QCD jet cross sections at NNLO accuracy

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in collaboration with
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Outline

- The problem and our goals
- Our method: recipe for a general subtraction scheme at any order in perturbation theory
- Main difficulty: integrating the counter terms
- Light in the tunnel: cancellation of poles
- Conclusions

Problem

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$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m}$$

$$\equiv \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$$

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  - in σ^{RR} kinematical singularities as one or two partons become unresolved yielding ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$ after integration over phase space, no explicit ϵ-poles
  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$
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How to combine to obtain finite cross section?

personal opinion: general solution is not yet available

Approaches

- Sector decomposition

  Anastasiou, Melnikov, Petriallo et al 2004-

- Antennae subtraction

  Gehrmann, Gehrmann-De Ridder, Glover et al 2004-

- q_T-subtraction

  S. Catani, M. Grazzini et al 2007-

- Sector-improved phase space for real radiation (STRIPPER)

  Czakon et al 2010-

- Completely Local Subtractions for Fully Differential Predictions at NNLO (Colorful NNLO)

  Somogyi, TZ et al 2005-

- For details see: NNLO Ante Portas (LHCPhenonet Summer School in Hungary, June 2014)

  http://www.lhcphenonet.eu/debrecen2014/

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✓ option to constrain subtraction near singular regions (important check)

Recipe

Structure

of subtractions is governed by the jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

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RR,A_2 regularizes doubly-unresolved limits

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

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RR,A_1 regularizes singly-unresolved limits

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$\mathrm{RR,A_{12}}$ removes overlapping subtractions

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RV,A_1 regularizes singly-unresolved limits

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Use known ingredients

- Universal IR structure of QCD (squared) matrix elements
  - ϵ-poles of one-loop amplitudes:

$$|\mathcal{M}_m^{(1)}(\{p\})\rangle = \boldsymbol{I}_0^{(1)}(\{p\},\epsilon)|\mathcal{M}_m^{(0)}(\{p\})\rangle + \mathrm{O}(\epsilon^0)$$

$$\boldsymbol{I}_0^{(1)}(\{p\},\epsilon) = \frac{\alpha_\mathrm{s}}{2\pi}\sum_i \left[\frac{1}{\epsilon}\gamma_i - \frac{1}{\epsilon^2}\sum_{k\neq i}\boldsymbol{T}_i\cdot\boldsymbol{T}_k\left(\frac{4\pi\mu^2}{s_{ik}}\right)^\epsilon\right]$$

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  - ϵ-poles of two-loop amplitudes:

$$|\mathcal{M}_m^{(2)}(\{p\},\epsilon)\rangle = \boldsymbol{I}_0^{(1)}(\{p\})|\mathcal{M}_m^{(1)}(\{p\})\rangle + \boldsymbol{I}_0^{(2)}(\{p\})|\mathcal{M}_m^{(0)}(\{p\})\rangle + \mathrm{O}(\epsilon^0)$$

$$\begin{aligned}\boldsymbol{I}_{\mathrm{R.S.}}^{(2)}(\epsilon,\mu^2;\{p\}) &= -\frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\left(\boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\}) + 4\pi\beta_0\frac{1}{\epsilon}\right)\\ &+ \frac{e^{+\epsilon\psi(1)}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(2\pi\beta_0\frac{1}{\epsilon} + K\right)\boldsymbol{I}^{(1)}(2\epsilon,\mu^2;\{p\}) \qquad (19)\\ &+ \boldsymbol{H}_{\mathrm{R.S.}}^{(2)}(\epsilon,\mu^2;\{p\}) \ ,\end{aligned}$$

S. Catani 1998

G. Sterman, M.E.Tejeda-Yeomans 2003,S. Moch, M. Mitov 2007

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tree-level 3-parton splitting, double soft current:

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- Extension over whole phase space using momentum mappings (not unique):

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

Momentum mappings

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

- implement exact momentum conservation
- recoil distributed democratically

  $\Rightarrow$ can be generalized to any number s of unresolved partons
- different mappings for collinear and soft limits
  - collinear limit $p_i || p_r$: $\{p\}_{n+1} \xrightarrow{\mathsf{C}_{ir}} \{\tilde{p}\}_n^{(ir)}$
  - soft limit $p_s \to 0$: $\{p\}_{n+1} \xrightarrow{\mathsf{S}_s} \{\tilde{p}\}_n^{(s)}$

