NNLO antenna functions for heavy quark pair production

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Exzellenzcluster PRISMA Präzisionsphysik, fundamentale Wechselwirkungen und Struktur der Materie

PRISMA

MARIE CURIE ACTIONS

LHCphenonet

In collaboration with: W. Bernreuther and C. Bogner (arXiv:1105.0530, arXiv:1309.6887, arXiv:1409.3124)
G. Abelof and A. Gehrmann-De Ridder (arXiv:1210.5059)
W. Bernreuther, J. Ablinger, and J. Blümlein (In preparation)

Motivation

The exploration of heavy quark production, in particular $t\bar{t}$ and or single top production, is a central issue at high energy colliders

- Sensitivity to electroweak symmetry breaking
- Probe for new physics interactions at high scales.
  - For instance new heavy resonances decaying to $t\bar{t}$
- Background to various new physics searches
- ...

$\Rightarrow$ Next-to-next-to-leading order (NNLO) predictions for heavy quark pair production cross sections are desirable.

As a step towards a fully differential NNLO treatment of heavy $Q\bar{Q}$ production in antenna subtraction framework investigate

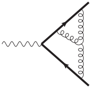
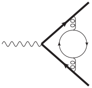
$$S \to Q\bar{Q} + X \quad \text{at NNLO QCD}$$

with uncolored initial state S, e.g. $e^+e^- \to \gamma/Z \to Q\bar{Q} + X$ or $H \to Q\bar{Q} + X$.

Distributions for $S \to Q\bar{Q} + X$: NNLO ingredients

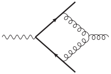
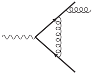
$$\sigma_{\text{NNLO}} = \int_{\Phi_2} d\sigma^{VV}_{\text{NNLO}} + \int_{\Phi_3} d\sigma^{RV}_{\text{NNLO}} + \int_{\Phi_4} d\sigma^{RR}_{\text{NNLO}}$$

- double virtual correction to $S \to Q\bar{Q}$: 2-loop $\times$ Born and (1-loop)^2

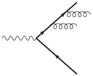
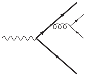
explicit infrared poles from loop integrals

- real virtual correction: One-loop correction to $S \to Q\bar{Q}g$

explicit infrared poles from loop integral
implicit implicit poles from soft/collinear emission

- double real correction: Tree-level matrix elements for $S \to Q\bar{Q}gg,\ Q\bar{Q}q\bar{q},\ Q\bar{Q}Q\bar{Q}$

implicit poles from double unresolved emission

Infrared (soft and collinear) poles only cancel in the sum (KLN-Theorem) .

Main Problem: Suitable methods to regularize and handle the infrared divergences

NNLO infrared subtraction

Generic structure of NNLO cross section with subtraction terms:

$$
\begin{aligned}
d\sigma_{\mathrm{NNLO}} &= \int_{\Phi_4} \left(d\sigma^{RR}_{\mathrm{NNLO}} - d\sigma^{S}_{\mathrm{NNLO}} \right) \\
&+ \int_{\Phi_3} \left(d\sigma^{RV}_{\mathrm{NNLO}} - d\sigma^{T}_{\mathrm{NNLO}} \right) \\
&+ \int_{\Phi_2} d\sigma^{VV}_{\mathrm{NNLO}} + \int_{\Phi_3} d\sigma^{T}_{\mathrm{NNLO}} + \int_{\Phi_4} d\sigma^{S}_{\mathrm{NNLO}}
\end{aligned}
$$

- $d\sigma^{S}_{\mathrm{NNLO}}$ coincides with $d\sigma^{RR}_{\mathrm{NNLO}}$ in all singular limits.
- $d\sigma^{T}_{\mathrm{NNLO}}$ coincides with $d\sigma^{VR}_{\mathrm{NNLO}}$ in all singular limits.

Each line is free of infrared poles and integration over the phase space can be carried out numerically in 4 dimensions.

