Higgs-boson production cross section at approximate N^3LO

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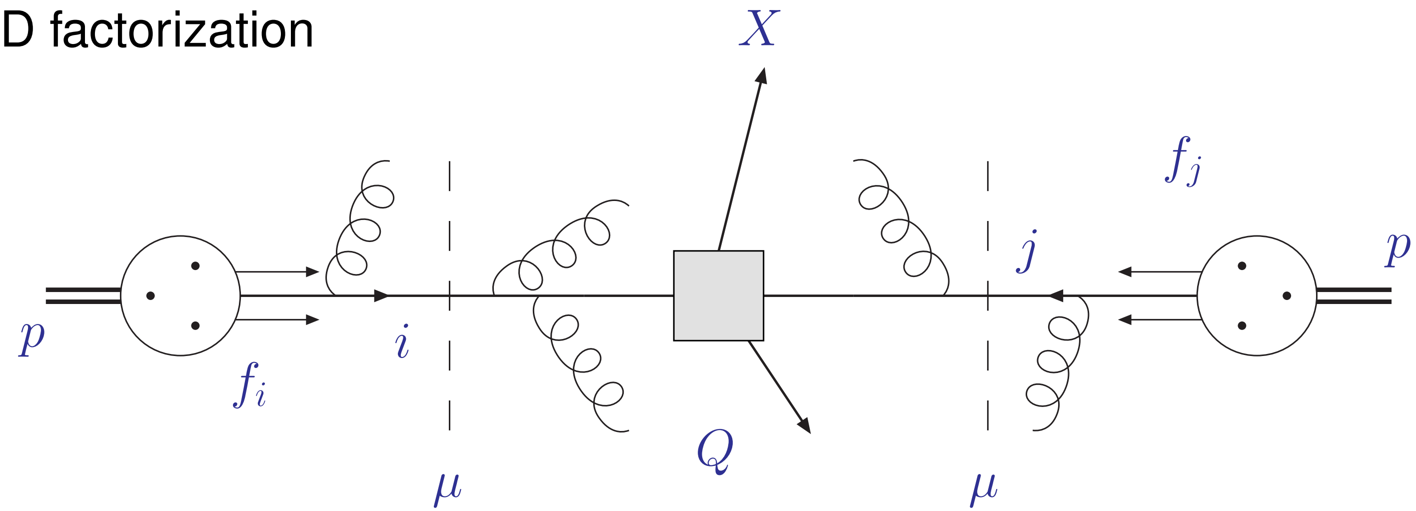
Work done in collaboration with:

- *Approximate N^{3}LO Higgs-boson production cross section using physical-kernel constraints*
  D. de Florian, J. Mazzitelli, S. M. and A. Vogt arXiv:1408.6277

QCD factorization

- QCD factorization

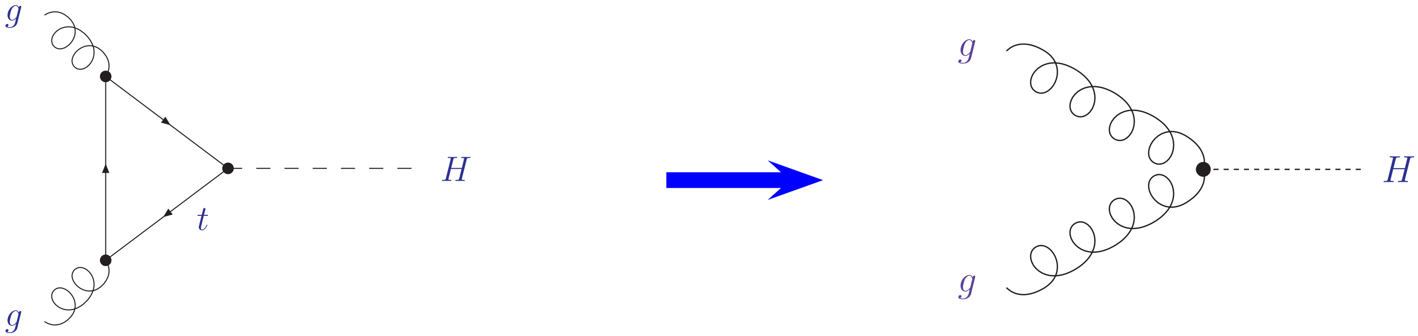


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, ...

Higgs production in gg-fusion

Effective theory



- Hard scattering cross section $\hat{\sigma}_{ij \to H+X}$ dominated by gluon-gluon fusion
  - typically treated in effective theory in limit $m_t \to \infty$; Lagrangian
    $\mathcal{L} = -\frac{1}{4}\,\frac{H}{v}\,C_H\,G^{\mu\nu\,a}\,G^a_{\mu\nu}$
- QCD corrections significant
  - NNLO corrections still large
    Harlander, Kilgore ‘02; Anastasiou, Melnikov ‘02; Ravindran, Smith, van Neerven ‘03
  - improvement with soft N^{3}LO corrections S.M., Vogt ‘05; Laenen, Magnea ‘05:
    in x-space ’$+$’-distributions $\alpha_s^k \ln^{2k-1}(1-x)/(1-x)$
  - NNLL resummation Catani, de Florian, Grazzini, Nason ‘03; Ahrens et al. ‘10; [. . .];
    Ahmed, Mahakhud, Rana, Ravindran ‘14
  - soft-virtual corrections completed: $\delta(1-x)$-term at N^{3}LO
    Anastasiou et al.‘14

Physical kernel

- Cross section for Higgs boson production function of $x = m_H^2/s$

$$\sigma(H) \quad = \quad \sigma_0 \sum_{a,b} f_a \otimes f_b \otimes c_{ab}$$

  - coefficient functions for hard scattering $c_{ab}(\alpha_s) = \alpha_s^k \, c_{ab}^{(k)}$:
    LO $c_{gg}^{(0)} = \delta(1-x)$, **NLO** $c_{ab}^{(1)}$, **NNLO** $c_{ab}^{(2)}$, **N^{3}LO** $c_{ab}^{(3)}$, $\ldots$
  - $c_{ab}(\alpha_s)$ in x-space show $x \to 1$ double logarithmic enhancement:
    '$+$'-distributions $\alpha_s^k \ln^l(1-x)/(1-x)$ with $0 \leq l \leq 2k-1$

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- Alternative factorization: partonic structure functions $\sigma(H) = \sum_{a,b} \sigma_0 \mathcal{F}_{ab}$
  - gg-channel dominant for $x \to 1$
  - evolution equation for $\mathcal{F}_{gg}$ defines physical kernel K_{gg}

$$\frac{d}{d\ln m_H^2}\mathcal{F}_{gg} \quad = \quad \underbrace{\left\{2P_{gg}(\alpha_s) + \beta(\alpha_s)\,\frac{dc_{gg}(\alpha_s)}{d\alpha_s} \otimes \left(c_{gg}(\alpha_s)\right)^{-1}\right\}}_{\textbf{scheme invariant}} \otimes \mathcal{F}_{gg}$$

$$\equiv \quad K_{gg}(\alpha_s) \otimes \mathcal{F}_{gg}$$

  - $K_{gg}(\alpha_s)$ for $x \to 1$ exhibits single logarithmic enhancement
    $\alpha_s^k \ln^l(1-x)/(1-x)$ with $0 \leq l \leq k \quad \longrightarrow \quad$ constraints on $c_{ab}(\alpha_s)$