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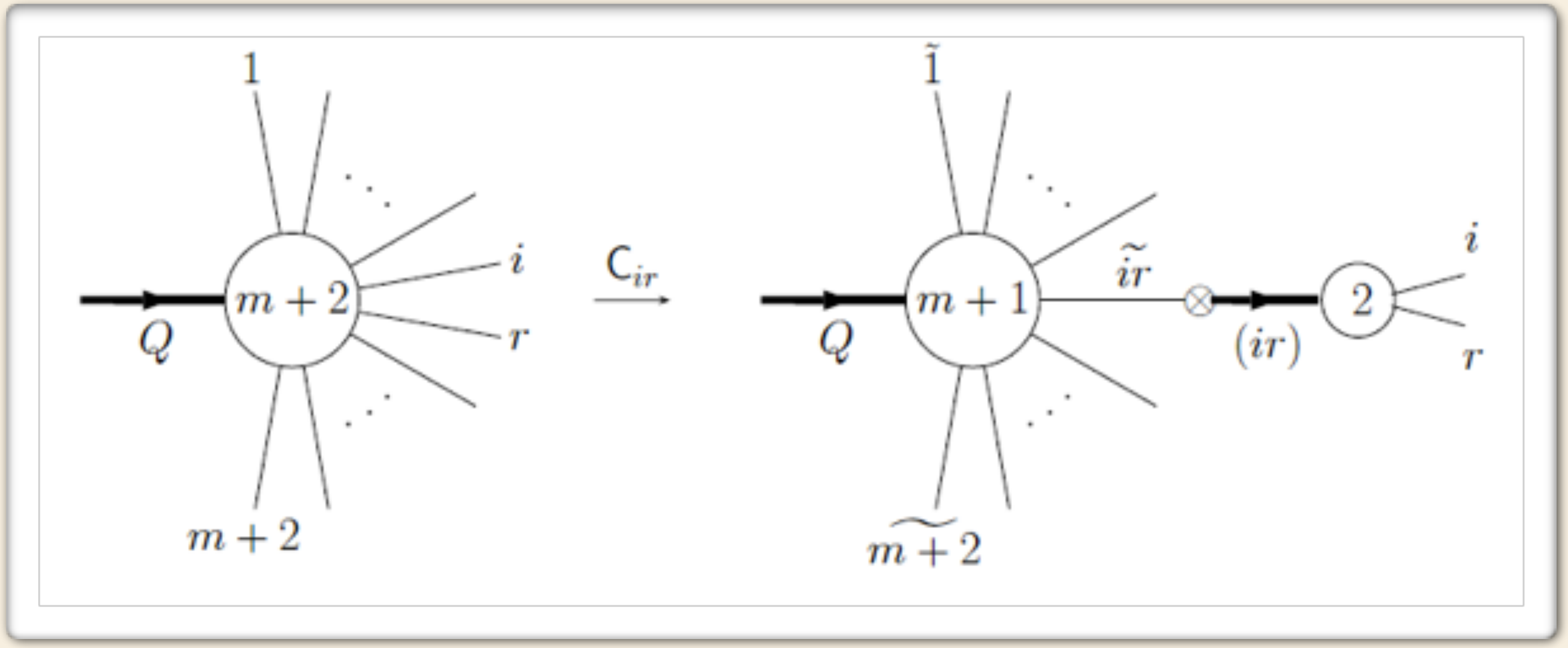

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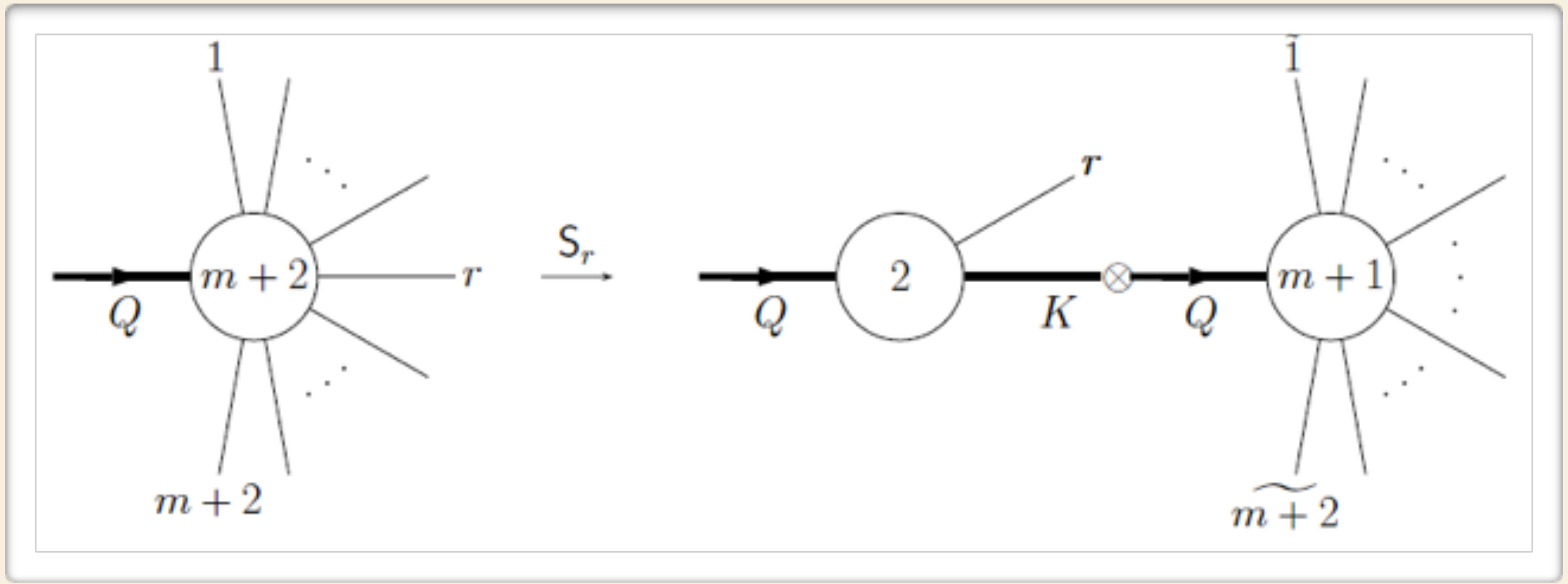

Momentum mappings

define subtractions

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Regularized RR and RV contributions

can now be computed by numerical Monte Carlo integrations

(implementation for general m in progress)