Several methods for constructing subtraction terms have been proposed at NNLO...

NNLO IR Subtraction Schemes

- sector-improved residue subtraction scheme [Czakon '11, Czakon, Heymes '14]
  - $Z \to ee$ [Boughezal, Melnikov, Petriello '11]
  - $pp \to Hj$ (gluons only) [Boughezal, Caola, Melnikov, Petriello, Schulze '13]
  - $pp \to t\bar{t}$ [Bärnreuther, Czakon, Fiedler, Mitov '13] , A_{FB} [Czakon, Fiedler, Mitov '14]
- q_T-subtraction for colorless high mass systems [Catani, Grazzini '07]
  - $pp \to H$ [Catani, Grazzini '07]
  - $pp \to V$ [Catani, Cieri, Ferrera, de Florian, Grazzini '09]
  - $pp \to VH$ [Ferrera, Grazzini, Tramontano '11]
  - $pp \to \gamma\gamma$ [Catani, Cieri, de Florian, Grazzini '11]
  - $pp \to ZZ, WW$ [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs '14]
  - $pp \to t\bar{t}$ $\to$ see talk by H. Sargsyan
- subtraction schemes by Del Duca, Somogyi, Trocsanyi et al.
- antenna subtraction [Kosower '97 ;Gehrmann-De Ridder, Gehrmann, Glover '05]
  - $e^+e^- \to 3j$ [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07, Weinzierl '08]
  - $pp \to 2j$ [Currie, Gehrmann-De Ridder, Gehrmann, Glover, Pires '13]
  - $pp \to H + jet$ [Chen, Gehrmann, Glover, Jaquier '14]
  - $pp \to t\bar{t}$ $(N_c^2, N_l N_c, N_l/N_c)$
    [Abelof, Gehrmann-De Ridder, '11 - '14, Abelof, Gehrmann-De Ridder, OD '12, Gabriel Abelof, Aude Gehrmann-De Ridder, Philipp Maierhöfer, Stefano Pozzorini '14]

Antenna Subtraction @ NLO

- Based on the universal factorization properties of $|\mathcal{M}_m|^2$ in all singular limits [Campbell, Glover '98; Catani, Grazzini '99 -'00; . . .]

$$|\mathcal{M}^0_{m+1}(\ldots,i,j,k,\ldots)|^2 \xrightarrow{p_j\to 0} S(i,j,k)\,|\mathcal{M}^0_m(\ldots,i,k,\ldots)|^2$$
$$|\mathcal{M}^0_{m+1}(\ldots,i,j,k,\ldots)|^2 \xrightarrow{j\|k} \frac{1}{s_{jk}} P_{jk\to K}\,|\mathcal{M}^0_m(\ldots,i,K,\ldots)|^2$$

Subtracted with

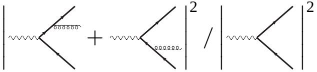
$$X^0_{ijk}\,|\mathcal{M}^0_m(\ldots,I,K,\ldots)|^2$$

Three-parton tree-level antenna function X^0_{ijk}

- Two hard particles i, k (hard radiatiors) and one unresolved parton j
- Give the right unresolved factor (splitting function, soft eikonal factor) in each limit
- Derived from physical matrix elements for tree-level $1 \to 3$ processes

$$X^0_{ijk} \propto \frac{|\mathcal{M}^0_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2}$$

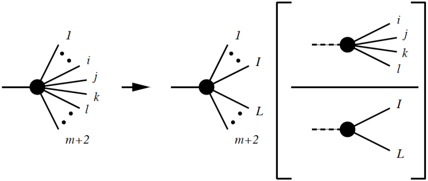
e.g. $A^0_3 \propto$



Antenna Subtraction @ NNLO

Double unresolved limits of $|\mathcal{M}_{m+2}|^2$

- Double soft limits
- Soft $q\bar{q}$ limits
- ...



