Matrix element/NLO calculations

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Terascale Monte Carlo School 2014

DESY, Hamburg, March 10–14, 2014

- Lecture 1: Relevance and calculation of NLO QCD corrections
- Lecture 2: Relevance and calculation of NLO electroweak corrections
- Lecture 3: NLO Calculations for Higgs Physics

Relevance and calculation of NLO QCD correctcions

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- Relevance of NLO (QCD) corrections
- Calculation of cross sections at NLO
- Conventional techniques based on Feynman diagrams
- Generalized unitarity techniques
- Example: $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

Relevance of NLO (QCD) corrections

Importance of NLO calculations for LHC

QCD corrections: substantial part of predictions

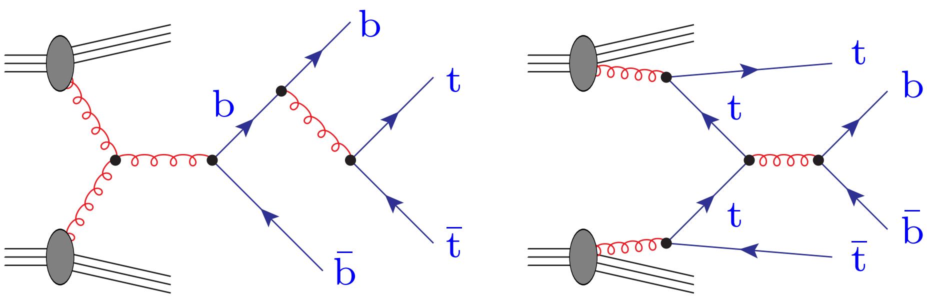
- leading-order (LO) predictions depend on $\alpha_s = \alpha_s(\mu)$
  renormalization scale μ free parameter
  $\Rightarrow$ large scale uncertainty (up to factor 2)
  $\Rightarrow$ often no quantitative prediction possible
  μ dependence due to missing higher orders
- next-to-leading-order (NLO) predictions: reduced scale uncertainty
  first real prediction
  $\Rightarrow$ needed for all scattering processes at the LHC
  $\mathcal{O}(\alpha_s) \times \log(\ldots) \sim 10\% - 100\%, \qquad \alpha_s(M_Z) \approx 0.12$
- next-to-next-to-leading-order (NNLO) predictions:
  scale uncertainty further reduced
  first real uncertainty estimate
  $\Rightarrow$ needed for selected processes like
  single W/Z production, $t\bar{t}$ production, Higgs production
  $\mathcal{O}(\alpha_s^2) \times \log^2(\ldots) \sim \text{few}\% - 20\%$

$\hookrightarrow$ NLO (NNLO) corrections important for reliable predictions

Example: scale dependence for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$

Background process to

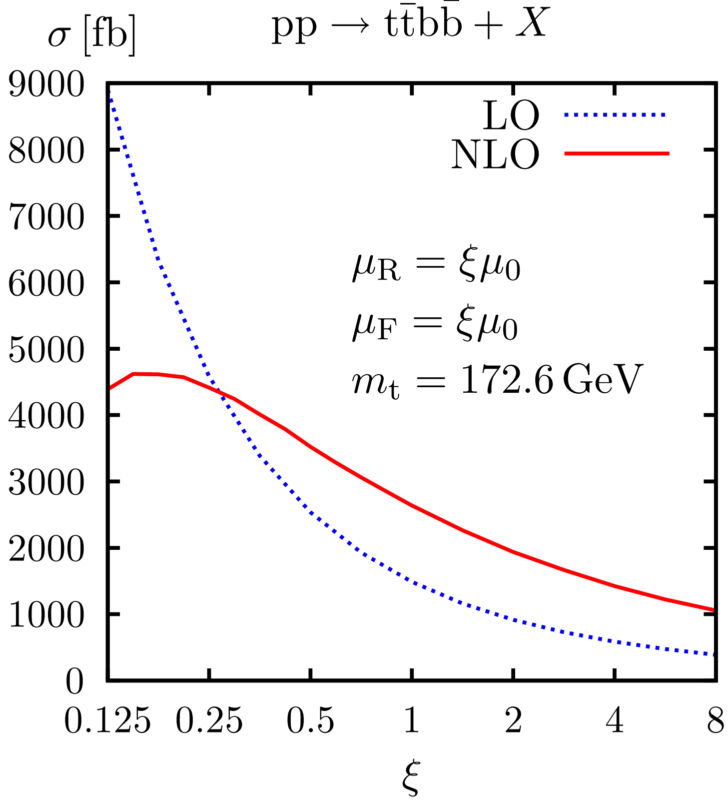
$\mathrm{pp} \to \mathrm{t\bar{t}H} + X \to \mathrm{t\bar{t}b\bar{b}} + X$



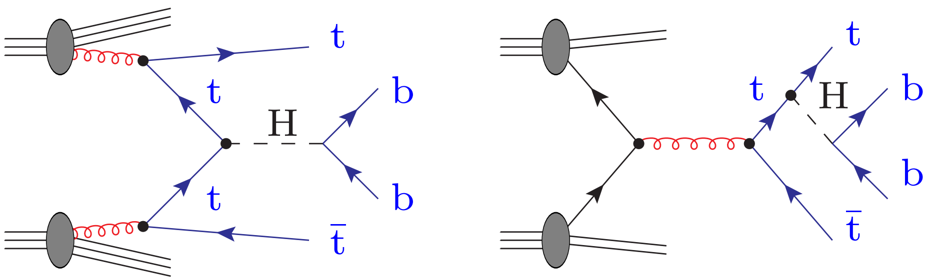
LO $\propto \alpha_{\mathrm{S}}(\mu_{\mathrm{R}})^4 \Rightarrow$ large scale uncertainty

μ_{R}	$m_{\mathrm{t}}/8$	$m_{\mathrm{t}}/4$	$m_{\mathrm{t}}/2$	m_{t}	$2m_{\mathrm{t}}$	$4m_{\mathrm{t}}$	$8m_{\mathrm{t}}$
$\alpha(\mu_{\mathrm{R}})$	0.151	0.133	0.119	0.108	0.098	0.091	0.084
$\frac{\alpha(\mu_{\mathrm{R}})}{\alpha(m_{\mathrm{t}})}$	1.40	1.24	1.11	1.00	0.91	0.84	0.78
$\left(\frac{\alpha(\mu_{\mathrm{R}})}{\alpha(m_{\mathrm{t}})}\right)^4$	3.88	2.34	1.49	1.00	0.70	0.50	0.37

Bredenstein, Denner, Dittmaier, Pozzorini '10



original (ATLAS) scale choice based on $\mathrm{t\bar{t}H}$

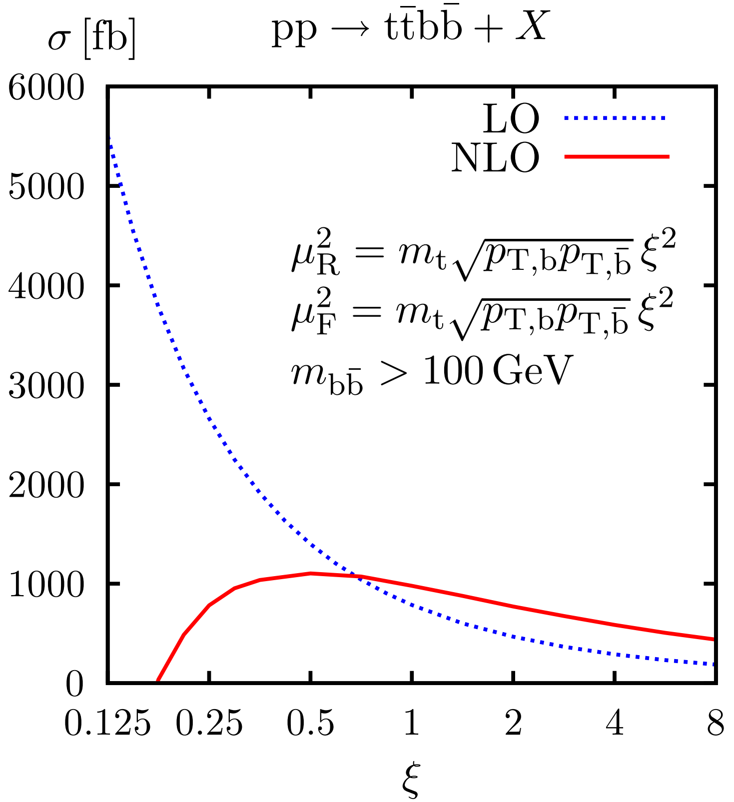


$$\mu_0 = E_{\mathrm{thr}}/2 = m_{\mathrm{t}} + m_{\mathrm{b\bar{b}}}/2$$

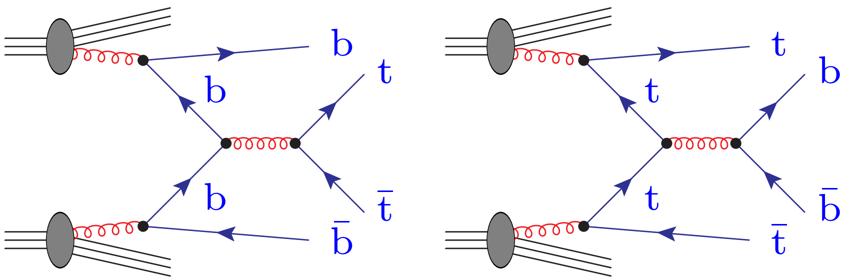
$\Rightarrow$ large K factor (1.8) and scale dependence (34%)

Example: scale dependence for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$

