Matching & Merging

Simon Plätzer

DESY

Outline.

1: Recaps on fixed order, logs and showers

Fixed order ingredients in a nutshell.

Consider an observable O starting in $\mathcal{O}(\alpha_s^n)$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{LO}} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{NLO}} + \mathcal{O}(\alpha_s^{n+2})$$

$$\begin{aligned}
\left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{LO}} &= \int_m \mathrm{d}\sigma_{\mathrm{Born}}(\phi_m)\delta(O - O(\phi_m)) \\
\left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{NLO}} &= \int_m \mathrm{d}\sigma_{\mathrm{Virtual}}(\phi_m)\delta(O - O(\phi_m)) \\
&+ \int_{m+1} \mathrm{d}\sigma_{\mathrm{Real}}(\phi_{m+1})\delta(O - O(\phi_{m+1}))
\end{aligned}$$

Definition of the observable as function of m final state momenta.
Number of jets, Z boson $p_\perp$, ...

Fixed order ingredients in a nutshell.

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{LO}} = \int_m \mathrm{d}\sigma_{\mathrm{Born}}(\phi_m)\delta(O - O(\phi_m))$$

$$\begin{aligned}\left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{NLO}} &= \int_m \mathrm{d}\sigma_{\mathrm{Virtual}}(\phi_m)\delta(O - O(\phi_m)) \\ &+ \int_{m+1} \mathrm{d}\sigma_{\mathrm{Real}}(\phi_{m+1})\delta(O - O(\phi_{m+1}))\end{aligned}$$

Infrared divergencies cancel between Virtual and Real contributions.
Ultraviolet divergencies in loop graphs are removed by renormalization.

Scale variations.

Renormalization scale dependence at LO:

$$\mathrm{d}\sigma_{\mathrm{LO}}(\xi\mu_R^2) = \alpha_s^n(\xi\mu_R^2)\ \mathrm{d}\tilde{\sigma}_{\mathrm{LO}} = \alpha_s^n(\mu_R^2)\ \mathrm{d}\tilde{\sigma}_{\mathrm{LO}} + \mathcal{O}(\alpha_s^{n+1}(\mu_R^2))$$

Scale variation in a LO cross section is a NLO effect.

Renormalization scale dependence at NLO:

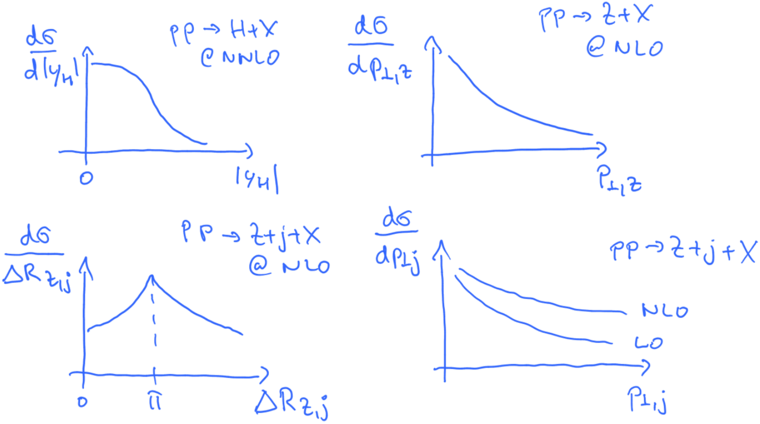
$$\mathrm{d}\sigma_{\mathrm{LO}}(\xi\mu_R^2) = \alpha_s^n(\mu_R^2)\ \mathrm{d}\tilde{\sigma}_{\mathrm{LO}} - n\ \alpha_s^{n+1}(\mu_R^2)\frac{\beta_0}{2\pi}\ln\xi\ \mathrm{d}\tilde{\sigma}_{\mathrm{LO}} + \mathcal{O}(\alpha_s^{n+2}(\mu_R^2))$$

$$\mathrm{d}\sigma_{\mathrm{Virtual}}(\xi\mu_R^2) = \mathrm{d}\sigma_{\mathrm{Virtual}}(\mu_R^2) + n\ \alpha_s^{n+1}(\mu_R^2)\frac{\beta_0}{2\pi}\ln\xi\ \mathrm{d}\tilde{\sigma}_{\mathrm{LO}} + \mathcal{O}(\alpha_s^{n+2}(\mu_R^2))$$

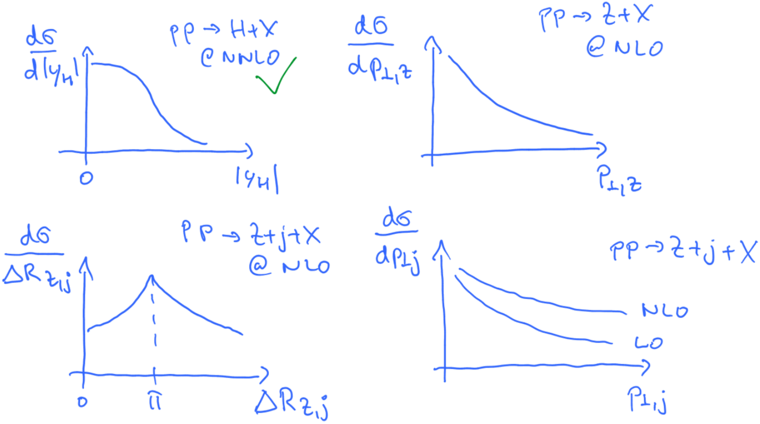
Scale variation in a NLO cross section is a NNLO effect.

Similar observations apply to factorization scale dependence.

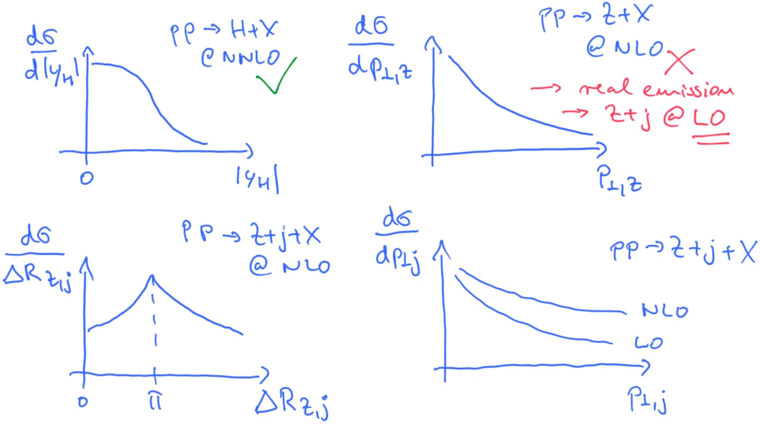
I've got $d\sigma/dX$ from Y@NNLO ... so it's NNLO.