Coefficient functions

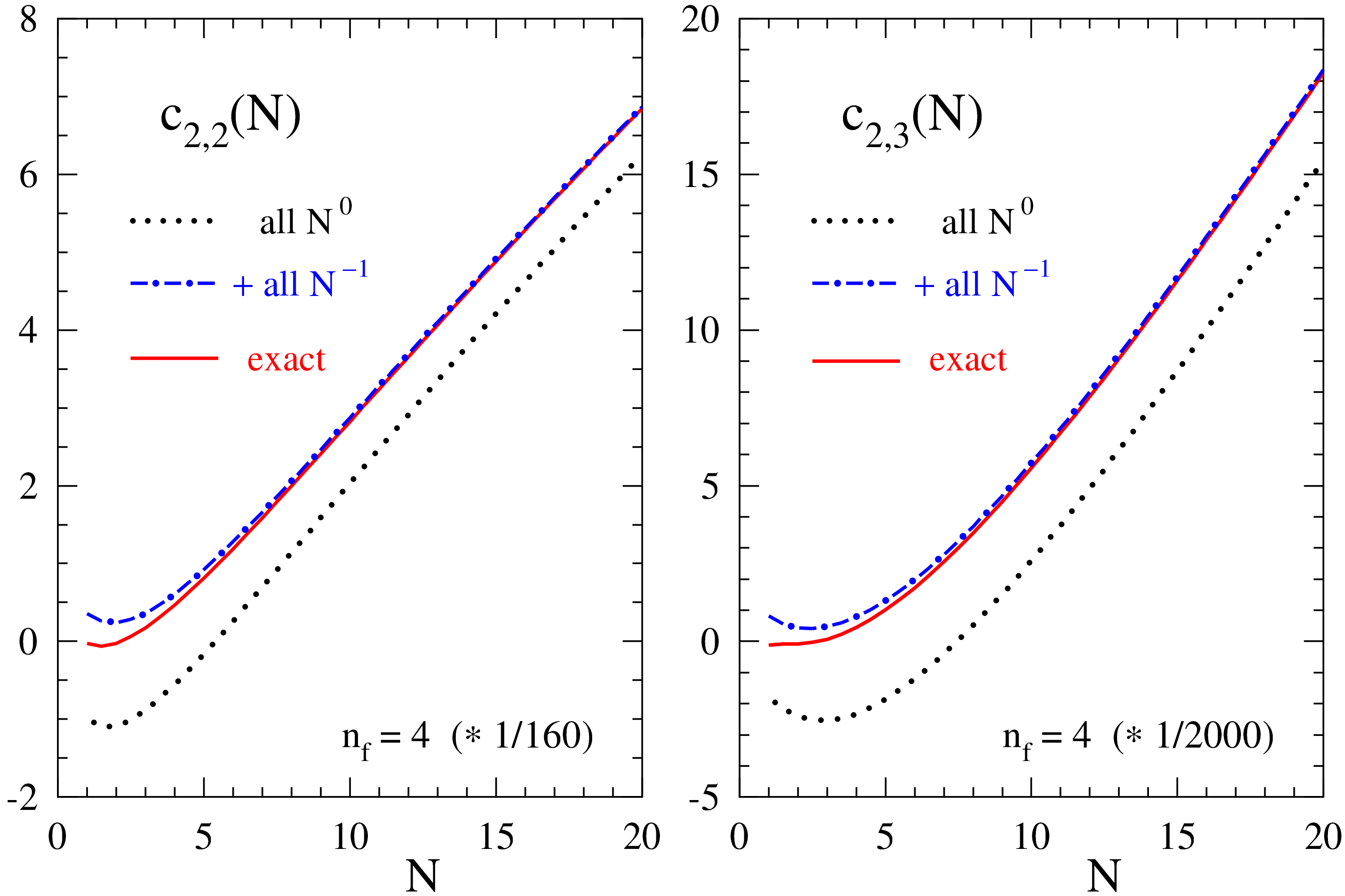
- Constraints from physical kernel $K_{gg}(\alpha_s)$ exact to all powers in $(1-x)^n$
  - access to power suppressed terms with logarithmic enhancement $\alpha_s^k \ln^l(1-x)$ with $1 \leq l \leq 2k$
- Coefficient function at **N^{3}LO** $c_{gg}^{(3)}$

$$c_{gg}^{(3)}(x) = c_{gg}^{(3)}(x)\Big|_{\mathcal{D}_k,\delta(1-x)} - 512 C_A^3 \ln^5(1-x) + \left\{1728 C_A^3 + \frac{640}{3} C_A^2 \beta_0\right\} \ln^4(1-x)$$

$$+ \left\{\left(-\frac{1168}{3} + 3584\zeta_2\right) C_A^3 - \left(\frac{2512}{3} + \frac{1}{3}\xi_H\right) C_A^2 \beta_0 - \frac{64}{3} C_A \beta_0^2\right\} \ln^3(1-x) + \ldots$$

  - unknown coefficient for $\ln^3(1-x)$ estimated $\xi_H = 300$
  - exact computation $\xi_H = 896/3$
    Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger arXiv:1411.3584
- Space of Mellin moments N:
  - leading contributions (SV): $\alpha_s^k \ln^l N$ with $1 \leq l \leq 2k$
  - sub-leading contributions (SV+1/N): $\alpha_s^k (\ln^l N)/N$ with $1 \leq l \leq 2k$

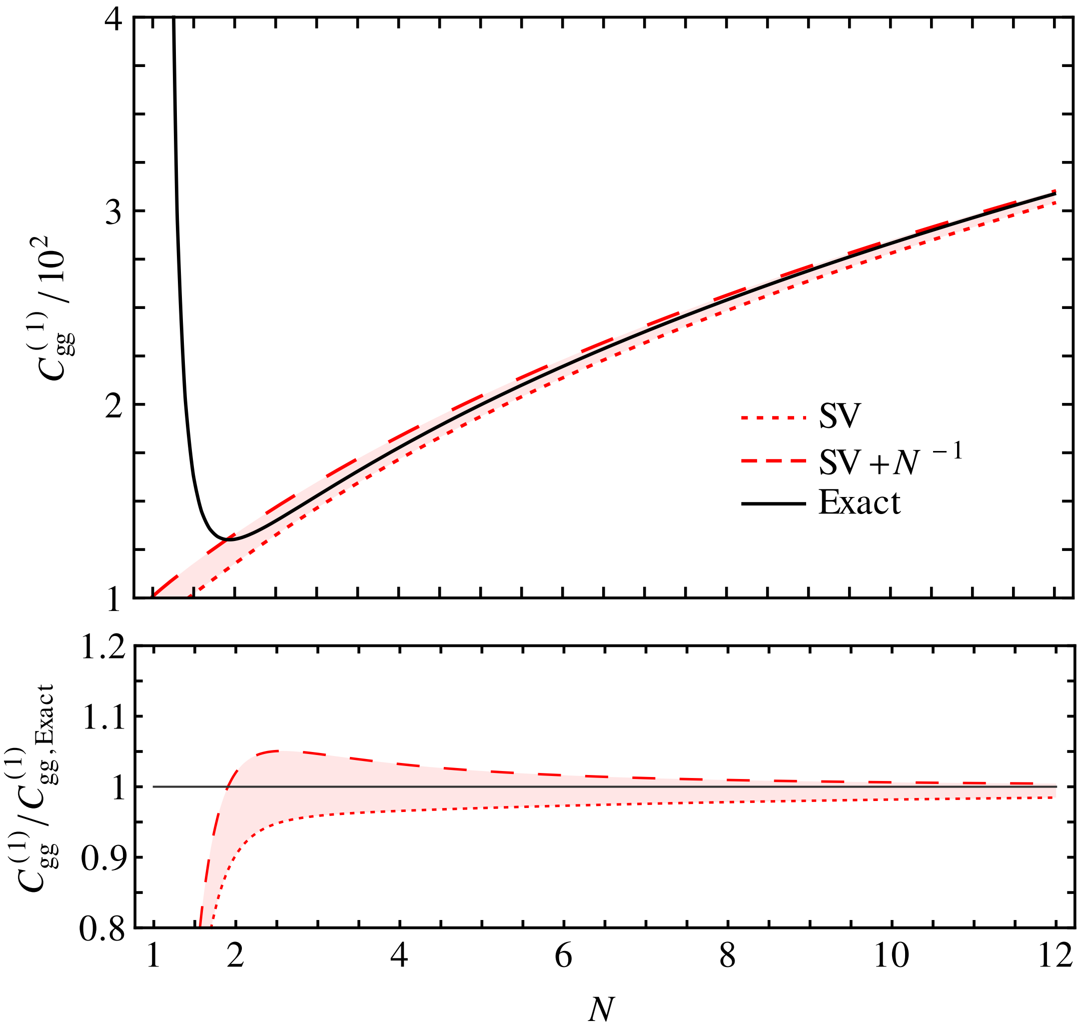
Quality check



- SV and SV+1/N approximations provide uncertainty band S.M., Vogt '09
  - tested for large number of observables
  - γ-DIS and ϕ-DIS known exactly to N^{3}LO
    S.M., Vermaseren, Vogt '05, Soar, S.M., Vermaseren, Vogt '09

