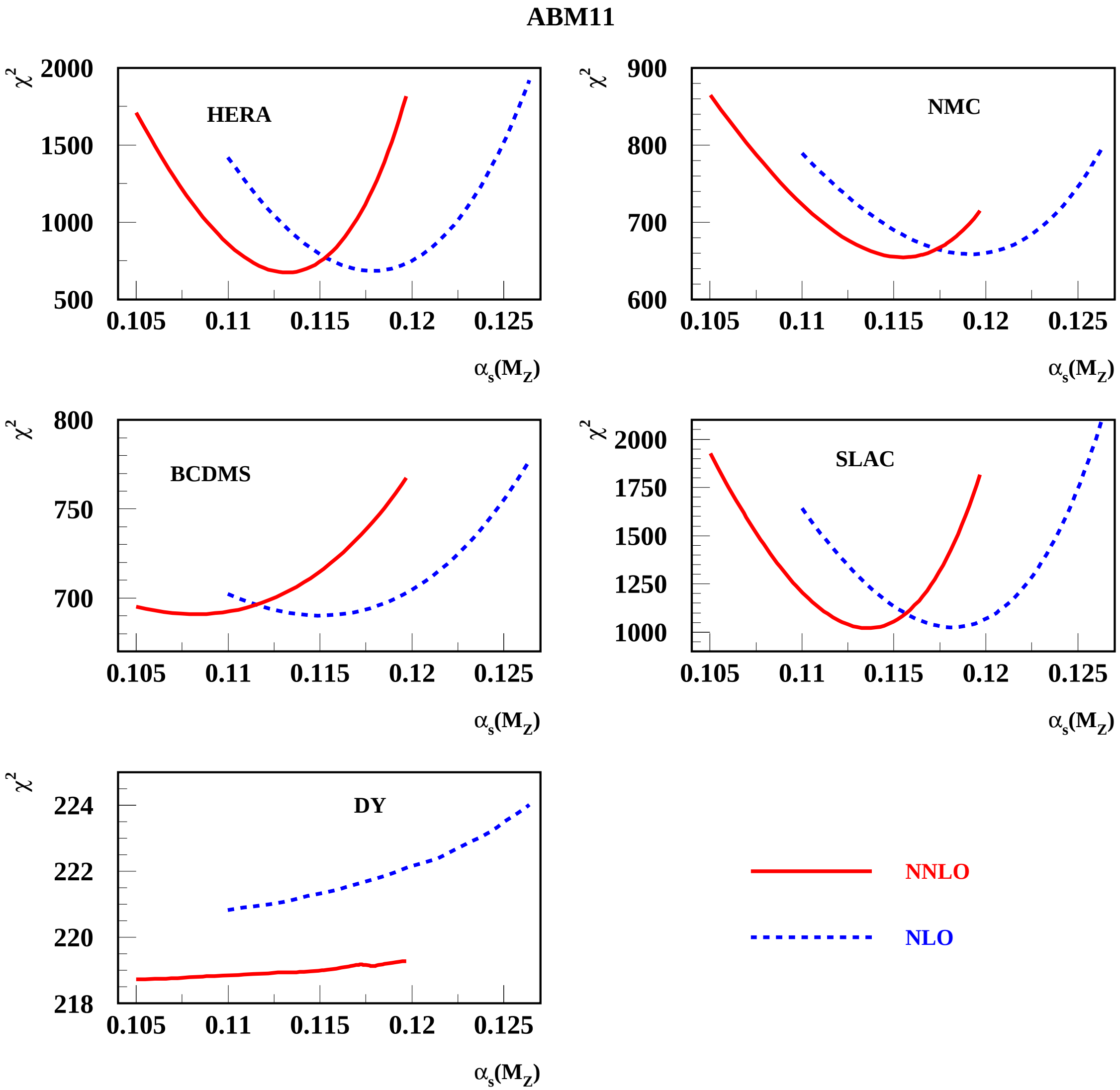
e^+e^- thrust	$0.1131 \begin{smallmatrix} +\ 0.0028 \\ -\ 0.0022 \end{smallmatrix}$	Gehrmann et al.	arXiv:1210.6945
e^+e^- thrust	0.1140 ± 0.0015	Abbate et al.	arXiv:1204.5746
CMS	0.1151 ± 0.0033	$t\bar{t}$	arXiv:1307.1907
NLO Jets ATLAS	$0.111 \begin{smallmatrix} +\ 0.0017 \\ -\ 0.0007 \end{smallmatrix}$		arXiv:1312.5694
NLO Jets CMS	0.1148 ± 0.0055		arXiv:1312.5694
3-jet rate	0.1175 ± 0.0025	Dissertori et al. 2009	arXiv:0910.4283
Z-decay	0.1189 ± 0.0026	BCK 2008/12 (N^{3}LO)	arXiv:1201.5804
τ decay	0.1212 ± 0.0019	BCK 2008	arXiv:0801.1821
τ decay	0.1204 ± 0.0016	Pich 2011	arXiv:1110.0016
τ-decay rate	0.325 ± 0.018 (at m_τ)	FOTP:	Jamin '13
τ-decay rate	0.374 ± 0.025 (at m_τ)	CIPT:	Jamin '13
lattice	0.1205 ± 0.0010	PACS-CS 2009 (2+1 fl.)	arXiv:0906.3906
lattice	0.1184 ± 0.0006	HPQCD 2010	arXiv:1004.4285
lattice	0.1200 ± 0.0014	ETM 2012 (2+1+1 fl.)	arXiv:1201.5770
lattice	0.1156 ± 0.0022	2012 (2+1 fl.)	arXiv:1205.6155
lattice	$0.1130 \pm 0.0010(stat)$	RBC-UKQCD (prel., 2014)	UKQCD '14
world average	0.1184 ± 0.0007	(2012)	arXiv:1210.0325

α_s *from DIS and PDFs*



- Significant spread of $\alpha_s(M_Z)$ values from DIS determinations
  Alekhin, Blümlein, S.M. ‘13

α_s *from DIS and PDFs*



- Profile of χ^2 for different data sets in ABM11 PDF fit Alekhin, Blümlein, S.M. '12

Comparison of α_s determinations

- Differences in α_s values:
  - result from different physics models and analysis procedures
    - target mass corrections (powers of nucleon mass M_N^2/Q^2)
    - higher twist $F_2^{\rm ht} = F_2 + ht^{(4)}(x)/Q^2 + ht^{(6)}(x)/Q^4 + \dots$
    - error correlations
- Effects for differences between ABM, MSTW and NN21 understood
  - variants of ABM with no higher twist etc. reproduce larger α_s values

	α_s at NNLO	target mass corr.	higher twist	error correl.
ABM11	0.1134 ± 0.0011	yes	yes	yes
NNPDF21	0.1166 ± 0.0008	yes	no	yes
MSTW	0.1171 ± 0.0014	no	no	no

Treatment of heavy-quarks

Light quarks

- Neglect "light quark" masses $m_u, m_d \ll \Lambda_{QCD}$ and $m_s < \Lambda_{QCD}$ in hard scattering process
  - scale-dependent u, d, s, g PDFs from mass singularities

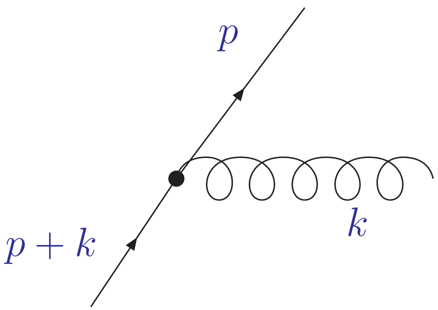
Heavy quarks

- No mass singularities for $m_c, m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
  - c and b PDFs for $Q \ggg m_c, m_b$ generated perturbatively
  - matching of two distinct theories
    $\longrightarrow n_f$ light flavors + heavy quark of mass m at low scales
    $\longrightarrow n_f + 1$ light flavors at high scales

Soft and collinear singularities

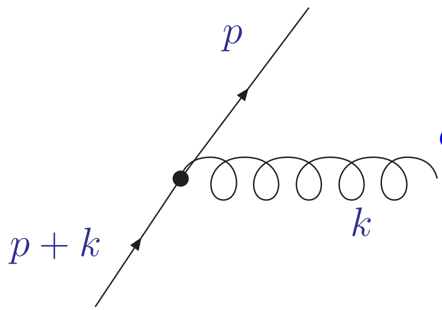
- Soft/collinear regions of phase space
  - massless partons

$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g(1-\cos\theta_{qg})}$$



Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



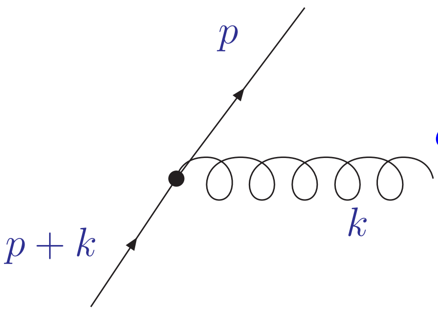
$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg}\, \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



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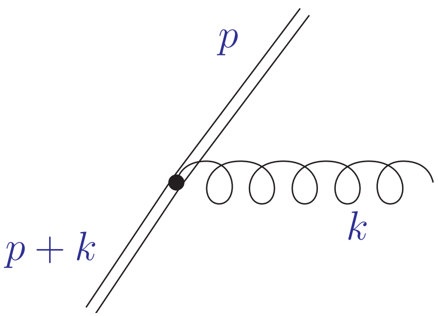
$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