Bredenstein, Denner, Dittmaier, Pozzorini '10



QCD dynamics of $\mathrm{t\bar{t}H}$ and $\mathrm{t\bar{t}b\bar{b}}$ different



various different channels for $\mathrm{t\bar{t}b\bar{b}}$

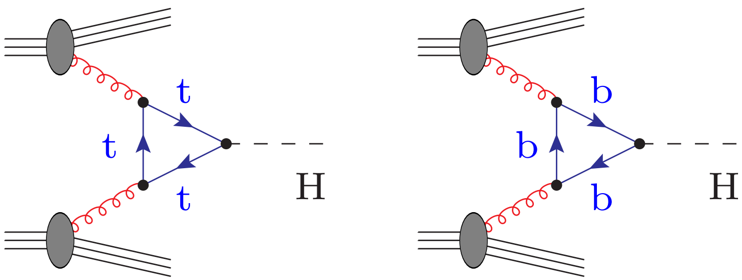
good central scale

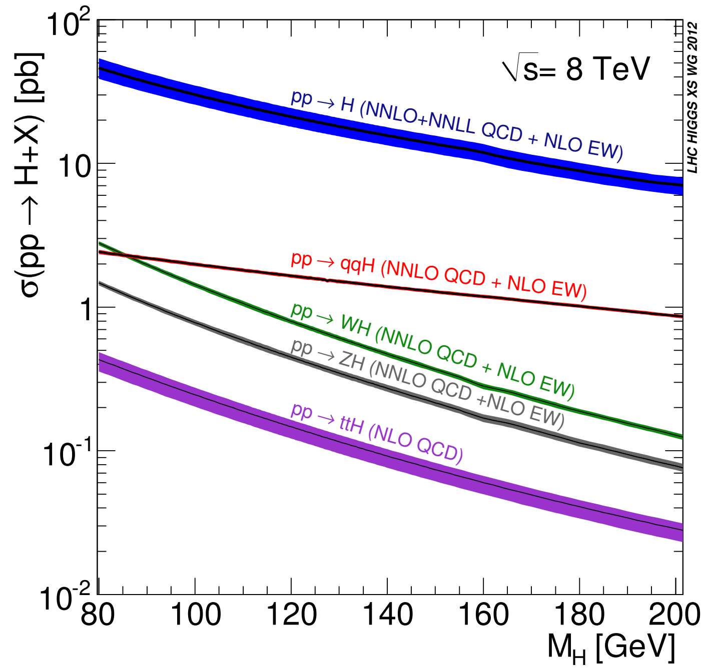
$$\mu_0^2 = m_t \sqrt{p_{T,b} p_{T,\bar{b}}}$$

one α_s at scale m_t
one α_s at scale of p_T of b quarks

- small correction and uncertainty: $K = 1.24 \pm 21\%$
- central scale close to a maximum

Example: Higgs production in gluon–gluon fusion





LHC HIGGS XS WG '12

- most important production channel at LHC
- $\sigma_{\mathrm{LO}} \propto \alpha_{\mathrm{s}}^2$
  strong dependence on factorization and renormalization scales (100%)
  $\Rightarrow$ higher-order corrections very important
- complete NLO: $80-100\%$
  virtual contribution $\pi\alpha_{\mathrm{s}} \sim 35\%$ $[\pi^2(\alpha_{\mathrm{s}}/\pi)]$
  real contribution $\sim 50\%$

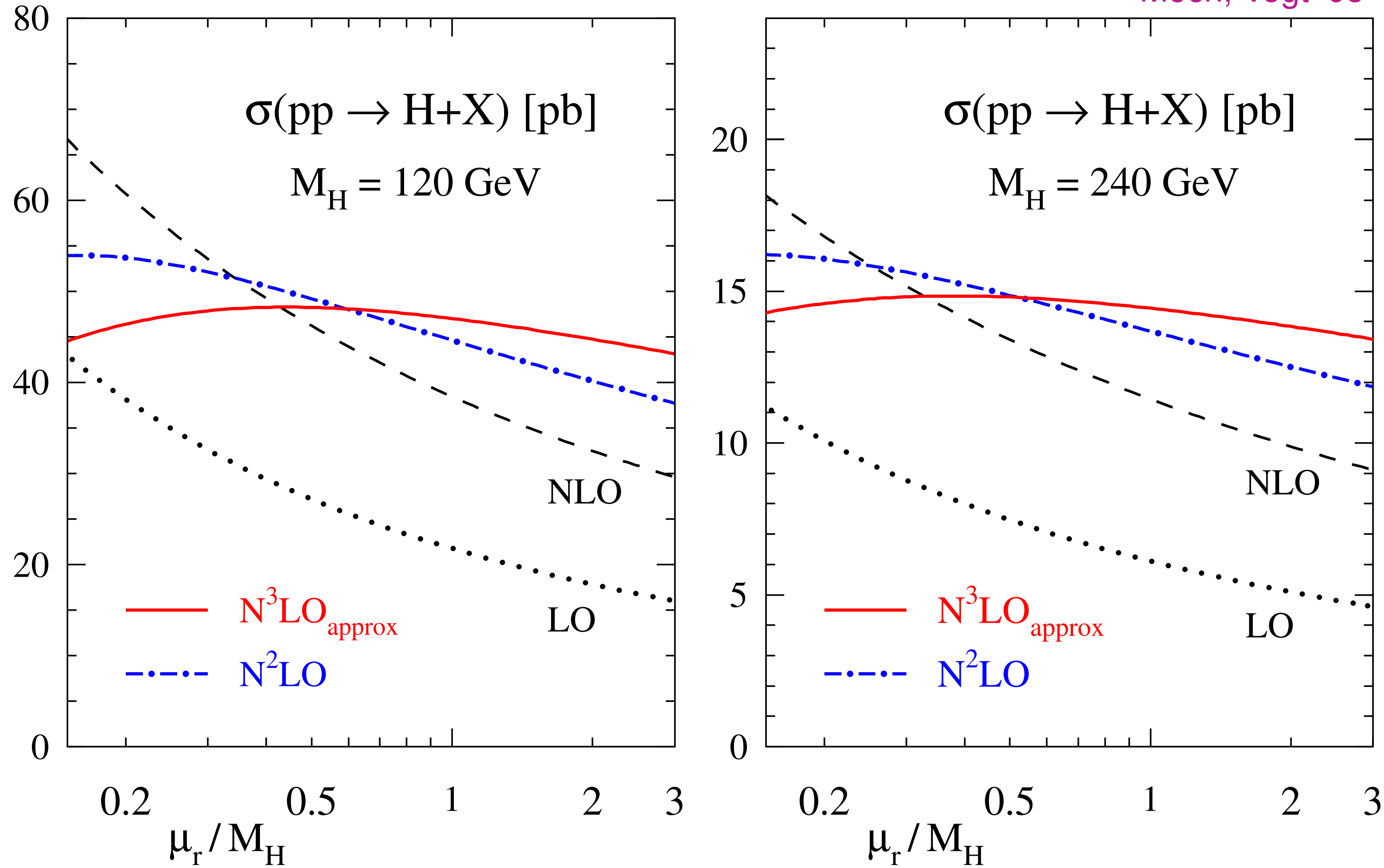
Graudenz, Spira, Zerwas '93
Djouadi, Graudenz, Spira, Zerwas '95

- NNLO: $\sim 25\%$

Harlander, Kilgore '01, '02; Catani, de Florian, Grazzini '01
Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03, '04
Ahrens, Becher, Neubert, Yang '08

QCD scale dependence for gg → H

Moch, Vogt ’05



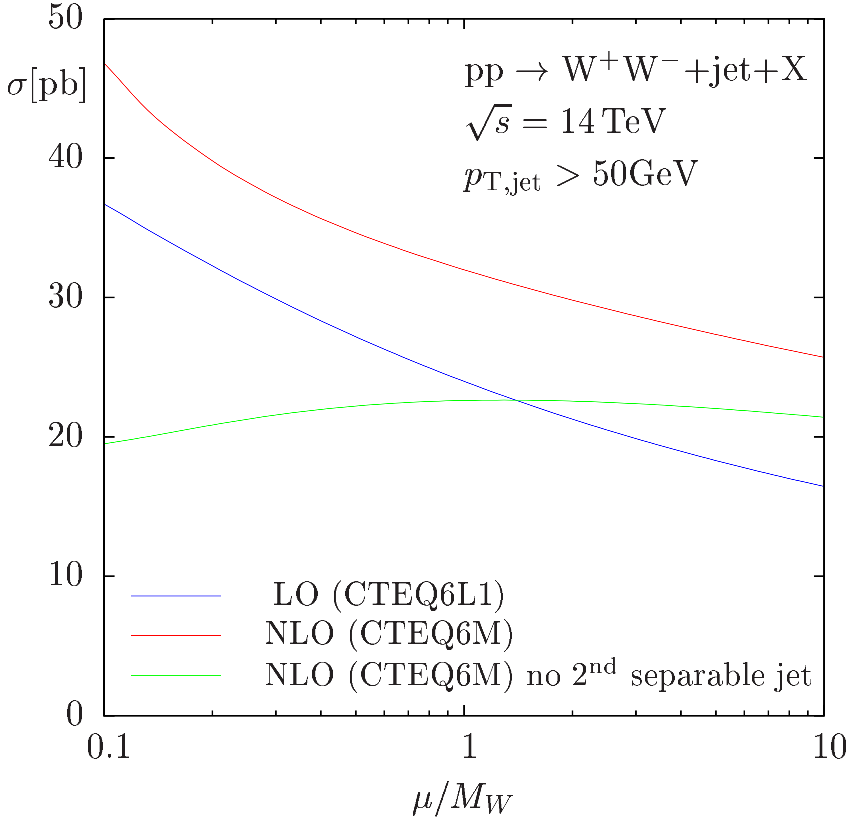
Reduction of renormalization scale dependence with increasing orders!