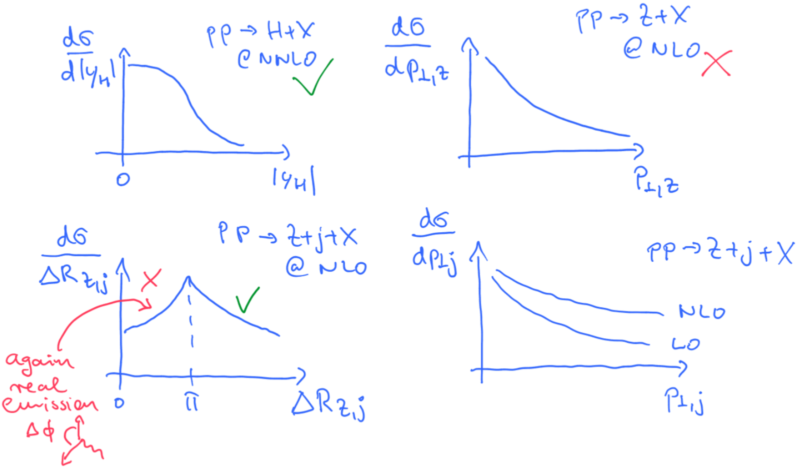
I've got dσ/dX from Y@NNLO ... so it's NNLO.



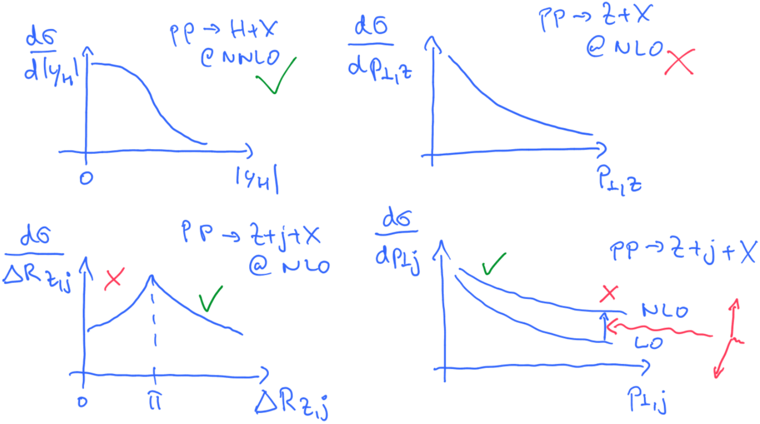
I've got $d\sigma/dX$ from Y@NNLO ... so it's NNLO.



I've got $d\sigma/dX$ from Y@NNLO ... so it's NNLO.



I've got $d\sigma/dX$ from Y@NNLO ... so it's NNLO.



I've got $\mathrm{d}\sigma/\mathrm{d}X$ from Y@NNLO ... so it's NNLO.

Not always easy to answer.
Check *uncertainties* order by order.

All that infrared mess.

(Renormalized) virtual contributions in dimensional regularisation:
$\rightarrow$ poles in ϵ from soft or collinear to external loop momenta.

Real contributions divergent for soft and/or collinear emission:
$\rightarrow$ poles in ϵ after phase space integration.

$$\begin{aligned}\left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{NLO}} &= \int_m \left[\mathrm{d}\sigma_{\mathrm{Virtual}}(\phi_m) + \int_1 \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{m+1})\right]_{\epsilon=0} \delta(O - O(\phi_m)) \\ &+ \int_{m+1} [\mathrm{d}\sigma_{\mathrm{Real}}(\phi_{m+1})_{\epsilon=0}\ \delta(O - O(\phi_{m+1})) \\ &\qquad - \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{m+1})_{\epsilon=0}\ \delta(O - O(\phi_m))]\end{aligned}$$

Use subtraction terms to handle the divergences.
Cannot generate events from an NLO cross section; real and subtraction term kinematics highly correlated.

Infrared safety.

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}O}\right|_{\mathrm{NLO}} = \int_m \left[\mathrm{d}\sigma_{\mathrm{Virtual}}(\phi_m) + \int_1 \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{m+1})\right]_{\epsilon=0} \delta(O - O(\phi_m))$$
$$+ \int_{m+1} [\mathrm{d}\sigma_{\mathrm{Real}}(\phi_{m+1})_{\epsilon=0}\ \delta(O - O(\phi_{m+1}))$$
$$- \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{m+1})_{\epsilon=0}\ \delta(O - O(\phi_m))]$$

Only finite for *infrared safe* obeservables:

- $O(\phi_{m+1}) \to O(\phi_m)$ for $E_g \to 0$
- $O(\phi_{m+1}) \to O(\phi_m)$ for $1 - \beta \cos\theta_{ij} \to 0$

No collinear divergences for massive partons.

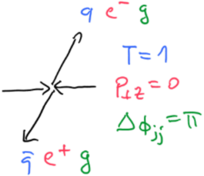
Infrared unsafe: highly sensitive to non-perturbative contributions.

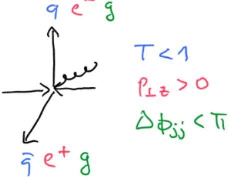
Infrared sensitivity.

Infrared sensitive observables are

- infrared safe, and so calculable in perturbation theory, but
- require a minimum amount of radiation for non-trivial values.

Prime examples: thrust in e^+e^-, Z at non-zero $p_\perp$, $\Delta\phi_{jj}$ in $pp \to$ jets.





Infrared sensitivity.

Infrared sensitive observables are divergent at any fixed order of perturbation theory, once the requirement on additional radiation is removed.

Roughly speaking: If an infrared sensitive observable requires n jets to be present, it will diverge at fixed order when going from n to $n-1$ jets.
$\rightarrow$ The divergence is entirely due to soft and/or collinear emissions.

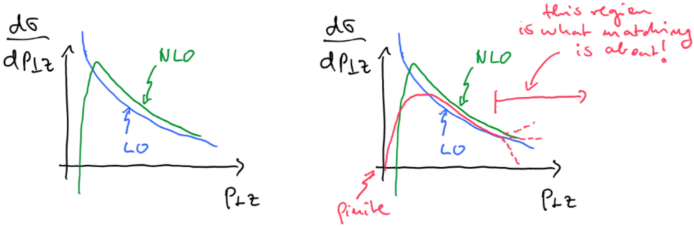
If $\tau \rightarrow 0$ is the divergent limit of an infrared sensitive observable τ, then it will behave as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim \sigma_0 \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} C_{n,m}\ \alpha_s^n \frac{\ln^{2n-m-1}\tau}{\tau}$$

Integrated quantities like jet rates have $2n$ and less large logarithms in each order α_s^n.