- Parton masses regulate collinear singularity

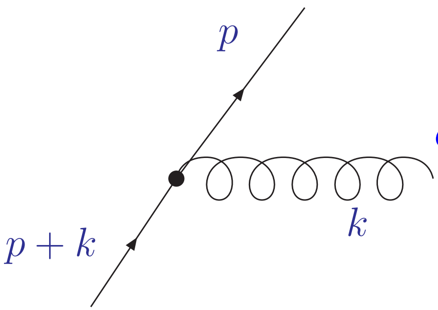


$$\frac{1}{(p+k)^2 - m_q^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g (1-\beta\cos\theta_{qg})}$$

with $\beta = \left(1 - \frac{m_q^2}{E_q^2}\right)^{1/2} < 1$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg} \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

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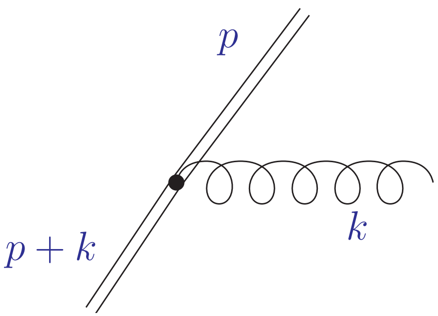
- Parton masses regulate collinear singularity



$$\frac{1}{(p+k)^2 - m_q^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_qE_g(1-\beta\cos\theta_{qg})}$$

with $\beta = \left(1 - \frac{m_q^2}{E_q^2}\right)^{1/2} < 1$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2 - m_q^2} \longrightarrow \alpha_s \frac{1}{\epsilon} \ln(m_q^2) \times (\dots)$$

Treatment of heavy-quarks

Charm structure function

- F_2^c at HERA (assume no “intrinsic charm”)
  - $Q \not\gg m_c$: Fixed flavor-number scheme FFNS
    u, d, s, g partons and massive charm coeff. fcts.
  - $Q \ggg m_c$: Zero-mass variable flavor-number scheme ZM-VFNS
    terms $m_c/Q \to 0$, $n_f = 4$ PDFs (matching), $m_c = 0$ coeff. fcts.
  - $Q \gg m_c$: General-mass variable flavor-number scheme GM-VFNS
    terms $m_c/Q \neq 0$, but quasi-collinear logs $\ln(Q/m_c)$ large
    $n_f = 4$ PDFs, “interpolating” coeff. fcts. (matching prescriptions)

FFNS

- Perturbative QCD predictions for F_2^c and F_L^c (neutral current)
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven '92
  - approximate expressions to NNLO
    Laenen, S.M. '98; Alekhin, S.M. '08; Lo Presti, Kawamura, S.M., Vogt '10
  - asymptotic NNLO terms at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein '09;
    Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock '14

VFNS

- Variable flavor number schemes $\longrightarrow$ matching of two distinct theories
  Aivazis, Collins, Olness, Tung '94; Thorne, Roberts '98;
  Buza, Matiounine, Smith, van Neerven '98
  $\longrightarrow$ n_f light flavors + heavy quark of mass m at low scales
  $\longrightarrow$ $n_f + 1$ light flavors at high scales
- Important aspects of variable flavor number schemes
  - mass factorization to be carried out before resummation
    $\longrightarrow$ mass factorization involves both heavy and light component of structure function
  - matching conditions required through NNLO
    Chuvakin, Smith, van Neerven '00
- Details of implementation matter in global fits

VFNS implementation

- GM-VFNS implementation using BSMN
  Buza, Matiounine, Smith, van Neerven '98
- DIS structure function F_2^h for heavy-quark h

$$F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2) =$$
$$= \quad F_2^{h,\mathrm{exact}}(N_f,x,Q^2) + F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2) - F_2^{h,\mathrm{asymp}}(N_f,x,Q^2)$$

  - $F_2^{h,\mathrm{exact}}$: massive heavy-quark structure function ($m \neq 0$)
  - $F_2^{h,\mathrm{ZMVFN}}$: DIS structure function with zero mass ($m = 0$)
  - $F_2^{h,\mathrm{asymp}}$: asymptotic expansion of heavy-quark structure function (logarithms $\ln(Q^2/m^2)$)

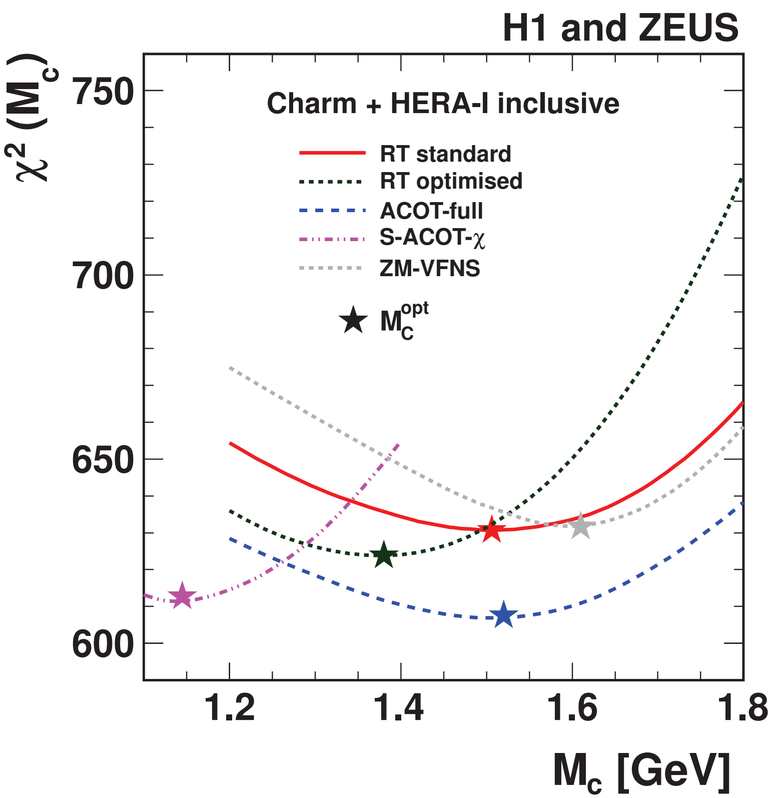
Heavy quark mass

- Data on F_2^c at HERA has correlation of m_c, $\alpha_S(M_Z)$, gluon PDF

  $\sigma_{c\bar{c}} \sim \alpha_s \, m_c^2 \, g(x)$

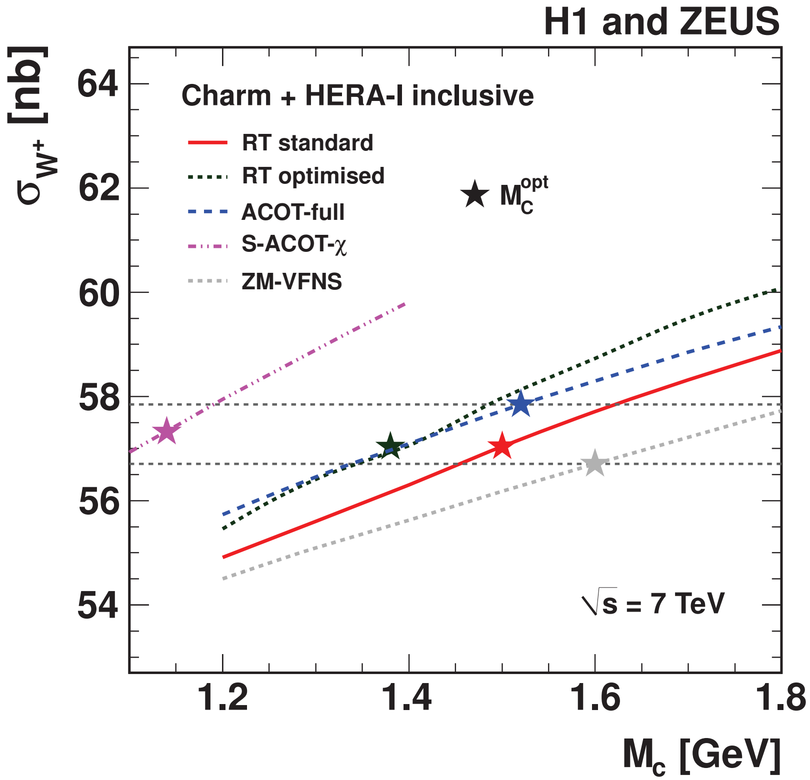
- Comparison of measured data with predictions in various VFNS schemes
  - data shows very good sensitivity to value of m_c
  - fit of value of m_c strongly dependent on particular choice of VFNS

    H1 coll. arxiv:1211.1182



Heavy quark mass

- Significant impact on cross section predictions at LHC
  - e.g., W^+-production



Quark masses in PDF fits

- Choice of value for heavy-quark masses part of uncertainty
- PDF fits assume pole mass scheme for heavy-quarks
  - numerical values systematically lower than those from PDG (2-loop conversion to pole mass)

[GeV]	PDG	ABKM	GJR	HERAPDF	MSTW	CT10	NNPDF21
m_c	1.66 $^{+0.09}_{-0.15}$	1.5 $^{+0.25}_{-0.25}$	1.3	1.4 $^{+0.25}_{-0.05}$	1.3	1.3	1.41
m_b	4.79 $^{+0.19}_{-0.08}$	4.5 $^{+0.5}_{-0.5}$	4.2	4.75 $^{+0.25}_{-0.45}$	4.75	4.75	4.75

PDG

- PDG quotes running masses:
  charm: $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV, bottom: $m_b(m_b) = 4.20^{+0.17}_{-0.07}$ GeV