Subtracted with (for color-connected configurations)

$$X^0_{ijkl}\,|\mathcal{M}^0_m(\ldots,I,L,\ldots)|^2 \qquad X^0_{ijkl} \propto \frac{|\mathcal{M}^0_{ijkl}|^2}{|\mathcal{M}^0_{IL}|^2}$$

Four-parton tree-level antenna function X^0_{ijkl}

- Two hard particles i, l (hard radiatiors) and two unresolved partons j, k
- Give the right unresolved factor in each limit
- Derived from physical matrix elements for tree-level $1 \to 4$ processes. Here:

$$|\mathcal{M}^0_{\gamma^*\to Q\bar{Q}gg}|^2 \propto \alpha_s^2 \left\{ N_c \left(A^0_4(1_Q,3_g,4_g,2_{\bar{Q}}) + A^0_4(1_Q,4_g,3_g,2_{\bar{Q}}) \right) - \frac{1}{N_c}\tilde{A}^0_4(1_Q,3_g,4_g,2_{\bar{Q}}) \right\}$$

$$|\mathcal{M}^0_{\gamma^*\to Q\bar{Q}q\bar{q}}|^2 \propto \alpha_s^2 \left\{ e_Q^2\, B^0_4(1_Q,3_q,4_{\bar{q}},2_{\bar{Q}}) + e_q^2\,(\ldots) + e_q e_Q\,(\ldots) \right\}$$

Integrated Double-Real Subtraction Terms

- factorization of double unresolved phase space

$$d\Phi_{m+2}(\ldots,p_i,p_j,p_k,p_l,\ldots;q) = d\Phi_m(\ldots,p_I,p_L,\ldots,;q) \times d\Phi_{X_{ijkl}}$$

$$d\sigma^S_{NNLO} \sim |\mathcal{M}^0_m(p_1,\ldots,p_I,p_L,\ldots,p_{m+2})|^2 d\Phi_m(\ldots,p_I,p_L,\ldots,;q) \times X^0_{ijkl}\, d\Phi_{X_{ijkl}}$$

- integrated antenna functions

$$\mathcal{X}^0_{ijkl} \propto \int d\Phi_{X_{ijkl}}\, X^0_{ijkl} \propto \int d\Phi_4 |\mathcal{M}^0_{ijkl}|^2$$

Reverse-unitarity

[Anastasiou, Melnikov '02]

- Introduce cut propagators [Cutkosky '60]

$$2\pi i\delta^+\left(p_i^2 - m_i^2\right) = \frac{1}{p_i^2 - m_i^2 + i0} - \frac{1}{p_i^2 - m_i^2 - i0} = \left[\frac{1}{D_i}\right]_c$$

- Integration-by-parts reduction [Chetyrkin, Tkachov '81, Laporta '00]

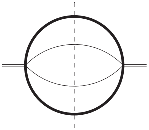
  `AIR` [Anastasiou, Lazopoulos '02], `FIRE` [Smirnov '08], `REDUZE(2)` [Studerus, v. Manteuffel '12]

- Differential equations [Kotikov '91; Remiddi '97; Gehrmann, Remiddi '00]

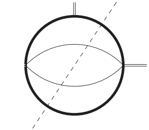
Double-Real Master Integrals

$p_1^2 = p_2^2 = m^2$
$p_3^2 = p_4^2 = 0$
$s_{ij} = 2p_i\, p_j$

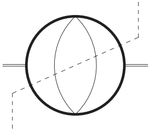
- $\mathcal{A}_4^0$, $\tilde{\mathcal{A}}_4^0$, $\mathcal{B}_4^0$ can be expressed in terms of 15 master integrals:



$\{1, s_{13}, s_{34}\}$



$\{1\}$



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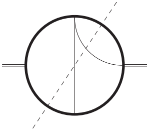