Why infrared sensitivity calls for showers.

The divergencies are all due to the fact that fixed order only accounts for a limited number of emissions (both real and virtual).



Once we resum any number of emissions, there will be a finite answer. The probability to emit *nothing* on top of a certain number of emissions is always less than one.

This is known as **Sudakov supression.**

What showers do.

Consider a shower with splitting kernels $P(\phi_n, q)$ to generate emissions off a patonic state ϕ_n at a scale q (we won't fix the evolution variable unless stated otherwise).

Shower action on events distributed according to a cross section $\mathrm{d}\sigma(\phi_n, Q)$ with hard scale Q:

$$\mathrm{PS}[\mathrm{d}\sigma(\phi_n, Q)] =$$

$$\Delta_n(\mu|Q)\mathrm{d}\sigma(\phi_n, Q) + \mathrm{PS}\left[\mathrm{d}\sigma(\phi_n, Q)\ P(\phi_n, q)\ \frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}\ \Delta_n(q|Q)\right]$$

Sudakov form factor: Probability for no emission between two scales.
Recursive algorithm: Generate next emission off the $n + 1$ parton state.
Observe the infrared cutoff μ: Transition to non-perturbative models, but still a ~ 1 GeV scale, *not* Λ_{QCD}.

What showers do.

Showers also have virtual and real emission contributions:

$$\mathrm{PS}[\mathrm{d}\sigma(\phi_n, Q)] =$$

$$\Delta_n(\mu|Q)\mathrm{d}\sigma(\phi_n, Q) + \mathrm{PS}[\mathrm{d}\sigma(\phi_n, Q)\ P(\phi_n, q)\ \frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}\ \Delta_n(q|Q)]$$

Showers preserve the total inclusive cross section: Unitarity.

$$\mathrm{d}\sigma(\phi_n, Q)\ P(\phi_n, q)\ \frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}\Delta_n(q|Q) = \frac{\partial}{\partial q}\Delta_n(q|Q) \qquad \Delta_n(Q|Q) = 1$$

Showers approximate matrix elements:

$$\mathrm{d}\sigma(\phi_{n+1}, q) \to \mathrm{d}\sigma(\phi_n, Q)\ P(\phi_n, q)\ \frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}$$

In the collinear limits, and in the soft limit for $N_c \to \infty$.

What showers do.

Showers use a running α_s, different for each emission, and typically at the $p_\perp$ of the splitting:

$$P(\phi_n, q) = \alpha_s(p_\perp(q))\hat{P}(\phi_n, q)$$

For a NLO expansion of the shower cross section, this doesn't matter:

$$\mathrm{PS}[\mathrm{d}\sigma(\phi_n, Q)] = \mathrm{d}\sigma(\phi_n, Q)$$
$$- \int_\mu^Q \mathrm{d}k P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n \mathrm{d}k}\mathrm{d}\sigma(\phi_n, Q) + \mathrm{d}\sigma(\phi_n, Q)P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n} + \mathcal{O}(\alpha_s^2)$$

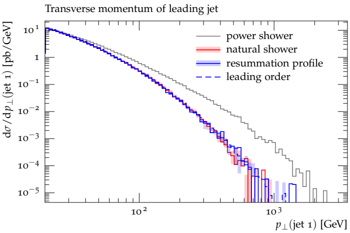
Virtual shower contributions are minus the integral of its real contributions.
$\rightarrow$ Cross sections after showering are preserved order by order.

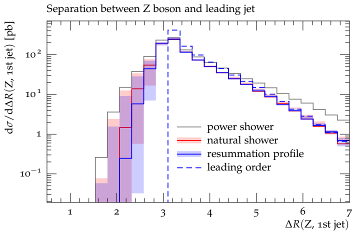
Where showers are unreliable.

"Away from soft and collinear regions."
So what's this exactly about? $\rightarrow$ Regions with no large logarithms.

Hard scale Q is a "typical hard scale" of the process: M_Z in DY, jet $p_\perp$, ...
Not unique though. Soft/collinear region is for emissions with $q \ll Q$.

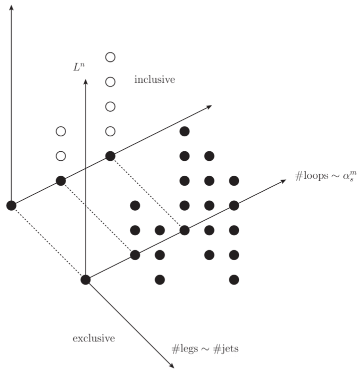
Generically expect $\ln(q/Q)$, so should not consider $q \gtrsim Q$ for showers.
$\rightarrow$ Vary Q as a first guess.



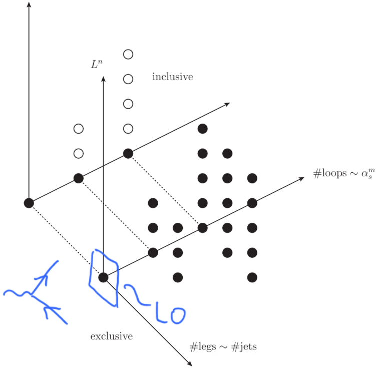


[Herwig++ dipole shower – SP & S. Gieseke 2011, 2013]

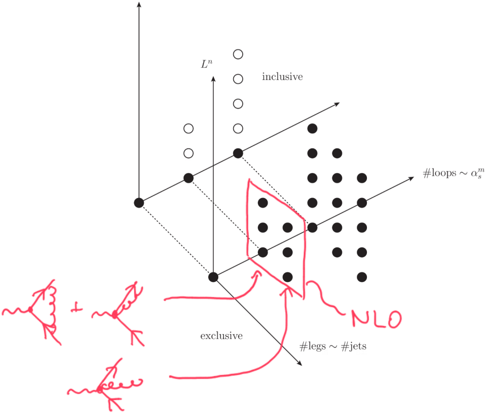
The logarithmic landscape of IR sensitive observables.



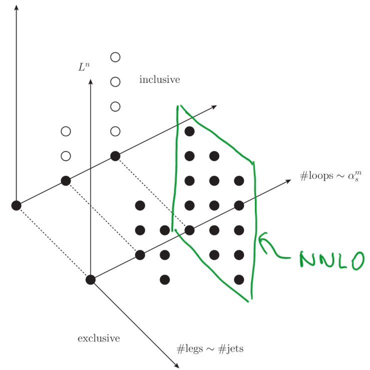
The logarithmic landscape of IR sensitive observables.



