Resummation and theoretical uncertainties

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Assessing and improving accuracy

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Theoretical uncertainties: fixed order

For fixed-order calculations we have two natural handles to evaluate theoretical uncertainties, the renormalisation and factorisation scales μ_R and μ_F

$$\alpha_s^n(x\mu_R) = \alpha_s^n + (n\,\beta_0 \ln x)\,\alpha_s^{n+1}(\mu_R)$$

By varying these scales we generate a higher-order contribution

The relevant questions here are

- What is the default choice for μ_R and μ_F ?
- What is the range over which we should vary these scales?
- How should we add uncertainties?

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- What is the range over which we should vary these scales?
- How should we add uncertainties?

Unfortunately, there is no theoretically sound answer to any of these questions

Theoretical uncertainties: central value

For an observable characterised by a single scale the dependence on the renormalisation scale appears in virtual corrections

$$R\left(\alpha_s(\mu), \frac{s}{\mu^2}\right) = 1 + R_1\alpha_s(\mu^2) + \left(R_1\beta_0 \ln\frac{s}{\mu^2} + R_2\right)\alpha_s^2(\mu^2) + \mathcal{O}(\alpha_s^3)$$

Choosing $\mu^2 = s$ gets rid of the term $\ln(\mu^2/s)$

For an observable with multiple scales at LO, like a jet cross section, one has

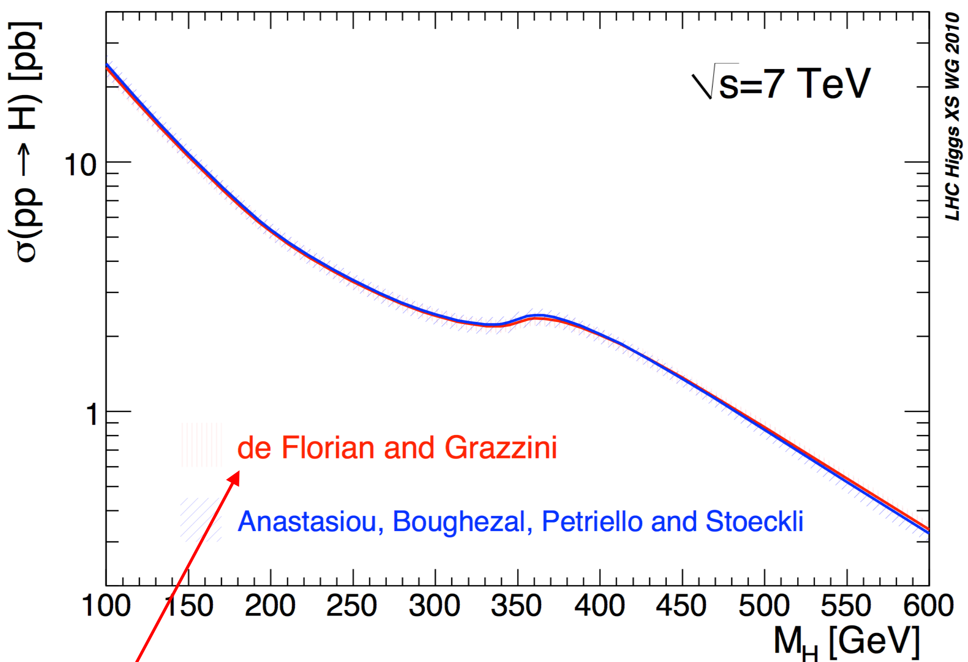
$$\sigma\left(\alpha_s(\mu^2), s_1, \ldots, s_n, \mu^2\right) \sim \sigma_0\alpha_s^n + \left(\beta_0 \ln\frac{s_1, \ldots, s_n}{\mu^{2n}}\sigma_0 + \sigma_1\right)\alpha_s^{n+1} + \mathcal{O}(\alpha_s^{n+2})$$

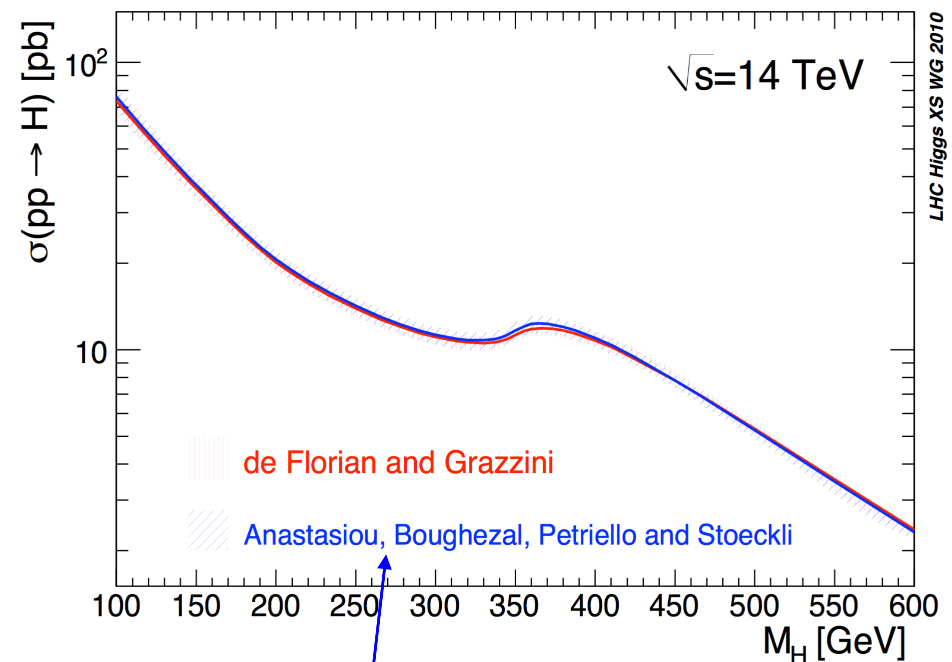
1. The choice that cancels that logarithm is $\mu^2 = (s_1\, s_2 \ldots s_n)^{1/n}$

2. It is not guaranteed that choosing that scale leads to a series that behaves better perturbatively. There might be for instance further scales coming from jet resolution parameters, kinematical cuts, etc.