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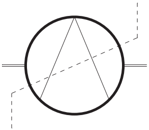
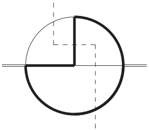


$\{1, s_{13}\}$



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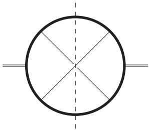

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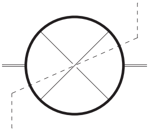

$\{1, s_{13}\}$

Double-Real Master Integrals

$p_1^2 = p_2^2 = m^2$
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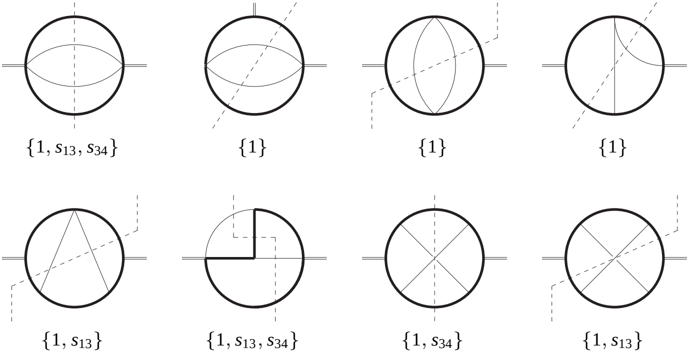

$$d\Phi_4(q\,;p_1,p_2,p_3,p_4) = \frac{1}{2\pi}\, dQ^2\, d\Phi_2(q\,;p_4,Q)\, d\Phi_3(Q\,;p_1,p_2,p_3)$$

- Compact expressions in terms of hypergeometric functions ${}_3F_2$
- Expansion around $d = 4$ yields harmonic polylogarithms (HPLs)

[Moch, Uwer, Weinzierl ‘01, Weinzierl ‘04] `HypExp(2)` [Huber, Maitre ‘05, ‘07]

Double-Real Master Integrals

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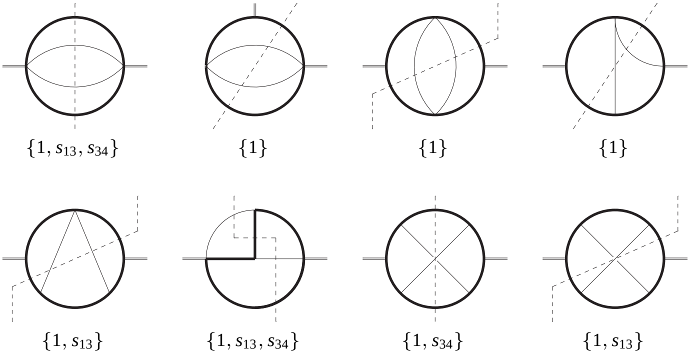

Differential equations method

[Kotikov '91; Remiddi '97; Gehrmann, Remiddi '00]

- Linear, inhomogeneous, first order DEs q^2 and $y = \dfrac{1-\beta}{1+\beta}$ $(\beta = \sqrt{1 - 4m^2/q^2})$
- Solve differential equation order by order in ϵ in terms of HPLs
- initial condition: Vanishing of phase space at threshold: $y \to 1$
  Massless limit: $y \to 0$ [Gehrmann-De Ridder, Gehrmann, Heinrich '04]

Double-Real Master Integrals

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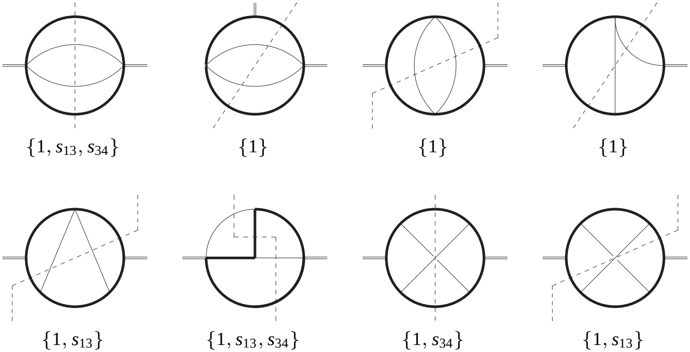


