Matrix element/NLO calculations

Ansgar Denner, Würzburg

Terascale Monte Carlo School 2014

DESY, Hamburg, March 10–14, 2014

- Lecture 1: Relevance and calculation of NLO QCD corrections
- Lecture 2: Relevance and calculation of NLO electroweak corrections
- Lecture 3: NLO Calculations for Higgs Physics

Relevance and calculation of NLO electroweak correctcions

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- Relevance of electroweak corrections
- RECOLA a generator for electroweak one-loop amplitudes
- Salient features of electroweak corrections
  photon effects on PDFs, photon-jet separation, unstable particles, . . .

Relevance of electroweak corrections

Situation at LEP

Electroweak (EW) processes

events

$1.5 \times 10^7 \ \ \mathrm{e^+e^-} \to \mathrm{Z} \to \mathrm{hadrons}, \qquad 1.7 \times 10^6 \ \ \mathrm{e^+e^-} \to \mathrm{Z} \to \mathrm{leptons}$

$4.8 \times 10^4 \ \ \mathrm{e^+e^-} \to \mathrm{W^+W^-}$

typical accuracies

$\mathrm{e^+e^-} \to \mathrm{Z} \to \mathrm{hadrons}$: $\Delta\sigma/\sigma \sim 0.09\%$

$\Delta \sin^2 \theta_{\mathrm{eff}}^{\mathrm{lept}} \sim 0.03\%, \qquad \Delta M_{\mathrm{W}}/M_{\mathrm{W}} \sim 0.05\%$

$\mathrm{e^+e^-} \to \mathrm{W^+W^-}$: $\Delta\sigma/\sigma \sim 1\%$

predictions are calculated within perturbation theory

expansion parameter: $\alpha/\pi \sim 0.23\%, \quad \alpha/(\pi \sin^2\theta_{\mathrm{w}}) \sim 1\%$

generic one-loop corrections: $\sim 2\%$

generic two-loop corrections: $\sim 0.05\% \sim (2\%)^2$

implications

complete one-loop electroweak corrections (EWC) required
for $\mathrm{e^+e^-} \to \mathrm{Z}$ (at least leading) two-loop EWC needed

Relevance of electroweak corrections

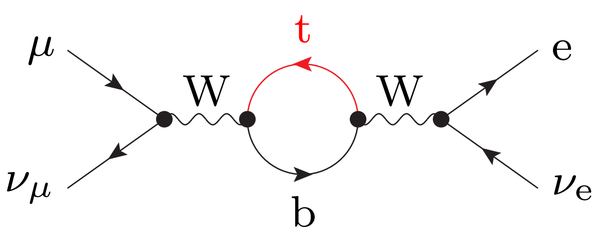
order of magnitude of electroweak corrections (EWC)

$$\sim \left[\frac{\alpha}{\pi} \dots \frac{\alpha}{\pi} \ln \frac{E^2}{m_{\mathrm{e}}^2}\right] \times \mathcal{O}(1) \sim [0.2\% \dots 6\%] \times \mathcal{O}(1) \text{ for } E = M_{\mathrm{Z}} \sim 90\,\mathrm{GeV}$$

$\Rightarrow$ mandatory for precision tests of the electroweak Standard Model (SM)

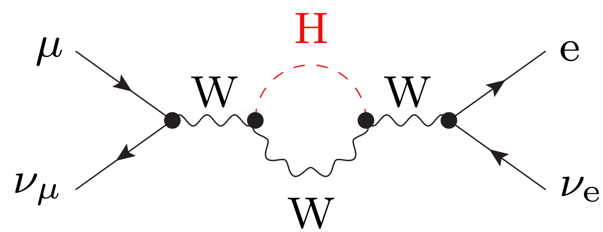
depend on all details of the theory

- top quark



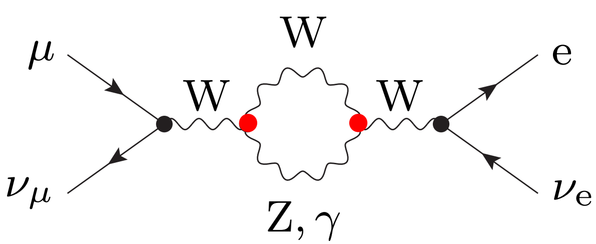
$$\sim \frac{\alpha}{\pi} \frac{m_{\mathrm{t}}^2}{s_{\mathrm{w}}^2 M_{\mathrm{W}}^2} \approx 0.05 \; (m_{\mathrm{t}} = 173\,\mathrm{GeV})$$

- Higgs boson

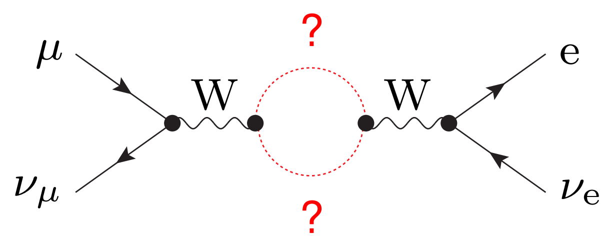


$$\sim \frac{\alpha}{\pi} \ln \frac{M_{\mathrm{H}}}{M_{\mathrm{W}}}$$

- gauge-boson self couplings



- new physics (supersymmetry) ?



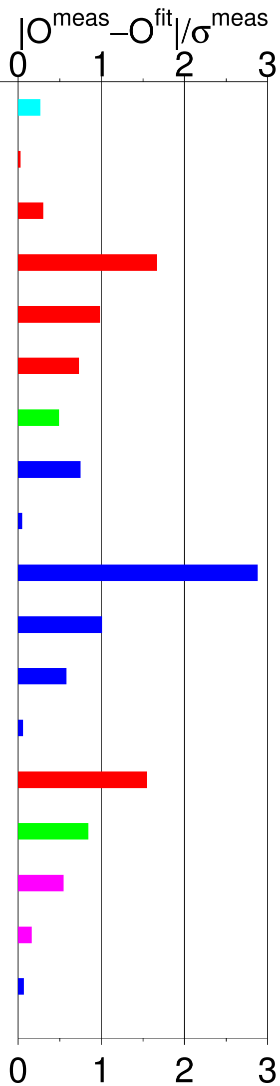
$\Rightarrow$ allow for indirect experimental tests of not directly accessible quantities

test the SM at its quantum level ($\propto \hbar$)

Global fit of LEP data

LEPEWWG ’12 (before Higgs discovery)

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2012

good agreement

best fit for Higgs-boson mass:

$M_H = 101\,^{+31}_{-25}$ GeV

$M_H < 152$ GeV @ 95% CL

direct search at LEP2

$(e^+e^- \not\to ZH)$

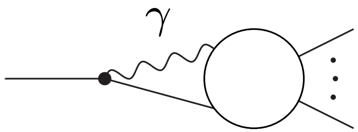
$M_H > 114.4$ GeV

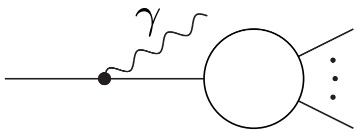
Leading EW corrections at LEP energies

leading electromagnetic logarithms

from ISR and FSR: (10%) QED

real and virtual corrections have to be added



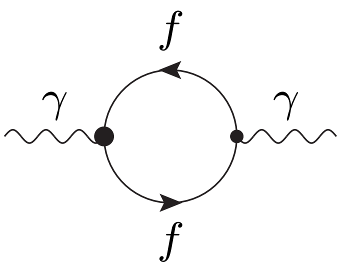


$$\sim \frac{\alpha}{\pi} \ln \frac{E^2}{m_\mathrm{e}^2} \sim 6\% \text{ at } E \sim M_\mathrm{Z}$$