Higgs boson production: coefficient function

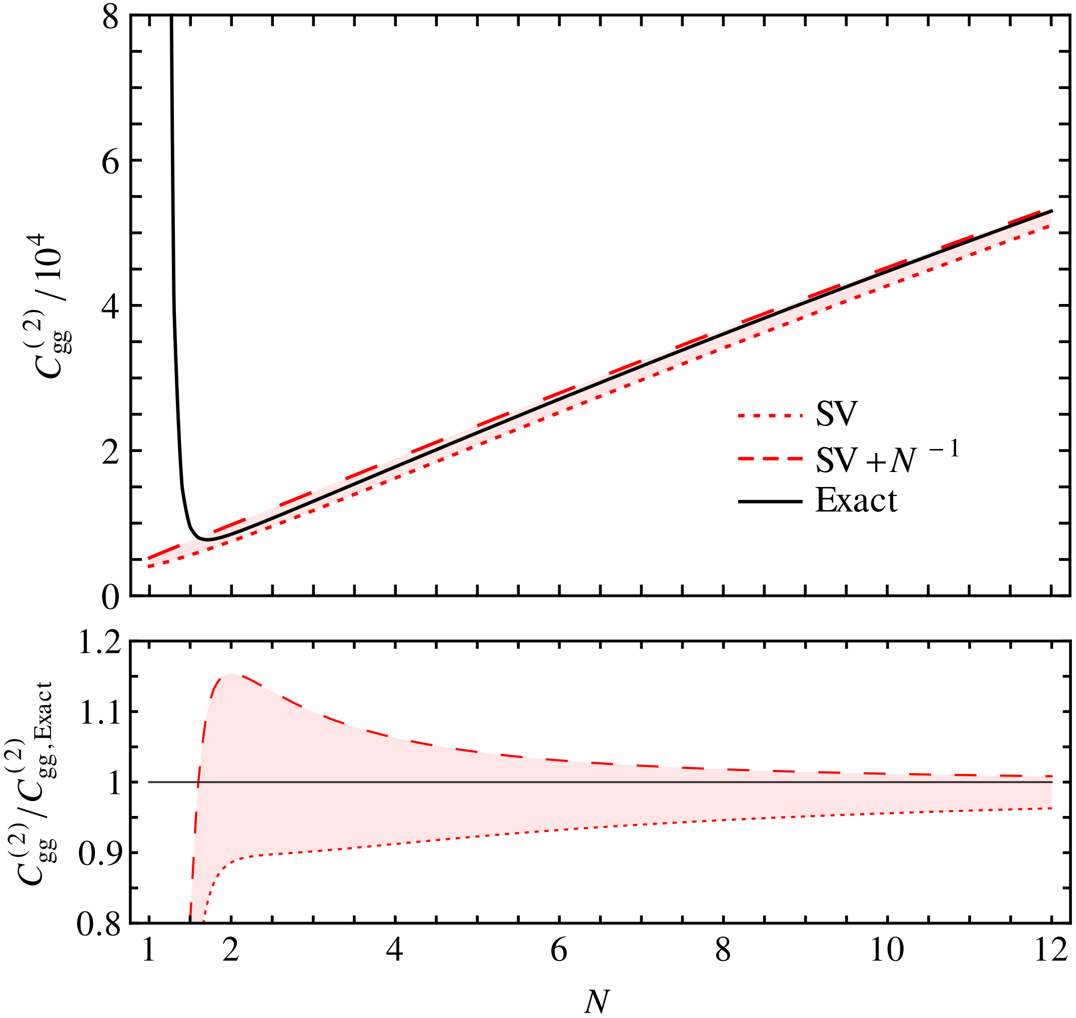
Coefficient function at NLO



- SV and SV+1/N approximations work even for N not so large

Higgs boson production: coefficient function

Coefficient function at NNLO



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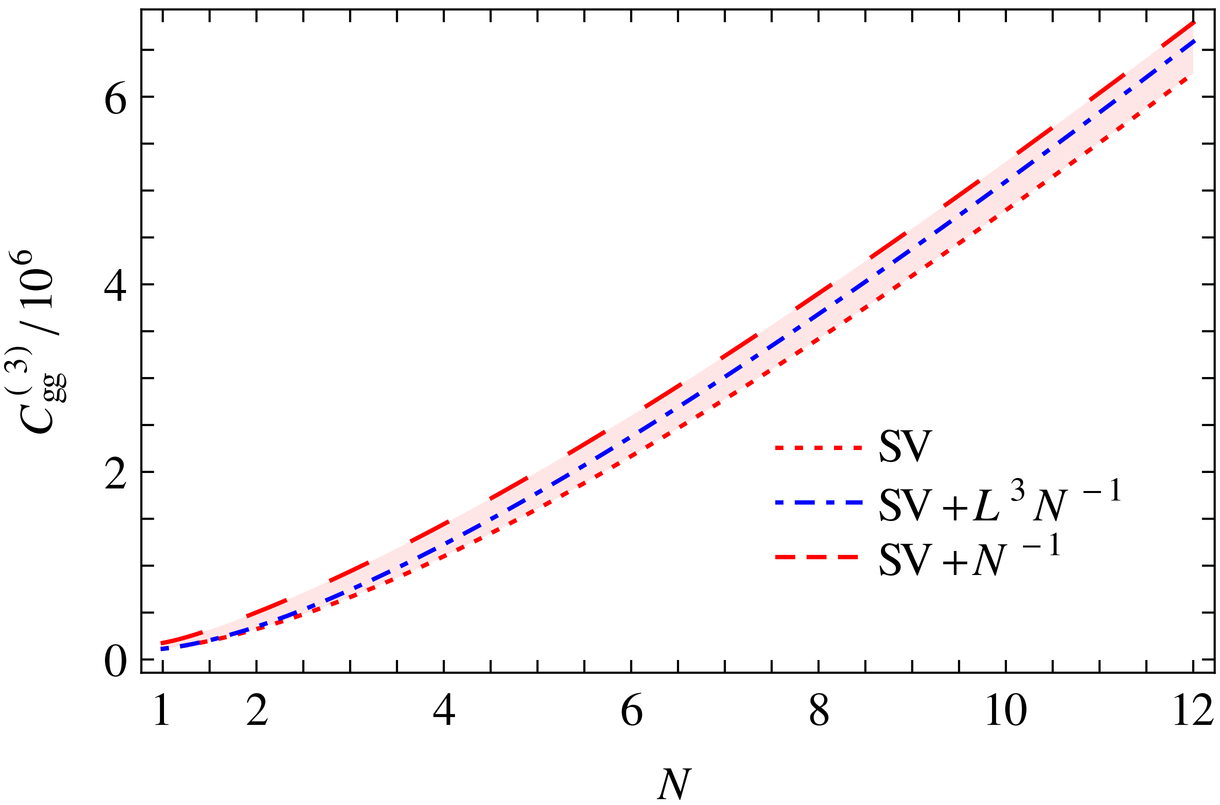
Approximate N^{3}LO: coefficient function



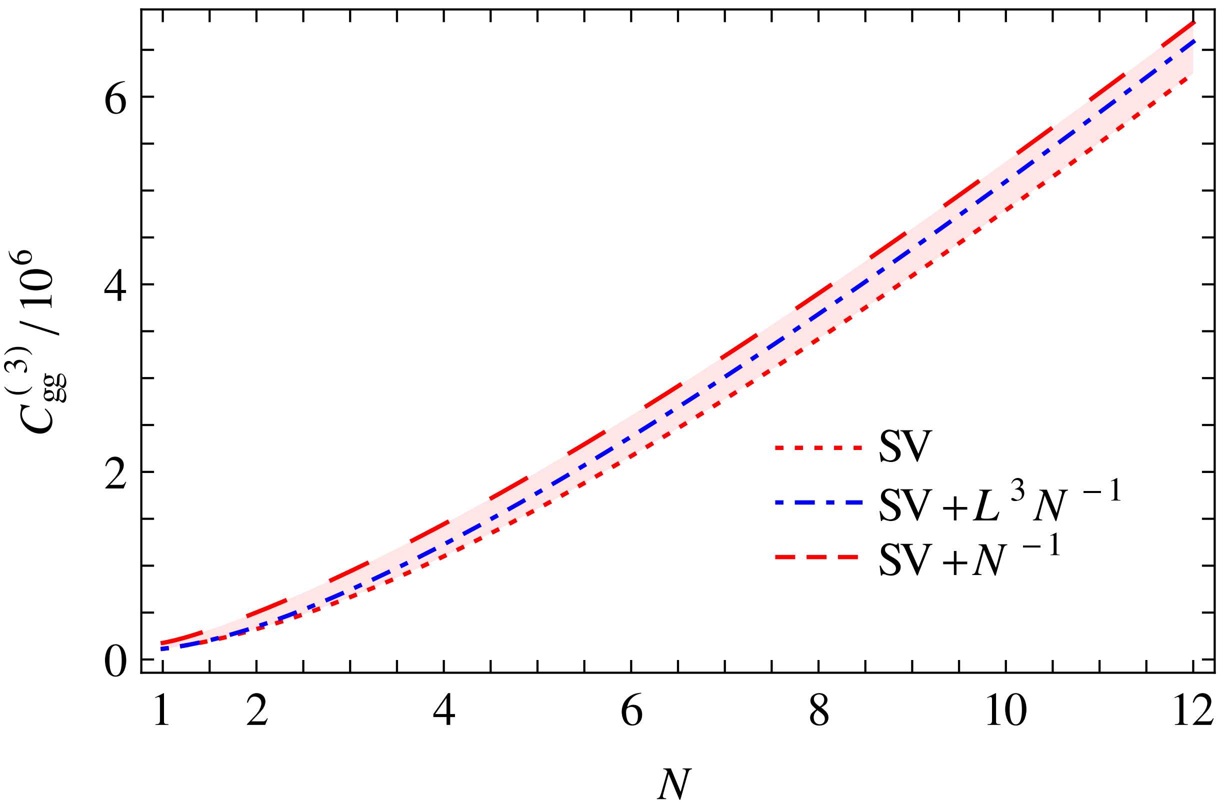
- Coefficient function at N^{3}LO (only subleading $1/N$ terms displayed)