⇒ residual scale uncertainty $\lesssim 5$–10%

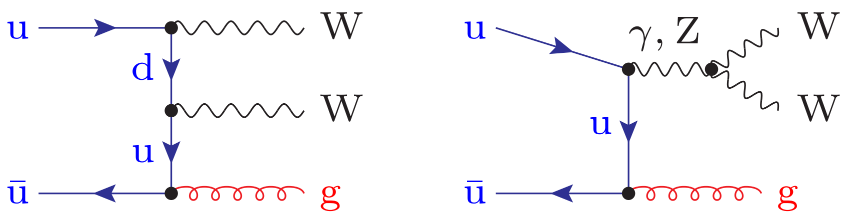
Example: scale dependence for pp $\to$ WWj + X

Appearance of new channels:

Dittmaier, Kallweit, Uwer '07

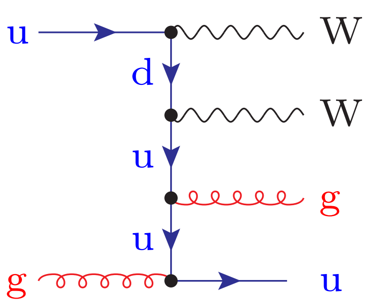


- $\sigma_{\mathrm{LO}} \propto \alpha_{\mathrm{s}}$

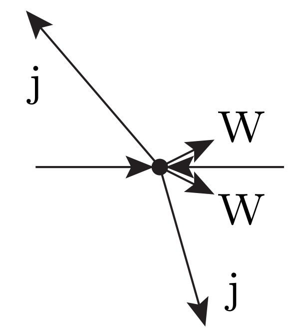


- scale dependence stabilises at NLO for genuine $\mathrm{WW} + \mathrm{j}$ production
- significant scale dependence is introduced by $\mathrm{WW} + 2\,\mathrm{j}$ (difference between green and red curves)

new diagram



new configuration



Importance of multiparticle processes at the LHC

Most signal processes involve few final-state particles:

- $2 \to 2 \quad \mathrm{pp} \to \mathrm{ll}, \mathrm{W}\gamma, \mathrm{WW}, \mathrm{tt}, \ldots + X$
- $2 \to 3 \quad \mathrm{pp} \to \mathrm{Hjj}, \mathrm{WW}\gamma, \ldots + X$

however,

- heavy particles $(\mathrm{W}, \mathrm{Z}, \mathrm{t}, \ldots)$ decay into jets, leptons, photons
  $\mathrm{pp} \to \mathrm{WW} \to \mathrm{ll}\nu_l\nu_l + X$, $\mathrm{pp} \to \mathrm{tt} \to \mathrm{be}\nu_\mathrm{e}\mathrm{b}\mu\nu_\mu + X$
- irreducible backgrounds involve genuine multiparticle final states
  $\mathrm{pp} \to \mathrm{ll}\nu_l\nu_l + X$, $\mathrm{pp} \to \mathrm{be}\nu_\mathrm{e}\mathrm{b}\mu\nu_\mu + X$
  (backgrounds often not fully accessible to measurements)
- large fraction of final states contains additional jets
  $\mathrm{pp} \to \mathrm{WWj} + X$, $\mathrm{pp} \to \mathrm{WWjj} + X, \ldots$
- interesting signals with many particles
  e.g. WW scattering, $\mathrm{pp} \to \mathrm{WWjj} \to \mathrm{e}\nu_\mathrm{e}\mu\nu_\mu\mathrm{jj}$

$\Rightarrow$ Need reliable predictions for multiparticle processes!

Status of NLO QCD calculations

NLO calculations

- $2 \to 2$ trivial (textbook)
- $2 \to 3$ standard (many groups)
- $2 \to 4$ state of the art (several groups)

  first $2 \to 4$ electroweak (EW) calculation:

  $\mathrm{e^+e^-} \to f_1 \bar{f}_2 f_3 \bar{f}_4$ Denner, Dittmaier, Roth, Wieders '05

  first $2 \to 4$ QCD calculations:

  $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ Bredenstein, Denner, Dittmaier, Pozzorini '09, Bevilacqua et al. '09

  $\mathrm{pp} \to \mathrm{W}jjj$ Berger et al. '09; R.K.Ellis et al. '09

  NLO QCD exists for $\gtrsim 20$ LHC $2 \to 4$ processes

- $2 \to \geq 5$ only very few (few groups)

  $\mathrm{pp} \to \mathrm{W/Z} + 4j$ Berger et al. '10/'11

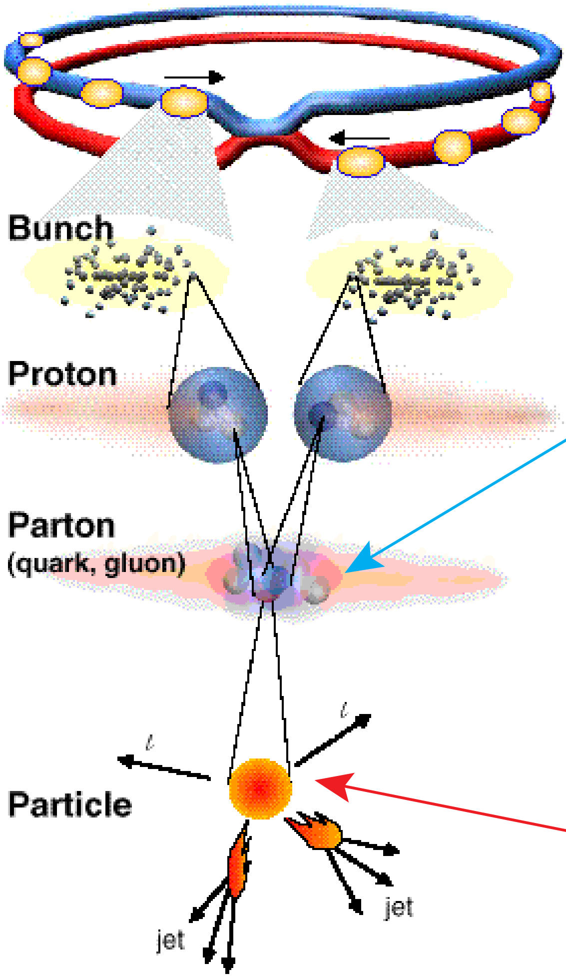
  $\mathrm{e^+e^-} \to 7j$ Becker et al. '11

  $\mathrm{pp} \to 5j$ Badger et al. '13

  $\mathrm{pp} \to \mathrm{W} + 5j$ Bern et al. '13

Calculation of cross sections at NLO

Hadronic cross sections



parton content of the proton:

valence quarks uud,

sea quarks $\mathrm{u, d, c, s, (+b,)}$ + antiquarks

gluons g (+ photons γ)

"parton distribution functions" (PDFs) $f_{i/\mathrm{p}}(x, \mu_\mathrm{F})$

probability for parton i to have fraction x

of momentum p at "factorization scale" μ_F

= non-perturbative input (from experiment)

process independent

hard interaction of partons

$\hookrightarrow$ perturbative QFT applicable,

model for hard interaction

(except QCD/QED) only enters here

$$\sigma_{\mathrm{pp}\to F+X}(p_1, p_2) = \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \sum_{a,b} f_{a/\mathrm{p}}(x_a, \mu)\, f_{b/\mathrm{p}}(x_b, \mu_\mathrm{F})\, \hat{\sigma}_{ab\to F}(x_a p_1, x_b p_2, \mu_\mathrm{F})$$

LO partonic cross section

LO partonic cross section for a $2 \to n$ process can be written as

$$\mathrm{d}\hat{\sigma}_{\mathrm{LO}} = \frac{1}{2s} \int \mathrm{d}\Phi_n |\mathcal{M}_{\mathrm{LO}}|^2$$

$$\int \mathrm{d}\Phi_n = (2\pi)^4 \delta^{(4)}\left(P - \sum_{i=1}^{n} q_i\right) \prod_{i=1}^{n} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}$$ n-particle phase space

$\mathcal{M}_{\mathrm{LO}}$: LO matrix element (contains model for hard interaction)

$s = P^2 = (\hat{p}_1 + \hat{p}_2)^2$ square of centre-of-mass energy of hard process ($\hat{p}_i = x_i p_i$)

Integration over phase space by Monte Carlo methods $\Rightarrow$

- any distribution can be determined simultaneously
- Monte Carlo events can be unweighted

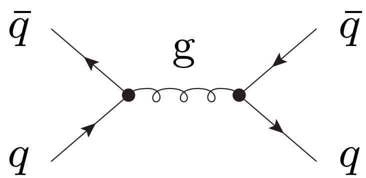
Many generic codes exist at LO:

- MADGRAPH Alwall, Herquet, Maltoni, Mattelaer, Stelzer
- WHIZARD Kilian, Ohl, Reuter
- SHERPA Höche, Krauss, Schuhmann, Siegert, Winter
- HELAC Papadopoulos, Worek
- . . . many more

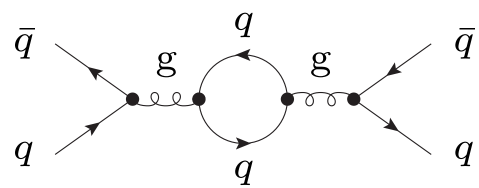
IR singularities in NLO corrections

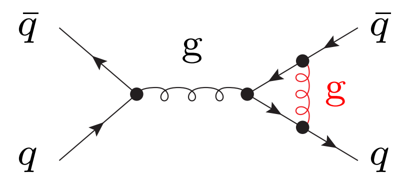
NLO corrections consist of Feynman diagrams of higher order to the same process:

tree diagrams



loop diagrams

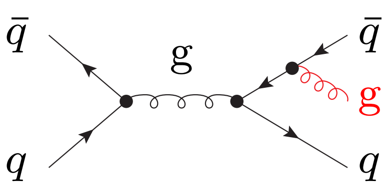




loop diagrams contain infrared singularities
from internal gluons soft or collinear to quarks

Process with an additional gluon has to be added to cancel IR singularities!