$\Rightarrow$ dedicated codes: ISR structure functions, Photos for FSR

running of α: $\sim 6\%$ "QED"

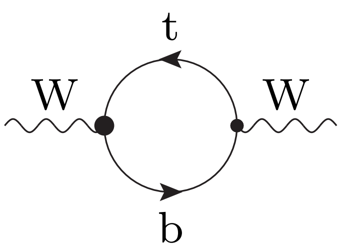
renormalization of electromagnetic coupling $\Rightarrow$ absorb in running α

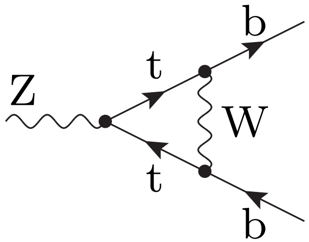


$$\sim \frac{\alpha}{3\pi} \sum_f Q_f^2 \ln \frac{E^2}{m_f^2}, \quad E \sim M_\mathrm{Z}$$

corrections $\propto m_\mathrm{t}^2/M_\mathrm{W}^2$: $\sim 3\%$

renorm. of weak mixing angle, ... $\Rightarrow$ absorb in effective mixing angle, ...

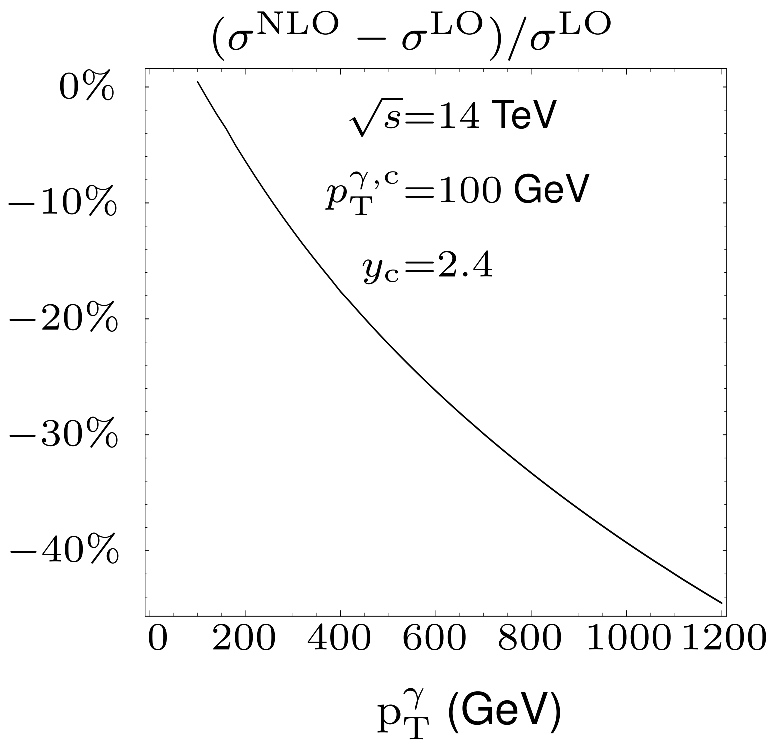




$$\sim \frac{\alpha}{\pi} \frac{m_\mathrm{t}^2}{s_\mathrm{w}^2 M_\mathrm{W}^2} \sim 0.05$$

Relevance EW NLO corrections at LHC

- generically: $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2) \sim \text{few}\%$
- EW corrections can be enhanced by large energy or kinematic effects
  example: electroweak corrections to $\mathrm{pp} \to \mathrm{Z}\gamma$ Hollik, Meier '04



small p_{T}

- corrections of $\mathcal{O}(\alpha) \sim 1\%$

$p_{\mathrm{T}} > 100\,\mathrm{GeV}$

- large negative corrections $\gg 1\%$
- increase with p_{T}
- -40% at $p_{\mathrm{T}} \sim 1\,\mathrm{TeV}$!

- leading NNLO EW corrections might be relevant for some processes
  (Drell-Yan, Z+jet, W+jet) $(40\%)^2 = 16\%$

Origin of large electroweak corrections

Energy scale $\gg$ characteristic scale of EW corrections:

e.g. $E \gg M_\mathrm{W} \approx 80\,\mathrm{GeV}$

$\Rightarrow$ large double logarithms

$$\ln^2\left(\frac{E^2}{M_\mathrm{W}^2}\right) \sim 25 \qquad \text{at} \quad E \sim 1\,\mathrm{TeV}$$

typical size of corrections:

$$\frac{\alpha}{\pi s_\mathrm{w}^2}\ln^2\left(\frac{E^2}{M_\mathrm{W}^2}\right) \sim 25\% \qquad \text{at} \quad E \sim 1\,\mathrm{TeV}$$

general feature of hard scattering processes!

Large EW logarithms can be related to mass singularities:

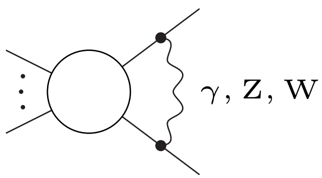
$$M_\mathrm{W}/E \ll 1 \quad \Rightarrow \quad E \to \infty \quad \text{or} \quad M_\mathrm{W} \to 0$$

EW logarithms can be calculated with process-independent methods.

Universal origin of leading EW logarithms

Large EW logarithms are of universal origin:

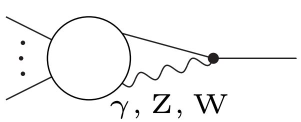
- infrared logarithms ⇔ external particles of the process
  - soft and collinear virtual gauge bosons (cancel in QED/QCD)



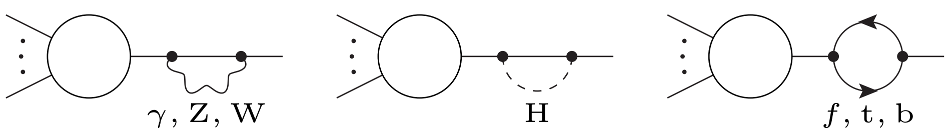
double logarithms $\alpha \ln^2 \frac{E^2}{M_{\mathrm{W}}^2}$

no complete cancellation with real $\mathrm{W/Z}$ radiation (Bloch–Nordsiek violation)

  - collinear or soft virtual gauge bosons, wave-function renormalization (ISR/FSR in QED)



single logarithms $\alpha \ln \frac{E^2}{M_{\mathrm{W}}^2}$



- ultraviolet logarithms ⇔ parameter renormalization at scale $M_{\mathrm{W}} \ll E$
  ⇒ running of electroweak couplings from M_{W} to E

  single logarithms $\alpha \ln \frac{E^2}{M_{\mathrm{W}}^2}$

⇒ (relatively) simple expressions for logarithmic corrections

Summary on electroweak logarithms

- studied by many people

  M. Ciafaloni, P. Ciafaloni, Comelli; Beccaria, Renard, Verzegnassi; Beenakker, Werthenbach; Denner, Pozzorini; Melles; Fadin, Lipatov, Martin; Hori, Kawamura, Kodaira; Jantzen, Kühn, Penin, Smirnov; Chiu, Fuhrer, Golf, Kelley, Manohar, ...