$$c_{gg}^{(3)}(N) \sim (108 \ln^5 N + 615.7 \ln^4 N + 2041.2 \ln^3 N + \underbrace{4968 \ln^2 N + 1944 \ln N + 972}_{\text{estimate from } \phi\text{-DIS}})/N$$

- Leading logarithm agrees with estimate by factorization of splitting function $P_{gg}^{(0)} \quad \longrightarrow \quad c_{gg}^{(3)}(N) \sim 108 \ln^5 N/N$

  Krämer, Laenen, Spira ‘96; Ball, Bonvini, Forte, Marzani, Ridolfi ´13

Approximate N^{3}LO: coefficient function



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- Exact computation

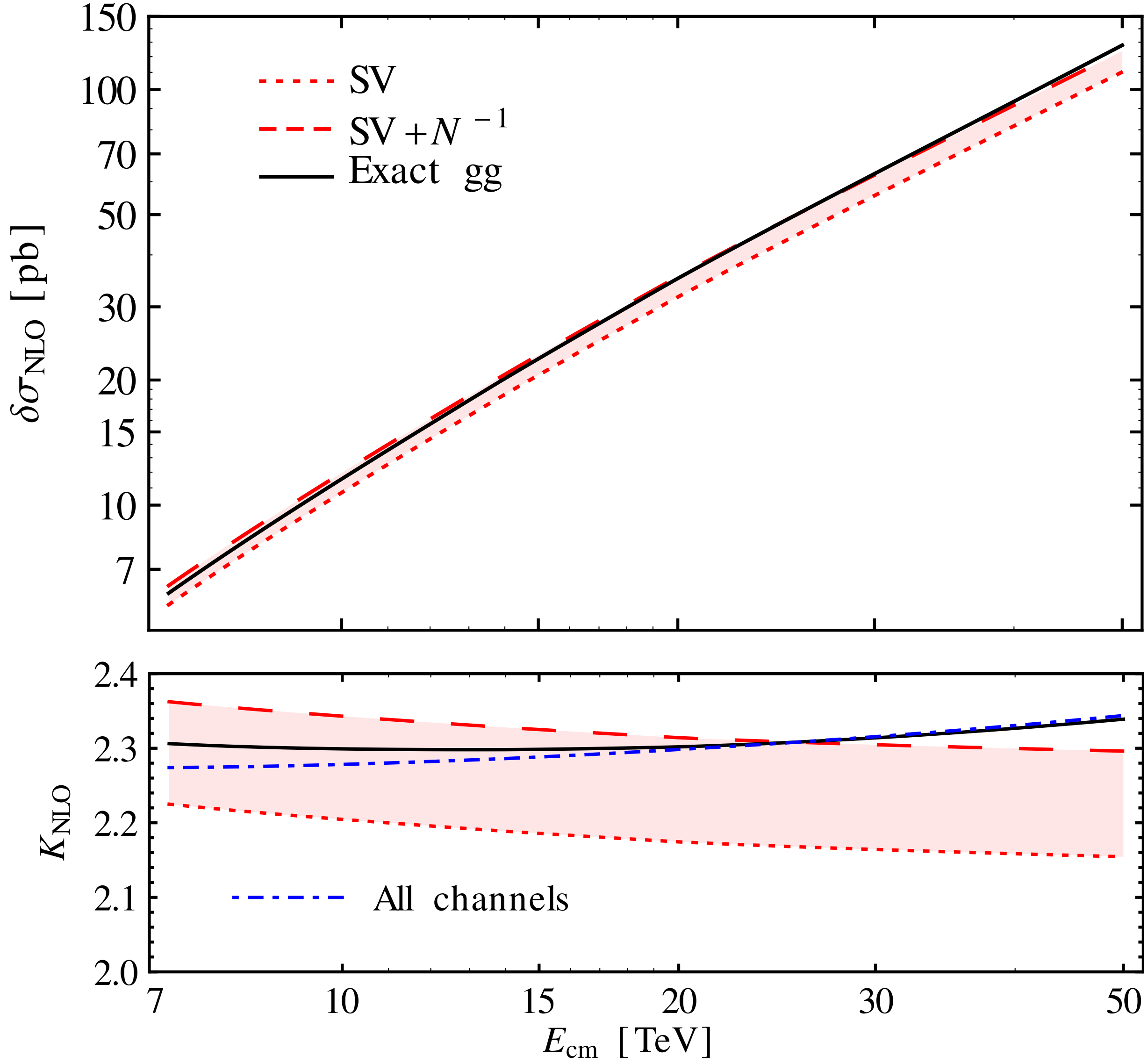
  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger arXiv:1411.3584

$$c_{gg}^{(3)}(N) \sim (108 \ln^5 N + 615.7 \ln^4 N + 2036.4 \ln^3 N + 3305 \ln^2 N + 3459 \ln N + 703)/N$$