Theoretical uncertainties: central scale

Since for one emission $\alpha_s = \alpha_s(k_t)$, a good practice is to try to estimate the typical scale for gluon radiation: this might depend on the observable





NNLL threshold resummation
central scale m_H

NNLO fixed order
central scale $m_H/2$

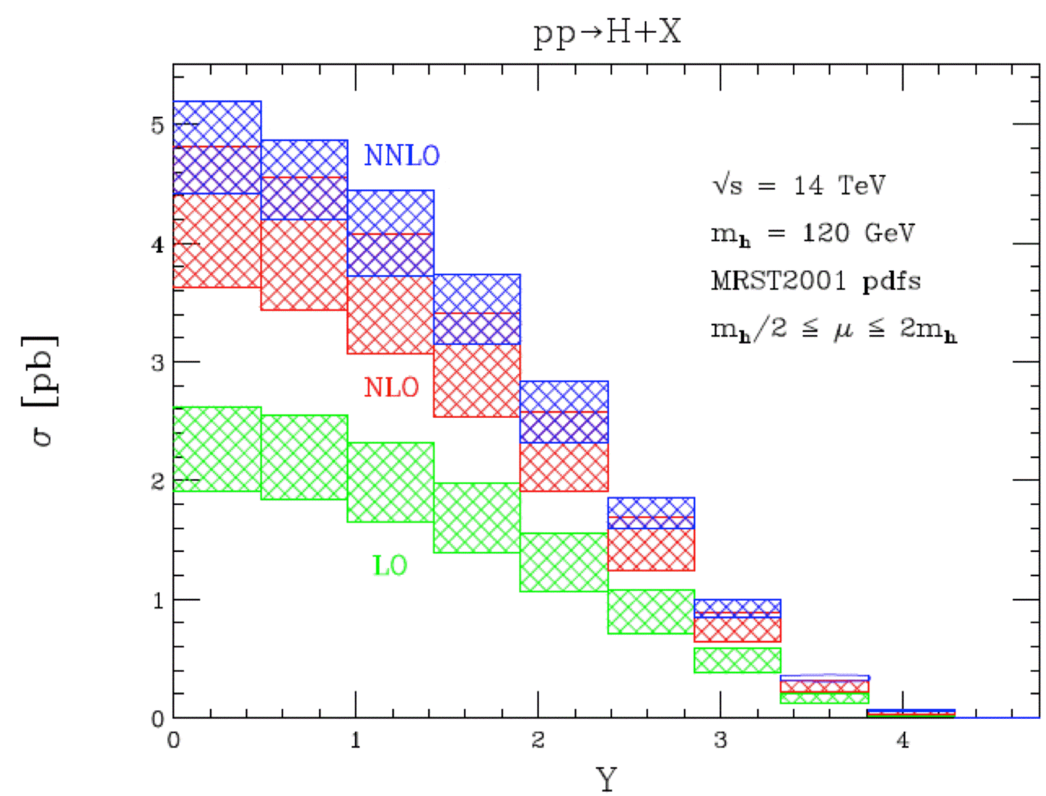
One can find an "optimal" scale for the fixed order by requiring that the K-factors are minimised, this gives the choice $m_H/2$

Theoretical uncertainties: scale variation

Only after one has identified a "central" scale does it make sense to take scale variations of a factor of two, so as not to generate large logarithms

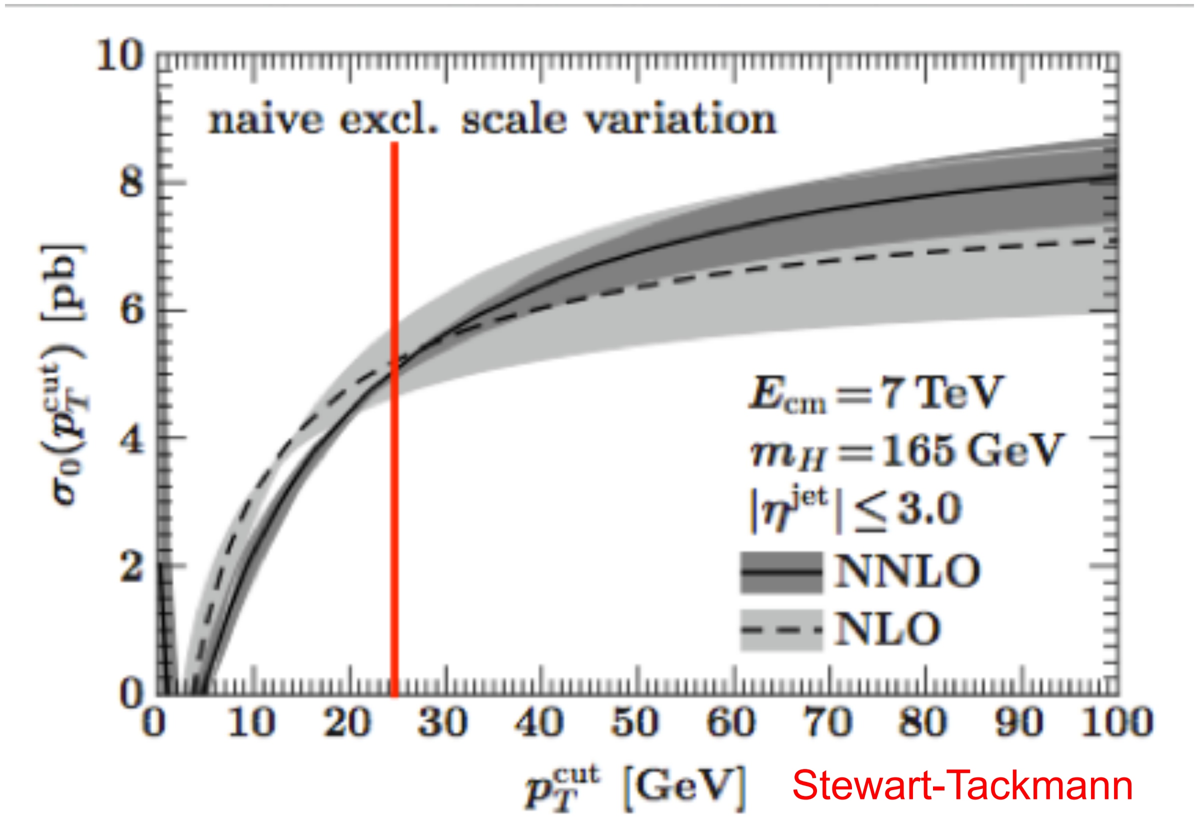
$$\alpha_s^n(x\mu_R) = \alpha_s^n + (n\,\beta_0 \ln x)\,\alpha_s^{n+1}(\mu_R)$$

Is it a robust method? It is if the prediction at the next order overlaps with the uncertainty band at the previous order. This is not always the case, especially with large K-factors. This is why having higher orders is so important