- results are valid in Sudakov regime:
  $s_{ij} = 2k_ik_j \gg M_\mathrm{W}^2$ *for all momenta* $k_i \neq k_j$
- Sudakov logarithms at NLO
  - known for arbitrary processes (not mass suppressed at LO)
  - provide simple estimate for one-loop corrections at level of 5–10%
  - non-logarithimc corrections not included
  - photonic radiation effects needed in addition
- Sudakov logarithms beyond NLO
  - resummation methods exist for higher-order EW logarithms
  - useful to estimate electroweak two-loop corrections
  - useful to estimate theoretical uncertainties
  - typically cancellations between leading and subleading logarithms

EWC beyond Sudakov logarithms

- at LHC often sizeable contributions from energies below $1\,\mathrm{TeV}$
- Sudakov regime not always relevant
  - Sudakov approximation not applicable to processes with resonances ($s_{\mathrm{res}} \not\gg M_{\mathrm{W}}^2$)
  - Sudakov approximation not applicable to processes dominated by t-channel diagrams ($t \not\gg M_{\mathrm{W}}^2$)
    $\Rightarrow$ potentially large logarithms of the form $\log(t/s) \sim 2\log\theta$
  - relevance depends on observable
    e.g. Drell-Yan at LHC: large $p_{\mathrm{T,l}}$ probes Sudakov regime
    large M_{ll} receives sizeable contributions from small t Dittmaier et al. '10
- real corrections should be included
  - real photon radiation $\Rightarrow$ large effects
  - real massive vector-boson radiation Baur '06, Bell et al. '10

  $\Rightarrow$ partial cancellation of enhanced logarithmic corrections
  strongly dependent on W/Z reconstruction and separation

$\Rightarrow$ exact calculations of NLO EWC preferable if possible

RECOLA

a generator for electroweak one-loop amplitudes

Calculation of NLO matrix elements

General form of one-loop amplitudes (free of unphysical singularities)

$$\delta\mathcal{M}^{\text{1-loop}} = \sum_j \sum_{R_j} c^{(j,R_j,N_j)}_{\mu_1\cdots\mu_{R_j}} T^{\mu_1\cdots\mu_{R_j}}_{(j,R_j,N_j)} = \sum_j d^{(j,N_j)} T_{(j,0,N_j)}$$

tensor integrals

$$T^{\mu_1\cdots\mu_{R_j}}_{(j,R_j,N_j)} = \frac{(2\pi\mu)^{4-D}}{\mathrm{i}\pi^2}\int \mathrm{d}^D q \, \frac{q^{\mu_1}\cdots q^{\mu_{R_j}}}{D_{j,0}\cdots D_{j,N_j-1}}, \qquad D_{j,a} = (q+p_{j,a})^2 - m^2_{j,a}$$

tensor coefficients

$c^{(j,R_j,N_j)}_{\mu_1\cdots\mu_{R_j}}$ free of unphysical singularities, $d^{(j,N_j)}$ involve unphysical singularities

proposal of van Hameren '09:
calculate $c^{(j,R_j,N_j)}_{\mu_1\cdots\mu_{R_j}}$ numerically in a recursive way

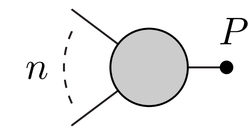
implemented for full Standard Model in RECOLA — Actis, Denner, Hofer, Scharf, Uccirati
(Recursive computation of one-loop amplitudes)

evaluation of tensor integrals by COLLIER — Denner, Dittmaier, Hofer, in preparation
(Complex one loop library in extended regularizations)

Off-shell currents at tree level

Basic building blocks of tree-level recursion:

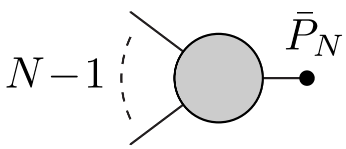
off-shell current of particle P with n external legs

$w(P, \mathcal{C}, \{l_1, \ldots, l_n\}) \quad =$ 

- w is a scalar, spinor or vector corresponding to P
- $\mathcal{C}$ represents the colour
- $\{l_1, \ldots, l_n\}$ list of primary external legs
- off-shell currents for external legs are wave functions

 $= u_\lambda(p)$,  $= \bar{u}_\lambda(p)$,  $= \epsilon_\lambda(p)$,  $= 1$

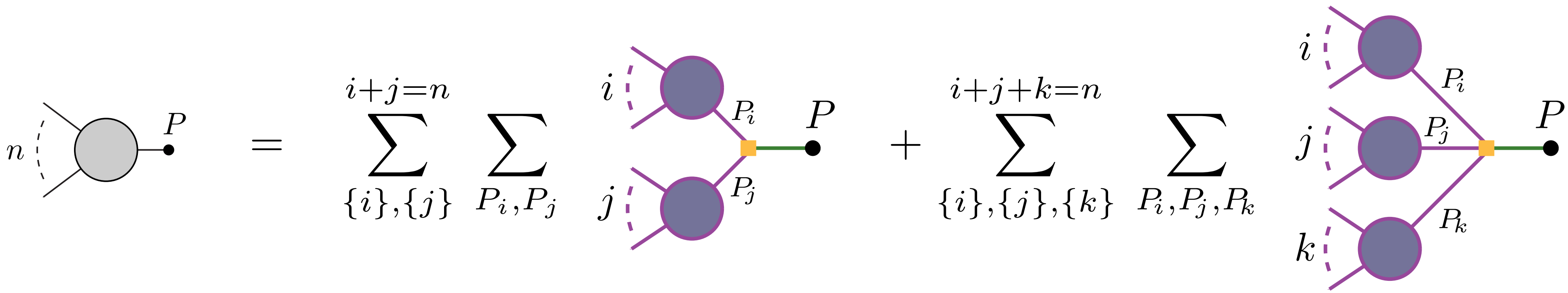
Amplitude for process with N external particles:

$\mathcal{M} \quad =$  $\times$ (propagator of $\bar{P}_N)^{-1} \times$  $\bar{P}_N$

amputate off-shell line and multiply with wave function

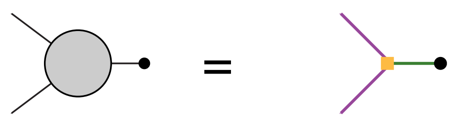
Tree-level recursion

Recursion relation



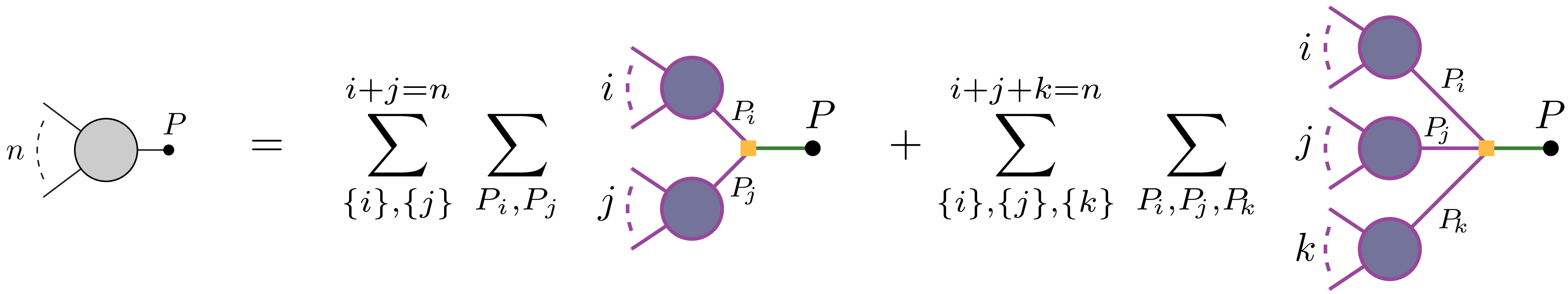
incoming currents × **vertex** × **propagator**

2-leg currents:



Tree-level recursion

Recursion relation



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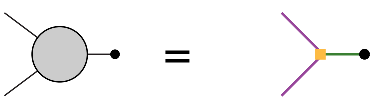
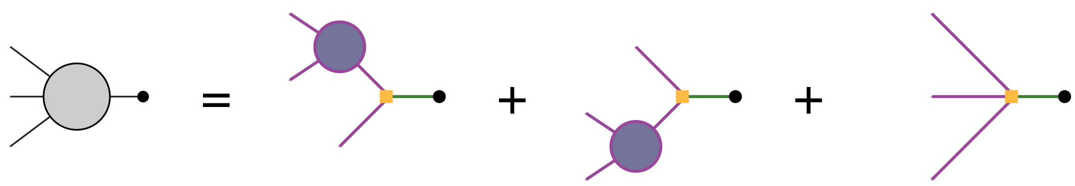
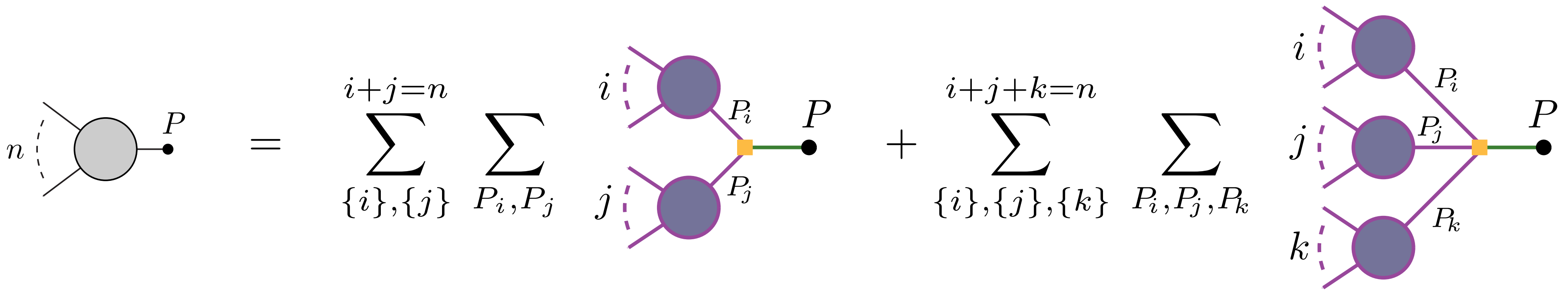


3-leg currents:



Tree-level recursion

Recursion relation



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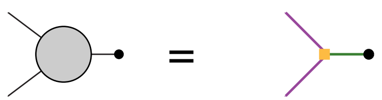

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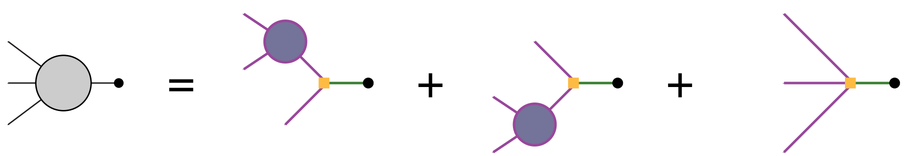
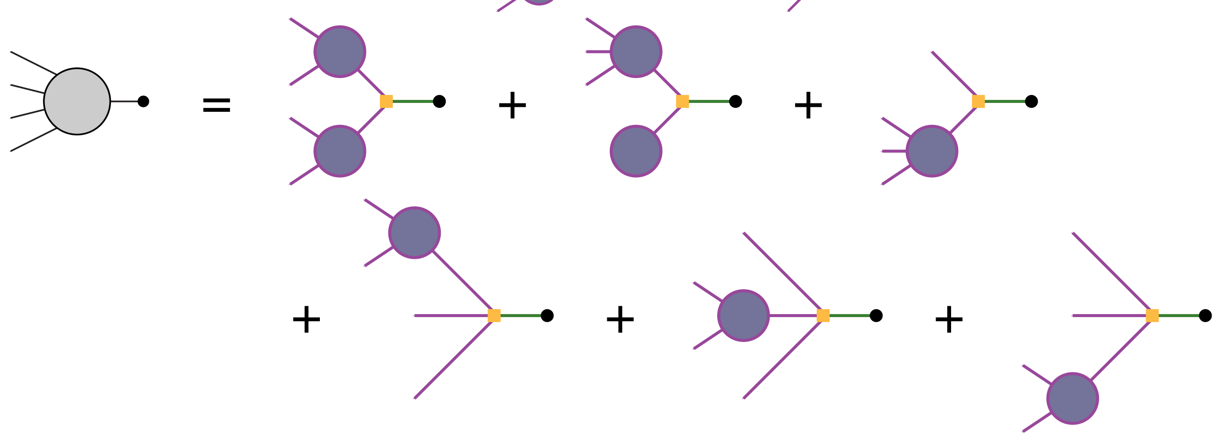
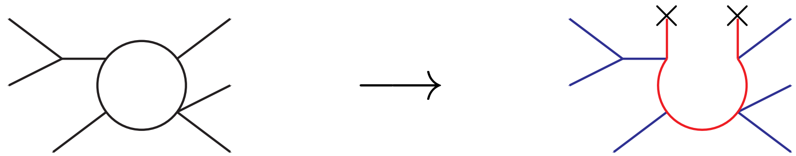


4-leg currents:



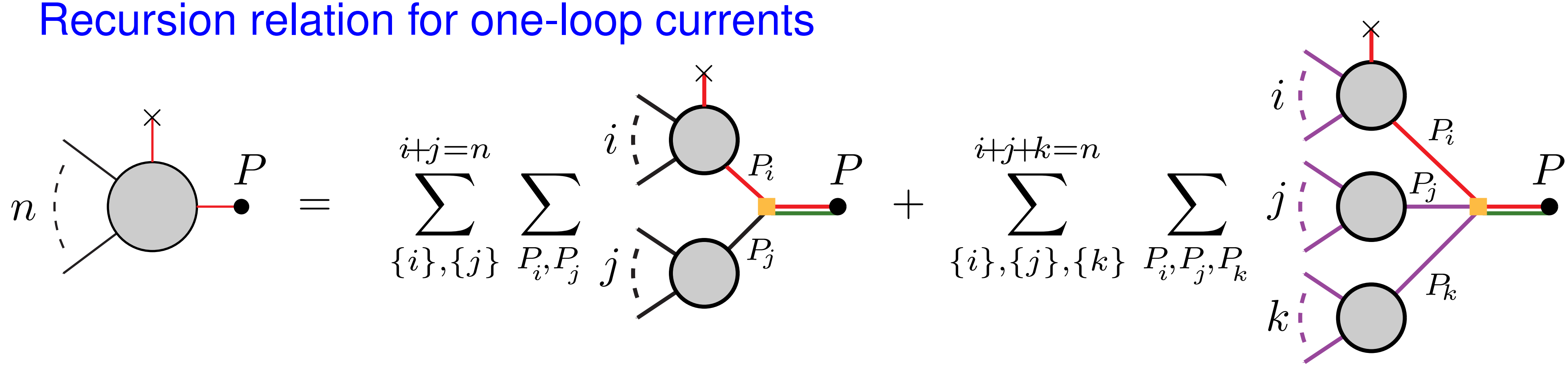
Recursion at one loop

Cut loop line and consider tree diagrams with two more legs



relation can be defined uniquely

Recursion relation for one-loop currents



$$(\textbf{vertex}) \times (\textbf{propagator}) = \frac{a^\mu q_\mu + b}{(q+p)^2 - m^2}$$

$q =$ loop momentum

$$\textbf{loop current} = \sum_{r=0}^{k} a^{(k,r)}_{\mu_1\cdots\mu_r} \frac{q^{\mu_1}\cdots q^{\mu_r}}{\prod_{h=1}^{k}[(q+p_h)^2 - m_h^2]}$$

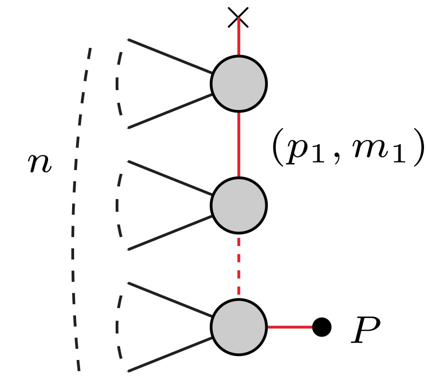
computed recursively

goes in TIs

One-loop off-shell currents

Basic building blocks for one-loop recursion:
one-loop off-shell currents

$$w_{i_k}(P, \mathcal{C}, \{l_1, \ldots, l_n\}, \{p_1, \ldots, p_k\}, \{m_1, \ldots, m_k\}) =$$



- $\{p_1, \ldots, p_k\}$, $\{m_1, \ldots, m_k\}$: sequence of momenta and masses in loop propagators
- i_k multi-index representing k, r and $\mu_1, \ldots, \mu_r$: $w_{i_k} = a^{(k,r)}_{\mu_1 \cdots \mu_r}$
- currents for the tree lines are the same as at tree level
- suitable wave functions for first and last loop line:

  i ×—▶—• $= \psi_i,$ i ×—◀—• $= \bar{\psi}_i,$ i ×∿∿• $= \epsilon_i,$ i × - - • $= 1$

  cutted lines are reconnected via polarization sums

$$\sum_{i=1}^{4} (\bar{\psi}_i)_\alpha (\psi_i)_\beta = \delta_{\alpha\beta}, \qquad \sum_{i=1}^{4} \epsilon_i^\mu \epsilon_i^\nu = \delta^{\mu\nu},$$

- coefficients $a^{(k,r)}_{\mu_1 \cdots \mu_r}$ of the last current equal tensor coefficients $c^{(j,R_j,N_j)}_{\mu_1 \cdots \mu_{R_j}}$

Treatment of colour

Colour-flow representation

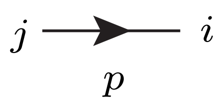
Maltoni, Paul, Stelzer, Willenbrock ’02

Gluon field : $\sqrt{2}\,A^a_\mu\,(\lambda^a)^i_j = (\mathcal{A}_\mu)^i_j$

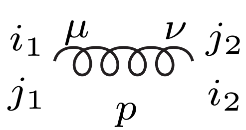
“usual” gluon with colour index $a = 1,\ldots,8$

gluon with colour-flow ${}^i_j$ $i,j = 1,2,3$

Feynman rules



$$= \delta^i_j \times \frac{\mathrm{i}(\not{p}+m)}{p^2-m^2}$$



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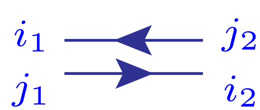
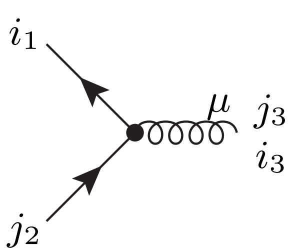
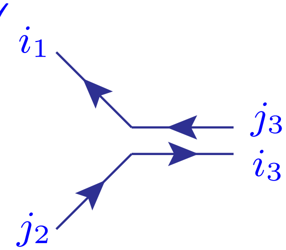


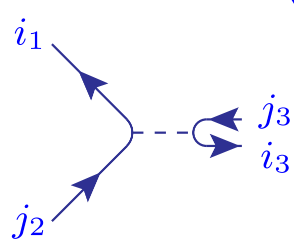
$$\times \frac{-\mathrm{i}\,g_{\mu\nu}}{p^2} = \delta^{i_1}_{j_2}\delta^{i_2}_{j_1} \times \frac{-\mathrm{i}\,g_{\mu\nu}}{p^2}$$



$= \Big($



$- \dfrac{1}{N_\mathrm{c}}$



$$\Big) \times \frac{\mathrm{i}\,g_\mathrm{s}}{\sqrt{2}}\gamma^\mu$$

$$= \left(\delta^{i_1}_{j_3}\delta^{i_3}_{j_2} - \frac{1}{N_\mathrm{c}}\delta^{i_1}_{j_2}\delta^{i_3}_{j_3}\right) \times \frac{\mathrm{i}\,g_\mathrm{s}}{\sqrt{2}}\gamma^\mu$$

Treatment of colour

Structure of the amplitude

$$\mathcal{A}^{i_1,\dots,i_n}_{j_1,\dots,j_n} = \sum_{P(j_1,\dots,j_n)} \delta^{i_1}_{j_1}\cdots\delta^{i_n}_{j_n}\,\mathcal{A}_P,$$

- Colour-dressed amplitudes:
  $\Rightarrow$ compute $\mathcal{A}^{i_1,\dots,i_n}_{j_1,\dots,j_n}$ for all possible colours (N_c^{2n})

  squared amplitude: $$\mathcal{M}^2 = \sum_{i_1\dots i_n,j_1,\dots,j_n} (\mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n})^*\mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n}$$

  requires colour-dressed off-shell currents

- Structure-dressed amplitudes:
  $\Rightarrow$ compute $\mathcal{A}_P$ for all possible P $(n!)$

  squared amplitude: $$\mathcal{M}^2 = \sum_{P,P'} \mathcal{A}^*_P\,C_{PP'}\,\mathcal{A}_{P'},$$

  requires structure-dressed off-shell currents
  efficiently obtained in recursive procedure