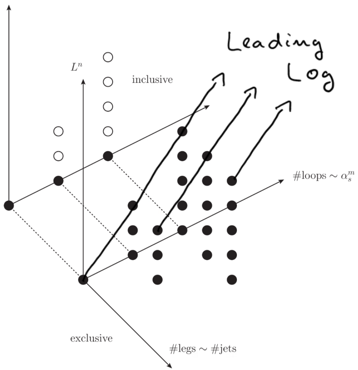
The logarithmic landscape of IR sensitive observables.



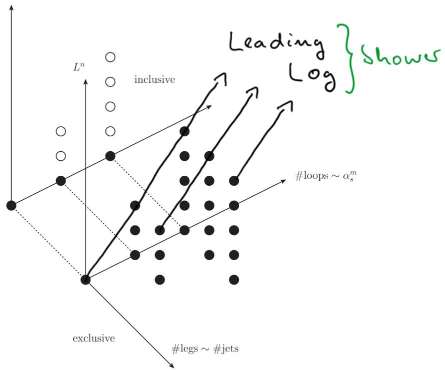
The logarithmic landscape of IR sensitive observables.



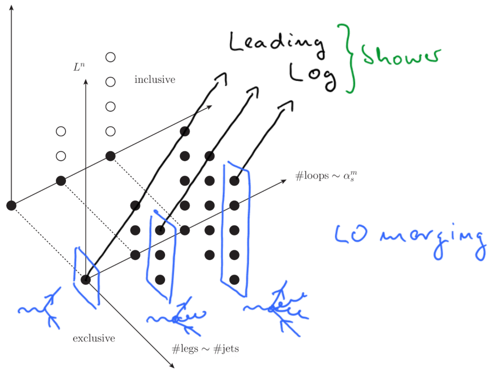
The logarithmic landscape of IR sensitive observables.



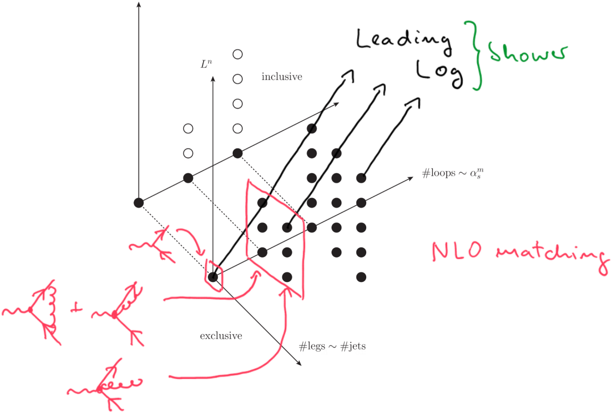
Landscaping: What we want to achieve.



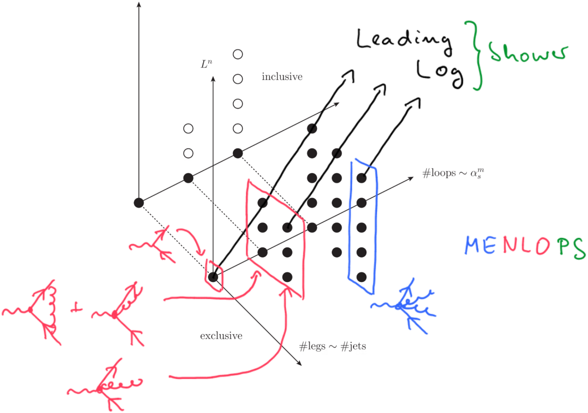
Landscaping: What we want to achieve.



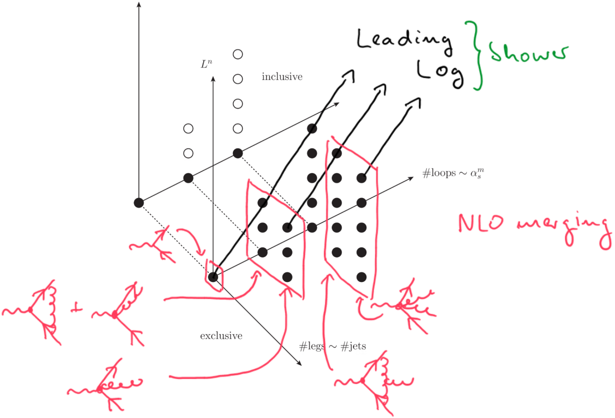
Landscaping: What we want to achieve.



Landscaping: What we want to achieve.



Landscaping: What we want to achieve.



Take-home messages.

It is not always straightforward to identify the accuracy at which an observable is described from a given fixed order. We will need to combine several calculations to improve the description.

Fixed order calculations diverge at the boundary from $n \to n-1$ jet observables; Sudakov supression will regulate these divergences.

Showers are only reliable in soft and collinear regions, summing at least leading logarithms. They have a hard scale above which they should not be trusted. Varying this scale can provide a first guess on the uncertainties.

We will need matching to remove double counting in virtual and real corrections, when combining NLO calculations with parton showers.

Showers approximate real emission matrix elements, and merging is needed to describe physics of additional, hard jets.

Questions?

2: Matching

Matching, merging – egal: Hauptsache higher orders!?

'Mailand, Madrid – egal: Hauptsache Italien!' ['Milan, Madrid - doesn't matter if it's: Italy!'] – A. Möller, German football player

Matching:

- Generally: Combine resummation with a fixed order.
- Here: Combine a parton shower and a NLO calculation.
- Applicable only where the fixed order calculation is reliable.

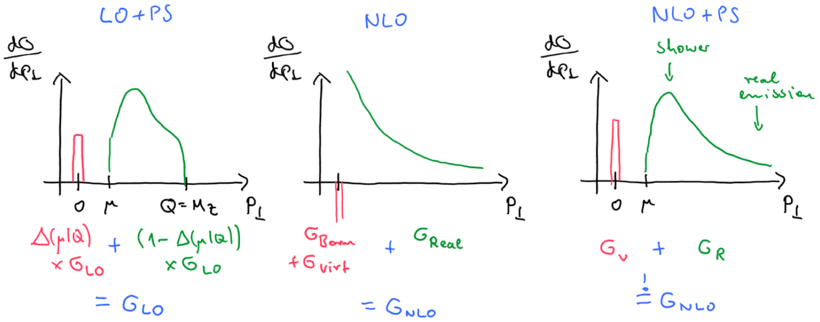
Merging:

- Combine calculations for different hard jet multiplicities.
- Add parton shower on top.
- Applicable when crossing 'jet bins'.
- At low scales typically only parton shower predictions.