(soft or collinear gluons cannot be separated experimentally!)



Square of bremsstrahlung matrix element of the same order as interference between LO diagrams and loop diagrams!

IR singularities cancel in sufficiently inclusive quantities! (KLN theorem)

Some collinear singularities from the initial state do not cancel but can be absorbed by a renormalization of the PDFs $\Rightarrow$ collinear counter term

Structure of NLO computation

NLO partonic cross section can be written as

$$\mathrm{d}\hat{\sigma}_{\mathrm{NLO}} = \int \mathrm{d}\Phi_n \left[|\mathcal{M}_{\mathrm{LO}}|^2 + 2\,\mathrm{Re}\{\mathcal{M}^*_{\mathrm{LO}}\mathcal{M}_{\mathrm{NLO,V}}\} + C \right]$$

$$+ \int \mathrm{d}\Phi_{n+1} \left[|\mathcal{M}_{\mathrm{NLO,R}}|^2 \right]$$

$\int \mathrm{d}\Phi_{n(+1)}$: n or $n+1$ particle phase space

$\mathcal{M}_{\mathrm{LO}}$, $\mathcal{M}_{\mathrm{NLO,V}}$, $\mathcal{M}_{\mathrm{NLO,R}}$: matrix elements for LO, virtual and real NLO

C collinear counter term from renormalization of PDFs
needed to cancel left-over collinear singularities from initial state

infrared singularities cancel only after phase-space integration
numerical phase-space integration impossible or inaccurate
$\Rightarrow$ use dedicated treatment of infrared singularities

Structure of NLO computation

NLO partonic cross section can be written as in subtraction method

$$\mathrm{d}\hat{\sigma}_{\mathrm{NLO}} = \int \mathrm{d}\Phi_n \left[|\mathcal{M}_{\mathrm{LO}}|^2 + 2\,\mathrm{Re}\{\mathcal{M}^*_{\mathrm{LO}}\mathcal{M}_{\mathrm{NLO,V}}\} + C + \int \mathrm{d}\Phi_1 \sum_j S_j \right]$$
$$+ \int \mathrm{d}\Phi_{n+1} \left[|\mathcal{M}_{\mathrm{NLO,R}}|^2 - \sum_j S_j \right]$$

$\int \mathrm{d}\Phi_{n(+1)}$: n or $n+1$ particle phase space

$\mathcal{M}_{\mathrm{LO}}$, $\mathcal{M}_{\mathrm{NLO,V}}$, $\mathcal{M}_{\mathrm{NLO,R}}$: matrix elements for LO, virtual and real NLO

C collinear counter term from renormalization of PDFs

$\sum S_j$: subtraction terms for real corrections

$\int \mathrm{d}\Phi_1 \sum S_j$: (analytically) integrated subtraction terms

subtraction terms cancel but render individual integrals finite $\Rightarrow$ stable numerical integration

$\mathcal{M}_{\mathrm{NLO,R}}$: tree-level matrix elements
subtraction terms S_j: colour-weighted tree-level matrix elements
virtual corrections $\mathrm{Re}\{\mathcal{M}^*_{\mathrm{LO}}\mathcal{M}_{\mathrm{NLO,V}}\}$ (loop diagrams): require different methods

Methods for virtual corrections

Until ∼2005: Virtual corrections were the bottleneck of NLO calculations.

- Feynman diagrams: worse than factorial complexity
- relied on process-specific algebraic calculations, no full automation

NLO revolution: Ossola, Papadopoulos, Pittau '06, Bern, Dixon, Kosower, Britto, Cachazo, Feng, Ellis, Giele, Melnikov, . . .

- unitarity-cut technique: polynomial complexity of rank 9: $\mathcal{O}(n^9)$ Giele, Zanderighi '08
- automation immediately performed by different groups

extension of recursion-relation technique to NLO:

- exponential complexity: $\mathcal{O}(n^4 2^n)$ van Hameren '09 (pure gluon amplitudes)
- asymptotic behaviour not necessarily relevant for practical purposes
- basis for automation of EW corrections with RECOLA Actis et al. '12

combination of Feynman diagrams and recursion relations:

- OPENLOOPS Cascioli, Maierhöfer, Pozzorini '12
  NLO-QCD matrix elements for many LHC processes, linked with SHERPA
- Feynman diagrams allow efficient summation over colours and helicities

Conventional techniques based on Feynman diagrams

Number and complexity of diagrams

Tree level:

- Feynman diagrams: double factorial complexity $[2n!! = 2n(2n-2)(2n-4)...]$

  # diagrams for pure gluon (scalar) processes

external gluons	4	5	6	7	8	9	...	n
# diags w/ only 3-g vertices	3	15	105	945	10395	135135		$(2n-5)!!$
# diags w/ 3-g and 4-g vert.	4	25	220	2485	34300	559405		

- recursion relation technique: polynomial complexity of rank 4: $\mathcal{O}(n^4)$

  Berends, Giele '88; Kleiss, Kuijf '89

One-loop level: specific examples

process	# LO diags	# NLO diags	# real NLO diags
$qq \to \mathrm{H} + 2\,\mathrm{jets}\ (\mathrm{EW})$	1	~ 200	9
$qq/\mathrm{gg} \to \mathrm{t\bar{t}b\bar{b}}$	7/36	188/1003	64/341
$qq/\mathrm{gg} \to \mathrm{WWb\bar{b}}$	14/31	280/788	90/222

asymptotic behaviour not necessarily relevant for practical multiplicities

Feynman rules of QCD

Lorentz tensor structures

colour matrices

propagator denominators

coupling constants

Propagators

quark: 

$= \frac{\mathrm{i}(\not{p}+m)_{\alpha\beta}\delta_{ij}}{p^2-m^2+\mathrm{i}\varepsilon}$

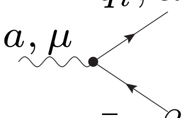
gluon: 

$= \frac{-\mathrm{i}g_{\mu\nu}\delta^{ab}}{p^2+\mathrm{i}\varepsilon}$

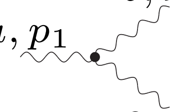
ghost: 

$= \frac{\mathrm{i}\delta^{ab}}{p^2+\mathrm{i}\varepsilon}$

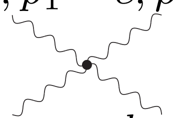
Vertices



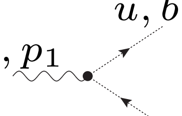
$= \mathrm{i}g(\gamma_\mu)_{\alpha\beta}T^a_{ij}$



$$= gf^{abc}[g_{\mu\nu}(p_1-p_2)_{\mu_3} + g_{\nu\rho}(p_2-p_3)_{\mu_1} + g_{\rho\mu}(p_3-p_1)_{\mu_2}]$$



$$= -\mathrm{i}g^2[f^{abh}f^{cdh}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho}) + f^{ach}f^{dbh}(g_{\mu\sigma}g_{\rho\nu}-g_{\mu\nu}g_{\rho\sigma}) + f^{adh}f^{bch}(g_{\mu\nu}g_{\sigma\rho}-g_{\mu\rho}g_{\sigma\nu})]$$



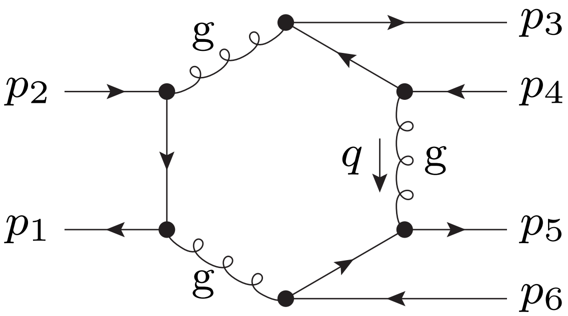
$= -gf^{abc}p_{2,\mu}$

colour matrices factorise apart from 4-gluon vertex

A typical 6-point diagram

$q\bar{q} \to q\bar{q}q\bar{q} \quad (\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ or $\mathrm{e^+e^-} \to 4f)$

$$q_2(p_2,\sigma_2) + \bar{q}_1(p_1,\sigma_1) \;\to\; q_3(p_3,\sigma_3) + \bar{q}_4(p_4,\sigma_4) + q_5(p_5,\sigma_5) + \bar{q}_6(p_6,\sigma_6)$$

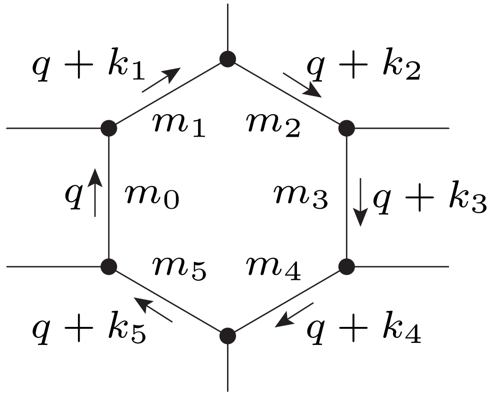


p_k momenta
σ_k helicities
i_k colour indices

analytic expression (colour factorises)

$$\mathcal{M}^{\sigma_1\ldots\sigma_6}_{i_1\ldots i_6}(p_1,\ldots,p_6) = g^6 (T^aT^b)_{i_1i_2}(T^bT^c)_{i_3i_4}(T^cT^a)_{i_5i_6}$$
$$\times \int \mathrm{d}^D q \, \frac{\bar{u}(p_5,\sigma_5)\gamma^\mu(\not{q}+\not{p}_5+m_5)\gamma^\nu v(p_6,\sigma_6)}{(q^2)[(q+p_5)^2-m_5^2]}$$
$$\times \frac{\bar{v}(p_1,\sigma_1)\gamma^\nu(\not{q}+\not{p}_5+\not{p}_6-\not{p}_1+m_1)\gamma^\lambda u(p_2,\sigma_2)}{[(q+p_5+p_6)^2][(q+p_5+p_6-p_1)^2-m_1^2]}$$
$$\times \frac{\bar{u}(p_3,\sigma_3)\gamma^\lambda(\not{q}-\not{p}_4+m_3)\gamma^\mu v(p_4,\sigma_4)}{[(q-p_3-p_4)^2][(q-p_4)^2-m_3^2]}$$