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

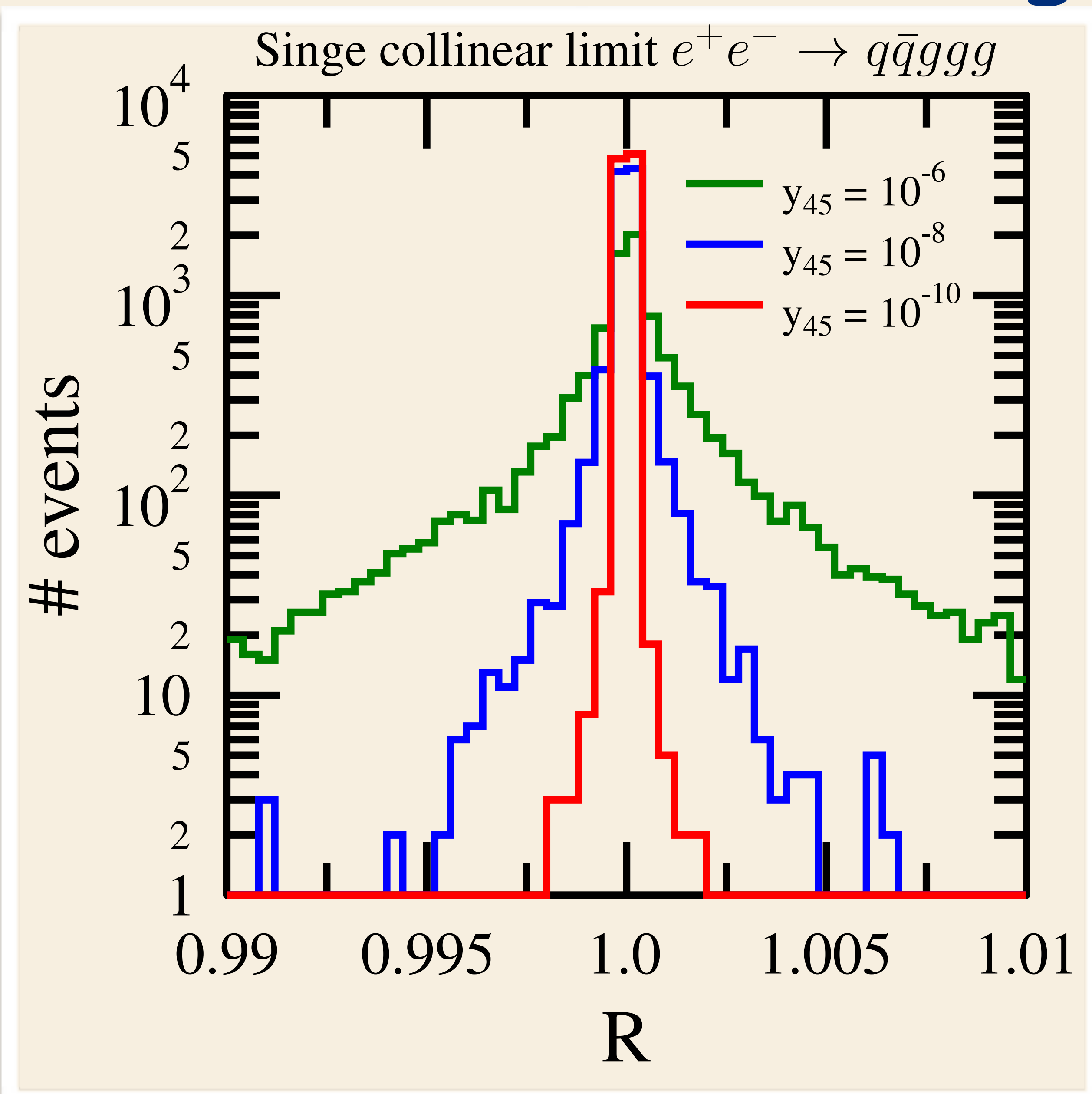
$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

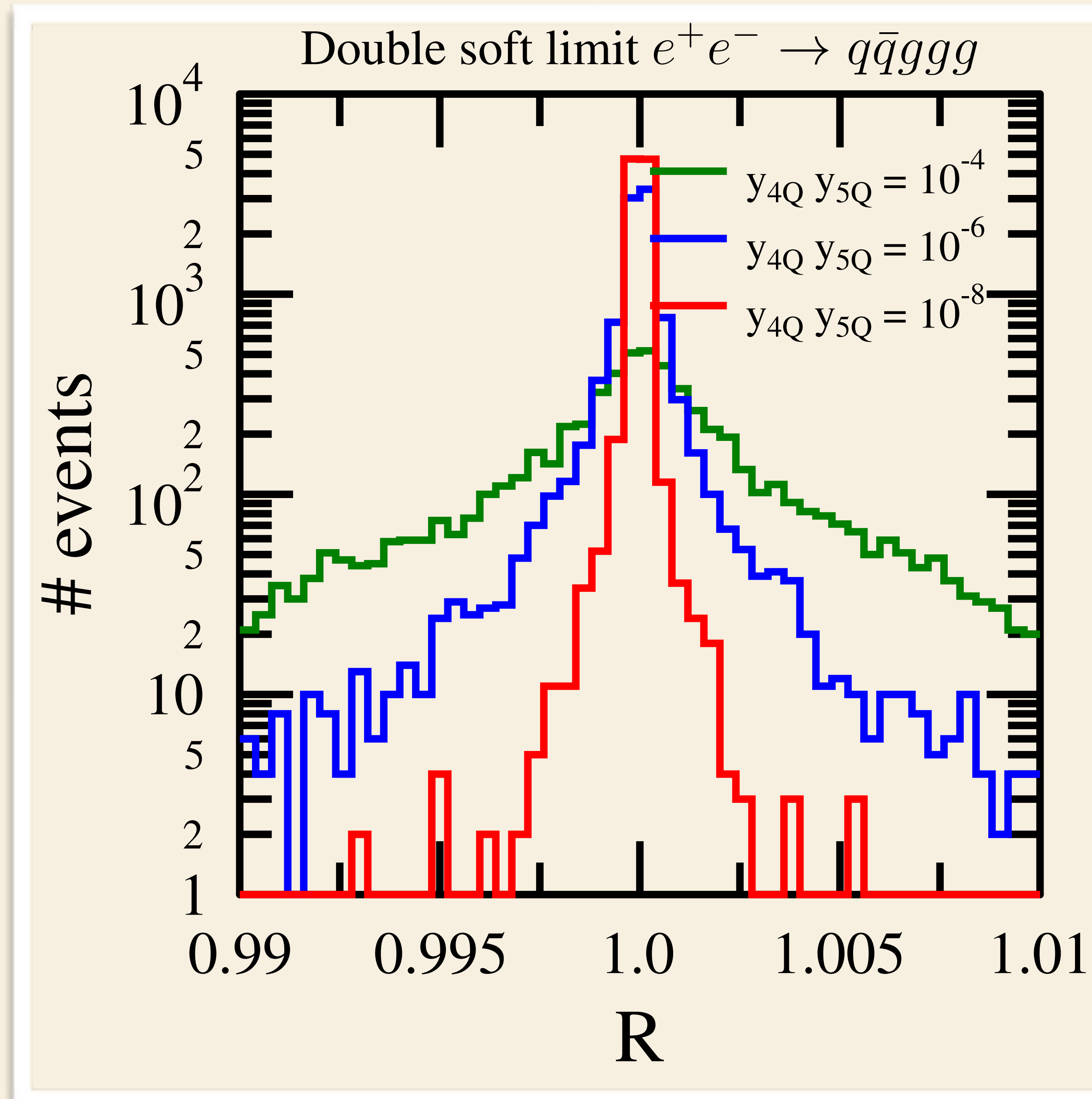
$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Kinematic singularities cancel