Matching.

Stating the problem of NLO matching:

- The total inclusive cross section is given by the NLO calculation.
- The first additional jet is described at LO ($\leftrightarrow$ NLO real emission).
- The $1 \to 0$ additional jets limit has the proper Sudakov supression.



The matching condition.

[NLO from now on is LO plus NLO correction]

$$\mathrm{PS}[\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{matched}}] = \mathrm{d}\sigma_{\mathrm{NLO}} + \mathcal{O}(\alpha_s^{n+2})$$

$$\int \mathrm{PS}[\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{matched}}] = \sigma_{\mathrm{NLO}}^{\mathrm{matched}} = \sigma_{\mathrm{NLO}}$$

This fully specifies any NLO matching algorithm.

Solving the matching condition.

From now on use a generic observable $u(\phi_n)$.

$$\mathrm{PS}[\mathrm{d}\sigma_{\mathrm{NLO}}] =$$

$$\mathrm{d}\sigma_{\mathrm{Born}}(\phi_n, Q)u(\phi_n)$$

$$+ \left(\mathrm{d}\sigma_{\mathrm{Virtual}}(\phi_n, Q) + \int_1 \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{n+1}, Q)\right) u(\phi_n)$$

$$- \ \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{n+1}, Q)u(\phi_n) \qquad + \mathrm{d}\sigma_{\mathrm{Real}}(\phi_{n+1}, Q)u(\phi_{n+1})$$

$$- \int_\mu^Q \mathrm{d}k \ P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n \mathrm{d}k}\mathrm{d}\sigma_{\mathrm{Born}}(\phi_n, Q)u(\phi_n) \qquad + \mathrm{d}\sigma_{\mathrm{Born}}(\phi_n, Q)P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}u(\phi_{n+1}) + \mathcal{O}(\alpha_s^2)$$

$$\mathrm{PS}[\mathrm{d}\sigma_{\mathrm{NLO}}] = \mathrm{d}\sigma_{\mathrm{NLO}} + \mathrm{PS}[\mathrm{d}\sigma_{\mathrm{LO}}]|_{\mathcal{O}(\alpha_s)} + \mathcal{O}(\alpha_s^2)$$

Solving the matching condition.

Double counting identified!

$$\mathrm{PS}[\mathrm{d}\sigma_{\mathrm{NLO}}] = \mathrm{d}\sigma_{\mathrm{NLO}} + \mathrm{PS}[\mathrm{d}\sigma_{\mathrm{LO}}]|_{\mathcal{O}(\alpha_s)} + \mathcal{O}(\alpha_s^{n+2})$$

$$\mathrm{PS}\left[\mathrm{PS}[\mathrm{d}\sigma_{\mathrm{LO}}]|_{\mathcal{O}(\alpha_s)}\right] = \mathrm{PS}[\mathrm{d}\sigma_{\mathrm{LO}}]|_{\mathcal{O}(\alpha_s)} + \mathcal{O}(\alpha_s^2) \qquad \int \mathrm{PS}[\mathrm{d}\sigma_{\mathrm{LO}}]|_{\mathcal{O}(\alpha_s)} = 0$$

The master formula of NLO matching:

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{matched}} = \mathrm{d}\sigma_{\mathrm{NLO}} - \mathrm{PS}[\mathrm{d}\sigma_{\mathrm{LO}}]|_{\mathcal{O}(\alpha_s)}$$

$$\mathrm{PS}[\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{matched}}] = \mathrm{d}\sigma_{\mathrm{NLO}} + \mathcal{O}(\alpha_s^{n+2}) \qquad \int \mathrm{PS}[\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{matched}}] = \sigma_{\mathrm{NLO}}$$

Now let's look at this in more detail ...

Solving the matching condition.

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{matched}} =$$

$$\mathrm{d}\sigma_{\mathrm{Born}}(\phi_n, Q)u(\phi_n)$$

$$+ \left(\mathrm{d}\sigma_{\mathrm{Virtual}}(\phi_n, Q) + \int_1 \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{n+1}, Q)\right) u(\phi_n)$$

$$- \ \mathrm{d}\sigma_{\mathrm{Sub}}(\phi_{n+1}, Q)u(\phi_n) \qquad\qquad + \mathrm{d}\sigma_{\mathrm{Real}}(\phi_{n+1}, Q)u(\phi_{n+1})$$

$$+ \int_\mu^Q \mathrm{d}k \ P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n \mathrm{d}k}\theta(q-\mu) \ \mathrm{d}\sigma_{\mathrm{Born}}(\phi_n, Q)u(\phi_n) \qquad - \mathrm{d}\sigma_{\mathrm{Born}}(\phi_n, Q)P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}\theta(q-\mu) \ u(\phi_{n+1})$$

Not discussing generation cuts.
Separate classes of S events (left) and H events (right) – finite weights?

Solving the matching condition.

$$\mathrm{d}\sigma^{\text{matched}}_{\text{NLO}} =$$

$$\mathrm{d}\sigma_{\text{Born}}(\phi_n, Q)u(\phi_n)$$

$$+ \left(\mathrm{d}\sigma_{\text{Virtual}}(\phi_n, Q) + \int_1 \mathrm{d}\sigma_{\text{Sub}}(\phi_{n+1}, Q)\right) u(\phi_n)$$

$$- \ \mathrm{d}\sigma_{\text{Sub}}(\phi_{n+1}, Q)u(\phi_n) \qquad + \mathrm{d}\sigma_{\text{Real}}(\phi_{n+1}, Q)u(\phi_{n+1})$$

$$+ \ \mathrm{d}\sigma_{\text{Bridge}}(\phi_{n+1}, Q)\theta(\mu - q)\ u(\phi_n) \qquad - \mathrm{d}\sigma_{\text{Bridge}}(\phi_{n+1}, Q)\theta(\mu - q)\ u(\phi_{n+1})$$

$$+ \ P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}\theta(q - \mu)\ \mathrm{d}\sigma_{\text{Born}}(\phi_n, Q)u(\phi_n) \qquad - \mathrm{d}\sigma_{\text{Born}}(\phi_n, Q)P(\phi_n, q)\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n}\theta(q - \mu)\ u(\phi_{n+1})$$

Auxiliary cross section below cutoff.
Here we did add a power correction $\mathcal{O}(\mu^2/Q^2)$ for IR safe observables.

[see review by Webber and Nason & SP PhD thesis]

Matching variants.