6-point tensor integrals



denominator factors:

$$N_i = (q+k_i)^2 - m_i^2 + \mathrm{i}\epsilon, \quad k_0 = 0$$
$$i = 0, \ldots, 5$$

integral definition

$$F^{\mu_1 \cdots \mu_P}(k_1, \ldots, k_5, m_0, \ldots, m_5) = \frac{(2\pi\mu)^{4-D}}{\mathrm{i}\pi^2} \int \mathrm{d}^D q \, \frac{q^{\mu_1} \cdots q^{\mu_P}}{N_0 N_1 \ldots N_5}$$

decomposition of tensor integral into covariants: (scalar integral $F = F_0$)

$$F^{\mu} = \sum_{i=1}^{5} k_i^{\mu} F_i, \qquad F^{\mu\nu} = \sum_{i,j=1}^{5} k_i^{\mu} k_j^{\nu} F_{ij} + g^{\mu\nu} F_{00}$$

$$F^{\mu\nu\rho} = \sum_{i,j,k=1}^{5} k_i^{\mu} k_j^{\nu} k_k^{\rho} F_{ijk} + \sum_{i=1}^{5} (g^{\mu\nu} k_i^{\rho} + g^{\nu\rho} k_i^{\mu} + g^{\rho\mu} k_i^{\nu}) F_{00i}$$

F_i, F_{ij}, F_{ijk}, F_{00i} tensor coefficients

$\Rightarrow$ separation between tensors and integrals

Structure of 1-loop contributions

Structure of diagram after insertion of tensor-integral decomposition

$$\mathcal{M}_{i_k}^{\sigma_k}(p_k) = \underbrace{\mathcal{C}_{i_k}}_{\text{factorised colour structure}} \sum_m \mathcal{F}_m(\{p_a \cdot p_b\}) \underbrace{\hat{\mathcal{M}}_m^{\{\lambda_k\}}(\{p_k\})}_{\text{standard matrix elements}}$$

colour structures $\mathcal{C}_{i_k}$

- factorises for most diagrams (3 colour structures for diagrams with 4-g vertex)
- low computational cost in Feynman-diagram approach

standard matrix elements $\hat{\mathcal{M}}_m^{\{\lambda_k\}}(\{p_k\})$

- comprise all tensorial and spinorial objects, purely kinematical
- depend on helicities of external particles
- examples:

$$\bar{v}(p_1,\sigma_1)\gamma^\nu \not{p}_6 \gamma^\lambda u(p_2,\sigma_2) \times \bar{u}(p_3,\sigma_3)\gamma^\lambda \not{p}_1 \gamma^\mu v(p_4,\sigma_4) \times \bar{u}(p_5,\sigma_5)\gamma^\mu \not{p}_3 \gamma^\nu v(p_6,\sigma_6)$$

$$\bar{v}(p_1,\sigma_1)\gamma^\mu u(p_2,\sigma_2) \times \bar{u}(p_3,\sigma_3)\gamma_\mu v(p_4,\sigma_4) \times \bar{u}(p_5,\sigma_5)\not{p}_2 v(p_6,\sigma_6)$$

Colour and helicity (sums) factorise from loop integrals!

Structure of 1-loop contributions II

invariant functions $\mathcal{F}_m(\{p_a \cdot p_b\})$:

$$\mathcal{F}_m(\{p_a \cdot p_b\}) = \sum_{j_1 \cdots j_R} \mathcal{K}_{m,j_1 \ldots j_R}(\{p_a \cdot p_b\}) \overbrace{T_{j_1 \ldots j_R}(\{p_a \cdot p_b\})}^{\text{tensor loop coefficients}}$$

- linear combinations of tensor coefficients $T_{j_1 \ldots j_R}$ $(F_0,\, F_i,\, F_{ij},\, F_{ijk})$
- coefficients $\mathcal{K}_{m,j_1 \ldots j_R}(\{p_a \cdot p_b\})$ depend on invariants, masses, couplings

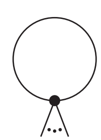
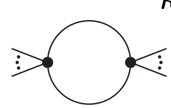
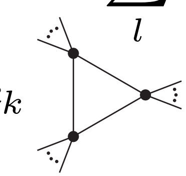
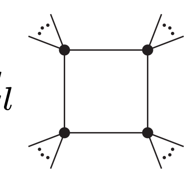
examples:

$$\mathcal{F}_1^{\rho\sigma\tau} = g^6 F_{334}$$
$$\mathcal{F}_2^{\rho\sigma\tau} = -g^6 (F_3 + F_{34} + F_{23} + F_{234})$$

tensor coefficients $T_{j_1 \ldots j_R}(\{p_a \cdot p_b\})$

- (numerically) reduced to scaler A_0, B_0, C_0 and D_0 (1-, 2-, 3-, 4-point) functions:

$$T_{j_1 \ldots j_R} = \sum_i a_i A_0(i) + \sum_j b_j B_0(j) + \sum_k c_k C_0(k) + \sum_l d_l D_0(l)$$

$$= \sum_i a_i$$  $$+ \sum_j b_j$$  $$+ \sum_k c_k$$  $$+ \sum_l d_l$$ 

with coefficients a_i, b_j, c_k, d_l depending on invariants and masses

$\Rightarrow$ Every one-loop amplitude can be reduced to A_0, B_0, C_0 and D_0.

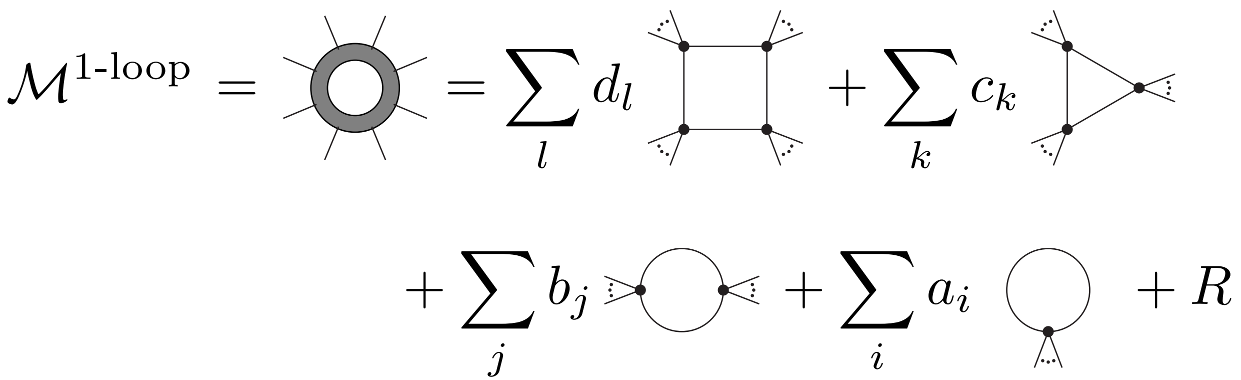
Further aspects of one-loop calculations

- (dimensional) regularization:
  needed to deal with divergent expressions
  $\Rightarrow$ rational terms = finite polynomial remnants of regularization
- treatment of ultraviolet singularities:
  renormalization, counterterms $\Rightarrow$ finite one-loop contribution
- treatment of infrared singularities:
  subtraction of singularities (universal structure)
  combination of singular terms with real corrections $\Rightarrow$ analytic cancellation
- basic scalar integrals A_0, B_0, C_0, D_0 available in form of libraries
  Ellis, Zanderighi '07, van Hameren '10, Cullen et al. '11, Denner, Dittmaier, Hofer
- numerical stability
  - dedicated reduction methods near critical phase-space points
    Denner, Dittmaier '02, '05, R.K. Ellis et al. '05, Binoth et al '05, Diakonidis et al. '08, '09
  - use of quadruple (and higher) precision

Generalized unitarity techniques

Unitarity-inspired methods: idea

- Starting point: In $4+\epsilon$ dimensions, any one-loop amplitude can be represented by a linear combination of scalar one-loop integrals



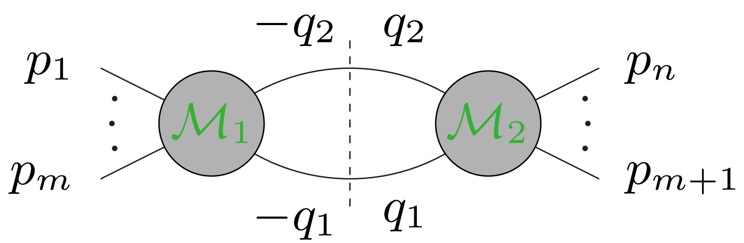
$$= \sum_l d_l D_0(l) + \sum_k c_k C_0(k) + \sum_j b_j B_0(j) + \sum_i a_i A_0(i) + R$$