ABM

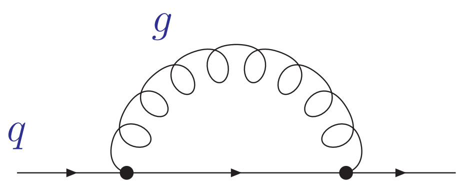
- ABM uses running masses, e.g., ABM11:
  charm: $m_c(m_c) = 1.27^{+0.08}_{-0.08}$ GeV, bottom: $m_b(m_b) = 4.19^{+0.13}_{-0.13}$ GeV

Quark mass renormalization

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2 = m_q^2}$$



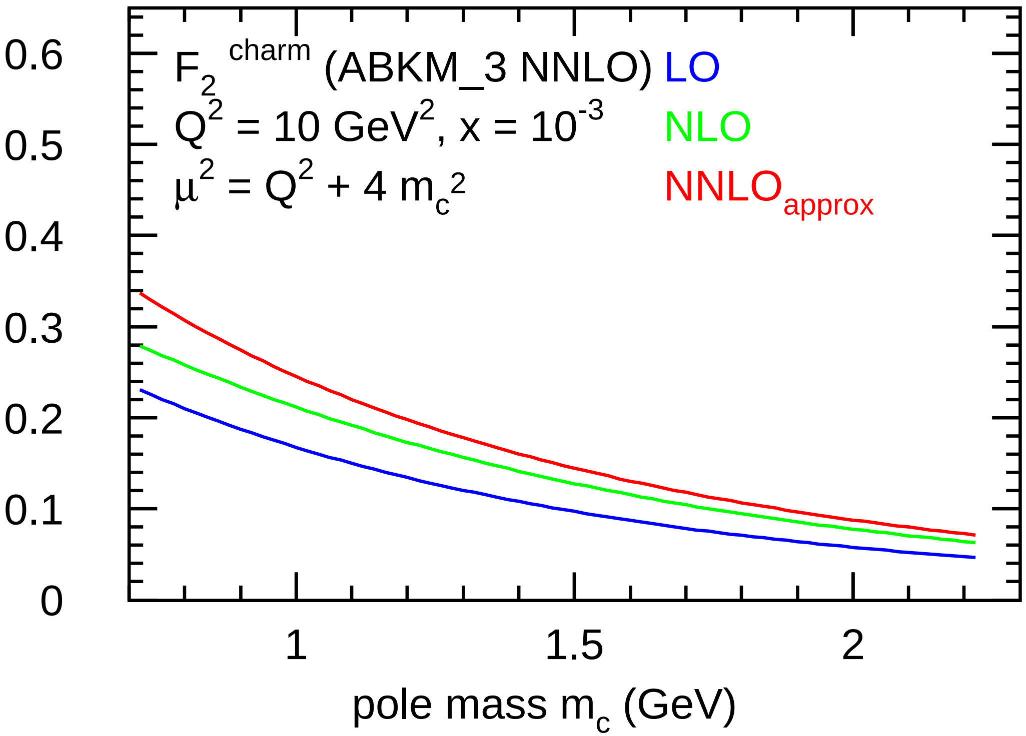
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Ambiguity Δm_q in definition of pole mass up to corrections $\mathcal{O}(\Lambda_{QCD})$
Bigi, Shifman, Uraltsev, Vainshtein '94; Beneke, Braun '94; Smith, Willenbrock '97
  - lattice QCD bound: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV Bauer, Bali, Pineda '11

Short distance mass

- Short distance masses ($\overline{MS}$, 1S Hoang, Teubner '99, PS Beneke '98, . . .) probe at scale of hard interaction: $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
- Conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ in perturbation theory
Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99

Running quark masses in DIS

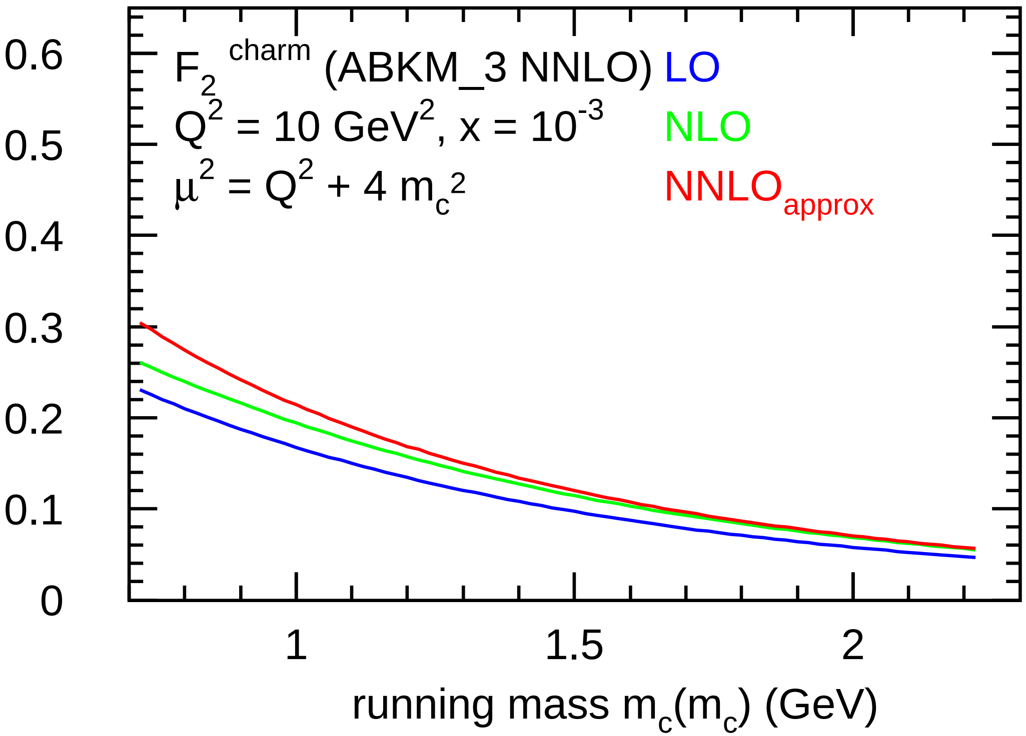
- Charm structure function



- Running quark masses in DIS
  - improved convergence
  - reduced scale dependence
- Comparison with pole mass scheme

Running quark masses in DIS

- Charm structure function



- Running mass
- Direct determination of $m_c(m_c)$ with all correlations

  Alekhin, Blümlein, Daum, Lipka, S.M. '12

  NLO

  $1.15 \pm 0.04\ (\mathrm{exp.})\ ^{+0.04}_{-0.00}(\mathrm{th.})$ GeV

  NNLO$_{\mathrm{approx}}$

  $1.24 \pm 0.03\ (\mathrm{exp.})\ ^{+0.03}_{-0.02}(\mathrm{th.})$ GeV

- PDG quotes running masses:

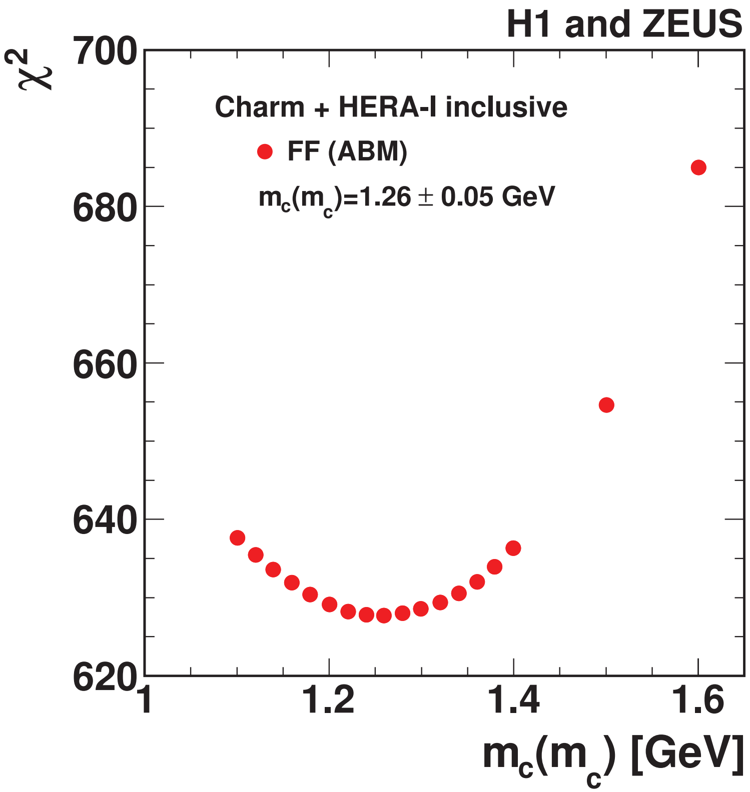
  $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV

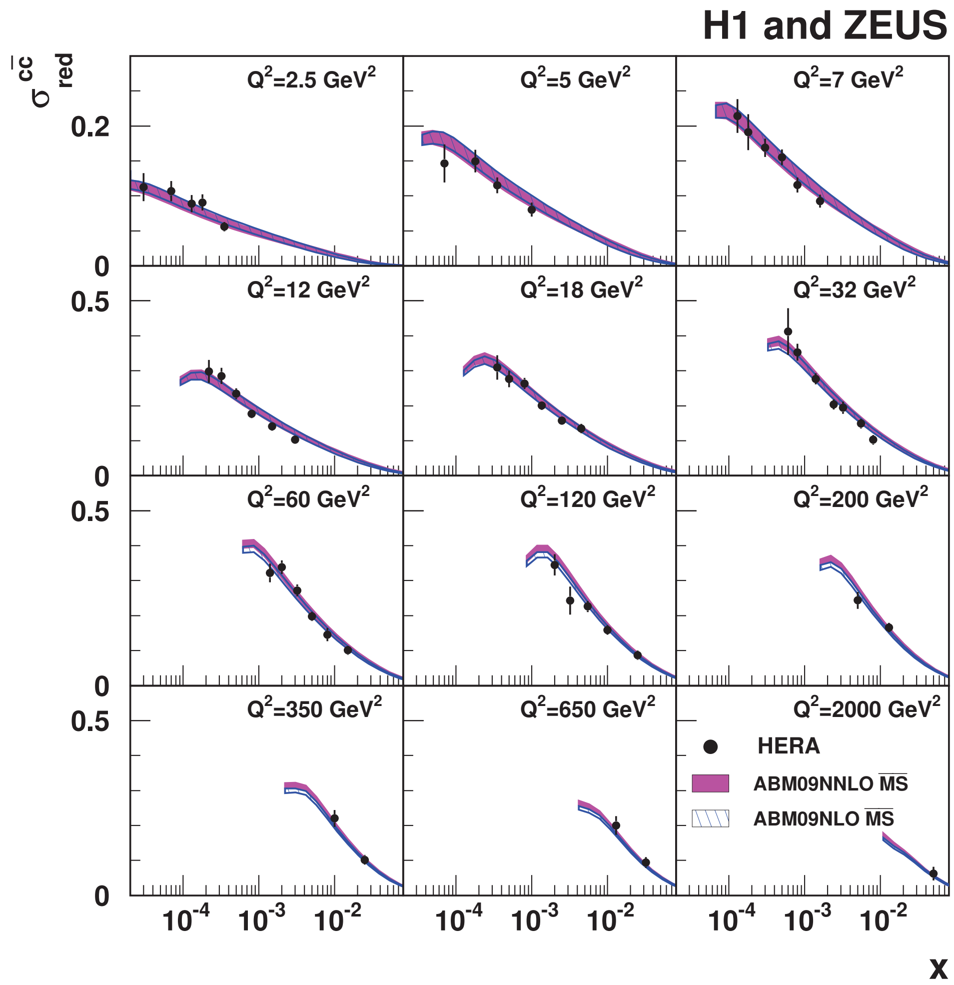
- Implicit $\alpha_s(M_Z)$ dependence in $m_c(m_c)$ determination from QCD sum rules

  Dehnadi, Hoang, Mateu, Zebarjad '11

Charm mass from HERA

- Determination of $\overline{\text{MS}}$-mass $m_c(m_c)$ in DIS H1 coll. arxiv:1211.1182
- Very good description of data



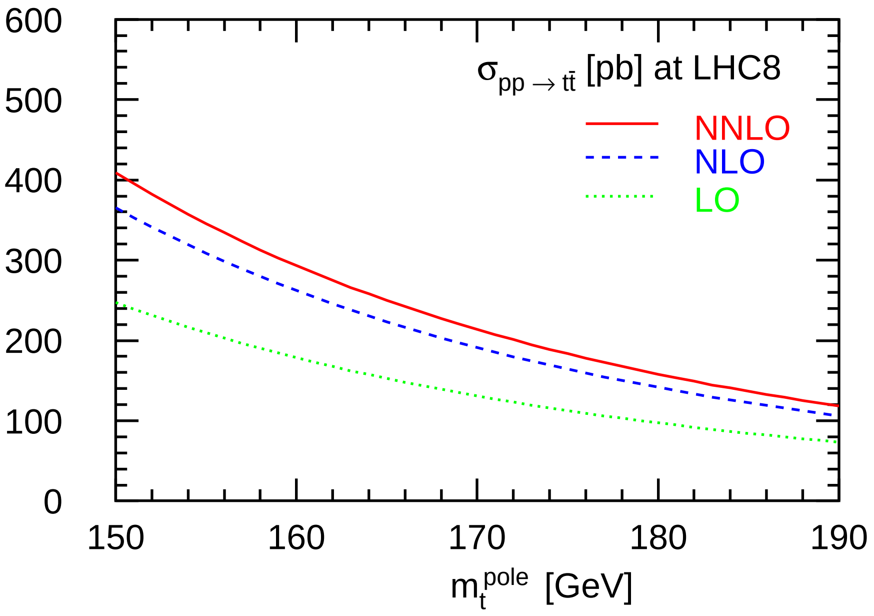


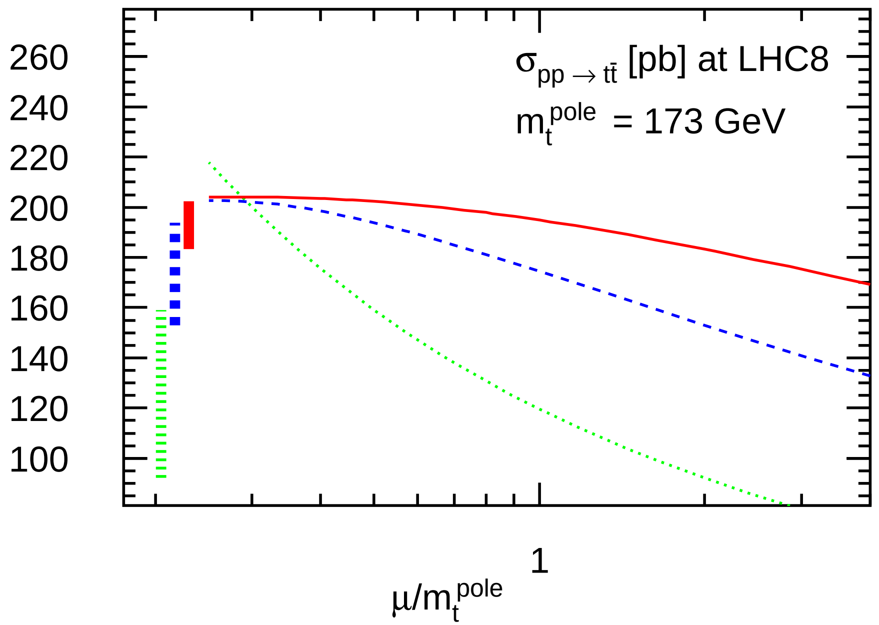
Top mass from total cross section

- Intrinsic limitation of sensitivity in total cross section

$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

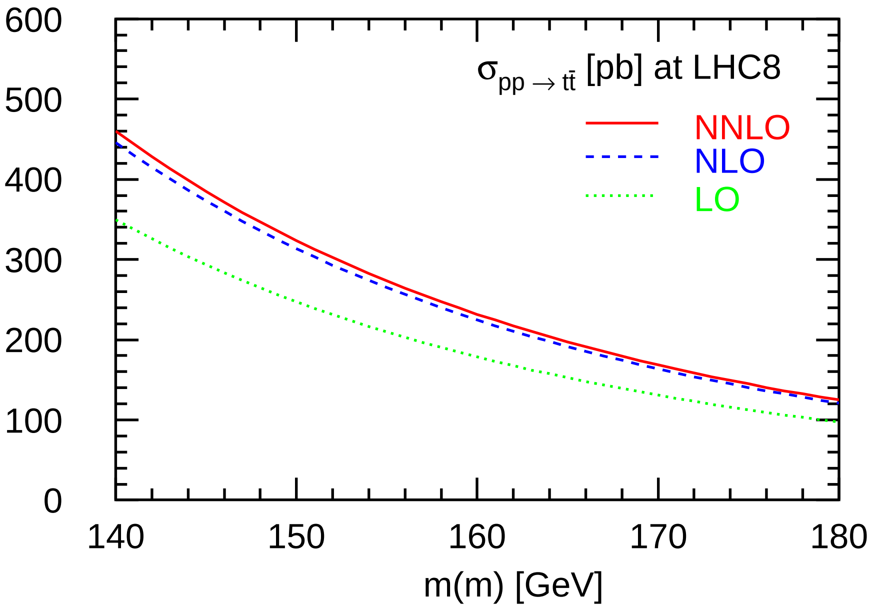
- NNLO QCD corrections Czakon, Fiedler, Mitov ‘13
  - K-factor (NLO $\rightarrow$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

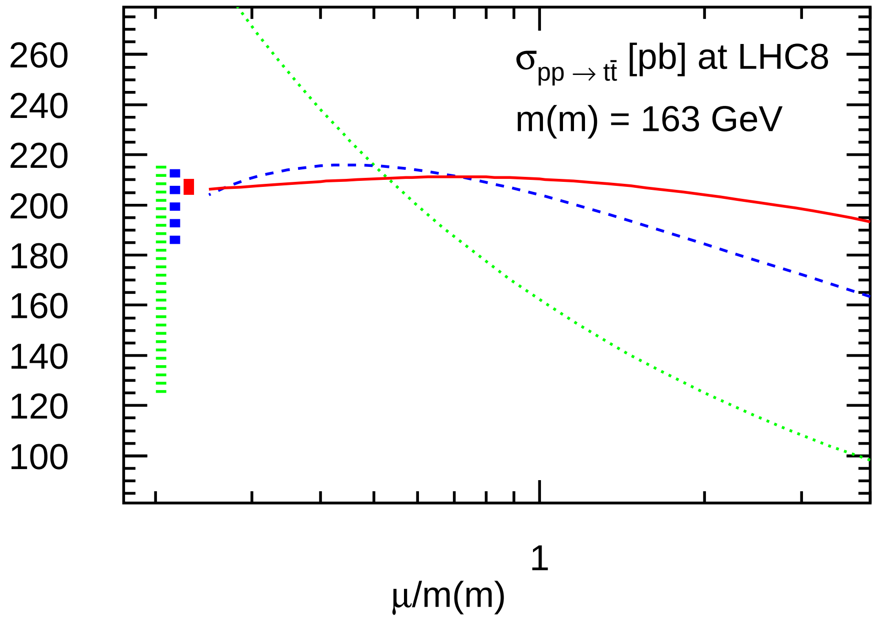




Total cross section with running mass

- Comparison pole mass vs. $\overline{MS}$ mass Dowling, S.M. '13
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity form scale variation