Depending on the precise choice of $\mathrm{d}\sigma_{\text{bridge}}$, the shower, and the subtraction terms there is a host of different matching implementations.

All attempts on simplifying the matched cross section or avoiding negative weights have turned out to be problematic for the most general case.

(a)MC@NLO [Webber, Frixione et al.]

$\rightarrow \mathrm{d}\sigma_{\text{Bridge}} = \mathrm{d}\sigma_{\text{Real}}, \mathrm{d}\sigma_{\text{Sub}} = \mathrm{d}\sigma_{\text{FKS}}, P = P_{\text{HERWIG}}, P_{\text{PYTHIA}}$

POWHEG [Nason, Frixione et al.]

$\rightarrow \mathrm{d}\sigma_{\text{Bridge}} = \mathrm{d}\sigma_{\text{Real}}, \mathrm{d}\sigma_{\text{Sub}} = \mathrm{d}\sigma_{\text{FKS}}, P = |\mathcal{M}_{n+1}|^2/|\mathcal{M}_n|^2 \times \text{damping}$

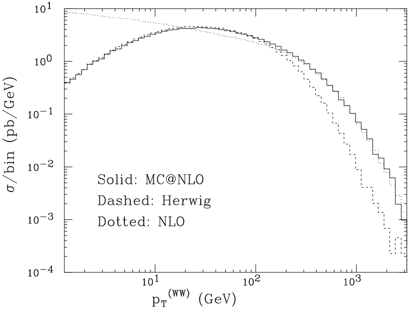
SHERPA MC@NLO [Hoeche, Krauss, Schönherr, Siegert]

$\rightarrow \mathrm{d}\sigma_{\text{Bridge}} = \mathrm{d}\sigma_{\text{Catani-Seymour}}, \mathrm{d}\sigma_{\text{Sub}} = \mathrm{d}\sigma_{\text{CS}}, P = D_{CS}/|\mathcal{M}_n|^2$

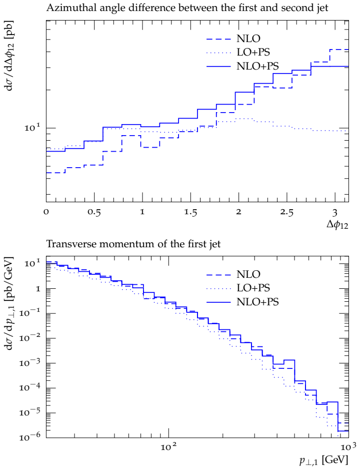
Matchbox MC@NLO [Gieseke, SP]

$\rightarrow \mathrm{d}\sigma_{\text{Bridge}} = \mathrm{d}\sigma_{\text{Real}}, \mathrm{d}\sigma_{\text{Sub}} = \mathrm{d}\sigma_{\text{CS}}, P = P_{Dipole}, P_{Herwig++}$

Examples.

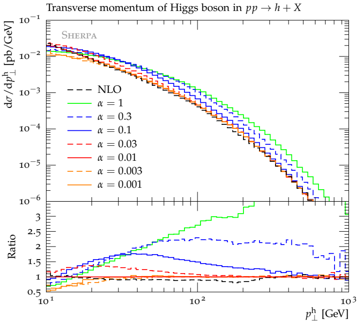


[Frixione, Webber, 2002]

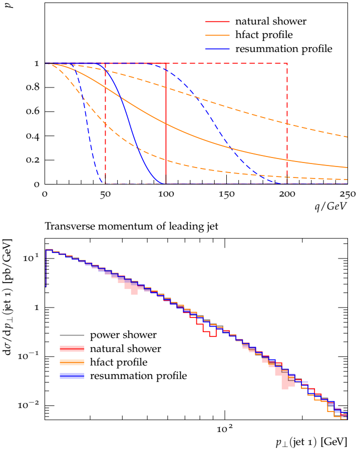


[Gieseke, SP, 2011]

Hard Scales & Uncertainties.

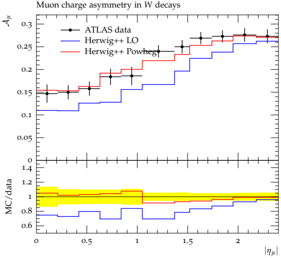


[Hoeche, Krauss, Schönherr, Siegert, 2011]

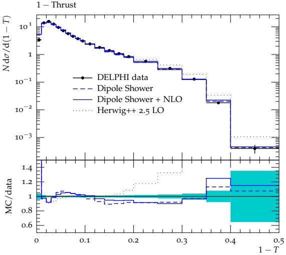


[SP, 2013]

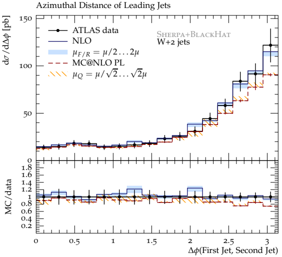
More examples.



[Hamilton, Richardson, Tully, 2009]



[Gieseke, SP, 2011]



[Hoeche, Krauss, Schönherr, Siegert, 2012]

Take-home messages.

Consistent combination of parton showers and NLO calculations.

The most general solution is to subtract the $\mathcal{O}(\alpha_s)$ contribution of the parton shower from the NLO calculation.
This yields a large family of matching strategies.

Still, NNLO effects can lead to strange features in distributions.
We need a smooth approach to the hard scale.

Watch out for uncertainties.
Still an active area of discussion, no ultimate conclusion here.

Watch out for generation cuts.

Mind the validity of the calculation.
(see discussion in the last lecture).

Questions?

3: Merging

LO Merging.

Parton showers approximate matrix elements and add resummation:

$$\mathrm{PS}\left[\mathrm{d}\sigma(\phi_0, q_0)\right] =$$

$$\sum_{k=0}^{N-1} \mathrm{d}\sigma(\phi_0, q_0)\frac{\mathrm{d}\phi_k}{\mathrm{d}\phi_0}\left(\prod_{i=1}^{k} P(\phi_{i-1}, q_i)\right)\Delta_k(\mu|q_k|\cdots|q_0) +$$

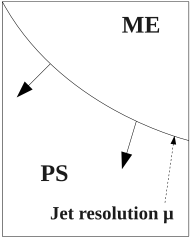
$$\mathrm{PS}\left[\mathrm{d}\sigma(\phi_0, q_0)\frac{\mathrm{d}\phi_N}{\mathrm{d}\phi_0}P(\phi_{N-1}, q_N)\cdots P(\phi_0, q_1)\Delta_{N-1}(q_N|\cdots|q_0)\right]$$

$$\Delta_k(\mu|q_k|\cdots|q_0) = \Delta_k(\mu|q_k)\cdots\Delta_0(q_1|q_0)$$

Merging in a nutshell:
Replace by exact matrix elements while keeping Sudakov form factors.
For efficiency divide into hard and soft region with a merging scale.