- R: rational terms = finite terms resulting from dimensional regularization $(D-4) \times 1/(D-4)$ terms
- calculation of amplitude $\Leftrightarrow$ determination of coefficients a_i, b_j, c_k, d_l and R
- coefficients a_i, b_j, c_k, d_l and R can be computed using (generalized) unitarity techniques

Unitarity cut

- Cut separates a one-loop diagram into two tree diagrams
- discontinuity can be computed by replacing cut-propagators by δ-functions under the loop integral   Cutcosky '60

$$\frac{\mathrm{i}}{q_1^2 - m_1^2} \to 2\pi\delta^+(q_1^2 - m_1^2), \qquad \frac{\mathrm{i}}{q_2^2 - m_2^2} \to 2\pi\delta^+(q_2^2 - m_2^2)$$



$$-\mathrm{i}\,\mathrm{Disc}\,\mathcal{M}^{\text{1-loop}}(p_1,\ldots,p_n) = \int [\mathrm{d}q]\, 2\pi\delta^+(q_1^2 - m_1^2)\, 2\pi\delta^+(q_2^2 - m_2^2)$$

$$\times\, \mathcal{M}_1^{\text{tree}}(p_1,\ldots,p_m,q_1,-q_2) \times \mathcal{M}_2^{\text{tree}}(p_{m+1},\ldots,p_n,-q_1,q_2)$$

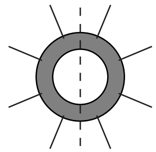
- relates one-loop amplitudes to products of tree amplitudes
- cut simplifies calculation loop integral

shorthands: $\delta^+(q^2 - m^2) = \delta(q^2 - m^2)\theta(q_0) \qquad [\mathrm{d}q] = \frac{\mathrm{d}^4 q}{(2\pi)^4}$

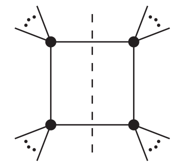
(Conventional) unitarity

- Conventional unitarity based on *double* cuts

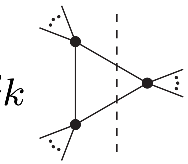
$$-\mathrm{i}\,\mathrm{Disc}\,\mathcal{M}^{\text{1-loop}} =$$



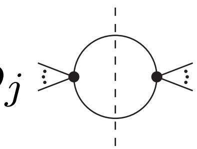
$$= \sum_l d_l$$



$$+ \sum_k c_k$$



$$+ \sum_j b_j$$



- compute discontinuity on both sides of equation
  discontinuities of basic scalar integrals known
  and characteristic for corresponding integrals

- reconstruct the coefficients from the cuts   Bern, Dixon, Dunbar, Kosower ’94
  need to consider all possible cuts in general

- problems:
  - same coefficient can contribute to different cuts
    $\Rightarrow$ problem of disentangling the coefficients
  - R and a_i cannot be determined from double cuts
    (a_i do not appear in massless theories)

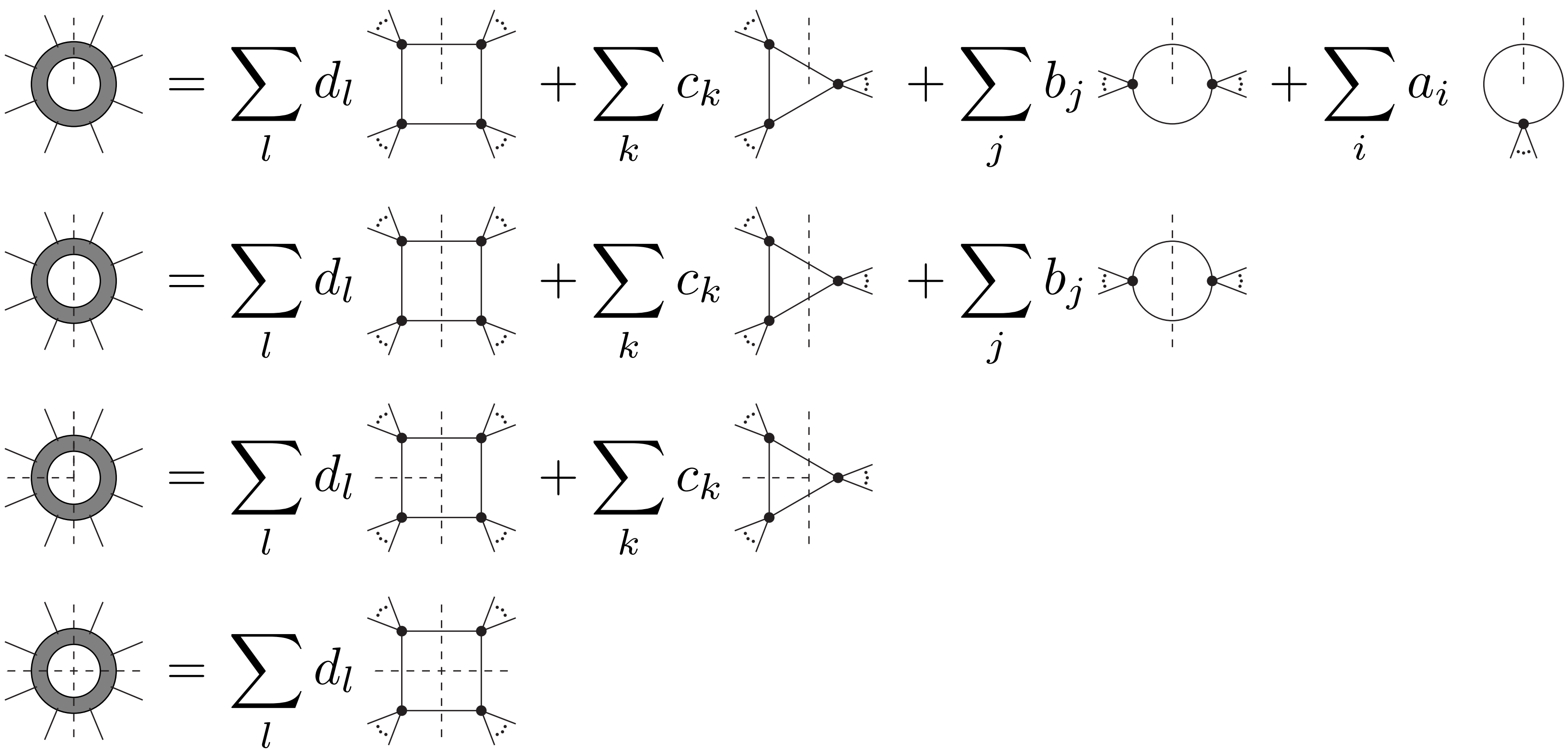
- virtue: one can directly work with amplitudes (no Feynman diagrams)

Generalized unitarity

- a unitarity cut is the replacement $\dfrac{\mathrm{i}}{q^2 - m^2} \to 2\pi\delta^+(q^2 - m^2)$

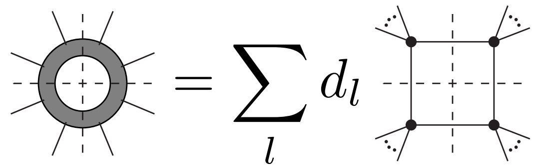
  cutting a line puts its momentum on-shell

- can apply more or less than two cuts $\Rightarrow$ multiple cuts



$\Rightarrow$ efficiently determine the coefficients of scalar integrals

Box coefficients and quadruple cuts



Britto, Cachazo, Feng ’05

- in a quadruple cut, 4 propagators are replaced by δ-functions

$$\frac{1}{q_1^2-m_1^2}\frac{1}{q_2^2-m_2^2}\frac{1}{q_3^2-m_3^2}\frac{1}{q_4^2-m_4^2}\to\delta(q_1^2-m_1^2)\delta(q_2^2-m_2^2)\delta(q_3^2-m_3^2)\delta(q_4^2-m_4^2)$$

$q_i = q + K_i, \quad q_i^2 = m_i^2,\ i = 1,\dots,4 \Rightarrow$ 4 constraints
$\Rightarrow$ cut momenta uniquely determined (in 4 dimensions): 2 solutions $q_\pm$
cut momenta $q_\pm$ are complex

- integrand of quadruple cut: product of 4 tree-level matrix elements

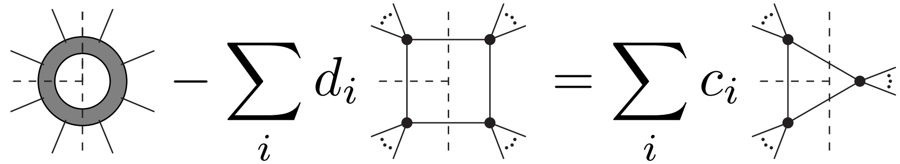
$$I_4(q) = \mathcal{M}_1^{\text{tree}} \times \mathcal{M}_2^{\text{tree}} \times \mathcal{M}_3^{\text{tree}} \times \mathcal{M}_4^{\text{tree}}$$

corresponding coefficient

$$d = \frac{1}{2}[I_4(q_+) + I_4(q_-)]$$

Other coefficients

- subtract contributions of boxes



- cut momentum for triangles depends on one free parameter
  triangle coefficients can be extracted by suitable projection methods
  Ossola, Papadopoulos, Pittau '06; Forde '08

- coefficients of bubbles and tadpoles can be calculated similarly

- rational part cannot be obtained from cuts in 4 dimensions
  - needs D-dimensional unitarity cuts   Ellis, Giele, Kunszt, Melnikov '08
  - conventional reduction methods   Binoth, Guillet, Heinrich '06; Xiao, Yang, Zhu '06
  - Feynman rules for rational terms   Draggiotis, Garzelli, Papadopoulos, Pittau '09
  - on-shell recursion relations   Bern, Dixon, Kosower '05