Higgs boson production: cross section

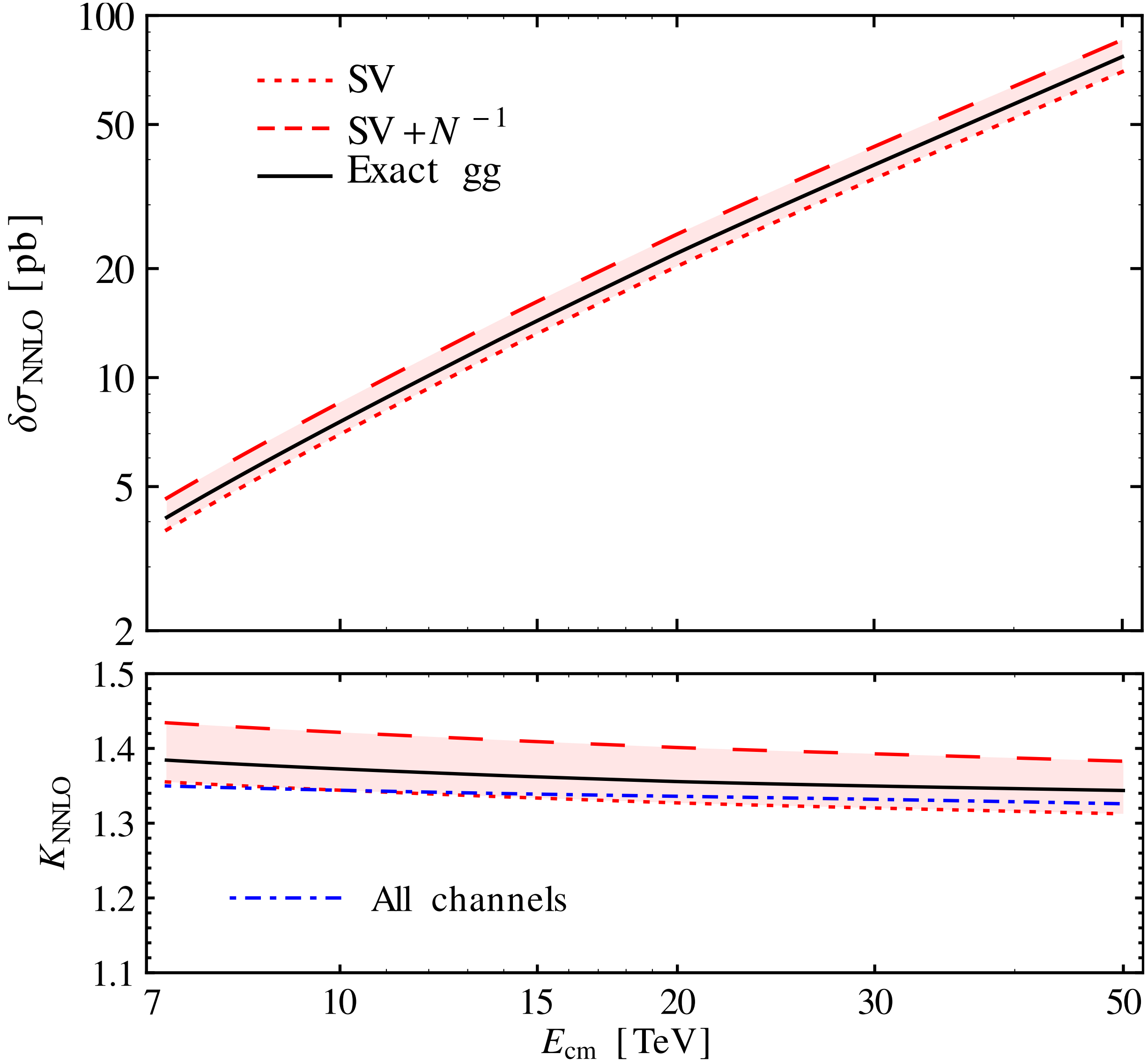
Cross section at NLO

- Approximation based on SV and SV+1/N works for current LHC energies $\sqrt{s} = 13$ TeV very well



Higgs boson production: cross section

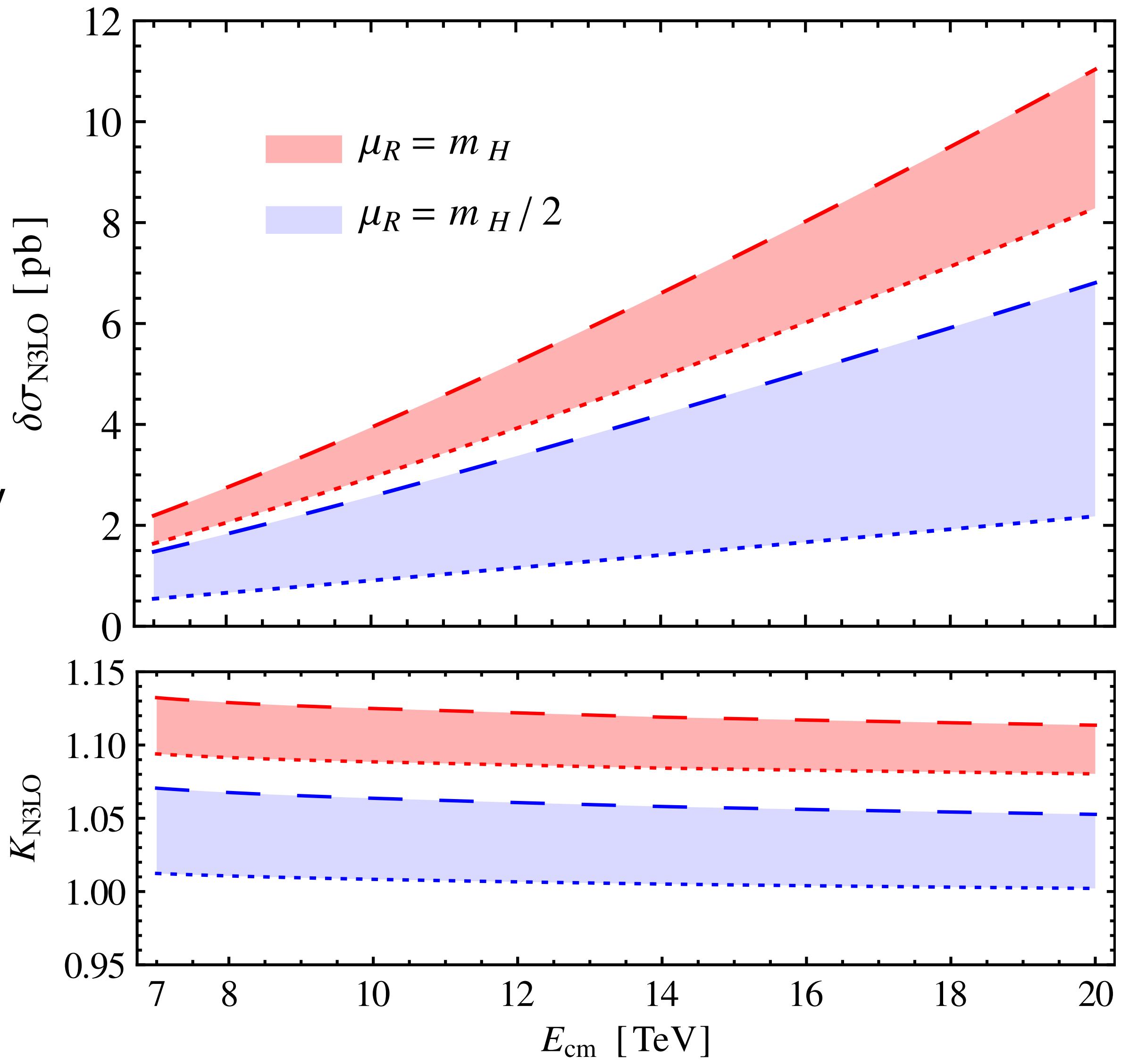
Cross section at NNLO



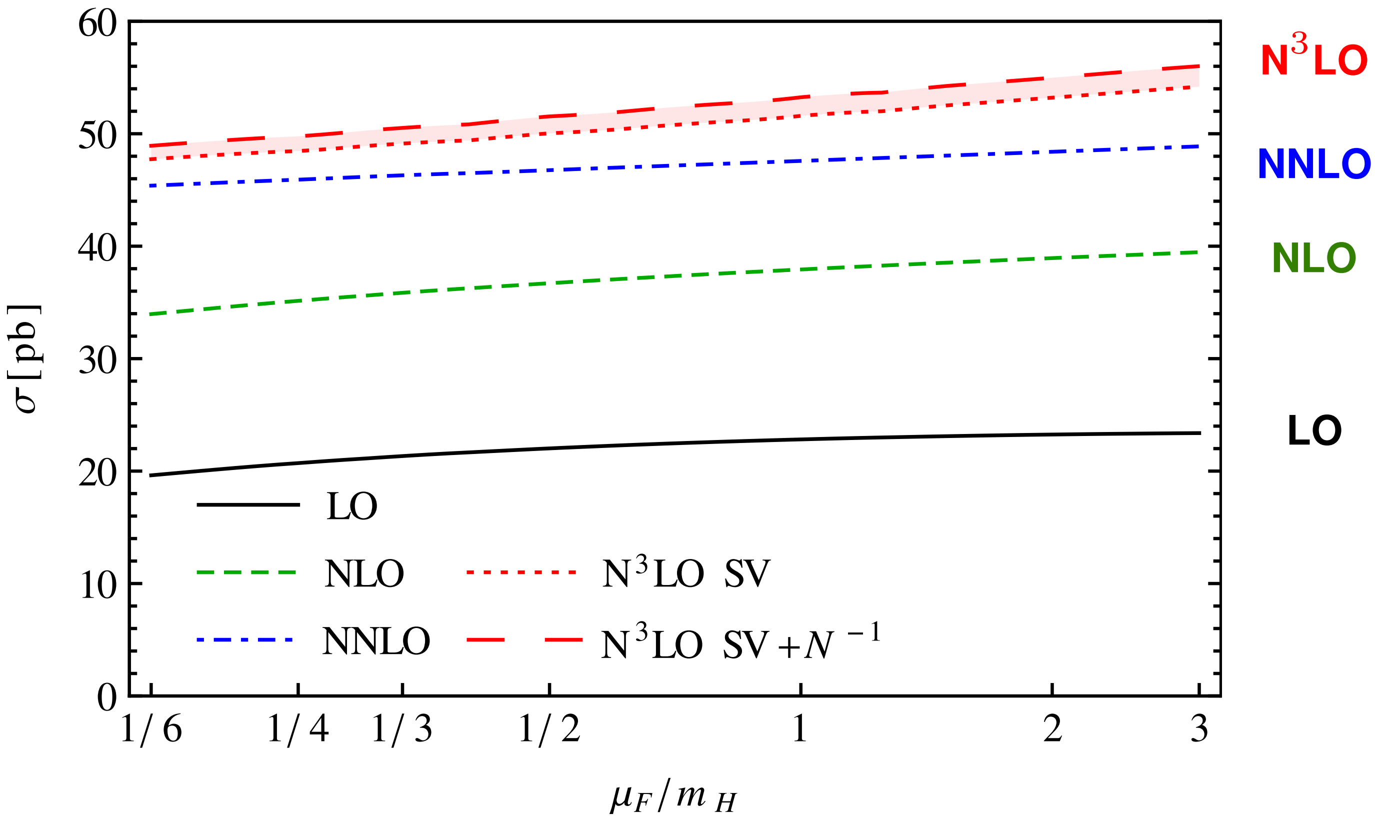
- Approximation based on SV and SV+1/N works for current LHC energies $\sqrt{s} = 13$ TeV very well