R = subtraction/SME

Difficulty

Integrated approximate xsections

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

After integrating over unresolved momenta & summing over unresolved colors and flavors, the subtraction terms can be written as products of insertion operators (in color space) and lower point cross sections:

$$\int_p {\rm d}\sigma^{\rm RR,A_p} = \boldsymbol{I}^{(0)}_p(\{p\}_n;\epsilon) \otimes {\rm d}\sigma^{\rm B}_n$$

Integrated approximate xsections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR,A}_p} = \int_p \left[\mathrm{d}\phi_{m+2}(\{p\}) \sum_R \mathcal{X}_R(\{p\}) \right]$$

$$= \int_p \left[\mathrm{d}\phi_n(\{\tilde{p}\}^{(R)})[\mathrm{d}p_p^{(R)}] \sum_R \left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p Sing_R(p_p^{(R)}) \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2 \right]$$

$$= \left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [\mathrm{d}p_p^{(R)}] Sing_R(p_p^{(R)}) \right] \otimes \mathrm{d}\phi_n(\{\tilde{p}\}^{(R)}) |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

$$= \underbrace{\left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [\mathrm{d}p_p^{(R)}] Sing_R(p_p^{(R)}) \right]}_{\boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon)} \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

the integrated counter-terms $[X]_R \propto \int_p [\mathrm{d}p_p^{(R)}] Sing_R(p_p^{(R)})$ are

independent of the process & observable

⇒ need to compute only once (admittedly cumbersome, though)

Summation over unresolved flavors

- integrated counter-terms $[X]_{f_i\ldots}$ carry flavor indices of unresolved patrons

  ⇒ need to sum over unresolved flavors:

  technically simple, though tedious, result can be summarized in flavor-summed integrated counter-terms

  P. Bolzoni, G. Somogyi, ZT arXiv:0905.4390

- symbolically:

$$\left(X^{(0)}\right)^{(j,l)\ldots}_{f_i\ldots} = \sum [X^{(0)}]^{(j,l)\ldots}_{f_k\ldots}$$

- and precisely, for instance, two-flavor sum:

$$\sum_{\{m+2\}} \frac{1}{S_{\{m+2\}}} \sum_t \sum_{k\neq t} [X^{(0)}_{kt}]^{(\ldots)}_{f_k f_t} \equiv \sum_{\{m\}} \frac{1}{S_{\{m\}}} \left(X^{(0)}_{kt}\right)^{(\ldots)}$$

Computing the integrals

See talk by Gabor tomorrow at noon

Status of (287) integrals

Int	status
$\mathcal{I}^{(k)}_{1\mathcal{C},0}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},3}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},4}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},5}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},7}$	✔
$\mathcal{I}_{1\mathcal{C},8}$	✔

Int	status
$\mathcal{I}_{1\mathcal{S},0}$	✔
$\mathcal{I}_{1\mathcal{S},1}$	✔
$\mathcal{I}_{1\mathcal{S},2}$	$(m > 3)$ ✘
$\mathcal{I}^{(k)}_{1\mathcal{S},3}$	✔
$\mathcal{I}_{1\mathcal{S},4}$	✔
$\mathcal{I}_{1\mathcal{S},5}$	✔
$\mathcal{I}_{1\mathcal{S},6}$	✔
$\mathcal{I}_{1\mathcal{S},7}$	✔

Int	status
$\mathcal{I}_{1\mathcal{CS},0}$	✔
$\mathcal{I}_{1\mathcal{CS},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{CS},2}$	✔
$\mathcal{I}_{1\mathcal{CS},3}$	✔
$\mathcal{I}_{1\mathcal{CS},4}$	✔

Int	status
$\mathcal{I}^{(k,l)}_{12\mathcal{C},1}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},3}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},5}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},7}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},8}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},9}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},10}$	✔