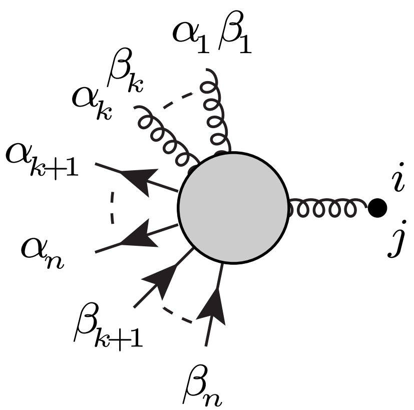
Structure-dressed off-shell currents

Colour structure: product of Kronecker δs
external currents:

β  $i \;=\; u_\lambda(p)\,\delta^i_\beta$ $\qquad$ α  $j \;=\; \bar{u}_\lambda(p)\,\delta^\alpha_j$ $\qquad$ $\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$  $\begin{smallmatrix}i\\ j\end{smallmatrix} \;=\; \epsilon_\lambda(p)\,\delta^i_\beta\delta^\alpha_j$

α, β: colour indices of external particles, i, j "open" colour indices

colour structure of off-shell current:



$\rightarrow$

saturated part: $\delta^{\alpha_1}_{\beta_1} \ldots \delta^{\alpha_{n-1}}_{\beta_{n-1}}$ $\qquad$ open part: $\delta^{\;i}_{\beta_n}\;\delta^{\;\alpha_n}_{\;j}$

$$w(\,\mathcal{P}\;,\;\mathcal{C}\;,\;\{l_1,\ldots l_n\}\;)$$

with all possible permutations of $\beta_1, \ldots \beta_n, j$

recursion procedure:

- saturated parts of incoming currents multiply
- open parts of incoming currents are contracted

optimization: compute currents differing just by the colour structure only once

Performance of RECOLA

- Memory for executables, object files and libraries: negligible
- RAM: less than 2 Gbyte also for complicated processes
- CPU time (processor Intel(R) Core(TM) i5-2400 CPU 3.10 GHz):

Process	t_{TIs} (COLLIER)	t_{gen} t_{TCs} (single helicity)	t_{gen} t_{TCs} (partial hel. sum)	t_{gen} t_{TCs} (helicity sum)
$\mathrm{u\bar{u} \to W^+W^-g}$ (QCD)	2.8 ms	0.3 s 0.6 ms (hel: - + - + -)	0.4 s 1.3 ms (hel: S S - + S)	1.6 s 9.8 ms (hel: S S S S S)
$\mathrm{u\bar{d} \to W^+ g\,g\,g}$ (QCD)	130 ms	14 s 14 ms (hel: - + - - - -)	25 s 76 ms (hel: S S - S S S)	52 s 221 ms (hel: S S S S S S)
$\mathrm{ug \to u\,g\,Z}$ (EW)	8.2 ms	0.5 s 1.4 ms (hel: - - - - -)	1.0 s 6.7 ms (hel: S S S S -)	2.2 s 20.2 ms (hel: S S S S S)
$\mathrm{ug \to u\,g\,l^+l^-}$ (EW)	28 ms	1.3 s 2.5 ms (hel: - - - - - +)	2.0 s 14.2 ms (hel: S S S S - +)	3.8 s 29.0 ms (hel: S S S S S S)

Features of RECOLA

- Full Standard Model (QCD + EW)
  - tree level and one-loop amplitudes
  - Feynman rules for counter terms   Denner '93
  - Feynman rules for rational terms (R_2) Garzelli, Malamos, Pittau '10
  - selection of resonances,   e.g. $qg \to qgZ \to qgl^+l^-$
- complex-mass scheme for unstable particles
- mass- and dimensional regularization supported for IR singularities
- renormalization
  - EW sector: on-shell renormalization
  - α_s: $\overline{\mathrm{MS}}$ renormalization
- selection of powers of α_s at matrix-element or cross-section level
- colour- and spin-correlated amplitudes for dipole subtraction
- needs external library for tensor integrals $\Rightarrow$ COLLIER

Salient features of EWC

Input-parameter scheme

- Natural input parameters: α, M_W, M_Z, m_f, M_H, α_s
- weak mixing angle: on-shell definition $\sin\theta_\mathrm{w} = \sqrt{1 - M_\mathrm{W}^2/M_\mathrm{Z}^2}$
- alternative input parameter sets: G_μ instead of M_W or α
  G_μ no fundamental parameter, but precisely measured in μ decay
- definition of α
  - on-shell: $\alpha(0)$
    appropriate for external photons
  - $\alpha(M_\mathrm{Z})$, $\alpha(\sqrt{s})$: $\frac{\alpha(M_\mathrm{Z})}{\alpha(0)} \approx 1.06$
    absorbs running of α from $Q = 0$ to EW scale
    appropriate for internal photons and weak bosons
  - G_μ scheme: $\alpha_{G_\mu} = \sqrt{2} G_\mu M_\mathrm{W}^2 (1 - M_\mathrm{W}^2/M_\mathrm{Z}^2)/\pi$: $\frac{\alpha_{G_\mu}}{\alpha(0)} \approx 1.03$
    absorbs running of α from $Q = 0$ to EW scale and $\Delta\rho$ in $W f \bar{f}'$ coupling
    appropriate for processes with W bosons (Z bosons)

  suitable choice of α reduces missing higher-order corrections
  effects can amount to several 10% for high powers of α

gauge invariance demands unique input-parameter set! [$\alpha(0)^n \alpha_{G_\mu}^m$ possible]

QED corrected PDFs

Consistent treatment of EW corrections $\Rightarrow$ QED corrected parton distributions analogous to QCD-improved parton model

- photon as parton in hadrons
- collinear photon emission from initial-state quarks and collinear photon splitting $\gamma \to q\bar{q}$
  $\Rightarrow$ mass singularities in EW corrections
  $\Rightarrow$ absorb into quark and photon distribution functions Dittmaier, Huber '10
  $\Rightarrow$ dependence on factorization scale $\mu_{\mathrm{fact,QED}}$
- usual form of DGLAP equations for one scale $\Rightarrow$ $\mu_{\mathrm{fact,QED}} = \mu_{\mathrm{fact,QCD}}$

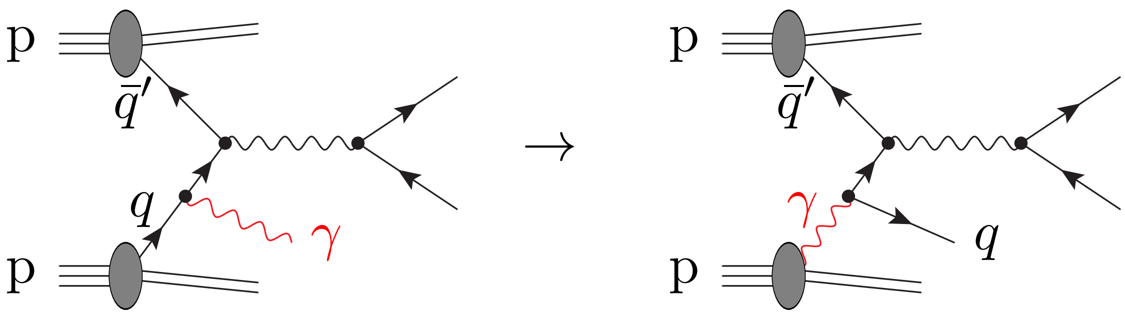
PDFs including QED corrections:

- MRST2004QED Martin et al. '04, only set until 2013, outdated
- NNPDF2.3QED Ball et al. '13
  $\Rightarrow$ currently best PDF set (N)NLO QCD + NLO EW
- DIS factorization scheme should be used for EW NLO corrections
- QED corrections $\lesssim 0.1\%$ for $x < 0.1$, per-cent effects for $x > 0.1$

Photon-induced processes

Photons as partons $\Rightarrow$ photon-induced processes

- result from crossing of photonic bremsstrahlung corrections
  (in all qq, $q\bar{q}$, $\bar{q}\bar{q}$ initiated processes)



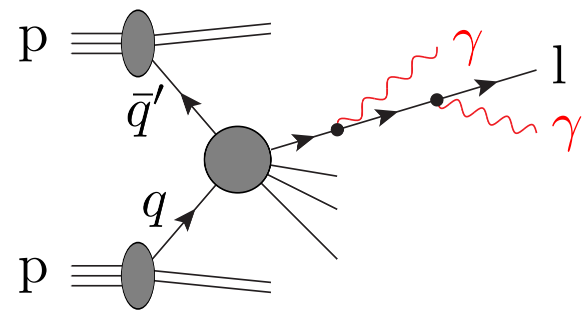
- LO contribution from $\gamma\gamma$ for production of charged particles in gg, $q\bar{q}$ channels (e.g. in $\mu^+\mu^-$, $\mathrm{W^+W^-}$ production)

relevance

- $q\gamma$ contributes at per cent level if $q\mathrm{g}$ channels exist
  (gluon PDFs are larger than photon PDFs)
- more significant contributions if no QCD counterparts exist
  e.g. $\gamma\gamma$ contributions up to 10% in certain regions of phase space for
  $\gamma\gamma \to \mathrm{l^+l^-}$ Dittmaier, Huber '10; Carloni Calame et al. '07, Boughezal, Li, Petriello '13
  $\gamma\gamma \to \mathrm{W^+W^-}$ Bierweiler et al. '12; Baglio, Ninh, Weber '13; Billoni et al. '13

Collinear final-state radiation off leptons

Soft/collinear photon emission
from initial/final-state leptons
$\Rightarrow$ logarithmically enhanced corrections
$\propto \alpha^n \ln^n(m_l^2/Q^2)$
$\Rightarrow$ resummation needed



- structure-function approach — Beenakker et al. '96, Arbuzov '99

$$\sigma_{\mathrm{LL,FSR}} = \int \underbrace{\mathrm{d}\sigma^{\mathrm{LO}}(k_l)}_{\text{hard scattering}} \int_0^1 \mathrm{d}z \underbrace{\Gamma_{\mathrm{ll}}^{\mathrm{LL}}(z,Q^2)}_{\text{structure function}} \theta_{\mathrm{cut}}(zk_l)$$

$\mathcal{O}(\alpha)$ approximation

$$\Gamma_{\mathrm{ll}}^{\mathrm{LL},1}(z,Q^2) = \frac{\alpha Q_l^2}{2\pi}\left[\ln\left(\frac{Q^2}{m_l^2}\right)-1\right]\left(\frac{1+z^2}{1-z}\right)_+$$

neglects transverse momenta, Q factorization scale

- dedicated photonic parton showers, e.g. PHOTOS
  Placzek, Jadach '03; Carloni Calame et al. '04; Golonka, Was '07

Treatment of final-state photon radiation

Final-state photon radiation off leptons:
2 options

- "calorimetric/dressed leptons" (typical for electrons)
  recombination of leptons with (soft/collinear) photons (inclusive treatment)
  $\Rightarrow$ IR-safe observables, no logarithmic enhancement

$$\int \mathrm{d}\sigma^{\mathrm{LO}}(k_l) \int_0^1 \mathrm{d}z\, \Gamma_{ll}^{\mathrm{LL}}(z, Q^2) = 0$$

  predictions depend on photon-recombination scheme
- "bare leptons" (typical for muons)
  photons are separated from leptons
  collinear singularities regularised by lepton mass
  $\Rightarrow$ logarithmically enhanced corrections
  subtraction-method formulation: Dittmaier, Kabelschacht, Kasprzik '08

full FSR not universal, in general not separable from other EW corrections
separation introduces conventions

Photon–jet separation

$F + \text{jet} + \gamma$ production contributes to (F some set of final-state particles)

- real EW NLO corrections to $F + \text{jet}$ production
- real QCD NLO corrections to $F + \gamma$ production

separation of $F + \text{jet}$ production and $F + \gamma$ production for collinear γ and jet at NLO requires photon–jet separation that

- guarantees cancellation of all IR singularities
- is experimentally feasible

natural separation criterion: based on
electromagnetic energy fraction inside a shower/jet $\quad z = \frac{E_{\mathrm{em}}}{E_{\mathrm{em}}+E_{\mathrm{had}}}$

- $z < z_{\mathrm{cut}}$: $\Rightarrow$ jet
- $z > z_{\mathrm{cut}}$: $\Rightarrow$ photon

cut affects collinear region
$\Rightarrow$ incomplete cancellation of collinear singularities from (anti-)quarks

Methods for photon–jet separation

- Quark-to-photon fragmentation function

  $D_{q\to\gamma}(z)$: probability of a quark fragmenting into a jet containing a photon with fraction z of total jet energy (similar concept as PDF)

  $D_{q\to\gamma}(z)$ consists of non-perturbative and perturbative part
  collinear singularities absorbed in bare fragmentation function

  Glover, Morgan '94; Denner et al. '10

  dependence of cross sections on $D_{q\to\gamma}(z)$ typically weak

- Frixione isolation   Frixione '98

  define isolated photon such that only soft partons can become collinear
  transverse energy in a cone around photon has to vanish with cone size δ:

$$\sum_i E_{\mathrm{T},i}\,\theta(\delta - R_{i\gamma}) \le \chi(\delta) \quad \text{for} \quad \delta \le \delta_0$$

  $R_{i\gamma} = \sqrt{(\eta_i - \eta_\gamma)^2 + (\phi_i - \phi_\gamma)^2}$, $\chi(\delta) \to 0$ for $\delta \to 0$

  collinear singularity (fragmentation function) shifted to the jet
  experimental implementation unclear (small cone sizes!)