Implementations

- OPP: Ossola, Papadopoulos, Pittau, . . . reduction at integrand level
  4-dimensional unitarity and counterterms for rational parts
- *Blackhat*: Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre
  4-dimensional unitarity and various additional techniques
  rational parts calculated via on-shell recursion relations
- *Rocket*: Ellis, Kunszt, Melnikov, Giele, Zanderighi
- Lazopoulos; Giele, Winter
- *Samurai*: Mastrolia, Ossola, Reiter, Tramontano
- *NGluon*: Badger, Biedermann, Uwer, Yundin

(Rocket, Lazopoulos; Giele, Winter, Samurai, NGluon:) D-dimensional unitarity

treatment of colour:

- unlike in Feynman-diagram approach no analytic treatment possible
- split amplitude in colour-ordered amplitudes $\Rightarrow$ each one must be evaluated separately
  (natural for pure gluon amplitudes, complicated for quark and colourless particles)
- coloured amplitudes in colour-flow representation $\Rightarrow$ each one must be evaluated separately

treatment of helicity: separate calculation of helicity amplitudes

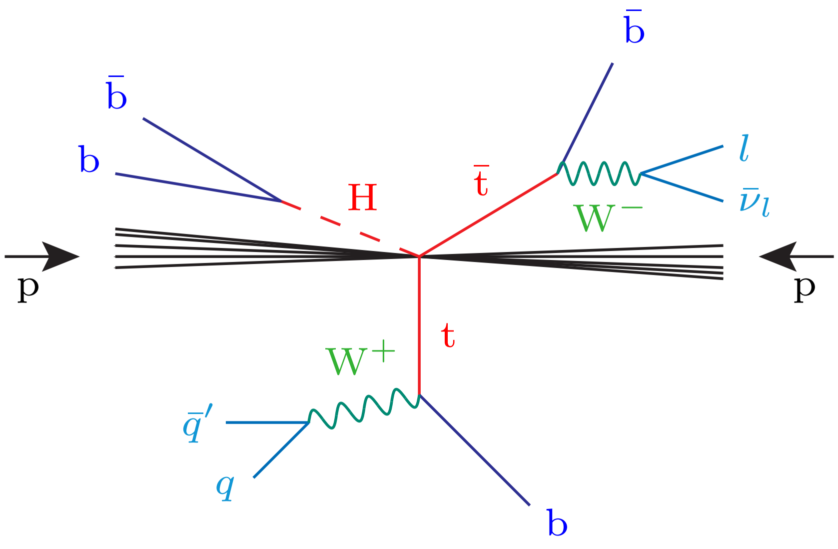
$\Rightarrow$ often Monte Carlo over colours and/or helicities

An example process

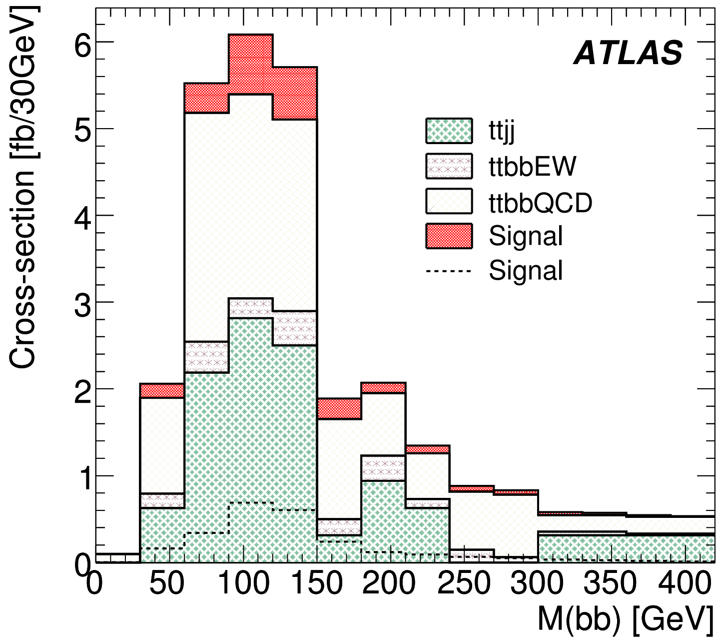
$$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$$

Example: $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

Background to $\mathrm{pp} \to \mathrm{t\bar{t}H}(\to \mathrm{b\bar{b}})$



"CSC book", CERN-OPEN-2008-020



- relevance: direct experimental access to $\mathrm{t\bar{t}H}$-Yukawa-coupling
- problem: control of background via $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}, \mathrm{t\bar{t}} + \mathrm{jets}$

  need: ▶ improved analysis methods (fat jets, boosted Higgs)

  ▶ NLO predictions for background processes

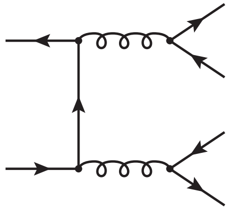
NLO QCD corrections for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

First complete NLO calculation for a $2 \to 4$ hadron-collider process

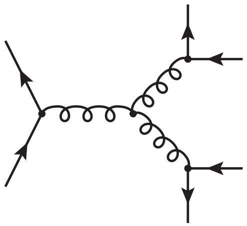
$q\bar{q} \to \mathrm{t\bar{t}b\bar{b}}$ 5% of cross section

Bredenstein, Denner, Dittmaier, Pozzorini '08

LO: 7 diagrams

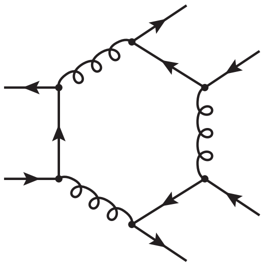


7 trees

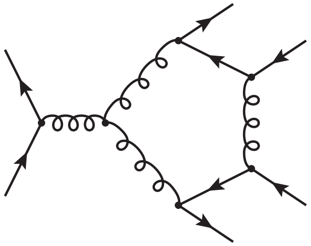


7 trees

NLO: 188 diagrams



8 hexagons



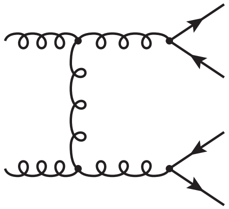
24 pentagons

bremsstrahlung diagrams: 64

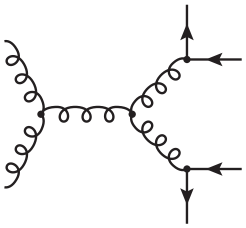
$\mathrm{gg} \to \mathrm{t\bar{t}b\bar{b}}$ 95% of cross section

Bredenstein, Denner, Dittmaier, Pozzorini '09

LO: 36 diagrams

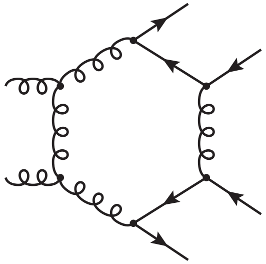


36 trees

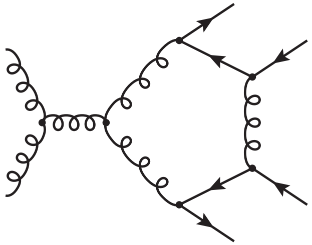


36 trees

NLO: 1003 diagrams



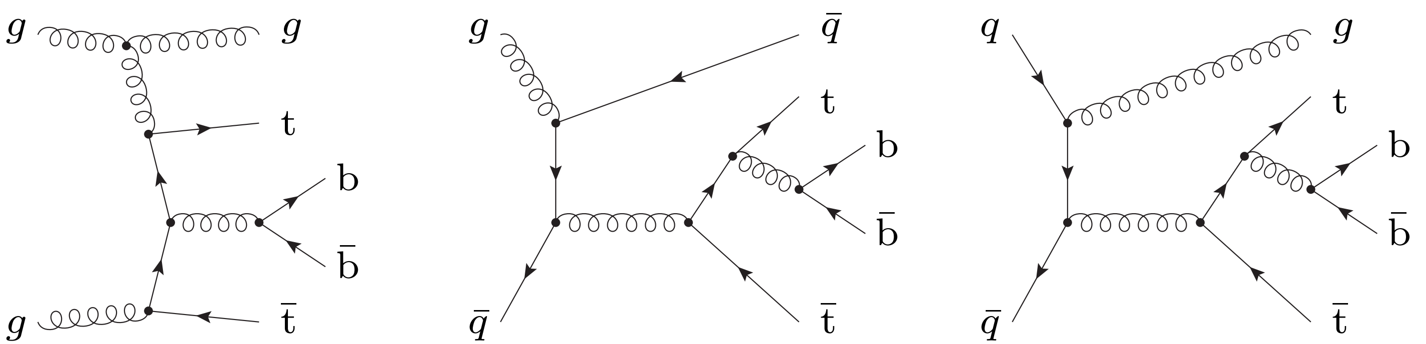
40 hexagons



114 pentagons

bremsstrahlung diagrams: 341

Corrections from real radiation: $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}j}$