Int	status
$\mathcal{I}_{2\mathcal{S},1}$	✔
$\mathcal{I}_{2\mathcal{S},2}$	✔
$\mathcal{I}_{2\mathcal{S},3}$	✔
$\mathcal{I}_{2\mathcal{S},4}$	✔
$\mathcal{I}_{2\mathcal{S},5}$	✔
$\mathcal{I}_{2\mathcal{S},6}$	✔
$\mathcal{I}_{2\mathcal{S},7}$	✔
$\mathcal{I}_{2\mathcal{S},8}$	✔
$\mathcal{I}_{2\mathcal{S},9}$	✔
$\mathcal{I}_{2\mathcal{S},10}$	✔
$\mathcal{I}_{2\mathcal{S},11}$	✔
$\mathcal{I}_{2\mathcal{S},12}$	✔
$\mathcal{I}_{2\mathcal{S},13}$	✔
$\mathcal{I}_{2\mathcal{S},14}$	✔
$\mathcal{I}_{2\mathcal{S},15}$	✔
$\mathcal{I}_{2\mathcal{S},16}$	✔
$\mathcal{I}_{2\mathcal{S},17}$	✔
$\mathcal{I}_{2\mathcal{S},18}$	✔
$\mathcal{I}_{2\mathcal{S},19}$	✔
$\mathcal{I}_{2\mathcal{S},20}$	✔
$\mathcal{I}_{2\mathcal{S},21}$	✔
$\mathcal{I}_{2\mathcal{S},22}$	✔
$\mathcal{I}_{2\mathcal{S},23}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{S},1}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},3}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},5}$	✔
$\mathcal{I}_{12\mathcal{S},6}$	✔
$\mathcal{I}_{12\mathcal{S},7}$	✔
$\mathcal{I}_{12\mathcal{S},8}$	✔
$\mathcal{I}_{12\mathcal{S},9}$	✔
$\mathcal{I}_{12\mathcal{S},10}$	✔
$\mathcal{I}_{12\mathcal{S},11}$	✔
$\mathcal{I}_{12\mathcal{S},12}$	✔
$\mathcal{I}_{12\mathcal{S},13}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{CS},1}$	✔
$\mathcal{I}_{12\mathcal{CS},2}$	✔
$\mathcal{I}_{12\mathcal{CS},3}$	✔

Int	status	Int	status
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},1}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},1}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},2}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},2}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},3}$	✔	$\mathcal{I}^{(2)}_{2\mathcal{CS},2}$	✔/✘
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},4}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},3}$	✔
$\mathcal{I}^{(-1,-1,-1,-1)}_{2\mathcal{C},5}$	✔/✘	$\mathcal{I}^{(k)}_{2\mathcal{CS},4}$	✔
$\mathcal{I}^{(k,l)}_{2\mathcal{C},6}$	✔	$\mathcal{I}^{(k)}_{2\mathcal{CS},5}$	✔

✓: pole coefficients are known analytically, finite numerically

✗: pole coefficients known analytically $O(\epsilon^{-1})$, rest numerically

Structure of insertion operators

recall general form for Born sections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR,A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

Insertion operators involve all possible color connections with given number of unresolved patrons with kinematic coefficients

for 1 unresolved parton on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_1^{(0)}(\{p\}_{m+1};\epsilon) = \frac{\alpha_\mathrm{s}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \sum_i \left[\mathrm{C}_{1,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(0),(i,k)} \boldsymbol{T}_i \boldsymbol{T}_k\right]$$

kinematic functions contain poles starting from $O(\epsilon^{-2})$ for collinear and from $O(\epsilon^{-1})$ for soft

G. Somogyi, ZT hep-ph/0609041

Structure of insertion operators

recall general form for Born sections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

for 2 unresolved patrons on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_2^{(0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \left\{ \sum_i \left[\mathrm{C}_{2,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{C}_{2,f_i f_k}^{(0)} \boldsymbol{T}_k^2\right] \boldsymbol{T}_i^2 \right.$$

$$+ \sum_{j,l} \left[\mathrm{S}_2^{(0),(j,l)} C_{\mathrm{A}} + \sum_i \mathrm{CS}_{2,f_i}^{(0),(j,l)} \boldsymbol{T}_i^2\right] \boldsymbol{T}_j \boldsymbol{T}_l$$

$$\left. + \sum_{i,k,j,l} \mathrm{S}_2^{(0),(i,k)(j,l)} \{\boldsymbol{T}_i \boldsymbol{T}_k, \boldsymbol{T}_j \boldsymbol{T}_l\} \right\}$$

the iterated doubly-unresolved has the same color structure, kinematic coefficients differ

G. Somogyi et al arXiv:0905.4390, arXiv:1301.3504, arXiv:1301.3919

Structure of insertion operators

general form at one loop

$$\int_1 \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} = \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{V}} + \boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

for 1 unresolved parton on loop SME $|M^{(1)}|^2$:

$$\boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_s}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \sum_i \left[\mathrm{C}_{1,f_i}^{(1)} C_\mathrm{A} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(1),(i,k)} C_\mathrm{A} \boldsymbol{T}_i \boldsymbol{T}_k \right.$$

$$\left. + \sum_{\substack{k,l \\ k\neq l}} \mathrm{S}_1^{(1),(i,k,l)} \sum_{a,b,c} f_{abc} T_i^a T_k^b T_l^c \right]$$

present for m > 3 (four or more hard partons)

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

singly-unresolved integrated singly unresolved:

$$\int_1 \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A}_1} \right)^{\mathrm{A}_1} = \left[\frac{1}{2} \left\{ \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon), \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \right\} + \boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) \right] \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

with only non-abelian contributions on iterated I:

$$\boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2} \right)^\epsilon \right]^2 \sum_i \left[\mathrm{C}_{1,1,f_i}^{(0,0)} C_{\mathrm{A}} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_{1,2}^{(0,0),(i,k)} C_{\mathrm{A}} \boldsymbol{T}_i \boldsymbol{T}_k \right]$$

kinematic functions contain poles starting from $O(\epsilon^{-3})$ only

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

- the color structures are independent of the precise definition of subtractions (momentum mappings), only subleading coefficients of ϵ-expansion in kinematic functions may depend
- we computed all insertion operators analytically (defined in our subtraction scheme) up to $O(\epsilon^{-2})$ for arbitrary m

Light in the tunnel

Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2, see Gabor's talk tomorrow at noon