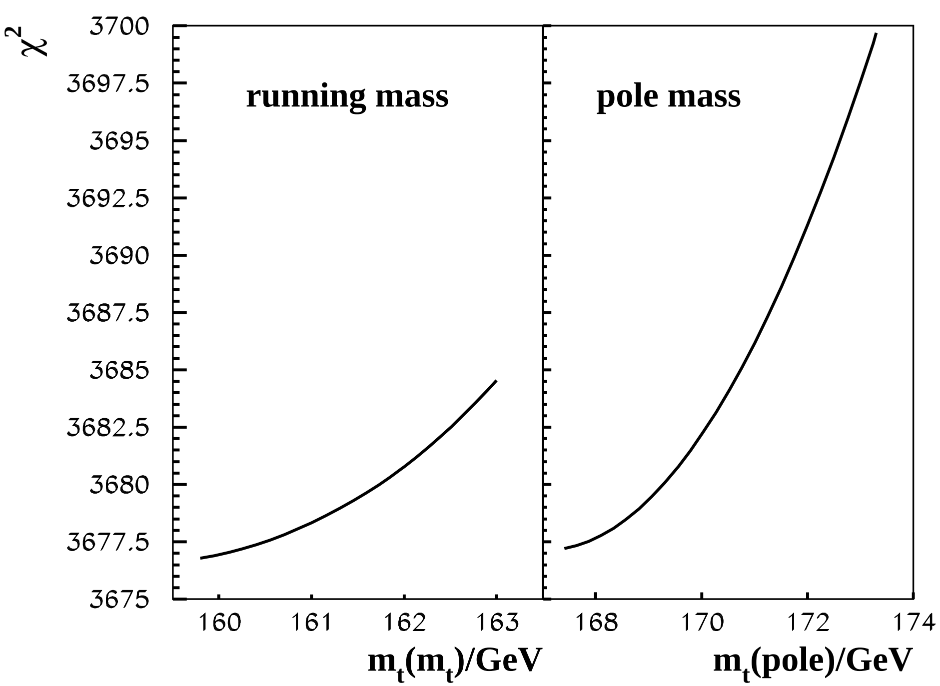


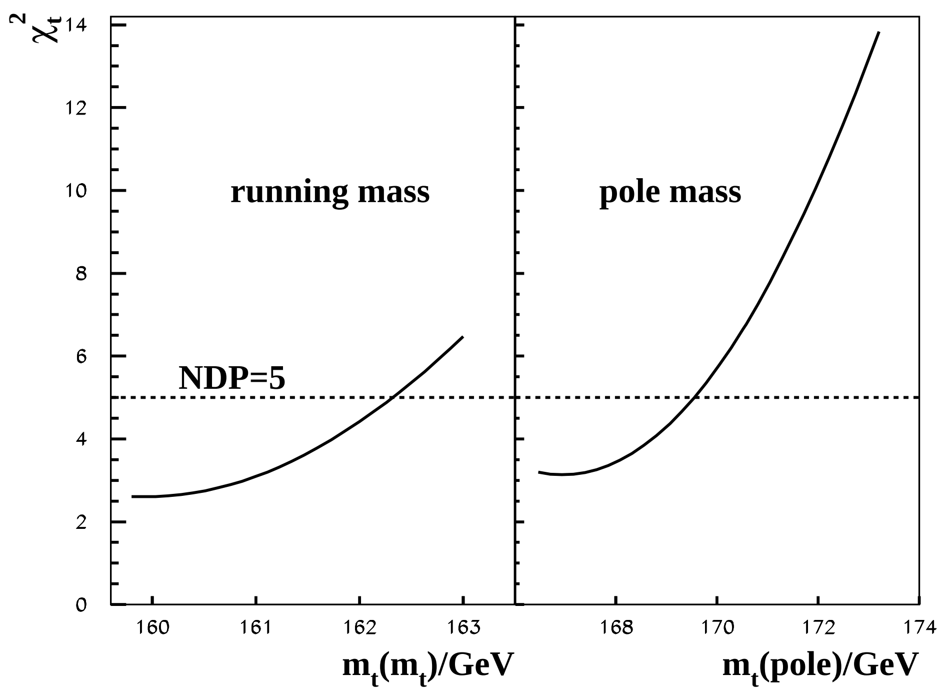
Correlations are essential

- Correlation of m_t, $\alpha_S(M_Z)$ and gluon PDF in cross section
  $\sigma_{t\bar{t}} \;\sim\; \alpha_s^2\, m_t^2\, g(x) \otimes g(x)$
- Effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$

Top cross section data in ABM12 fit

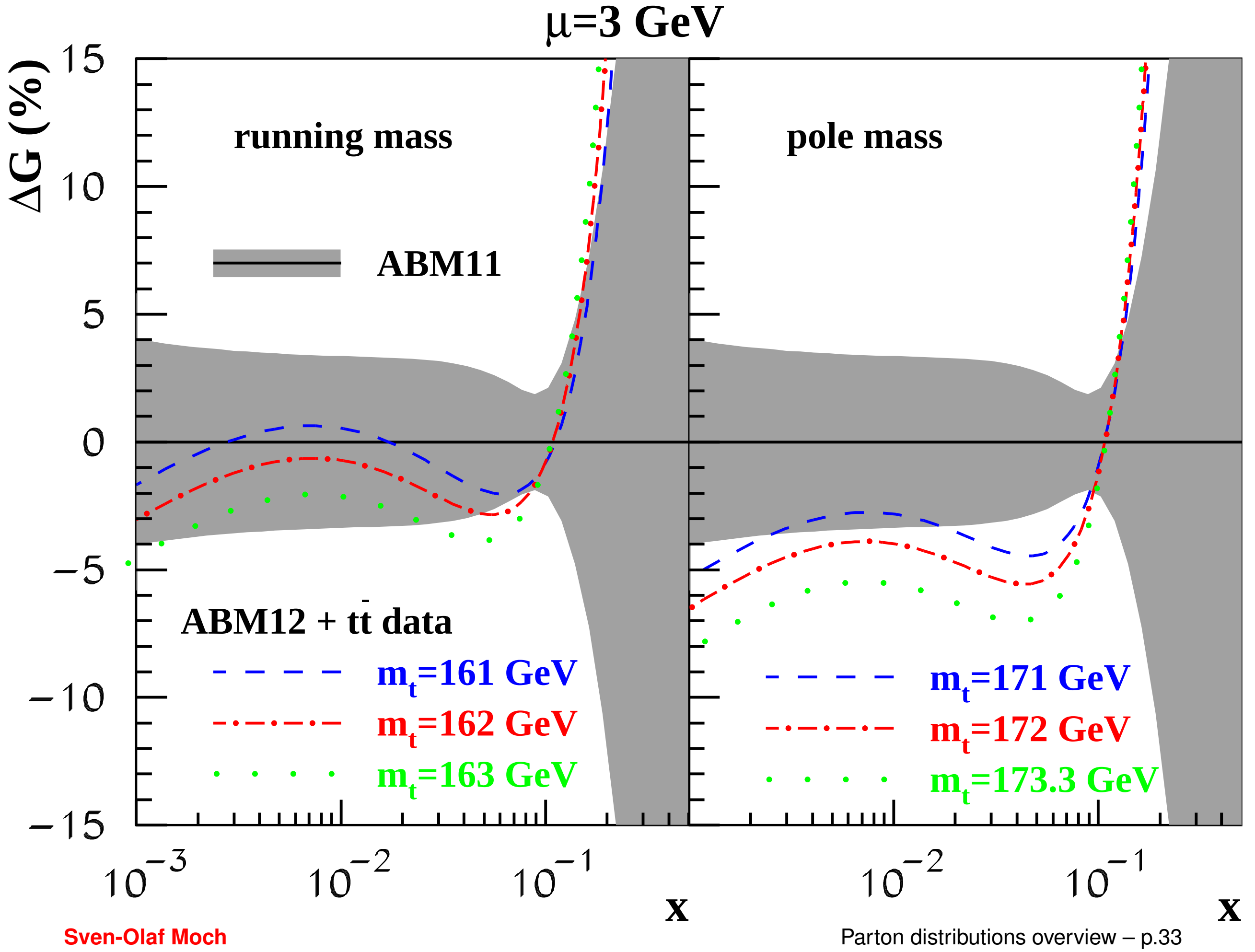
- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162.3 \pm 2.3$ GeV (equivalent to pole mass $m_t = 171.2 \pm 2.4$ GeV) Alekhin, Blümlein, S.M. '13
  - χ^2-profile steeper for pole mass (bigger impact of top-quark data and greater sensitivity to theoretical uncertainty at NNLO)