Approximate N^3LO: cross section

- N^3LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- Point of minimal sensitivity around $\mu = m_H/2$ with K-factor $1 \dots 5\%$ at $\sqrt{s} = 13$ TeV

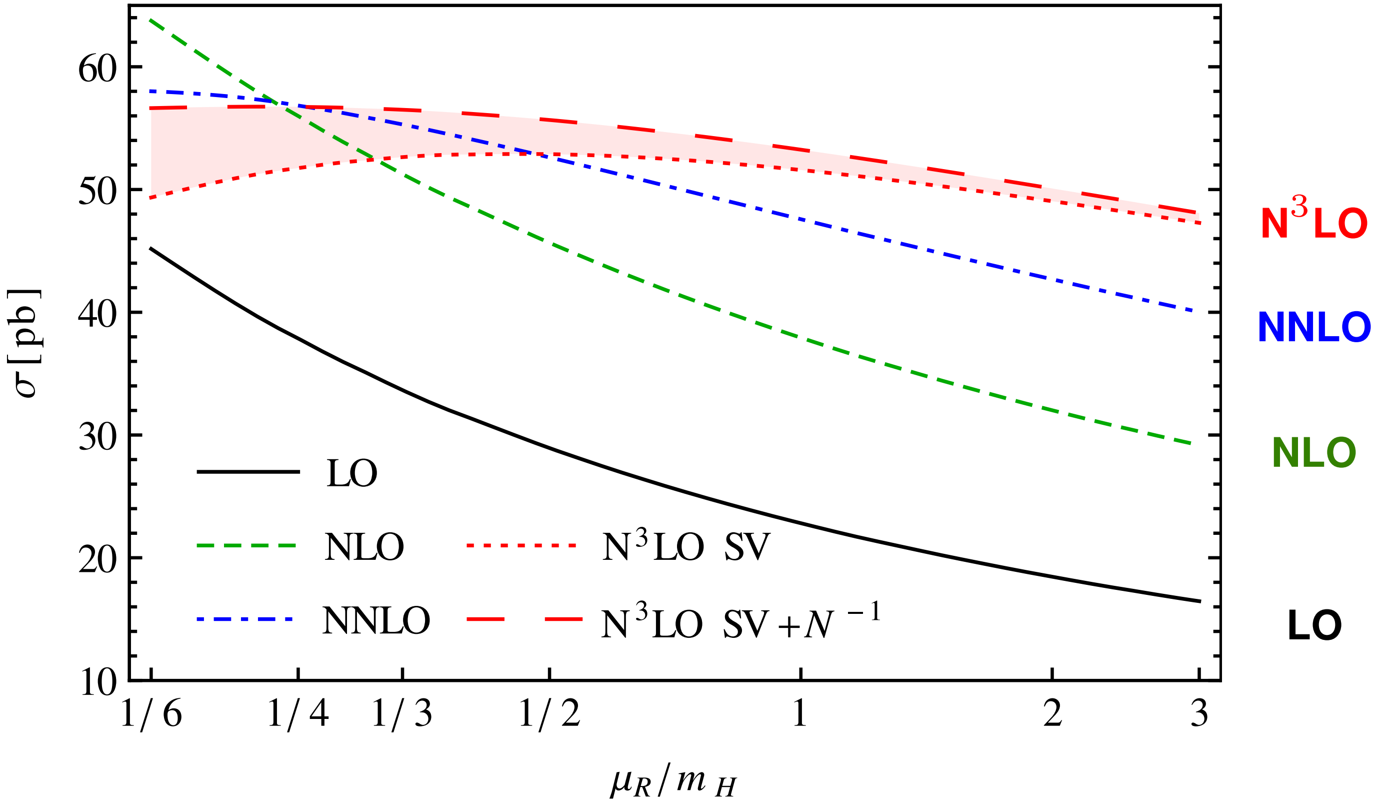


Factorization scale variation



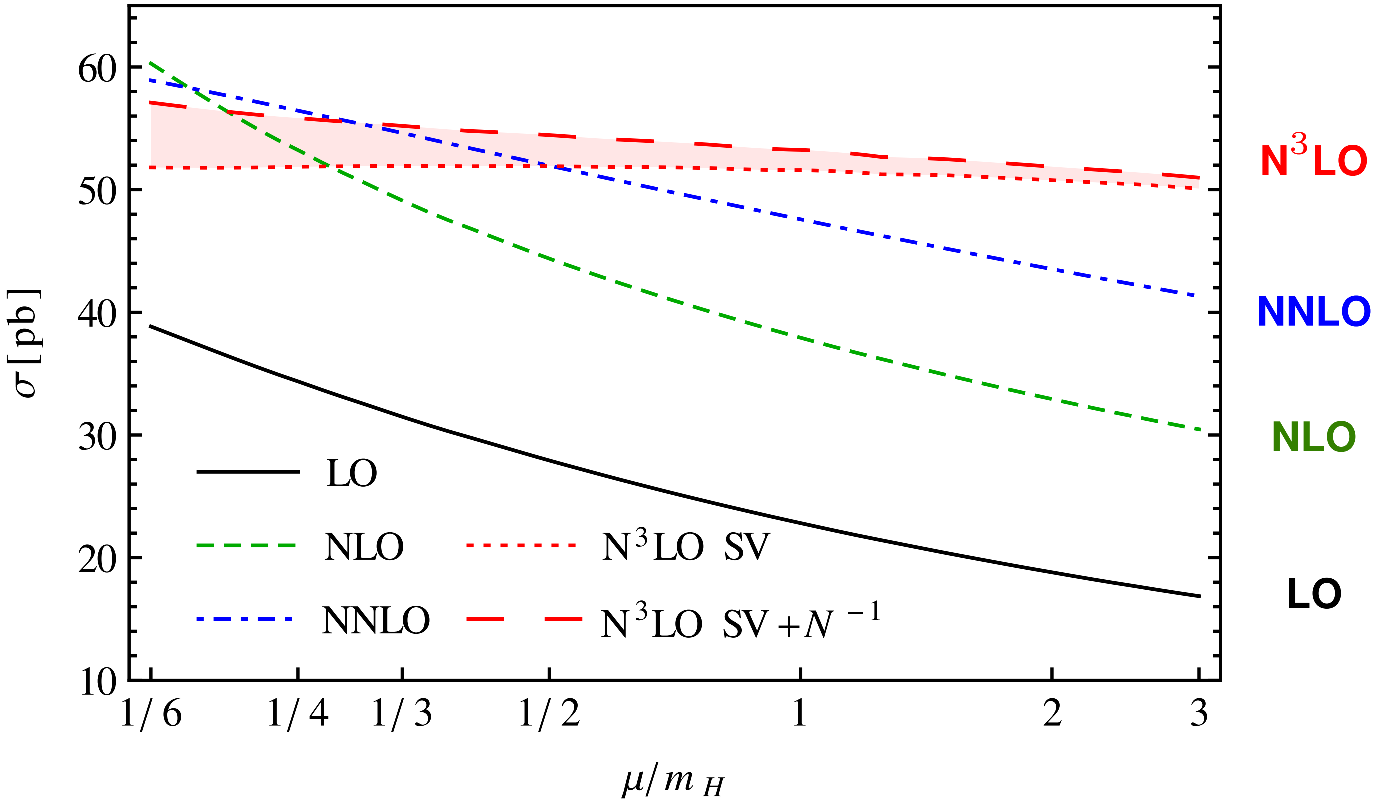
- Variation of factorization scale μ_F for fixed $\mu_R = m_H$ at $\sqrt{s} = 14$ TeV
- Dependence on μ_F at **N^{3}LO** larger than at **NNLO**
  - missing four-loop splitting functions $P_{ij}^{(3)}$
  - gluon PDF only known up to **NNLO**
  - omission of the quark-gluon and quark-(anti)quark channels