Theoretical uncertainties: scale variation

Scale uncertainties are however able to highlight pathological behaviours of cross sections, for instance infrared sensitivity



$$\sigma_{0-\text{jet}} = \sigma_{\text{tot}} - \sigma_{\geq 1-\text{jet}}$$

large K-factor

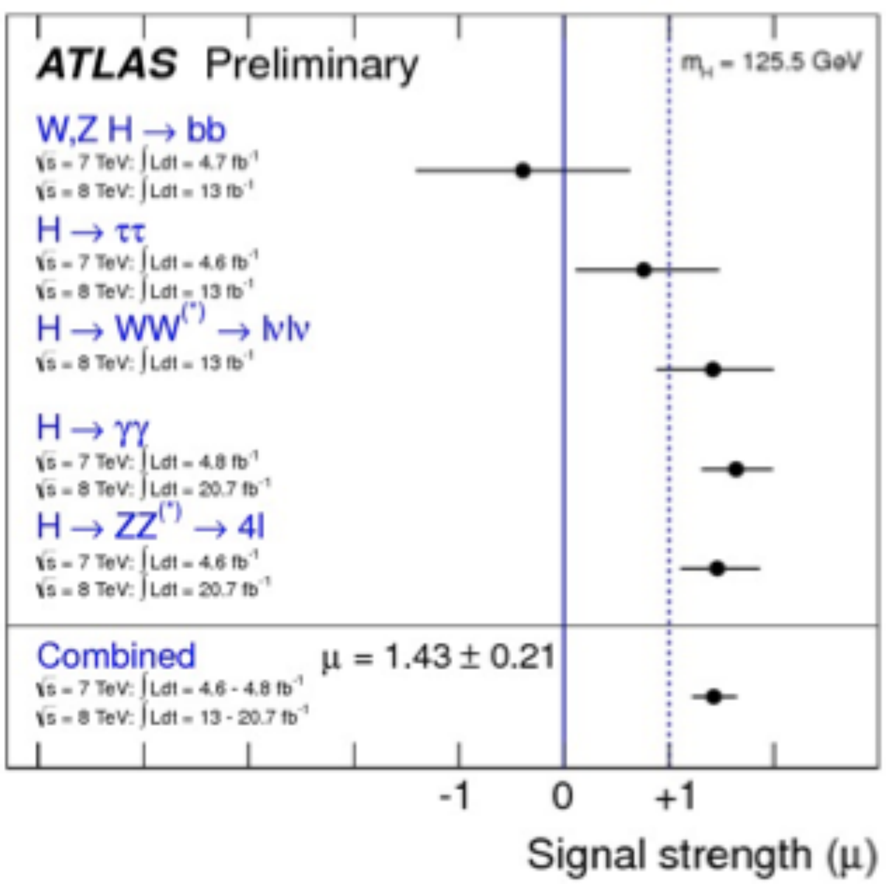
large logarithms

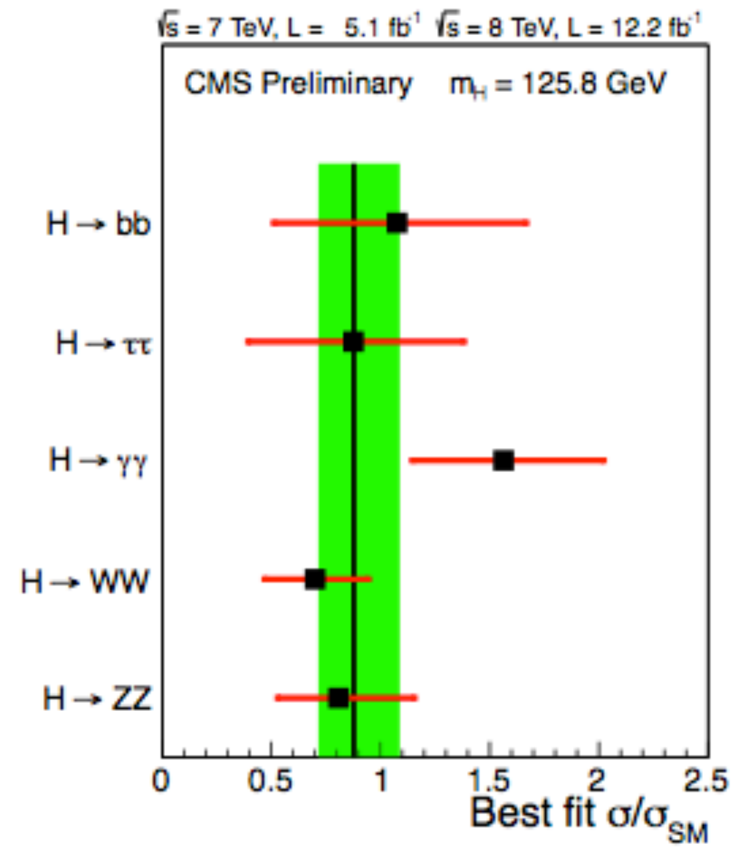
The cancellation of two large effects gives a spurious vanishing of scale uncertainties at low values of the jet-veto resolution $p_T^{\rm cut}$

A vanishing scale uncertainty is clearly not a good estimate of missing higher orders...