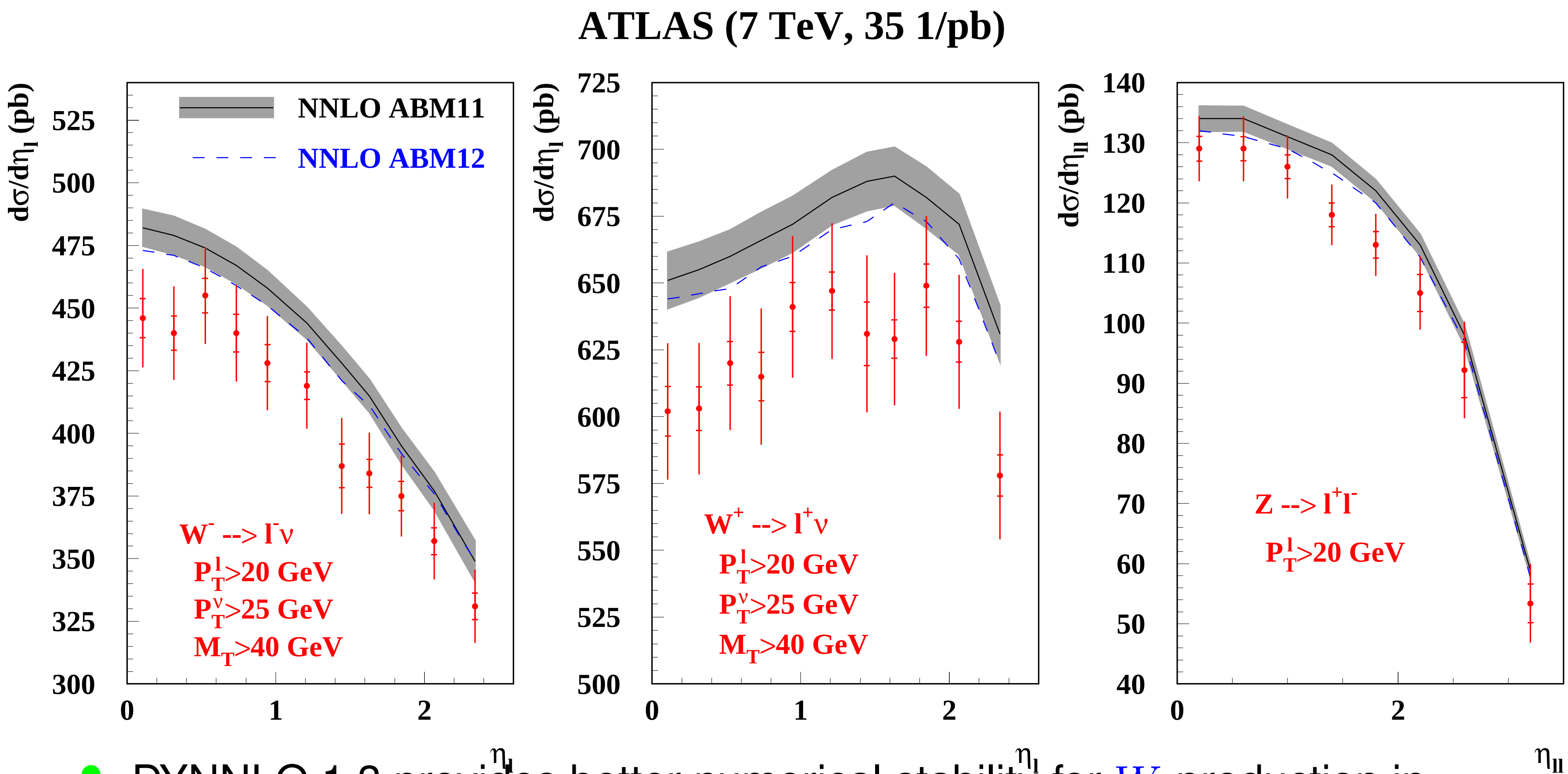
Top cross section data in ABM12 fit

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - correlation of gluon PDF with value of m_t
    (illustration of bias in analysis Czakon, Mangano, Mitov, Rojo '13)



Quark flavour constraints from LHC data

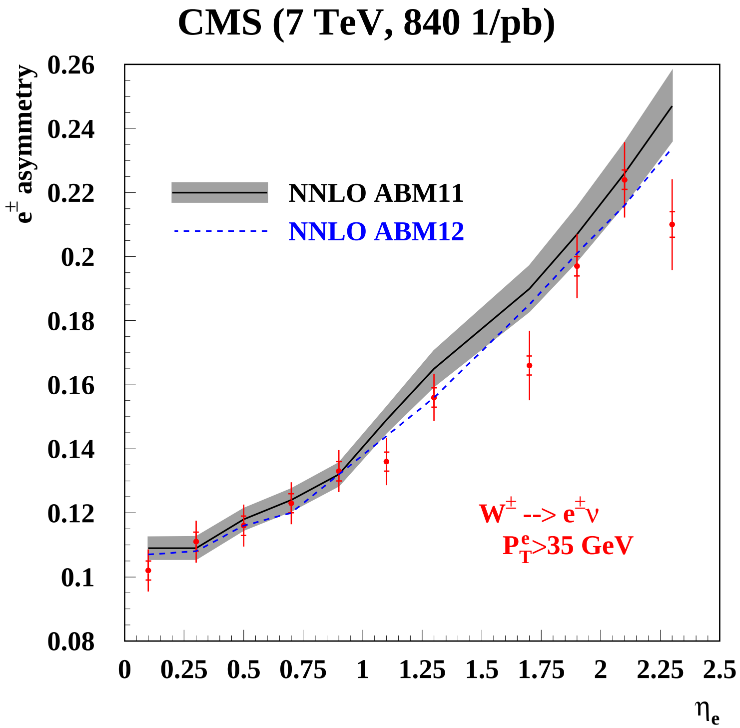
Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. '13

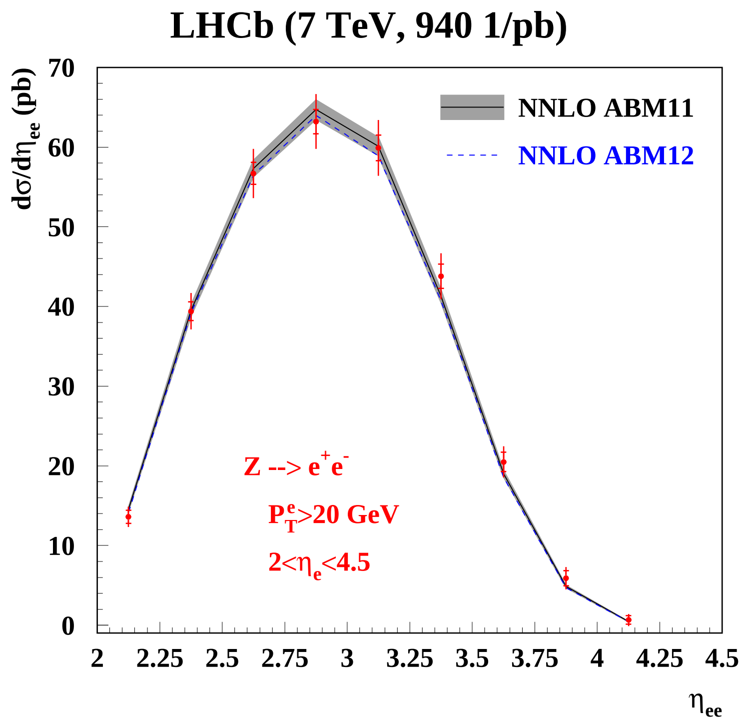


- DYNNLO 1.3 provides better numerical stability for W-production in central region ($\sim$ 200h) Catani, Cieri, Ferrera, de Florian, Grazzini '09
- FEWZ 3.1 more convenient/stable for estimation of PDF uncertainties ($\sim$ 2d x 24 processors) Li, Petriello '12
- Central values computed with DYNNLO and the PDF errors with FEWZ