Renormalization scale variation



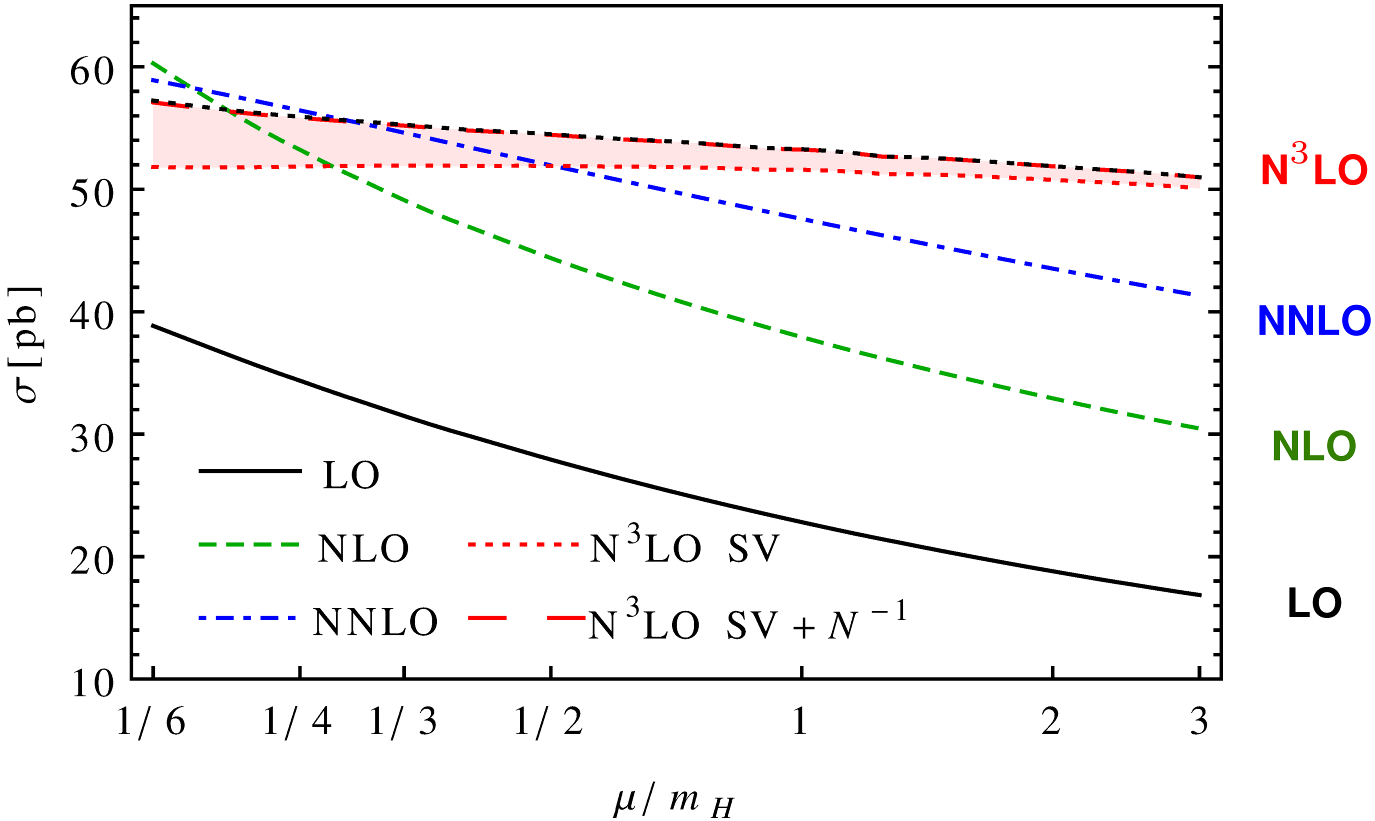
- Variation of renormalization scale μ_R for fixed $\mu_F = m_H$ at $\sqrt{s} = 14$ TeV
- Perturbative stability under renormalization scale variation
  - dependence on μ_R flattens off at **N^{3}LO**
  - four-loop beta function β_3 known
- Point of minimal sensitivity around $\mu_R = m_H/2$

Combined scale variation



- Simultaneous variation of scales $\mu_R = \mu_F$ around m_H at $\sqrt{s} = 14$ TeV
- Small improvement with respect μ_R variation for fixed μ_F
  - should not be trusted quantitatively
- Conservative error estimate: variation of μ_R for fixed μ_F
  - overall theoretical uncertainty $\Delta\sigma(H)$ less than 5%

Combined scale variation



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Benchmark cross section

- Cross section $\sigma(H)$ at LHC $\sqrt{s} = 13$ TeV
- N^{3}LO from average of SV and SV+1/N approximation
  - parameters: $m_H = 125$ GeV, $m_t = 172.5$ GeV pole mass, $\alpha_s(M_Z) = 0.1171$ with MSTW2008 PDFs (nnlo68cl)
  - scale choice $\mu_R = \mu_F = m_H$

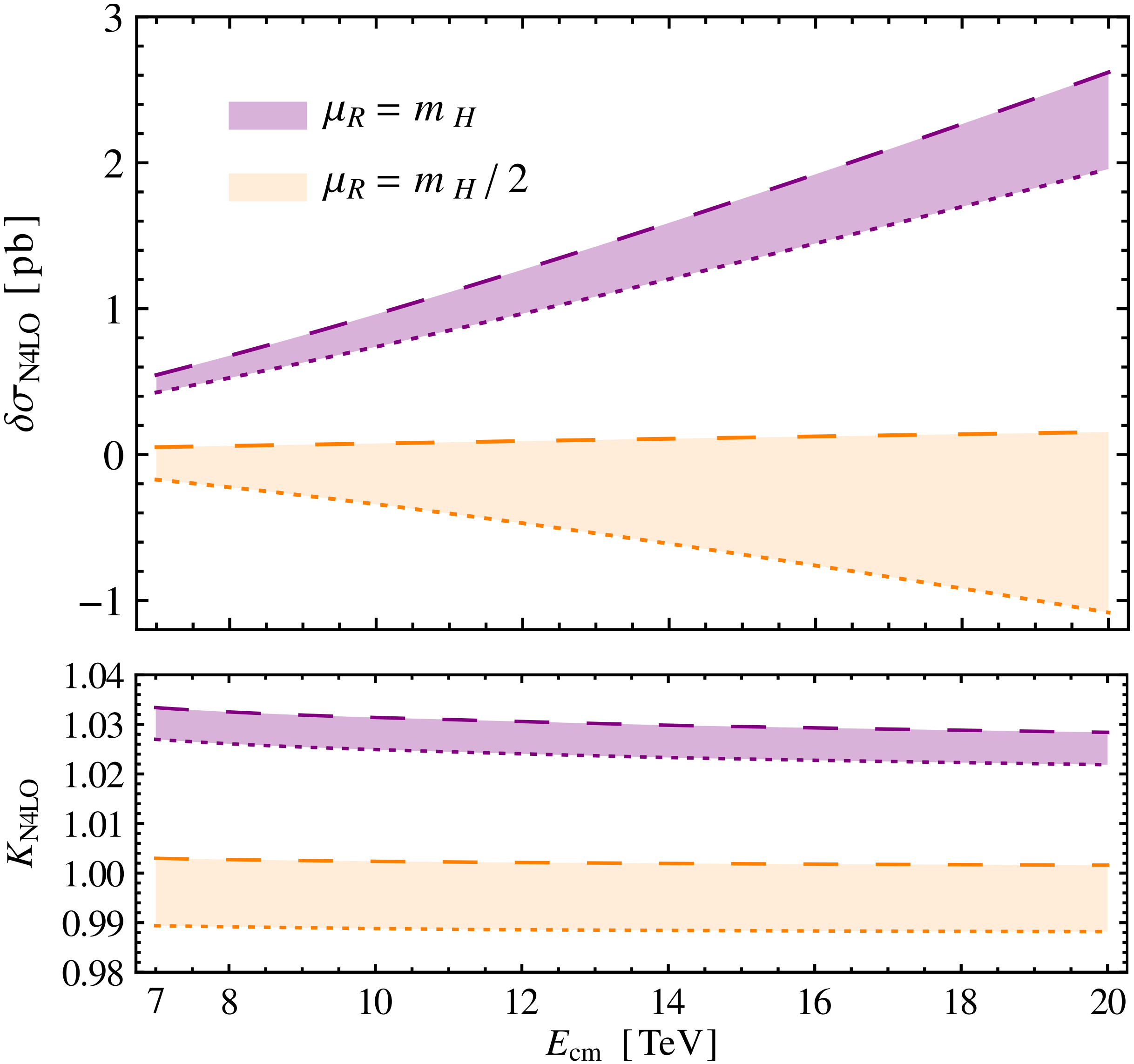
$\mu_0 = m_H$	$\mu_R/\mu_0 = 0.5$	$\mu_R/\mu_0 = 1$	$\mu_R/\mu_0 = 2$
$\mu_F/\mu_0 = 0.5$	48.05 ± 1.17	45.98 ± 0.70	–
$\mu_F/\mu_0 = 1$	48.91 ± 1.29	47.22 ± 0.76	44.63 ± 0.47
$\mu_F/\mu_0 = 2$	–	48.50 ± 0.82	46.07 ± 0.51