Higgs production with a jet-veto

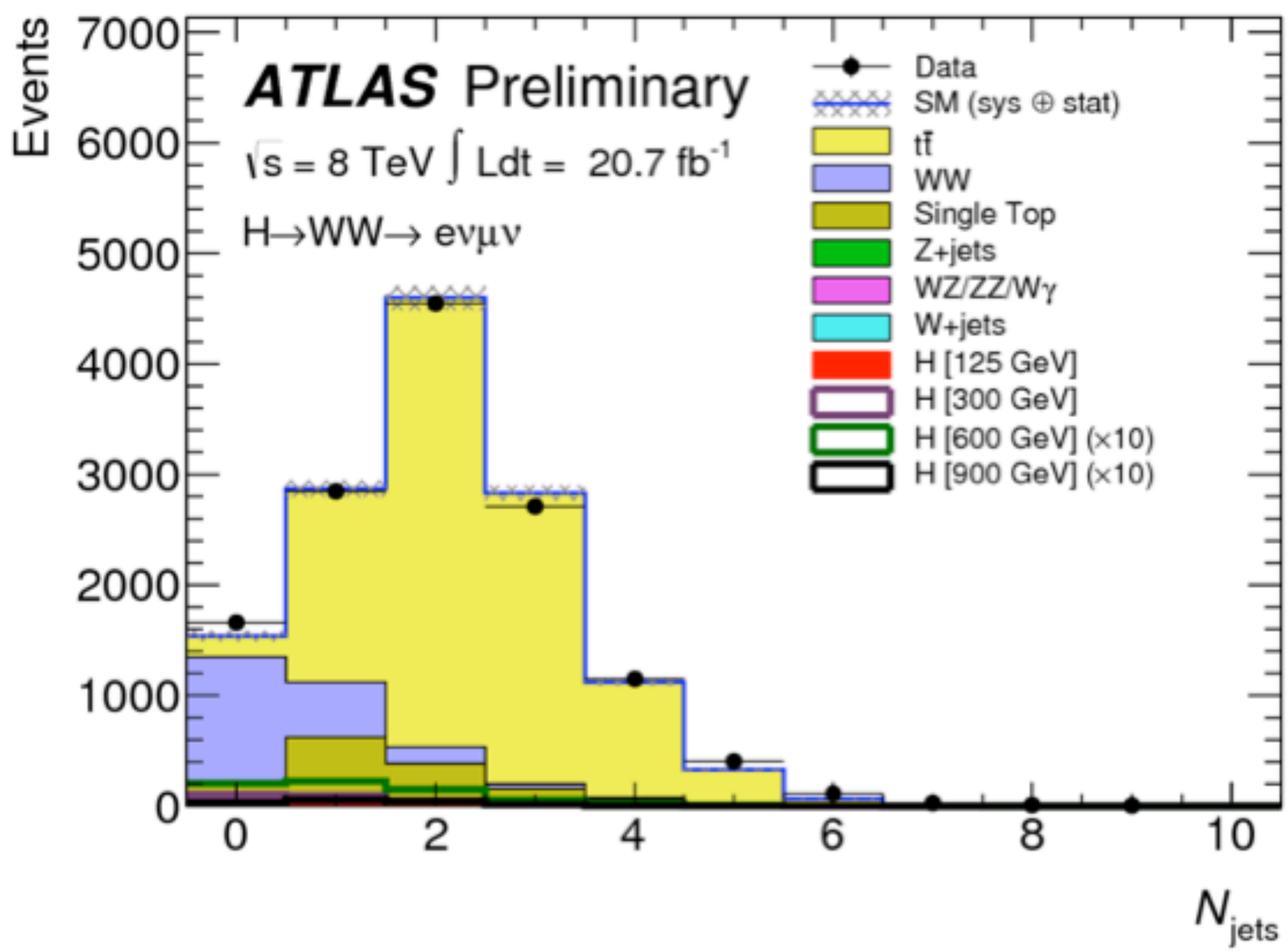
The main interest in jet-veto cross sections is to establish whether the new boson found at the LHC is the Standard Model Higgs





Higgs production with a jet-veto

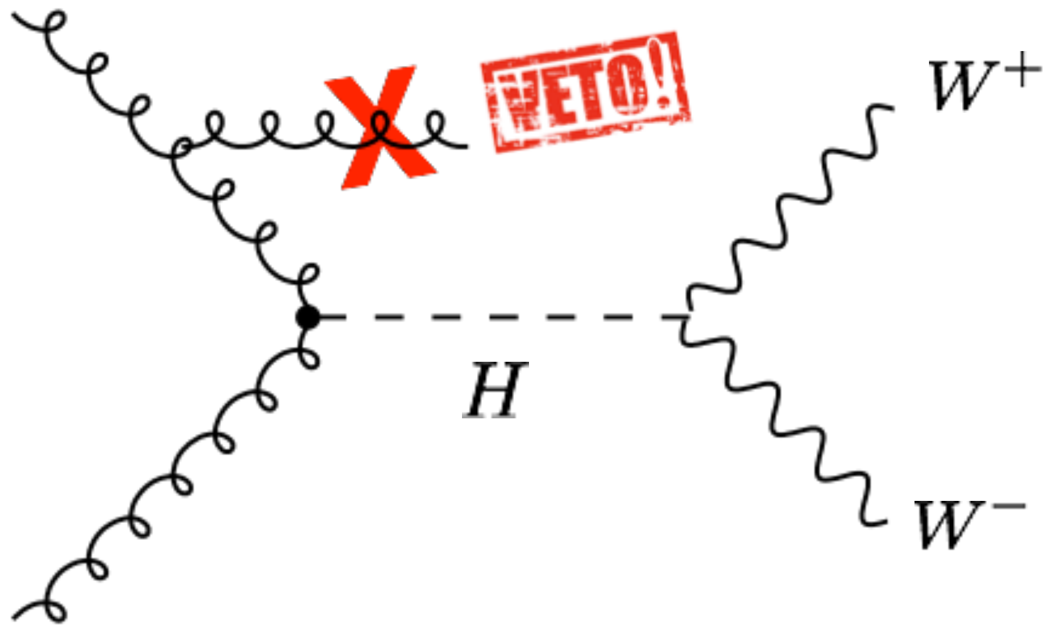
In order to suppress the large top-antitop background to $H \to WW$ we require that all jets have a transverse momentum less that a threshold value $p_{\mathrm{t,veto}}$



This works: the zero-jet cross section $\sigma_{0-\mathrm{jet}}$ is least contaminated by the huge (yellow) top-antitop background

Jet-veto as a two-scale problem

The 0-jet cross section is characterised by two scales, the Higgs mass m_H and the jet resolution $p_{\mathrm{t,veto}}$