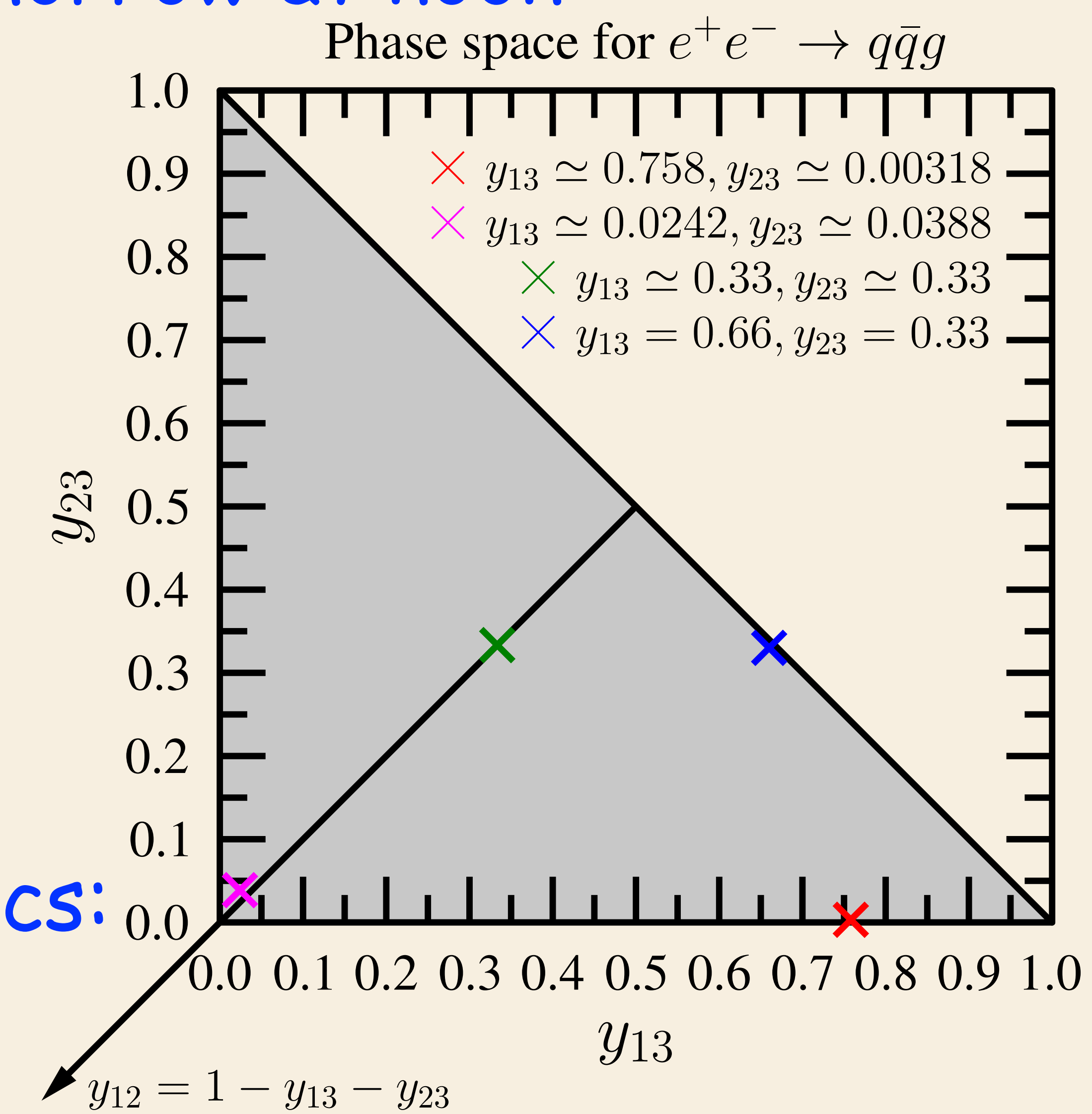
Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2, see Gabor's talk tomorrow at noon
- for m=3,
  - color algebra can be performed explicitly:

$$\boldsymbol{T}_1\boldsymbol{T}_2 = \frac{1}{2}C_{\mathrm{A}} - C_{\mathrm{F}}$$

$$\boldsymbol{T}_1\boldsymbol{T}_3 = \boldsymbol{T}_2\boldsymbol{T}_3 = -\frac{1}{2}C_{\mathrm{A}}$$

  - the insertion operators depend on 3-jet kinematics:



Example: $e^+e^- \rightarrow$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$d\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{F}inite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

$$\begin{aligned}
&\mathcal{P}oles\left(A_3^{(2\times 0)}(1_q,3_g,2_{\bar{q}}) + A_3^{(1\times 1)}(1_q,3_g,2_{\bar{q}})\right) \\
&= 2\Bigg[- \left(\boldsymbol{I}^{(1)}_{q\bar{q}g}(\epsilon)\right)^2 - \frac{\beta_0}{\epsilon}\boldsymbol{I}^{(1)}_{q\bar{q}g}(\epsilon) \\
&\quad + e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right)\boldsymbol{I}^{(1)}_{q\bar{q}g}(2\epsilon) + \boldsymbol{H}^{(2)}_{q\bar{q}g}\Bigg] A_3^0(1_q,3_g,2_{\bar{q}}) \\
&\quad + 2\,\boldsymbol{I}^{(1)}_{q\bar{q}g}(\epsilon)\, A_3^{(1\times 0)}(1_q,3_g,2_{\bar{q}})\ . \qquad (4.59)
\end{aligned}$$

$$\boldsymbol{I}^{(1)}_{q\bar{q}g}(\epsilon) = \mathcal{R}e\boldsymbol{I}^{(1)}_0(p_q,p_{\bar{q}},p_g;\epsilon)$$

$$\begin{aligned}
\boldsymbol{H}^{(2)}_{q\bar{q}g} &= \frac{e^{\epsilon\gamma}}{4\,\epsilon\,\Gamma(1-\epsilon)}\Bigg[\left(4\zeta_3 + \frac{589}{432} - \frac{11\pi^2}{72}\right)N^2 + \left(-\frac{1}{2}\zeta_3 - \frac{41}{54} - \frac{\pi^2}{48}\right) \\
&\quad + \left(-3\zeta_3 - \frac{3}{16} + \frac{\pi^2}{4}\right)\frac{1}{N^2} + \left(-\frac{19}{18} + \frac{\pi^2}{36}\right)NN_F + \left(-\frac{1}{54} - \frac{\pi^2}{24}\right)\frac{N_F}{N} + \frac{5}{27}N_F^2.\Bigg]\ . \qquad (4.61)
\end{aligned}$$