$\hookrightarrow$ Master integrals and integrated antenna functions $\mathcal{A}_4^0$, $\tilde{\mathcal{A}}_4^0$, $\mathcal{B}_4^0$ are obtained analytically to all relevant orders in terms of HPLs.
[Bernreuther, Bogner, OD '11, '13]

Real-Virtual Subtraction

Single unresolved limits of one-loop amplitudes: [Bern, Dixon, Dunbar, Kosower, Catani, Grazzini, ...]

$$Loop_{m+1} \longrightarrow Splitt_{tree} \times Loop_m + Splitt_{loop} \times Tree_m$$

Subtract single unresolved limits of $|\mathcal{M}^1_{m+1}(\ldots,i,j,k,\ldots)|^2 = 2\mathrm{Re}(\mathcal{M}^{0\dagger}_{m+1}\mathcal{M}^1_{m+1})$ with

$$X^0_{ijk}\,|\mathcal{M}^1_m(\ldots,I,K,\ldots)|^2 \;+\; X^1_{ijk}\,|\mathcal{M}^0_m(\ldots,I,K,\ldots)|^2$$

Three-parton one-loop antenna function X^1_{ijk}

- Give the right unresolved factor $Splitt_{loop}$ in each limit
- Derived from physical matrix elements for one-loop $1 \to 3$ processes. Here:

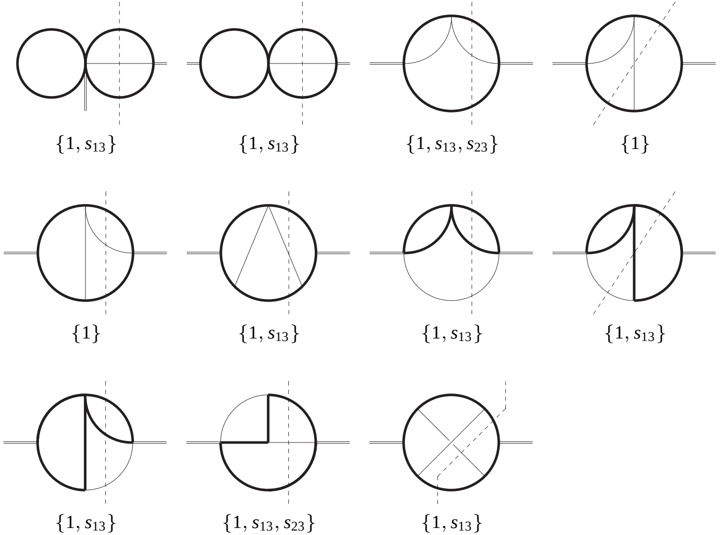
$$\begin{aligned}|\mathcal{M}^1_{\gamma^*\to Q\bar{Q}g}|^2 \;\propto\; \alpha_s^2 \Big\{ & N_c \left(A^0_3(1_Q,3_g,2_{\bar{Q}})\,|\mathcal{M}^1_{\gamma^*\to Q\bar{Q}}|^2 \;+\; A^1_3(1_Q,3_g,2_{\bar{Q}})\,|\mathcal{M}^0_{\gamma^*\to Q\bar{Q}}|^2 \right) \\ & - \frac{1}{N_c} \left(A^0_3(1_Q,3_g,2_{\bar{Q}})\,|\mathcal{M}^1_{\gamma^*\to Q\bar{Q}}|^2 \;+\; \tilde{A}^1_3(1_Q,3_g,2_{\bar{Q}})\,|\mathcal{M}^0_{\gamma^*\to Q\bar{Q}}|^2 \right) \\ & + 2T_R\,n_f\,\tilde{A}^1_{3,f}(1_Q,3_g,2_{\bar{Q}})\,|\mathcal{M}^0_{\gamma^*\to Q\bar{Q}}|^2 + 2T_R\,\tilde{A}^1_{3,F}(1_Q,3_g,2_{\bar{Q}})\,|\mathcal{M}^0_{\gamma^*\to Q\bar{Q}}|^2 \Big\}\end{aligned}$$