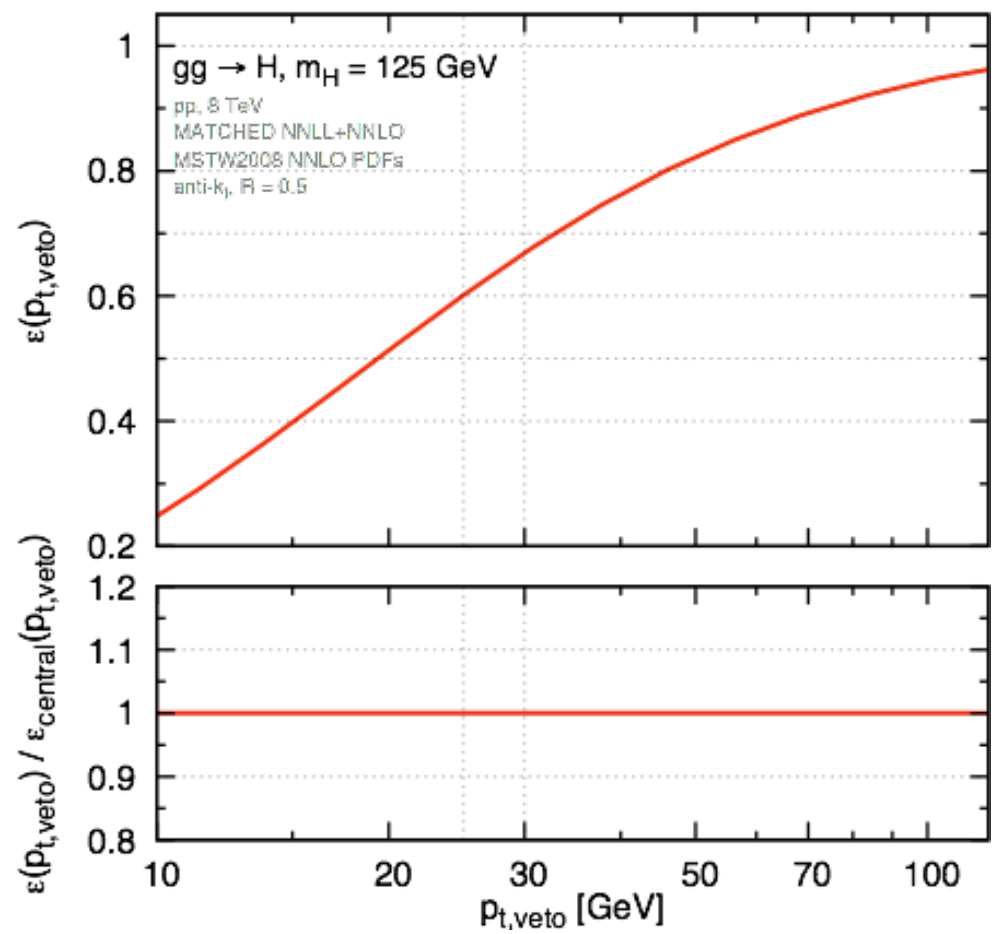


The jet-veto condition restricts the phase space available to gluons, so we expect logarithmically enhanced contributions $\ln(m_H/p_{\mathrm{t,veto}})$ at all orders

Does a resummation of large logarithms help solve the problem of the weird behaviour of scale-uncertainties?

Resummation uncertainties

Resummation has more handles to assess theoretical uncertainties

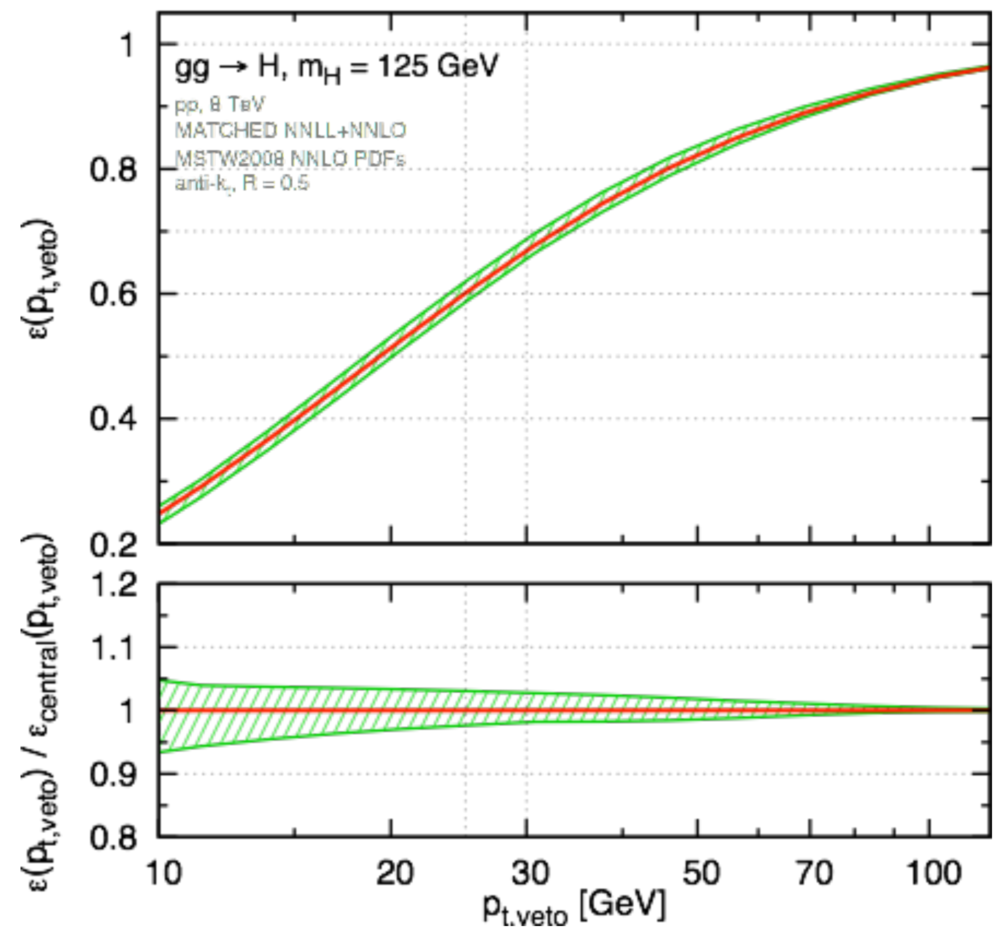


Resummation uncertainties

Resummation has more handles to assess theoretical uncertainties

1. "Traditional" variation of renormalisation and factorisation scale in the range

$$\frac{m_H}{4} \leq \mu_R, \mu_F \leq m_H \qquad \frac{1}{2} \leq \frac{\mu_F}{\mu_R} \leq 2$$



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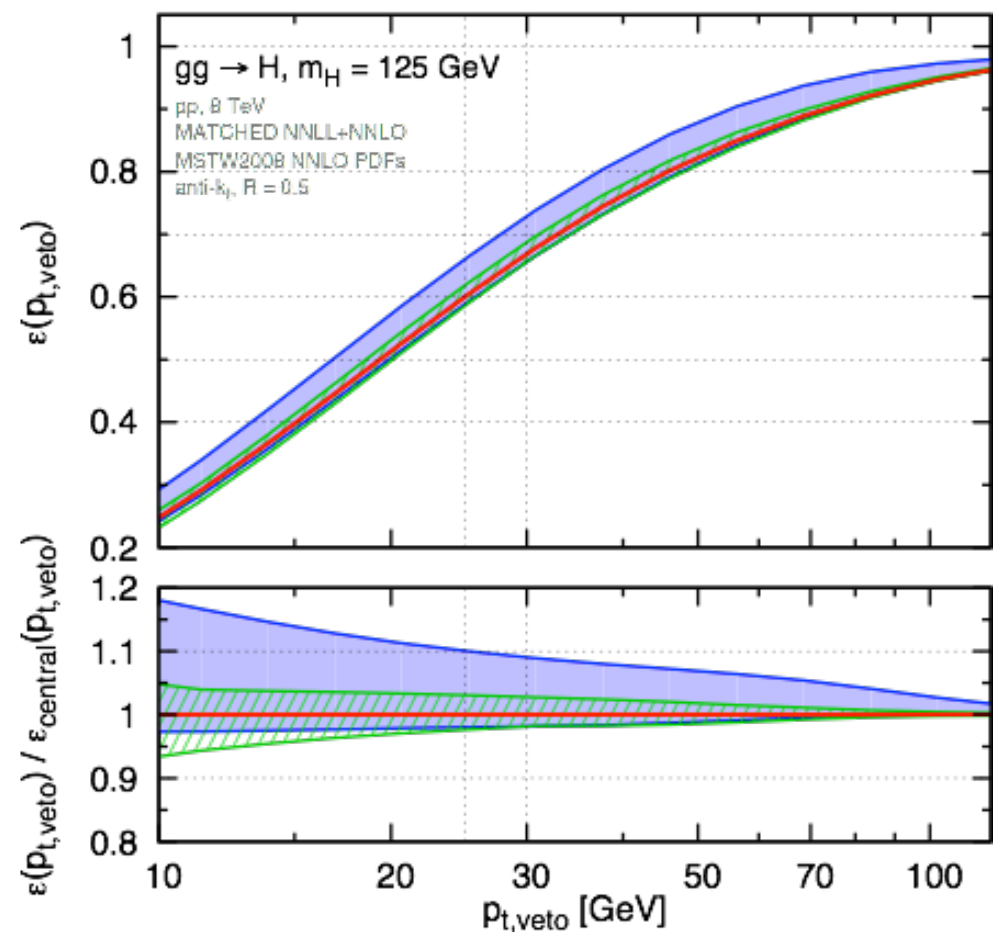
2. Resummation scale: change in the logs to be resummed, giving an idea of higher logarithmic corrections