A. Gehrmnn-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich arXiv:0710.0346

Example: $e^+e^- \longrightarrow$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \left[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$ {\rm d}\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{F}inite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

e.g. in symmetric point:

$$\begin{aligned}
\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)\left(y_{13} = \frac{1}{3}, y_{23} = \frac{1}{3}\right) = {\rm d}\sigma_3^{\rm B}\Bigg[& \frac{1}{\epsilon^4}\left(-2 + 2N_{\rm c}^2 + \frac{1}{2N_{\rm c}^2}\right) \\
& + \frac{1}{\epsilon^3}\left(-12.1028 + 13.8111 N_{\rm c}^2 + \frac{2.59861}{N_{\rm c}^2} - \frac{7}{6} N_{\rm c} n_{\rm f} + \frac{7 n_{\rm f}}{12 N_{\rm c}}\right) \\
& + \frac{1}{\epsilon^2}\left(-16.9786 + 12.8613 N_{\rm c}^2 + \frac{5.36423}{N_{\rm c}^2} - 2.79306 N_{\rm c} n_{\rm f} + \frac{1.16042 n_{\rm f}}{N_{\rm c}} + \frac{1}{9} n_{\rm f}^2\right) \\
& + \frac{1}{\epsilon}\left(29.6349 - 57.5088 N_{\rm c}^2 - \frac{1.59907}{N_{\rm c}^2} + 5.04531 N_{\rm c} n_{\rm f} - 1.51226 \frac{n_{\rm f}}{N_{\rm c}}\right)\Bigg] + {\rm O}\left(\epsilon^0\right)
\end{aligned}$$

A. Gehrmnn-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich arXiv:0710.0346

$e^+e^- \to$ 3 jets at symmetric point

$$\mathcal{P}oles\left(A_3^{(2\times 0)}+A_3^{(1\times 1)}\right)\left(y_{13}=\frac{1}{3},y_{23}=\frac{1}{3}\right)=\mathrm{d}\sigma_3^{\mathrm{B}}\left[\frac{1}{\epsilon^4}\left(-2+2N_c^2+\frac{1}{2N_c^2}\right)\right.$$
$$+\frac{1}{\epsilon^3}\left(-12.1028+13.8111N_c^2+\frac{2.59861}{N_c^2}-\frac{7}{6}N_c n_f+\frac{7n_f}{12N_c}\right)$$
$$+\frac{1}{\epsilon^2}\left(-16.9786+12.8613N_c^2+\frac{5.36423}{N_c^2}-2.79306N_c n_f+\frac{1.16042n_f}{N_c}+\frac{1}{9}n_f^2\right)$$
$$\left.+\frac{1}{\epsilon}\left(29.6349-57.5088N_c^2-\frac{1.59907}{N_c^2}+5.04531N_c n_f-1.51226\frac{n_f}{N_c}\right)\right]+\mathrm{O}\left(\epsilon^0\right)$$

$$\sum\int \mathrm{d}\sigma^{\mathrm{A}}\left(y_{13}=\frac{1}{3},y_{23}=\frac{1}{3}\right)=\mathrm{d}\sigma_3^{\mathrm{B}}\left[-\frac{1}{\epsilon^4}\left(-2+2N_c^2+\frac{1}{2N_c^2}\right)\right.$$
$$-\frac{1}{\epsilon^3}\left(-12.1028+13.8111N_c^2+\frac{2.59861}{N_c^2}-\frac{7}{6}N_c n_f+\frac{7n_f}{12N_c}\right)$$
$$-\frac{1}{\epsilon^2}\left(-16.9786+12.8613N_c^2+\frac{5.36423}{N_c^2}-2.79306N_c n_f+\frac{1.16042n_f}{N_c}+\frac{1}{9}n_f^2\right)$$
$$\left.-\frac{1}{\epsilon}\left(29.6364-57.5095N_c^2-\frac{1.59905}{N_c^2}+5.04529N_c n_f-1.51226\frac{n_f}{N_c}\right)\right]+\mathrm{O}\left(\epsilon^0\right)$$

$e^+e^- \to$ 3 jets at soft point

$$\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)(y_{13} = 0.0242, y_{23} = 0.0388) = \mathrm{d}\sigma_3^{\mathrm{B}}\left(\frac{1}{\epsilon^4}\left(-2 + 2N_\mathrm{c}^2 + \frac{1}{2N_\mathrm{c}^2}\right)\right.$$
$$+\frac{1}{\epsilon^3}\left(-14.8102 + 23.3602N_\mathrm{c}^2 + \frac{1.56502}{N_\mathrm{c}^2} - \frac{7}{6}N_\mathrm{c}n_\mathrm{f} + \frac{7n_\mathrm{f}}{12N_\mathrm{c}}\right)$$
$$+\frac{1}{\epsilon^2}\left(-35.5896 + 101.076N_\mathrm{c}^2 + \frac{0.605297}{N_\mathrm{c}^2} - 5.18033N_\mathrm{c}n_\mathrm{f} + \frac{0.643623n_\mathrm{f}}{N_\mathrm{c}} + \frac{1}{9}n_\mathrm{f}^2\right)$$
$$\left.+\frac{1}{\epsilon}\left(-10.0215 + 176.578N_\mathrm{c}^2 - \frac{3.75642}{N_\mathrm{c}^2} - 4.96733N_\mathrm{c}n_\mathrm{f} - \frac{0.260912n_\mathrm{f}}{N_\mathrm{c}}\right) + \mathrm{O}\left(\epsilon^0\right)\right)$$