Integrated real-virtual antenna functions

$$\mathcal{X}^1_{ijk} \;\propto\; \int d\Phi_{X_{ijk}}\, X^1_{ijk} \;\propto\; \frac{1}{|\mathcal{M}^0_{IK}|^2} \int d\Phi_3 |\mathcal{M}^1_{ijk}|^2 - \frac{|\mathcal{M}^1_{IK}|^2}{|\mathcal{M}^0_{IK}|^2} \int d\Phi_3\, X^0_{ijkl}$$

Real-Virtual Master Integrals

- $\mathcal{A}_3^1$, $\tilde{\mathcal{A}}_3^1$ can be expressed in terms of 22 master integrals: [Bernreuther, O.D. '14]



- Only the real part is relevant here.

$s_{ij} = 2p_i \cdot p_j$

Computation of Real-Virtual Master Integrals

- Derive differential equations for each MI in the same algorithmic fashion as before
- Solutions for 5 of the 9 topologies exhibit new structures:

$$\int_0^y dx \, \frac{2x+1}{x^2+x+1} \, H(\ldots, x)$$

- Consider Poincaré iterated integrals generated by

$$\left\{ \frac{1}{x}, \frac{1}{x-1}, \frac{1}{x+1}, \frac{1}{x^2+x+1}, \frac{x}{x^2+x+1} \right\}$$

$\hookrightarrow$ Cyclotomic harmonic polylogarithmns [Ablinger, Blümlein, Schneider '11 - '13]
[$\rightarrow$ `HarmonicSums`]

Constants of Integration

- boundary condition from an expansion of d-dim. one-loop integral and PS measure in ϵ and in $\beta = \sqrt{1 - 4m^2/q^2}$ prior to PS integration:

$$\int d\Phi_3 \, s_{13}^{\nu_1} \, s_{23}^{\nu_2} \, Z^0(q^2, s_{13}, s_{23}, \beta, \epsilon) \qquad Z^0 \in \{A^0, B^0, C^0, D^0\}$$

- Integration constants are given by values of cyclotomoic HPLs at y = 1:
  - Either: Numerical evaluation from integral representation
  - Or: Reduction in terms of transcendental numbers [Ablinger, Blümlein]

The $e^+e^- \to Q\bar{Q}$ cross section at NNLO

As a check and first application, we consider

$$R = \frac{\sigma(e^+e^- \to \gamma^* \to Q\bar{Q} + X)}{\sigma(e^+ e^- \to \gamma^* \to \mu^+ \mu^-)} = e_Q^2 \Bigg[N_c R^{(0)} + \left(\frac{\alpha_s(\mu^2)}{2\pi}\right) \left(N_c^2 - 1\right) R^{(1)}$$

$$+ \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 \left(N_c^2 - 1\right) \left(N_c R^{(2)}_{\text{LC}} - \frac{1}{N_c} R^{(2)}_{\text{SC}} + 2T_R n_f R_f^{(2)} + 2T_R R_F^{(2)}\right) + \ldots \Bigg]$$

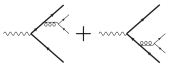
- Consider one heavy quark Q (charge e_Q, mass m) and n_f massless quark flavors
- By adding the contributions of the 2-parton, 3-parton and 4-parton final states, we use R to check that the individual IR singularities cancel in the sum (KLN-Theorem)
- Corrections were computed before numerically/partly analytically from imaginary part of vacuum polarization tensor [$\mathcal{O}(\alpha_s^2)$: Chetyrkin et al. '96, $\mathcal{O}(\alpha_s^3)$: Kiyo et al. '09, Marquard et al. '10]
- Our computation of $R^{(2)}_{\text{LC}}$, $R^{(2)}_{\text{SC}}$ and $R^{(2)}_f$ is exact to order α_s^2