$$\ln\left(\frac{m_H}{p_{\mathrm{t,veto}}}\right) \to \ln\left(\frac{Q}{p_{\mathrm{t,veto}}}\right)$$



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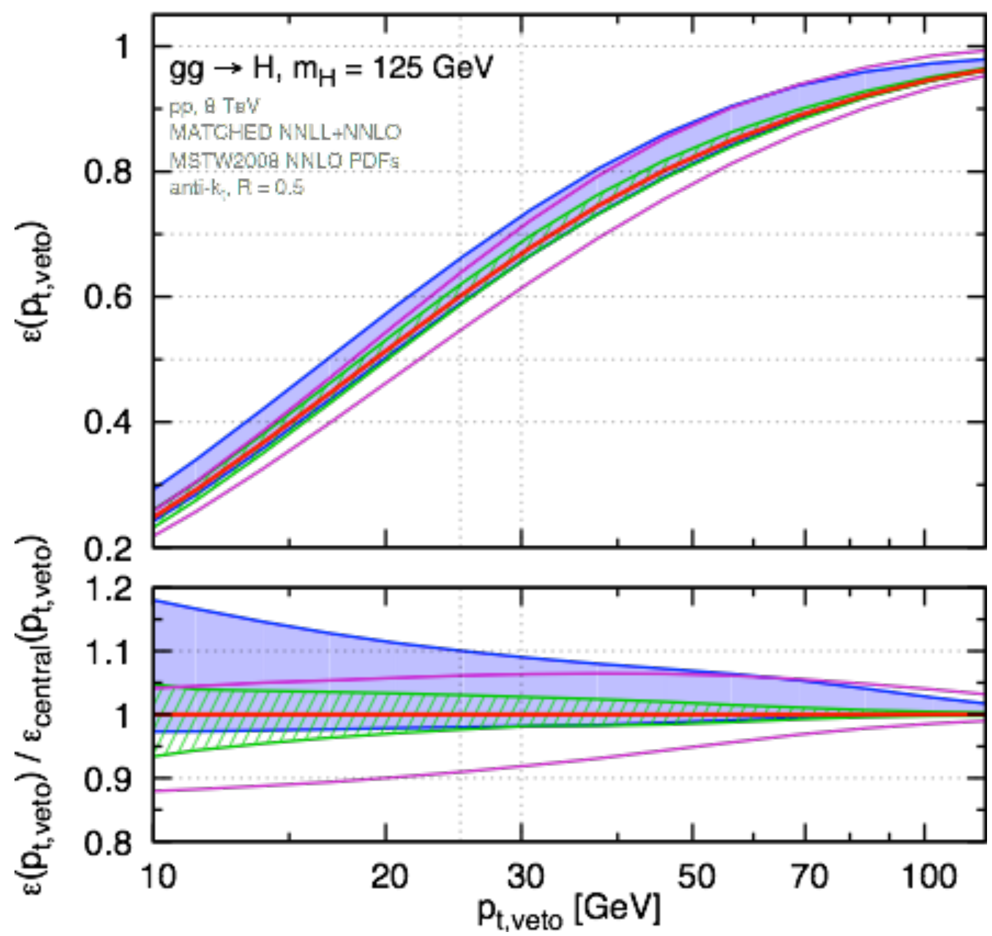
3. Variation of the scheme with which resummation is matched to exact fixed-order