$$\sum\int \mathrm{d}\sigma^{\mathrm{A}}(y_{13} = 0.0242, y_{23} = 0.0388) = \mathrm{d}\sigma_3^{\mathrm{B}}\left(-\frac{1}{\epsilon^4}\left(-2 + 2N_\mathrm{c}^2 + \frac{1}{2N_\mathrm{c}^2}\right)\right.$$
$$-\frac{1}{\epsilon^3}\left(-14.8102 + 23.3602N_\mathrm{c}^2 + \frac{1.56502}{N_\mathrm{c}^2} - \frac{7}{6}N_\mathrm{c}n_\mathrm{f} + \frac{7n_\mathrm{f}}{12N_\mathrm{c}}\right)$$
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$e^+e^- \to$ 3 jets at collinear point

$$\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)(y_{13} = 0.758, y_{23} = 0.00318) = \mathrm{d}\sigma_3^{\mathrm{B}}\Bigg(\frac{1}{\epsilon^4}\left(-2 + 2N_{\mathrm{c}}^2 + \frac{1}{2N_{\mathrm{c}}^2}\right)$$
$$+\frac{1}{\epsilon^3}\left(-16.6015 + 21.4723N_{\mathrm{c}}^2 + \frac{2.93269}{N_{\mathrm{c}}^2} - \frac{7}{6}N_{\mathrm{c}}n_{\mathrm{f}} + \frac{7n_{\mathrm{f}}}{12N_{\mathrm{c}}}\right)$$
$$+\frac{1}{\epsilon^2}\left(-61.1426 + 75.7281N_{\mathrm{c}}^2 + \frac{13.9679}{N_{\mathrm{c}}^2} - 4.70835N_{\mathrm{c}}n_{\mathrm{f}} + 1.32746\frac{n_{\mathrm{f}}}{N_{\mathrm{c}}} + \frac{1}{9}n_{\mathrm{f}}^2\right)$$
$$+\frac{1}{\epsilon}\left(-130.139 + 146.396N_{\mathrm{c}}^2 + \frac{20.8421}{N_{\mathrm{c}}^2} - 4.15223N_{\mathrm{c}}n_{\mathrm{f}} + \frac{3.5162n_{\mathrm{f}}}{N_{\mathrm{c}}}\right) + \mathrm{O}\left(\epsilon^0\right)$$

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$e^+e^- \to$ 3 jets at diagonal point

$$\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)(y_{13} = 0.33, y_{23} = 0.66) = \mathrm{d}\sigma_3^{\mathrm{B}}\left(\frac{1}{\epsilon^4}\left(-2 + 2N_{\mathrm{c}}^2 + \frac{1}{2N_{\mathrm{c}}^2}\right)\right.$$
$$+ \frac{1}{\epsilon^3}\left(-18.4429 + 12.465N_{\mathrm{c}}^2 + \frac{6.10517}{N_{\mathrm{c}}^2} - \frac{7}{6}N_{\mathrm{c}}n_{\mathrm{f}} + \frac{7n_{\mathrm{f}}}{12N_{\mathrm{c}}}\right)$$
$$+ \frac{1}{\epsilon^2}\left(-61.1965 + 5.55521N_{\mathrm{c}}^2 + \frac{34.2824}{N_{\mathrm{c}}^2} - 2.45653N_{\mathrm{c}}n_{\mathrm{f}} + \frac{2.9137n_{\mathrm{f}}}{N_{\mathrm{c}}} + \frac{1}{9}n_{\mathrm{f}}^2\right)$$
$$\left.+ \frac{1}{\epsilon}\left(-40.4299 - 57.8405N_{\mathrm{c}}^2 + \frac{102.509}{N_{\mathrm{c}}^2} + 5.55926N_{\mathrm{c}}n_{\mathrm{f}} + \frac{1.98203n_{\mathrm{f}}}{N_{\mathrm{c}}}\right) + \mathrm{O}\left(\epsilon^0\right)\right)$$

$$\sum\int \mathrm{d}\sigma^{\mathrm{A}}(y_{13} = 0.33, y_{23} = 0.66) = \mathrm{d}\sigma_3^{\mathrm{B}}\left(-\frac{1}{\epsilon^4}\left(-2 + 2N_{\mathrm{c}}^2 + \frac{1}{2N_{\mathrm{c}}^2}\right)\right.$$
$$- \frac{1}{\epsilon^3}\left(-18.4429 + 12.465N_{\mathrm{c}}^2 + \frac{6.10517}{N_{\mathrm{c}}^2} - \frac{7}{6}N_{\mathrm{c}}n_{\mathrm{f}} + \frac{7n_{\mathrm{f}}}{12N_{\mathrm{c}}}\right)$$
$$- \frac{1}{\epsilon^2}\left(-61.1965 + 5.55521N_{\mathrm{c}}^2 + \frac{34.2824}{N_{\mathrm{c}}^2} - 2.45653N_{\mathrm{c}}n_{\mathrm{f}} + \frac{2.9137n_{\mathrm{f}}}{N_{\mathrm{c}}} + \frac{1}{9}n_{\mathrm{f}}^2\right)$$
$$\left.- \frac{1}{\epsilon}\left(-40.4299 + 57.8405N_{\mathrm{c}}^2 + \frac{102.509}{N_{\mathrm{c}}^2} + 5.55924N_{\mathrm{c}}n_{\mathrm{f}} - \frac{1.98203n_{\mathrm{f}}}{N_{\mathrm{c}}}\right) + \mathrm{O}\left(\epsilon^0\right)\right)$$

Message:

$$\sigma_3^{\mathrm{NNLO}} = \int_3 \left\{ \mathrm{d}\sigma_3^{\mathrm{VV}} + \sum \int \mathrm{d}\sigma^{\mathrm{A}} \right\}_{\epsilon=0} J_3$$

indeed finite in d=4 dimensions

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- ✓ First application: see Gabor's talk tomorrow at noon