Light flavor correction $R_f^{(2)}$

$$R_f^{(2)} = R^{(0)} \left(\mathcal{B}_4^0\left(\epsilon, \mu^2/s; y\right) + \hat{\mathcal{A}}_{3,f}^1\left(\epsilon, \mu^2/s; y\right) + \hat{\mathcal{A}}_{2,f}^2\left(\epsilon, \mu^2/s; y\right)\right)$$

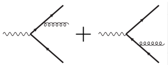
$Q\bar{Q}q\bar{q}$ final state

$$\mathcal{B}_4^0 \propto \int d\Phi_4 \left| \cdots + \cdots \right|^2 \sim \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$$



$Q\bar{Q}g$ final state (counterterm contribution)

$$\hat{\mathcal{A}}_{3,f}^1 \propto \frac{1}{\epsilon} \int d\Phi_3 \left| \cdots + \cdots \right|^2 \sim \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$$

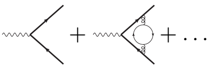


[Gehrmann-de Ridder, Ritzmann ‘09]

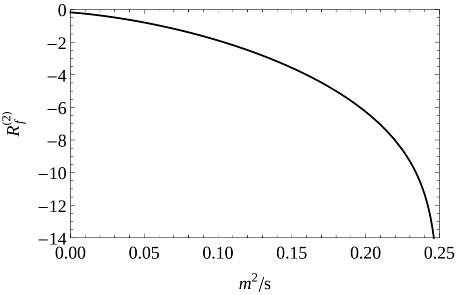
$Q\bar{Q}$ final state: heavy quark from factors

[Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi, ‘05; Gluza, Mitov, Moch, Riemann ‘09]

$$\hat{\mathcal{A}}_{2,f}^2 \propto \left| \cdots + \cdots + \ldots \right|^2 \sim \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$$



- ✓ Analytical cancellation of IR poles
- ✓ Analytical agreement with result of [Hoang, Kühn, Teubner ’95].

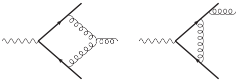


Leading Color Corrections $R_{LC}^{(2)}$

$$R_{LC}^{(2)} = R^{(0)} \left(\mathcal{A}_2^2\left(\epsilon, \mu^2/s; y\right) + \mathcal{A}_2^1\left(\epsilon, \mu^2/s; y\right) \mathcal{A}_3^0\left(\epsilon, \mu^2/s; y\right) + \mathcal{A}_3^1\left(\epsilon, \mu^2/s; y\right) + \mathcal{A}_4^0\left(\epsilon, \mu^2/s; y\right) \right)$$

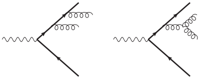
- $Q\bar{Q}$ final state: heavy quark from factors
  [Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi, '05; Gluza, Mitov, Moch, Riemann '09]
- $Q\bar{Q}g$ final state $\leftrightarrow$ $\mathcal{A}_3^1 + \mathcal{A}_2^1 \, \mathcal{A}_3^0$

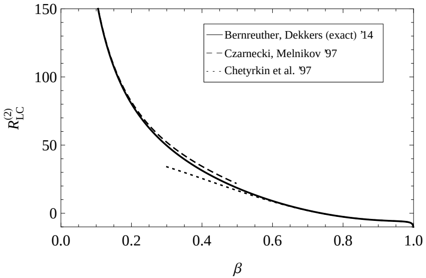
  e.g.



- $Q\bar{Q}gg$ final state $\leftrightarrow$ $\mathcal{A}_4^0(\epsilon; \mu^2/s, y)$

  e.g.