  - scale choice $\mu_R = \mu_R = m_H/2$

$\mu_0 = m_H/2$	$\mu_R/\mu_0 = 0.5$	$\mu_R/\mu_0 = 1$	$\mu_R/\mu_0 = 2$
$\mu_F/\mu_0 = 0.5$	48.66 ± 1.86	47.16 ± 1.04	–
$\mu_F/\mu_0 = 1$	48.73 ± 2.11	48.05 ± 1.17	45.98 ± 0.70
$\mu_F/\mu_0 = 2$	–	48.91 ± 1.29	47.22 ± 0.76

Approximate N^4LO: cross section

- Consistency check with approximate N^4LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- K-factor $\simeq 1\%$ for $\mu = m_H/2$ with at $\sqrt{s} = 13$ TeV



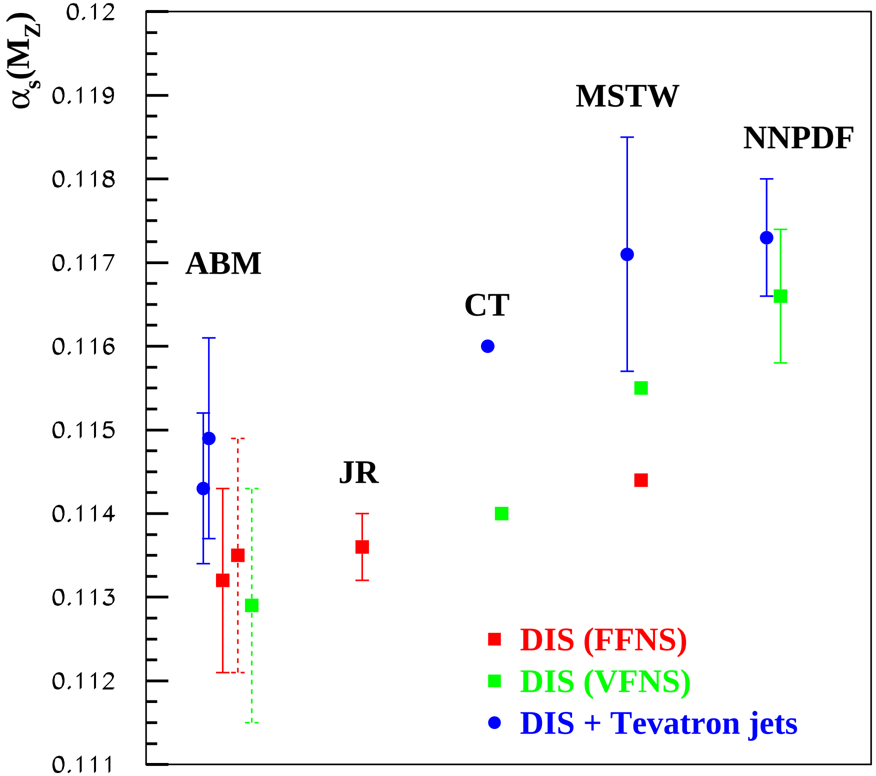
Dependence on parton distributions

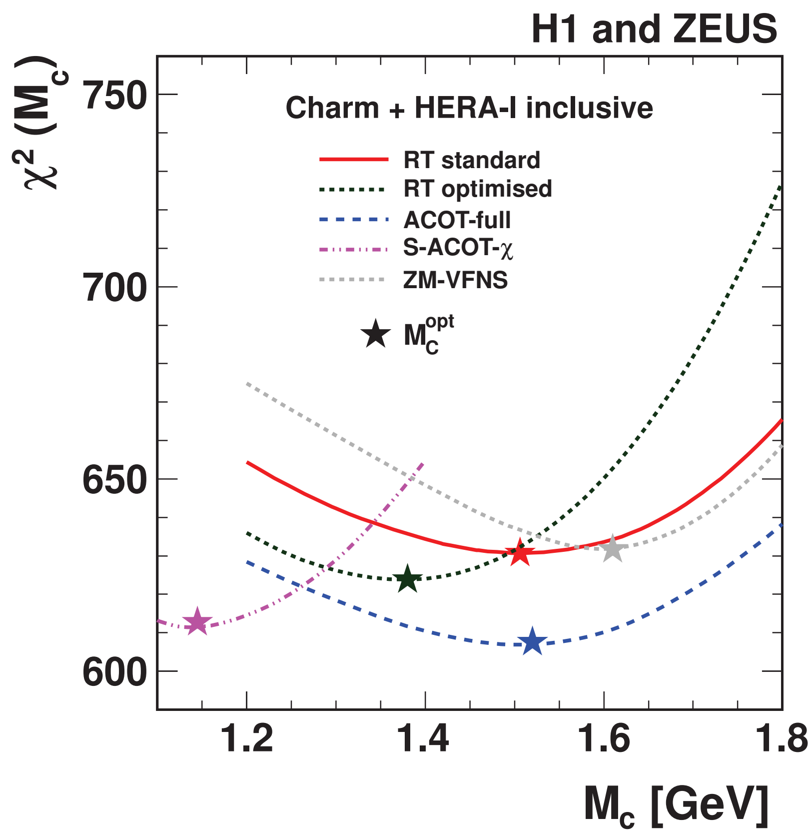
- Cross section $\sigma(H)$ at NNLO with uncertainties: $\sigma(H) + \Delta\sigma(\mathrm{PDF} + \alpha_\mathrm{s})$

ABM11	ABM12	CT10	MSTW	NN23
39.58 ± 0.77	39.70 ± 0.84	$41.84\ ^{+1.30}_{-1.69}$	$42.12\ ^{+0.44}_{-0.63}$	43.75 ± 0.41

- Comparison for PDF sets at NNLO
  - ABM11, ABM12 Alekhin, Blümlein, S.M. '13, CT10 Gao et al. '13, MSTW Martin, Stirling, Thorne, Watt '09, NNPDF (NN23) Ball et al. '12
- Large spread for predictions from different PDFs $\sigma(H) = 39.6 \ldots 43.8$
- PDF and α_s differences between sets amount to up to 10%
  - significantly larger than residual theory uncertainty due to incomplete N^{3}LO QCD corrections
- Observed spread due to differences in theory considerations and analysis procedures $\longrightarrow$ correlations between α_s, $g(x)$ and m_q
  - target mass corrections and higher twist in DIS
  - treatment of heavy quarks
  - error correlations among data sets
  - fits to compatible data sets

Non-perturbative parameters





- Differences in α_s values due to different physics models for heavy quarks
- Charm mass m_c “tuning” parameter for variable flavor number schemes
  H1 coll. arxiv:1211.1182
- Effect of Higgs cross section
  - linear rise in $\sigma(H) = 40.6 \ldots 43.8$ for $m_c = 1.05 \ldots 1.75$ with MSTW PDFs Martin, Stirling, Thorne, Watt ‘10

Summary

Inclusive Higgs cross section at the LHC

- Radiative corrections at higher orders in QCD well under control
  - approximate N^{3}LO in QCD with overall theoretical uncertainty $\Delta\sigma(H)$ less than 5%
  - good stability around point of minimal sensitivity $\mu = m_H/2$
  - conservative error estimate: variation of μ_R for fixed μ_F

Parton distributions, α_s and all that

- Precision determinations of non-perturbative parameters is dominating source of uncertainty
  - parton content of proton (PDFs)
  - coupling constants $\alpha_s(M_Z)$
  - masses m_c, m_b, m_t, m_H, ...
- Spread in predictions amounts currently up to 10%