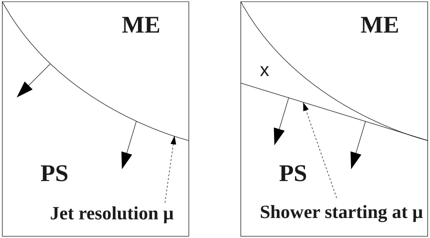
LO Merging.

Cut phasespace according to jet measure on emission-by-emission basis.



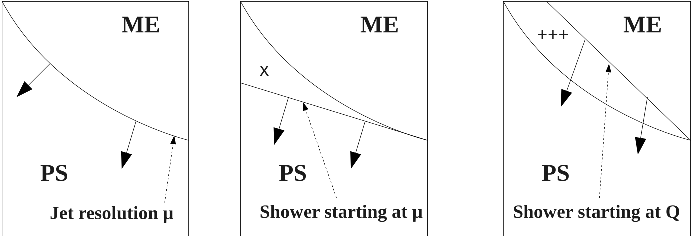
LO Merging.

Cut phasespace according to jet measure on emission-by-emission basis.



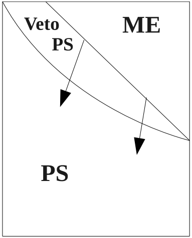
LO Merging.

Cut phasespace according to jet measure on emission-by-emission basis.



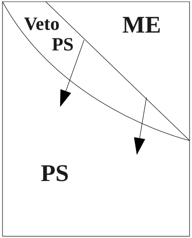
LO Merging.

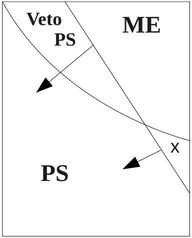
Cut phasespace according to jet measure on emission-by-emission basis.



LO Merging.

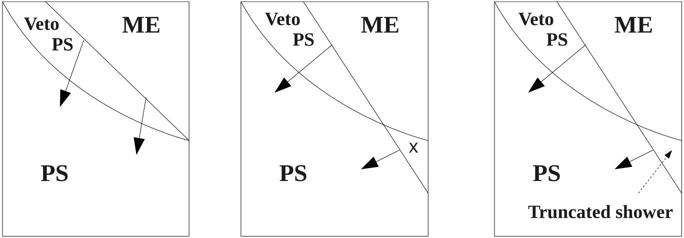
Cut phasespace according to jet measure on emission-by-emission basis.





LO Merging.

Cut phasespace according to jet measure on emission-by-emission basis.



LO Merging.

The literature most often refers to ‘matrix element merging’.
Let’s be more precise here: We merge cross sections.
$\rightarrow$ There is no NLO matrix element, only NLO cross sections.

Look at n parton shower emissions with $q > \rho$:
$\rightarrow$ approximate, exclusive LO n-jet cross section with veto logarithms.

$$\mathrm{d}\sigma(\phi_0, q_0)\frac{\mathrm{d}\phi_n}{\mathrm{d}\phi_0}P(\phi_{n-1}, q_n)\cdots P(\phi_0, q_1)\Delta_n(\rho|q_n|\cdots|q_0)$$
$$\times\,\theta(q_n-\rho)\theta(q_{n-1}-q_n)\cdots\theta(q_0-q_1)$$

We want this to be

$$\mathrm{d}\sigma_\rho^{\mathrm{LO}}(\phi_n, q_n)\times\theta(q_n-\rho)\theta(q_{n-1}-q_n)\cdots\theta(q_0-q_1)\times\Delta_n(\rho|q_n|\cdots|q_0)$$

LO Merging.

Exclusive n-jet cross section at LO:

$$\mathrm{d}\sigma(\phi_n, q_n) \times \theta(q_n - \rho)\theta(q_{n-1} - q_n) \cdots \theta(q_0 - q_1) \times \Delta_n(\rho|q_n| \cdots |q_0)$$

This we get from:

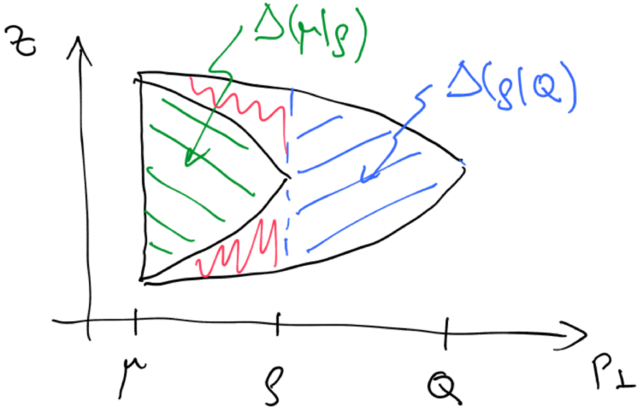
- Calculate LO cross sections
- Cluster back with a jet algorithm
- Reweight with Sudakov form factors (and running couplings)

Then we need to add a *vetoed* shower below ρ such that exclusive n-parton configurations are given by

$$\mathrm{d}\sigma(\phi_n, q_n) \times \theta(q_n - \rho)\theta(q_{n-1} - q_n) \cdots \theta(q_0 - q_1) \times \Delta_n(\mu|q_n| \cdots |q_0)$$

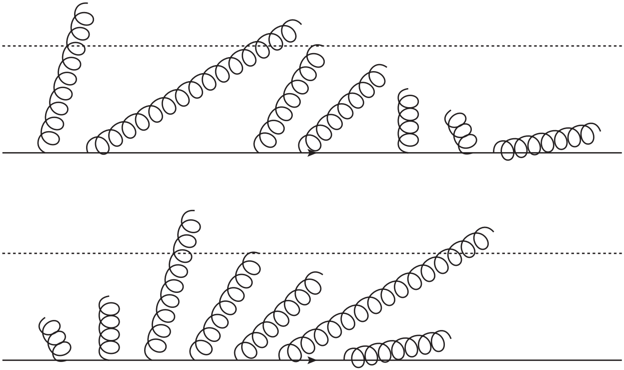
[Catani, Krauss, Kuhn, Webber 2001]

Why vetoed showers?



Truncated showers.