Chetyrkin, Kühn, Steinhauser '96: interpolate threshold and asymptotic exp. with Padé approx.

✓ IR poles (solely HPLs) cancel analytically

Analytical agreement with

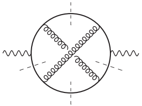
✓ Threshold exp. to $\mathcal{O}(\beta)$ [Czarnecki, Melnikov ; Beneke, Signer, Smirnov '97]

✓ Large-q^2 exp. through $\mathcal{O}((m^2/q^2)^6)$ [Chetyrkin, Harlander, Kuhn, Steinhauser '97]

Subleading Color Corrections $R_{SC}^{(2)}$

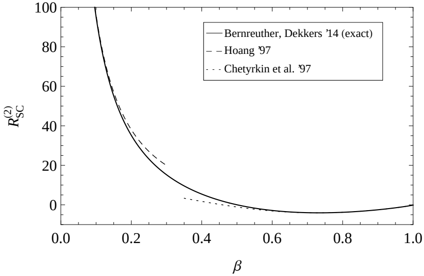
$$
\begin{aligned}
R_{SC}^{(2)} = R^{(0)} \Big(& \tilde{\mathcal{A}}_2^2\left(\epsilon;\mu^2/s,y\right) + \mathcal{A}_2^1\left(\epsilon;\mu^2/s,y\right)\mathcal{A}_3^0\left(\epsilon;\mu^2/s,y\right) \\
& + \tilde{\mathcal{A}}_3^1\left(\epsilon;\mu^2/s,y\right) + \frac{1}{2}\tilde{\mathcal{A}}_4^0\left(\epsilon;\mu^2/s,y\right) + 2\,\mathcal{C}_4^0\left(\epsilon;\mu^2/s,m^2/s\right)\Big)
\end{aligned}
$$

- $Q\bar{Q}$ final state: heavy quark from factors
- $Q\bar{Q}g$ final state $\leftrightarrow$ $\tilde{\mathcal{A}}_3^1(\epsilon;\mu^2/s,y)$
- $Q\bar{Q}gg$ final state $\leftrightarrow$ $\tilde{\mathcal{A}}_4^0(\epsilon;\mu^2/s,y)$
- $Q\bar{Q}Q\bar{Q}$ final state $\leftrightarrow$ $\mathcal{C}_4^0$ $\leftrightarrow$ E-Terms



$m \to 0$: log. enhanced and finite terms known [Catani, Seymour '99]

+ 2 more 4p cuts



✓ IR poles (solely HPLs) cancel analytically

✓ Agreement with threshold exp. through $\mathcal{O}(\beta)$ [Czarnecki, Melnikov; Beneke, Signer, Smirnov]

✓ Numerical comparison with high-energy exp. [Chetyrkin, Harlander, Kuhn, Steinhauser]

Summary and Outlook

Antenna subtraction extended to heavy quark pair production by uncolored initial states @ NNLO:

- Construction of (integrated) subtraction terms accomplished
- Analytic results for integrated massive antenna functions in terms of HPLs (double-real) and cyclotomic PolyLogs (real-virtual)
- allows for fully differential treatment of $S \to Q\bar{Q} + X$ at NNLO QCD

First application and check: Hadronic R-ratio

- Analytic cancellation of IR poles in $\mathcal{O}(\alpha_s^2)$-corrections
- Exact computation of $R_{LC}^{(2)}$, $R_{SC}^{(2)}$, $R_f^{(2)}$ in agreement with (approximate) results in the literature

Next steps:

- Numerical computation of differential cross sections and distributions for $e^+e^- \to \gamma/Z \to Q\bar{Q} + X$ at NNLO

Outlook: Construction of (integrated) antenna subtraction terms for $pp(p\bar{p}) \to t\bar{t}$ at NNLO

$\hookrightarrow$ Our antennae form part of these subtraction terms! [Abelof, Gehrmann-De Ridder, '11 - '14]

Thanks!