Unstable particles

Almost all interesting particles unstable $\mathrm{Z}, \mathrm{W}, \mathrm{t}, \mathrm{H}, \ldots$ (weak decays)

approximation of stable particles in general not adequate

- intrinsic errors of $\mathcal{O}(\Gamma/M) \sim$ few %
- no distributions of physical decay products

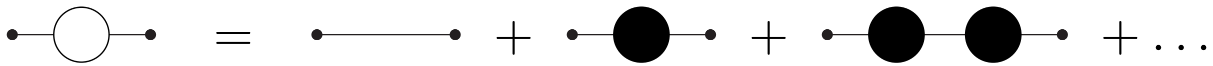
narrow-width approximation

$$\frac{1}{|k^2 - M^2 + \mathrm{i}M\Gamma|^2} \underset{\Gamma\to 0}{\sim} \frac{\pi}{M\Gamma}\delta(k^2 - M^2)$$

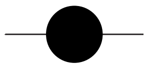
- distributions in decay products available
- spin correlations between production and decay can be included
- intrinsic errors of $\mathcal{O}(\Gamma/M) \sim$ few %
- can fail completely if large “non-resonant” contributions are present
  Berdine, Kauer, Rainwater ’07

Mass and width of unstable particles

propagator near resonance: (Dyson series for scalar example)



$$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2 - m^2} + \frac{\mathrm{i}}{p^2 - m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2 - m^2} + \ldots = \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)}$$

$-\mathrm{i}$  $= \Sigma(p^2) =$ renormalized self-energy, $m =$ ren. mass

stable particle: $\mathrm{Im}\{\Sigma(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2, physical mass = pole of propagator
renormalization condition for physical mass m: $\Sigma(m^2) = 0 \leftrightarrow \mathrm{Re}\,\Sigma(m^2) = 0$

unstable particle: $\mathrm{Im}\{\Sigma(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole shifted into complex p^2 plane

on-shell mass: $M_{\mathrm{OS}}^2 - m^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \stackrel{!}{=} 0$

M_{OS} gauge dependent at 2-loop order — Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

pole mass: $\mu^2 - m^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

complex pole position $\mu^2 \equiv M^2 - \mathrm{i}M\Gamma$ gauge independent

Propagators for unstable particles

- Dyson-resummed propagator $\frac{\mathrm{i}}{p^2-M^2+\Sigma(p^2)}$
  - $\Sigma(p^2)$ gauge dependent $\Rightarrow$ gauge-dependent results
  - might be reasonable approximation or completely off
- propagator with running width $\frac{\mathrm{i}}{p^2-M^2_{\mathrm{OS}}+\mathrm{i}p^2\Gamma_{\mathrm{OS}}/M_{\mathrm{OS}}\theta(p^2)}$
  - related to on-shell masses for W, Z bosons
  - basis for measurement of M_{Z}, M_{W} at LEP and Tevatron
  - violates $\mathrm{SU}(2)$ and $\mathrm{U}(1)$ gauge invariance
- propagator with fixed width $\frac{\mathrm{i}}{p^2-M^2+\mathrm{i}M\Gamma}$
  - results from pole masses $\Rightarrow$ consistent propagator
  - respects $\mathrm{U}_{\mathrm{em}}(1)$ gauge invariance
  - violates $\mathrm{SU}(2)$ gauge invariance “mildly”
  - relation to on-shell mass

$$M = \frac{M_{\mathrm{OS}}}{\sqrt{1+\Gamma^2_{\mathrm{OS}}/M^2_{\mathrm{OS}}}}, \qquad \Gamma = \frac{\Gamma_{\mathrm{OS}}}{\sqrt{1+\Gamma^2_{\mathrm{OS}}/M^2_{\mathrm{OS}}}}, \qquad M_{\mathrm{Z,OS}} - M_{\mathrm{Z}} \approx 34\,\mathrm{MeV}$$

Modifications of propagators not sufficient for consistent schemes!

Treatment of resonances

- factorization schemes: multiply full amplitudes without widths with factors $\dfrac{p^2 - m^2}{p^2 - m^2 + \mathrm{i}m\Gamma}$ for each potentially resonant propagator
  - preserves gauge invariance
  - introduces spurious factors of $\mathcal{O}(\Gamma/m)$, mistreats non-resonant terms
  - nontrivial for more complicated processes
- pole scheme: consistent expansion about resonance Stuart '91; Aeppli et al. '93, '94; ...

$$\mathcal{M} = \frac{R(p^2)}{p^2 - m^2} + N(p^2) \to \frac{R(m^2)}{p^2 - m^2 + \mathrm{i}m\Gamma} + \frac{R(p^2) - R(m^2)}{p^2 - m^2} + N(p^2)$$

  - gauge invariant
  - allows separation of resonant and non-resonant terms
    $\Rightarrow$ definition of signal and background, pseudo observables
  - applicable to higher orders (but cumbersome)
  - definition of $R(m^2)$ not unique and potentially problematic
  - not reliable in off-shell tails, near thresholds (presence of small scales)
  - NLO corrections involve factorisable and non-factorisable corrections

Treatment of resonances

- pole approximation: leading contributions in resonance expansion
  - non-resonant contributions are neglected
    $\Rightarrow$ only sensible for observables dominated by resonant contributions
- effective-field-theory approach: Beneke et al. '04, '07, Hoang, Reisser '04
  - field theoretically elegant formulation of pole expansion
  - possibilities for resummations, combination with other expansions
  - differential distributions and cuts difficult to implement
- complex-mass scheme: Denner et al. '99, '05
  use complex masses consistently everywhere: $\mu^2 = M^2 - \mathrm{i}M\Gamma$
  $\Rightarrow$ complex weak mixing angle $\cos^2\theta_\mathrm{w} = \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$
  - exact gauge invariance
  - perturbative calculations as usual (with complex masses and counterterms)
  - accurate everywhere in phase space (resonant and non-resonant regions)
  - spurious terms of $\mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$ relative to considered order

Combination of EWC with QCD and MC

Combination of NLO QCD and EW corrections:

- additive: $1+\delta^{\mathrm{NLO}} = 1+\delta^{\mathrm{NLO}}_{\mathrm{QCD}}+\delta^{\mathrm{NLO}}_{\mathrm{EW}}$
- multiplicative: $1+\delta^{\mathrm{NLO}} = (1+\delta^{\mathrm{NLO}}_{\mathrm{QCD}})(1+\delta^{\mathrm{NLO}}_{\mathrm{EW}})$

} variants differ at $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$

soft and collinear corrections factorise $\Rightarrow$ multiplicative combination preferable

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} = \frac{\mathrm{d}\sigma_{\mathrm{QCD}}}{\mathrm{d}x}\left(1+\delta^{\mathrm{NLO}}_{\mathrm{EW}}(x)\right) + \frac{\mathrm{d}\sigma_{\gamma}}{\mathrm{d}x}$$

- correction factor should capture dominant phase-space dependence
- used for Higgs production in VBF and WH/ZH to combine NLO EW and NNLO QCD Higgs cross-section working group ’13
- useful for including EW corrections in event generators
  Anderson et al ’10; Gieseke, Kasprzik, Kühn ’14
- treats photon radiation inclusively
  $\Rightarrow$ kinematic effects of hard emission neglected
- reweighting should be checked against fully differential results

Conclusion Lecture 2

Conclusion for Lecture 2

- Electroweak corrections (EWC) are relevant for LHC processes
- EWC particularly large in tails of distributions or near resonances
- Generator for EW NLO amplitudes exists
  $\Rightarrow$ full automation to come
- full EWC involve many new features and subtleties
  - parton distributions including EWC
  - photon-induced processes
  - photon–jet separation
  - treatment of resonances
  - interplay between QCD and EW corrections
  - parton showers with EW effects
- EWC can be included approximatively via correction factors in MC generators