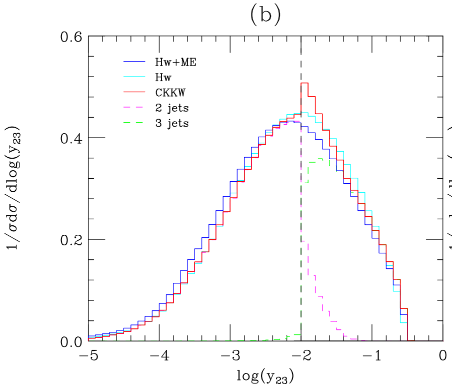
What if we choose a jet algorithm which is not the inverse of the shower?

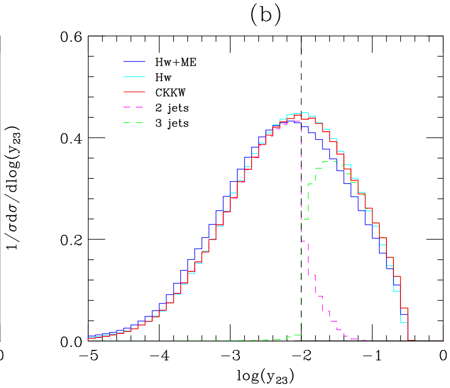


[Hamilton, Richardson, Tully 2009] [Hoeche, Krauss, Schumann, Siegert 2009]

Truncated showers.

Do they matter?





[Hamilton, Richardson, Tully 2009]

Inclusive cross sections.

Expectations from the shower:

$$\begin{array}{ll} = n & \mathrm{d}\sigma(\phi_0, q_0)\dfrac{\mathrm{d}\phi_n}{\mathrm{d}\phi_0}P(\phi_{n-1}, q_n)\cdots P(\phi_0, q_1)\Delta_n(\rho|q_n|\cdots|q_0) \\ \geq n & \mathrm{d}\sigma(\phi_0, q_0)\dfrac{\mathrm{d}\phi_n}{\mathrm{d}\phi_0}P(\phi_{n-1}, q_n)\cdots P(\phi_0, q_1)\Delta_{n-1}(q_n|\cdots|q_0) \end{array}$$

With LO merging have:

$$\begin{array}{lll} = n & & \mathrm{d}\sigma(\phi_n, q_n)\Delta_n(\rho|q_n|\cdots|q_0) \\ \geq n & & \mathrm{d}\sigma(\phi_n, q_n)\Delta_{n-1}(q_n|\cdots|q_0) \\ & + & \displaystyle\int_\rho^{q_n}\mathrm{d}q_{n+1}\left(\frac{\mathrm{d}\sigma(\phi_{n+1}, q_{n+1})}{\mathrm{d}q_{n+1}} - \frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}\phi_n\mathrm{d}q_{n+1}}P(\phi_n, q_{n+1})\mathrm{d}\sigma(\phi_n, q_n)\right)\Delta_n(q_{n+1}|\cdots|q_0) \end{array}$$

Aim to remove these contributions to prevent spurious logs.
Allows to use very small (IR cutoff) merging scales.

[Lönnblad, Prestel 2012] [SP 2012]

NLO merging.

Look at n parton shower emissions with $q > \rho$:
$\rightarrow$ approximate, exclusive LO n-jet cross section with veto logarithms.

$$\mathrm{d}\sigma(\phi_0, q_0)\frac{\mathrm{d}\phi_n}{\mathrm{d}\phi_0}P(\phi_{n-1}, q_n)\cdots P(\phi_0, q_1)\Delta_n(\rho|q_n|\cdots|q_0)$$
$$\times\,\theta(q_n-\rho)\theta(q_{n-1}-q_n)\cdots\theta(q_0-q_1)$$

We want this to be

$$\mathrm{d}\sigma_\rho^{\mathrm{NLO}}(\phi_n, q_n)\times\theta(q_n-\rho)\theta(q_{n-1}-q_n)\cdots\theta(q_0-q_1)\times\Delta_n(\rho|q_n|\cdots|q_0)$$

So subtracting double counting and adding a shower gives

$$\frac{\mathrm{d}\sigma(\phi_n, q_n)}{\Delta_n(\rho|q_n|\cdots|q_0)} = \mathrm{d}\sigma_\rho^{\mathrm{NLO}}(\phi_n, q_n) + \mathcal{O}(\alpha_s^{n+2})$$

for exclusive n-jet configurations. Plus a lot of details I can't dig into here.

NLO merging.

This is not a unique approach either, and subject to ongoing development.
NB All of these implementatinos can do LO merging, of course.

NL^3
available for Ariadne and Pythia 8 [Lavesson, Lönnblad 2008] [Lönnblad, Prestel 2012]

UNLOPS approaches – preserve inclusive cross sections
available for Pythia 8 and (soon) for Herwig++ dipole shower

[Lönnblad, Prestel 2012] [SP 2012] [Bellm, Gieseke, SP 2014]

MINLO merging – process specific, but allow for NNLO matching
based on POWHEG, shower independent [Hamilton, Nason, Oleari, Re, Zanderighi 2012]

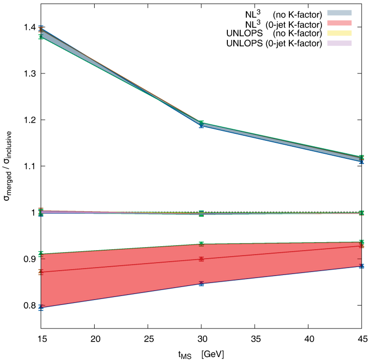
Sherpa MEPS@NLO
based on Sherpa MC@NLO and Catani-Seymour shower

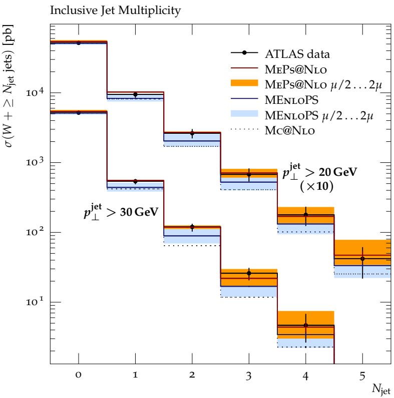
[Hoeche, Krauss, Schönherr, Siegert + Gehrmann 2012]

Vincia – Iterate matrix element corrections @ NLO
plugin to Pythia 8 [Skands et al. 2012]

Examples.

UNLOPS inclusive cross section and Sherpa MEPS@NLO





Take-home messages.

LO merging is an established technique, available in many event generators.

'Unitarized' approaches try to preserve inclusive cross sections. Allows for low merging scales, can be generalized to NLO.

NLO merging is still an active area of development.

Questions?

Thanks for your attention!

(and don't hesitate to bug me on more details)