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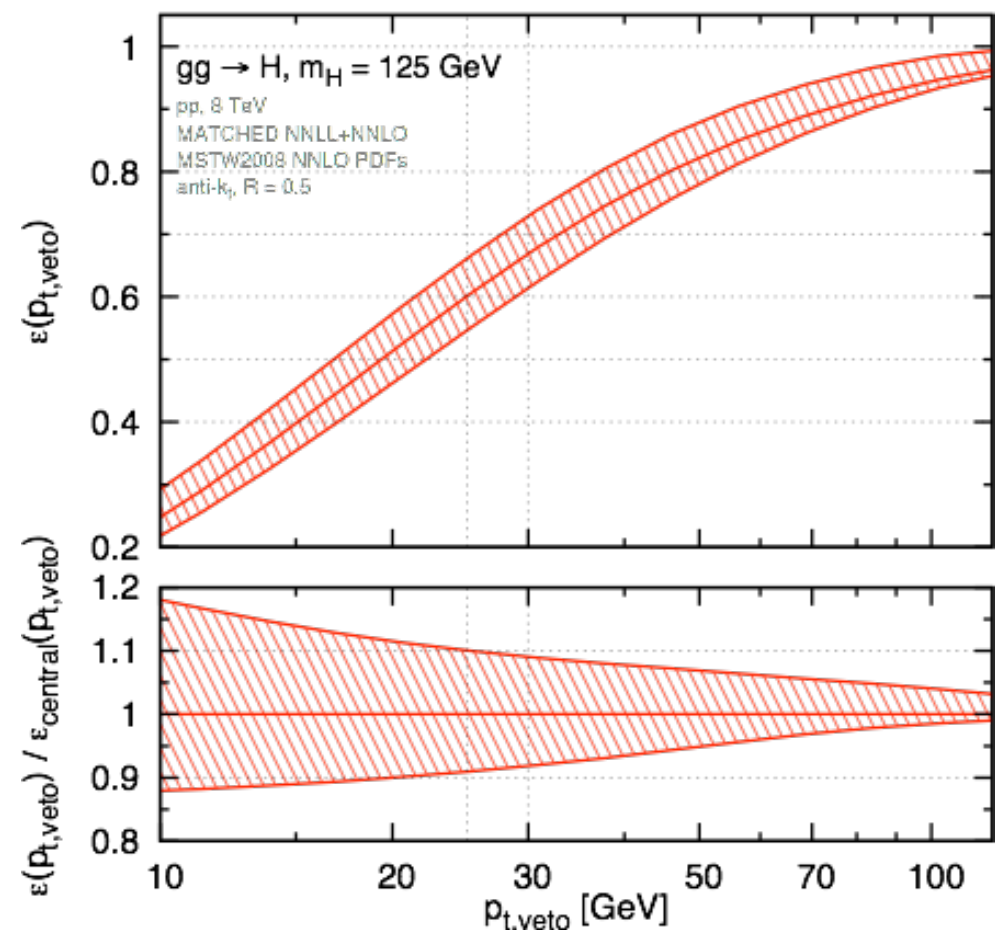
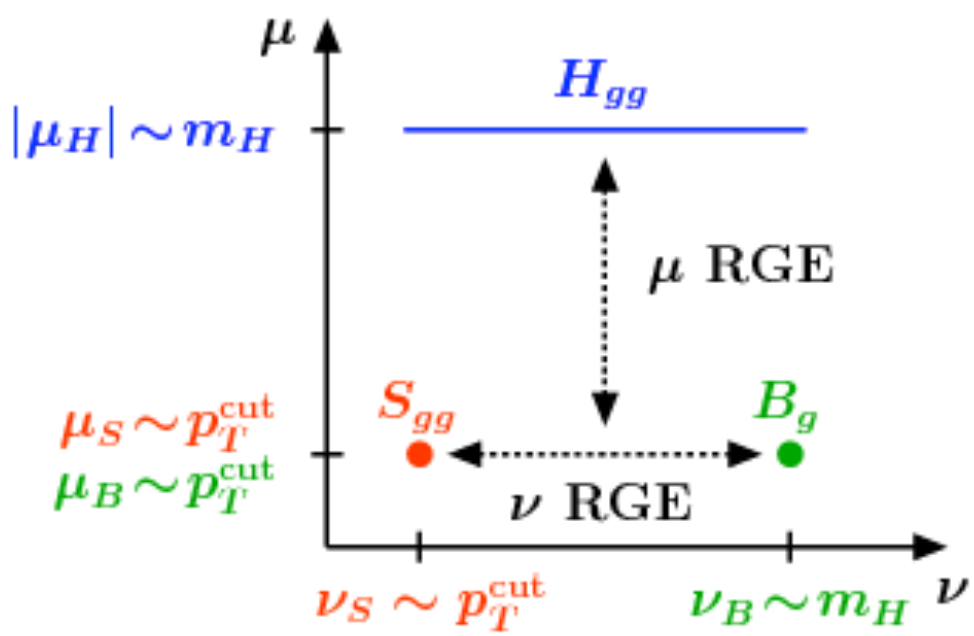
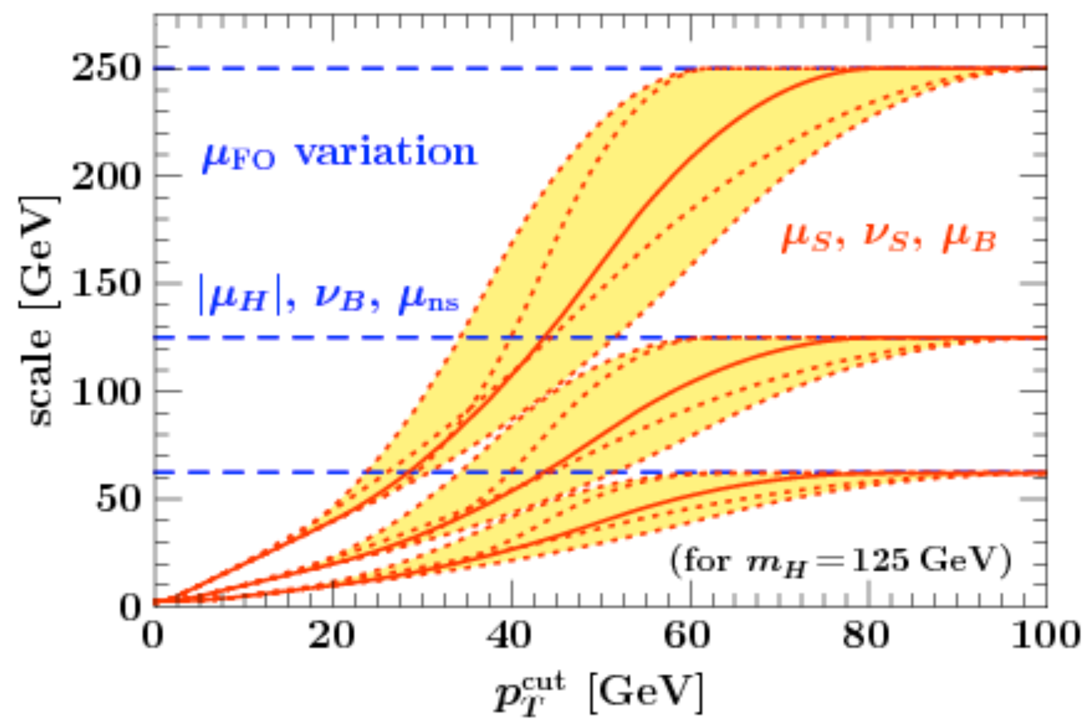


Total uncertainty: envelope of all these curves

Resummation uncertainties

Other resummed predictions have different central scales, a wider range of resummation scales, and the range of scale variation varies as a function of $p_{\mathrm{t,veto}}$