ABM PDFs with LHC data

Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. ‘13

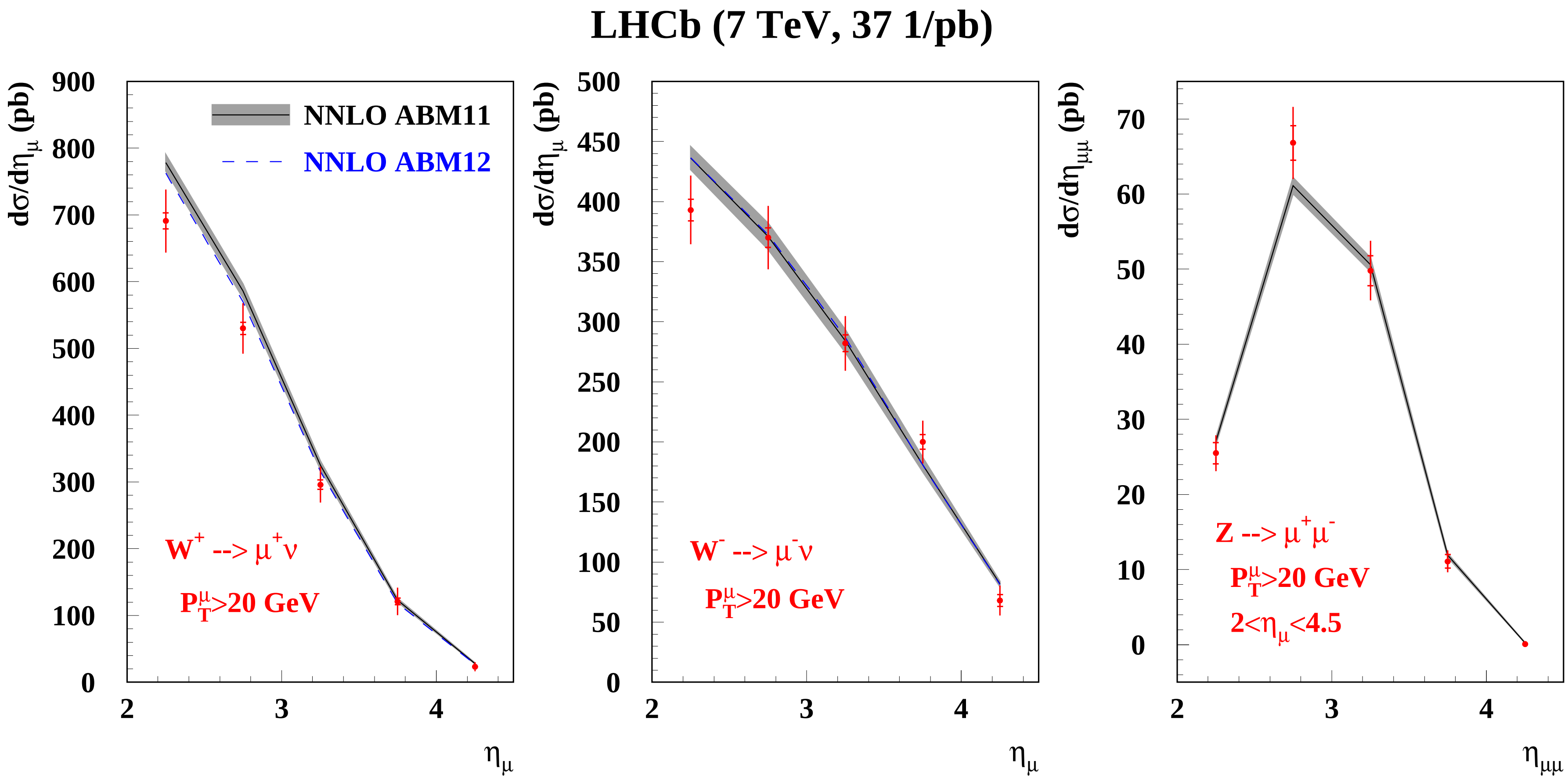




- Good overall agreement with data of CMS ‘10 and LHCb ‘12, ‘13

ABM PDFs with LHC data

Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. '13



- Good overall agreement with data of CMS '10 and LHCb '12, '13

Benchmarking of ABM PDFs

Experiment	ATLAS '11	CMS '12	LHCb '12	LHCb '12
Final states	$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to e^+\nu$ $W^- \to e^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$Z \to e^+e^-$
Luminosity (1/pb)	35	840	37	940
NDP	30	11	10	9
χ^2 (ABM11)	35.7(7.7)	10.6(4.7)	13.1(4.5)	11.3(4.2)
χ^2 (ABM12)	35.6	9.3	14.4	13.4

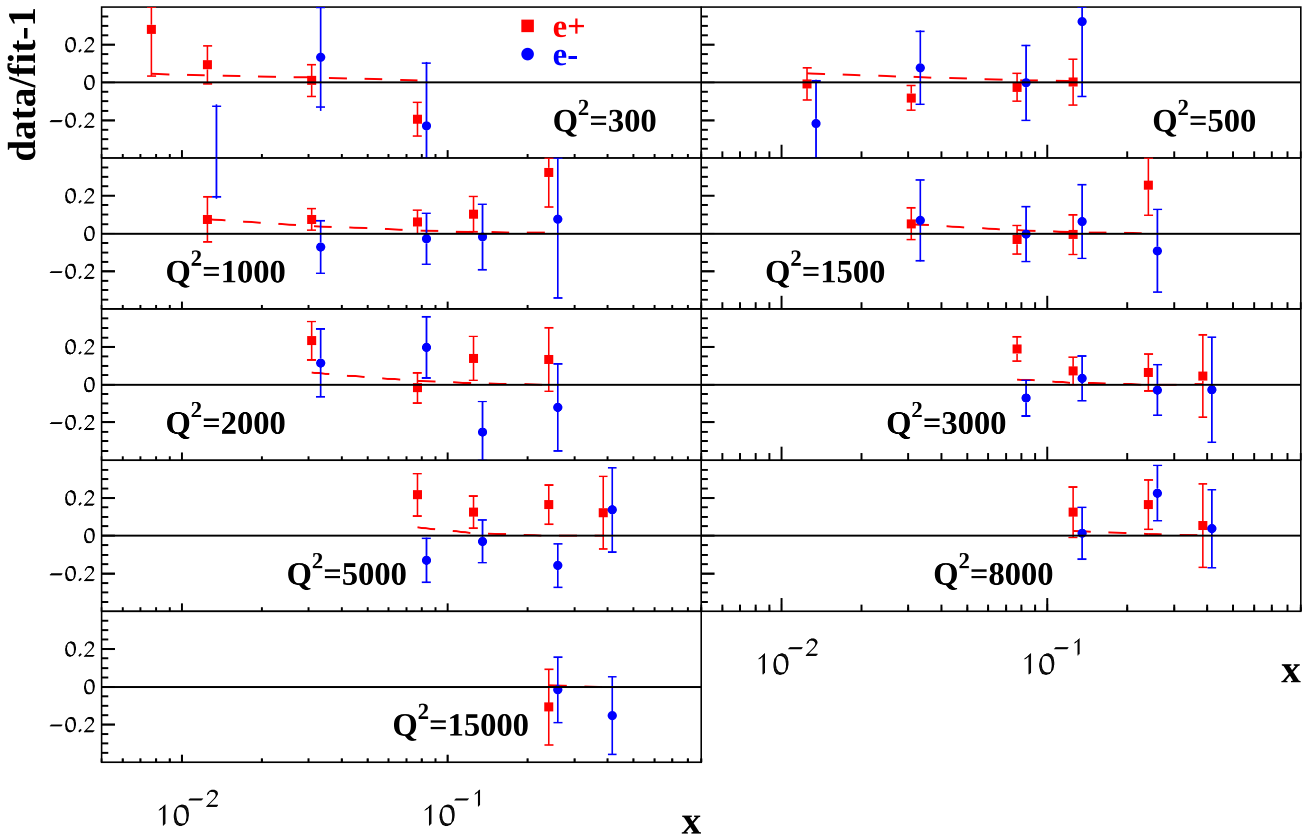
- value of χ^2 for Drell-Yan data at the LHC with NNLO ABM11 PDFs (+ 1σ standard deviation of χ^2 equal to $\sqrt{2NDP}$)
- Benchmarking in arXiv:1211.5142 reports wrong χ^2 values for PDF comparison to ABM11 (NLO theory with MCFM and K-factors, no PDF errors, shifted α_s)

Strange sea determination

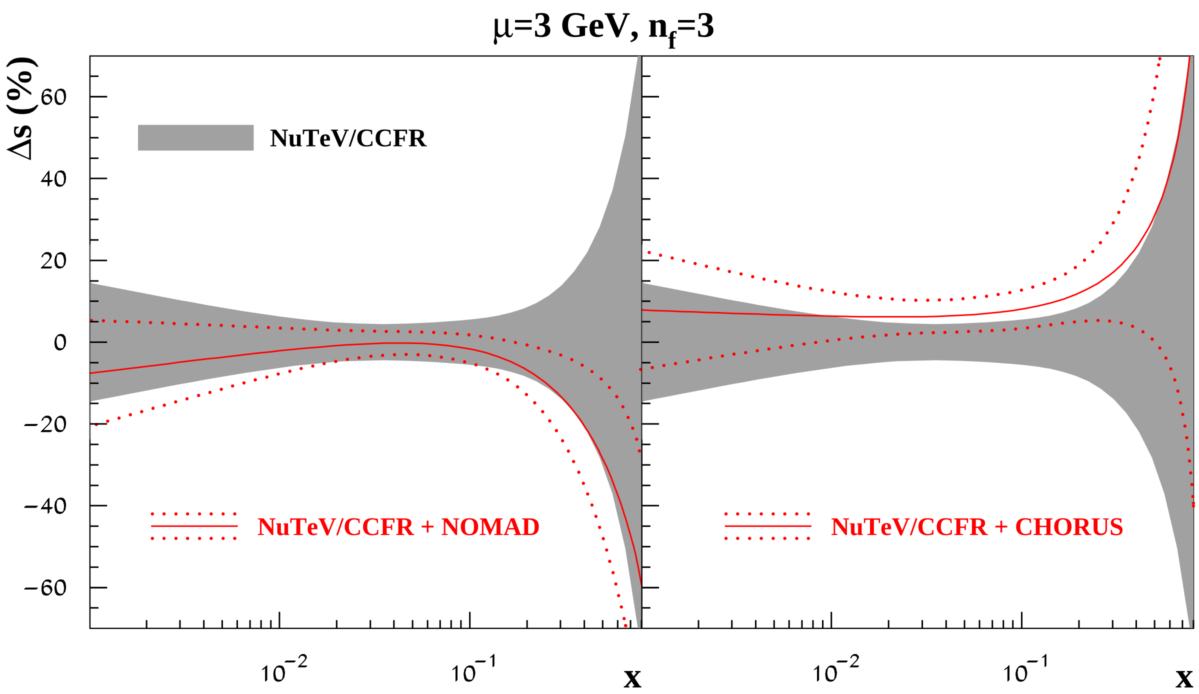
Charged current DIS

Alekhin, Bümlein, Caminada, Lipka, Lohwasser, S.M. Petti, Placakyte '14

- CC DIS inclusive data (HERA), CC DIS di-muon production data (NOMAD) and CC DIS charmed-hadron production data (CHORUS)
- Theory description with exact NLO QCD corrections and asymptotic NNLO terms at large $Q^2 \gg m^2$ Buza van Neerven '97