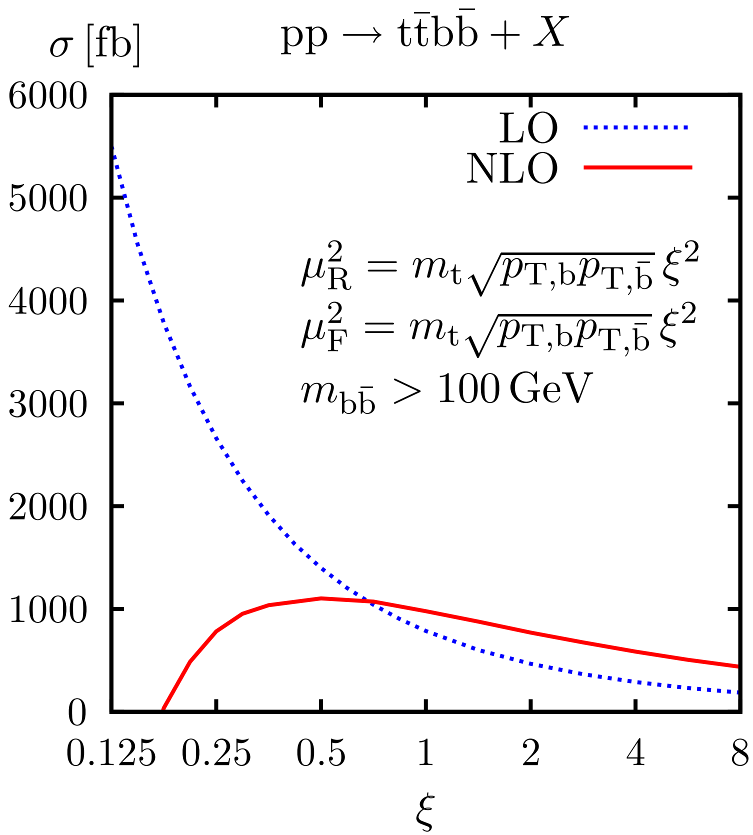


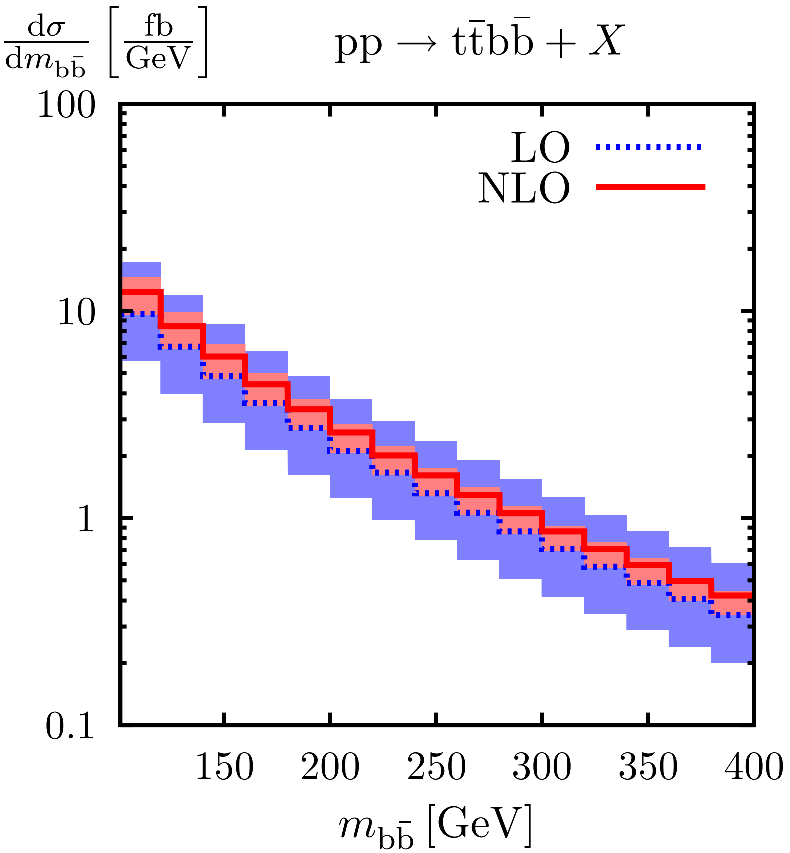
etc.

- channels: $\mathrm{gg} \to \mathrm{b\bar{b}t\bar{t}g}$, $q\mathrm{g} \to \mathrm{b\bar{b}t\bar{t}}q$, $\bar{q}\mathrm{g} \to \mathrm{b\bar{b}t\bar{t}}\bar{q}$, $q\bar{q} \to \mathrm{b\bar{b}t\bar{t}g}$
- numerical (Monte Carlo) integration over 11-dimensional phase space
- fast calculation of amplitudes (bremsstrahlung and LO) needed
- treatment of soft and collinear singularities via subtraction method
  $\Rightarrow$ 30 ($= 6 \times 5$) dipole subtraction terms per channel
  $\Rightarrow$ LO matrix element has to be calculated 30 times for each event
- run times: $\sim 50\,$h on single CPU for 10^7 events (2009)
  $\Rightarrow$ 0.5% accuracy for total integrated cross section

Exemplary results for $pp \to t\bar{t}b\bar{b}$

Bredenstein, Denner, Dittmaier, Pozzorini ’09





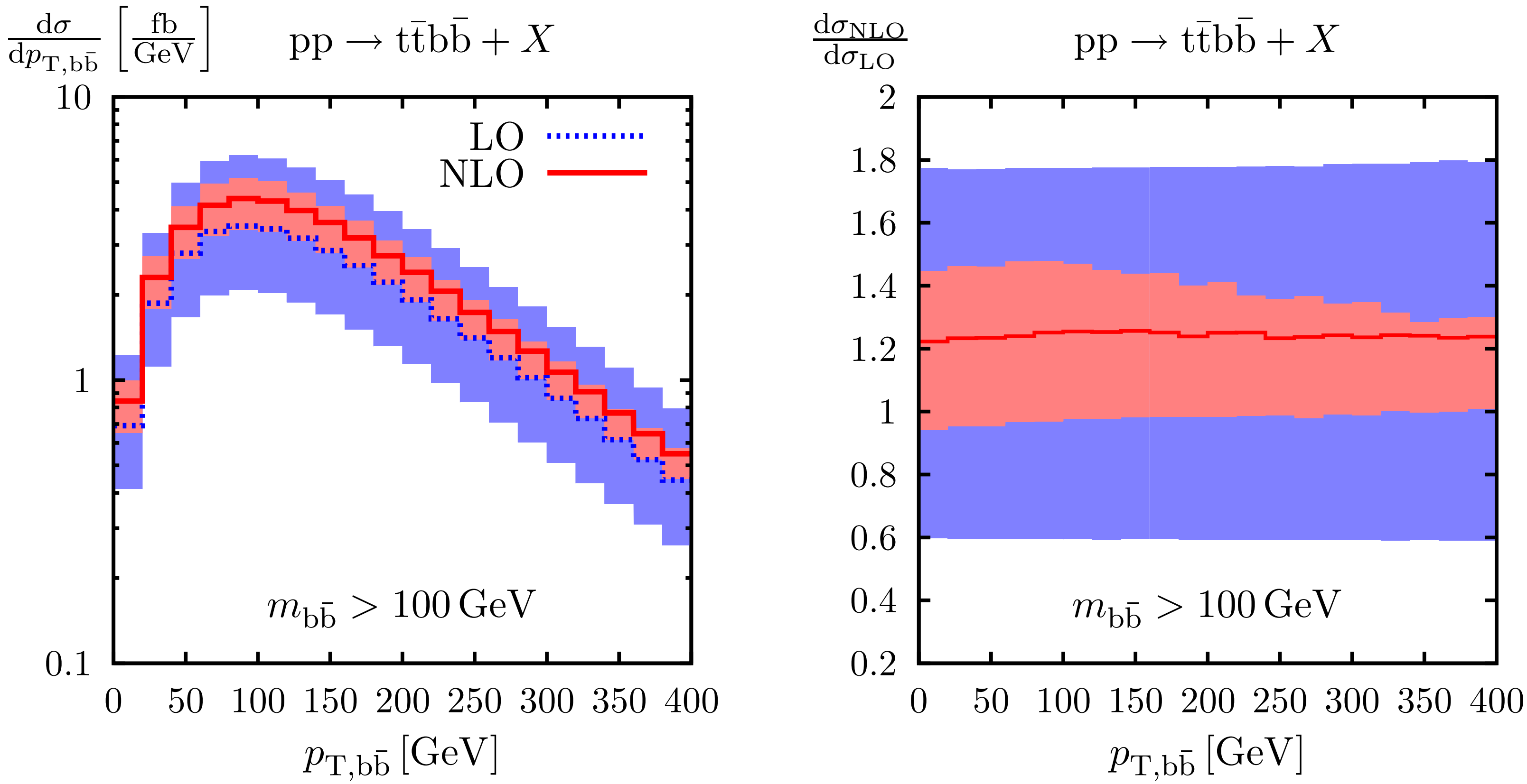
- small NLO correction $K \simeq 1.24$
- reduction of scale uncertainty

$$\Delta_{\text{LO}} \sim 100\% \quad \rightarrow \quad \Delta_{\text{NLO}} \sim 20\text{–}30\%$$

Exemplary results for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

Distribution in transverse momentum of $\mathrm{b\bar{b}}$ pair

Bredenstein, Denner, Dittmaier, Pozzorini ’09



- K-factor almost constant over wide $p_{\mathrm{T,b\bar{b}}}$ range for dynamical scale
- NLO-analysis enables suitable dynamic scale choice (depends on observable)
  $\Rightarrow$ improvement of LO prediction via rescaling

Conclusion
Lecture 1

Conclusion for Lecture 1

- NLO QCD corrections are crucial for LHC processes
- Calculational techniques for NLO QCD corrections well established
  - conventional Feynman-diagram techniques
  - generalized unitarity techniques
  - recursion-relation techniques (see lecture 2)
- Automatized tools for NLO QCD corrections exist
  - MadLoop Hirschi et al. and aMC@NLO Frederix et al.
  - Openloops Cascioli et al. and Sherpa Hoeche et al.
  - HELAC-1LOOP and HELAC-NLO Bevilacqua et al.
  - Gosam Cullen et al.
  - ...

Calculation of NLO QCD corrections considered solved

Les Houches wishlist 2013:
⇒ NNLO QCD and NLO EW corrections for various processes