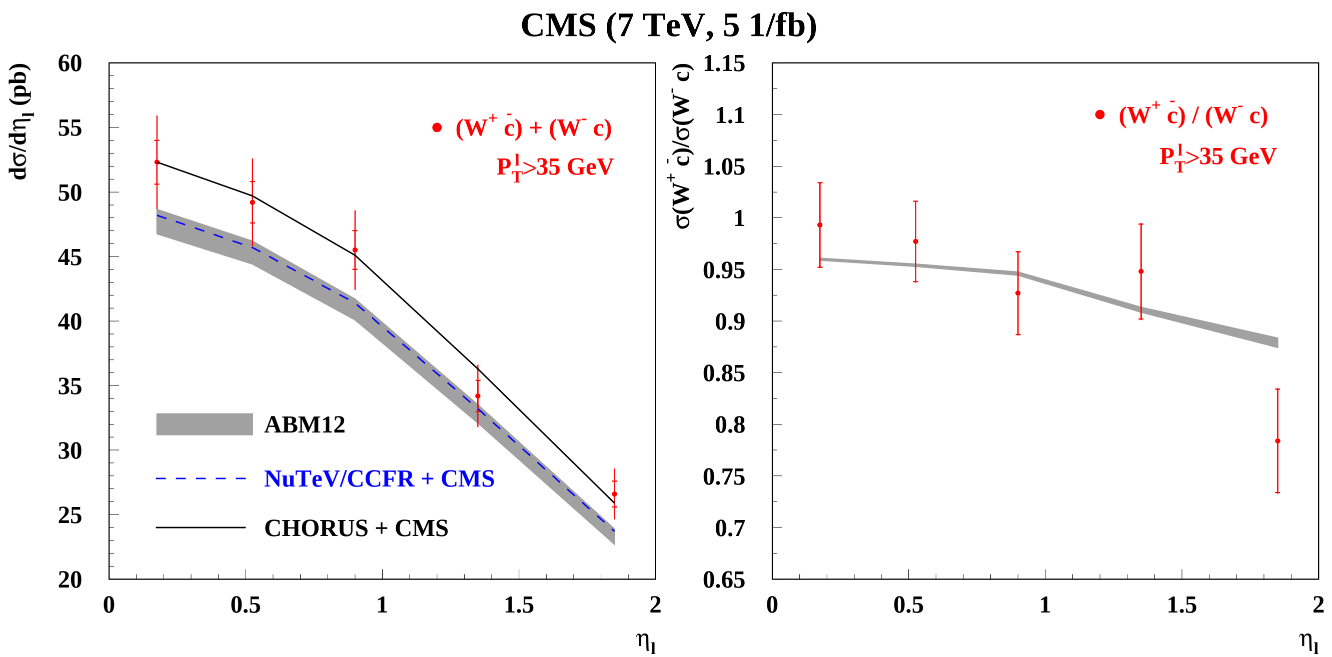
Strange sea from new fixed target data



- Nomad data on ratio of di-muon sample to incl. CC DIS with statistics of 15000 events (much more than CCFR and NuTeV samples)
  - systematics, nuclear corrections, etc. cancel in ratio
  - pull down strange quarks at $x > 0.1$; sizable reduction of uncertainty
  - $m_c(m_c) = 1.23 \pm 0.03(\mathrm{exp.})\mathrm{GeV}$
- Chorus data pull strangeness up
  - statistical significance the effect is poor

W+charm production at LHC

- Cross check with LHC data for W+charm production

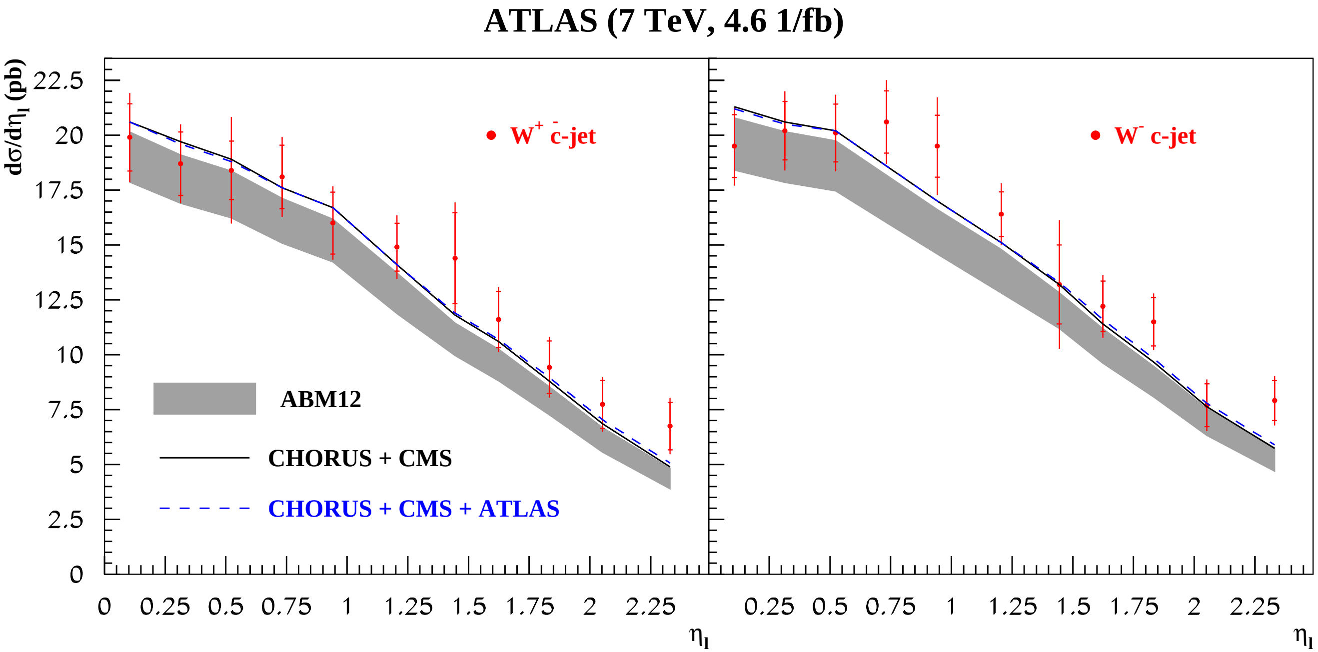


CMS

- CMS data above NuTeV/CCFR by 1σ
- Charge asymmetry in a good agreement with charge-symmetric strange sea

W +charm production at LHC

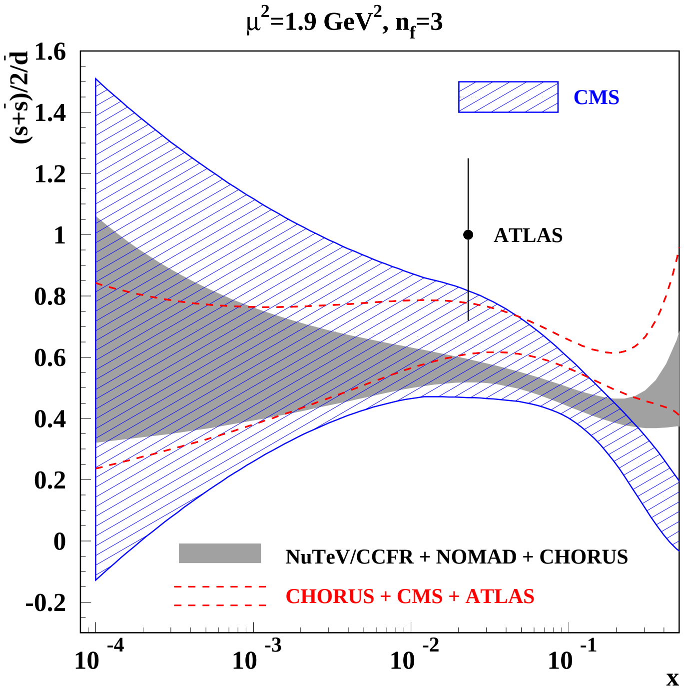
- Cross check with LHC data for W+charm production

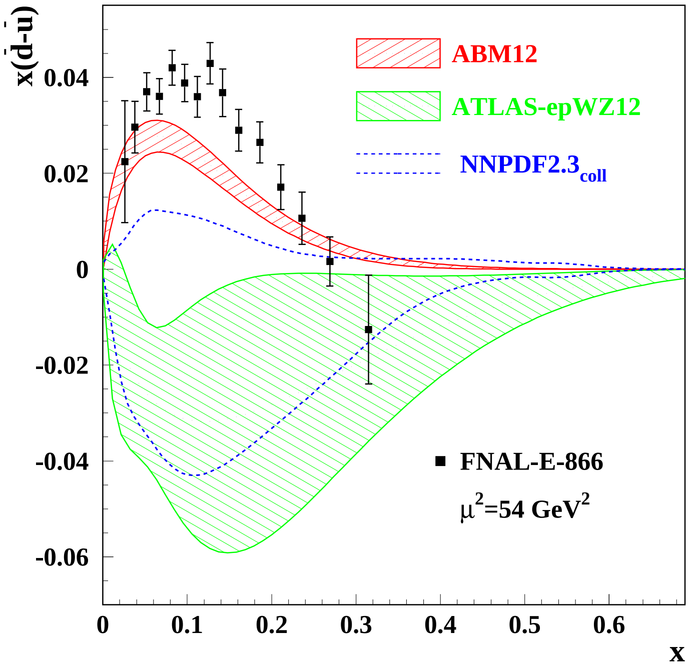


ATLAS

- ATLAS data in good agreement with NuTeV/CCFR
- Highest bin in η_l deviates

Comparision with earlier determinations





- ABM update (NuTeV/CCFR+NOMAD+CHORUS) in good agreement with CMS results
- ATLAS strange-sea in enhanced, but correlated with d-quark sea suppression (disagreement with the FNAL-E-866 data)
- Upper margin of ABM analysis (CHORUS+CMS+ATLAS) is lower than ATLAS

Summary

Parton distributions at the LHC

- Precision determinations of non-perturbative parameters is essential
  - parton content of proton (PDFs)
  - coupling constants $\alpha_s(M_Z)$
  - masses m_c, m_b, m_t, M_W, m_H, . . .

- Precision measurements require careful definition of observable
  - confronting LHC data requires continuous benchmarking
  - source of interesting observations

- Radiative corrections at higher orders in QCD and EW are mandatory
  - NNLO in QCD is *conditio sine qua non*
  - theory improvements driven by experimental precision

- Lots of challenging tasks in the future . . .