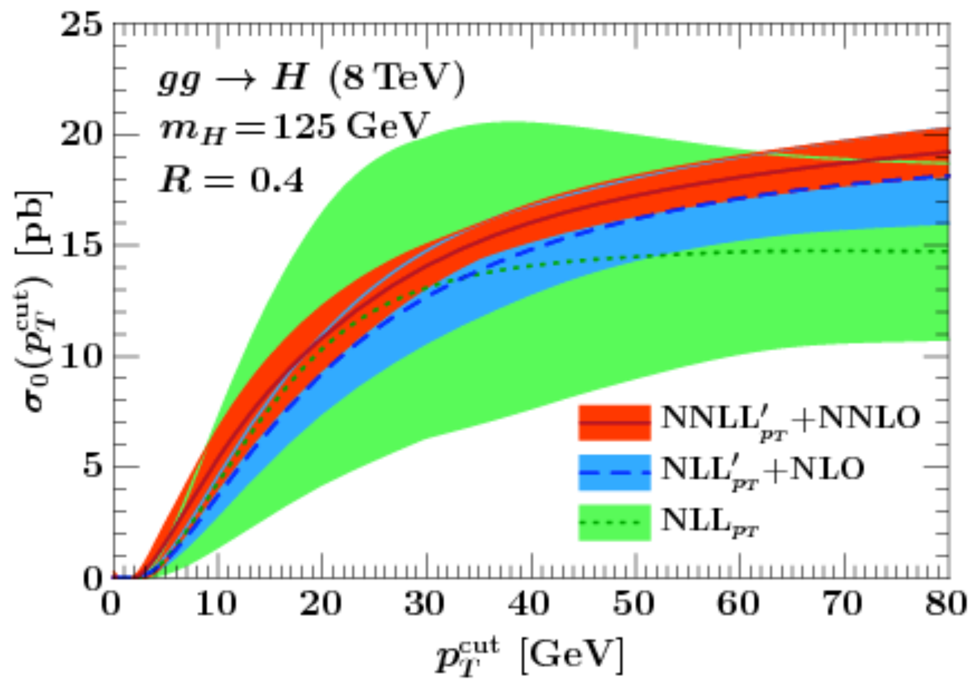
Stewart Tackmann Walsh Zuberi

These scales correspond to towers of logarithms to be resummed: they are small in the region where the resummation is important, and large in the region where the fixed-order is well behaved $\Longrightarrow$ smooth matching between resummation and NNLO

Resummation uncertainties

In all resummed calculations for the jet-veto cross section, uncertainties reduce consistently when increasing the resummation order

Stewart Tackmann Walsh Zuberi



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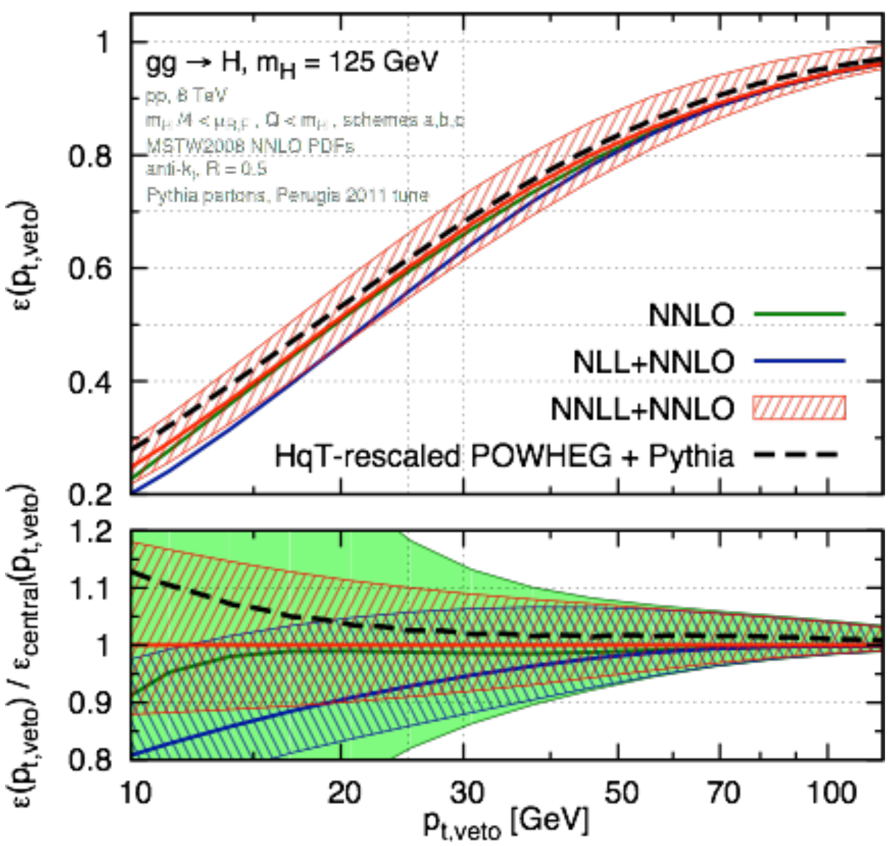
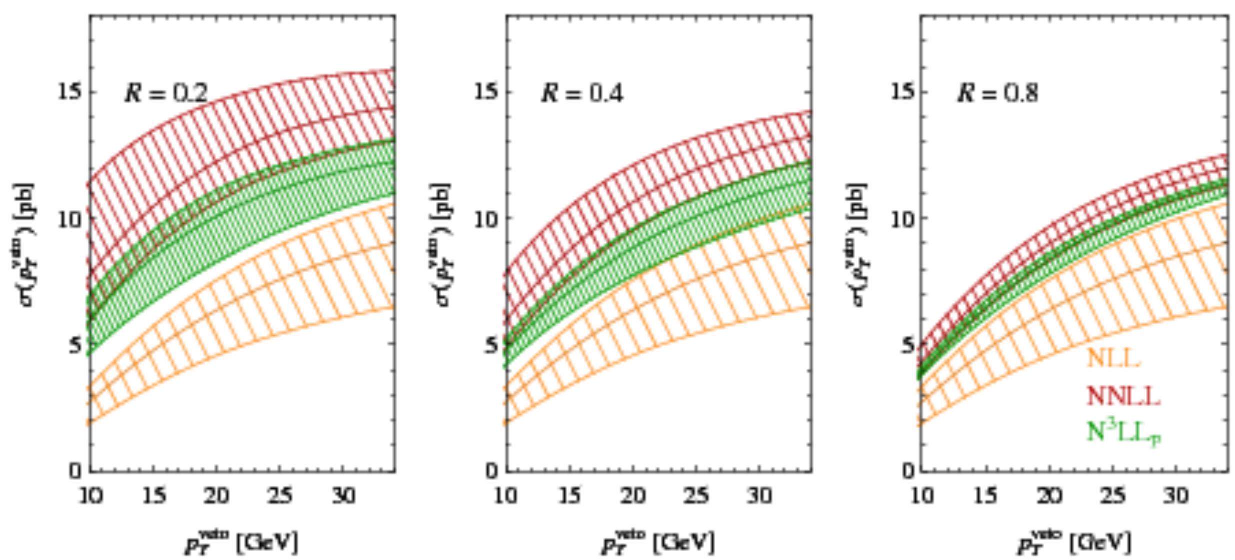
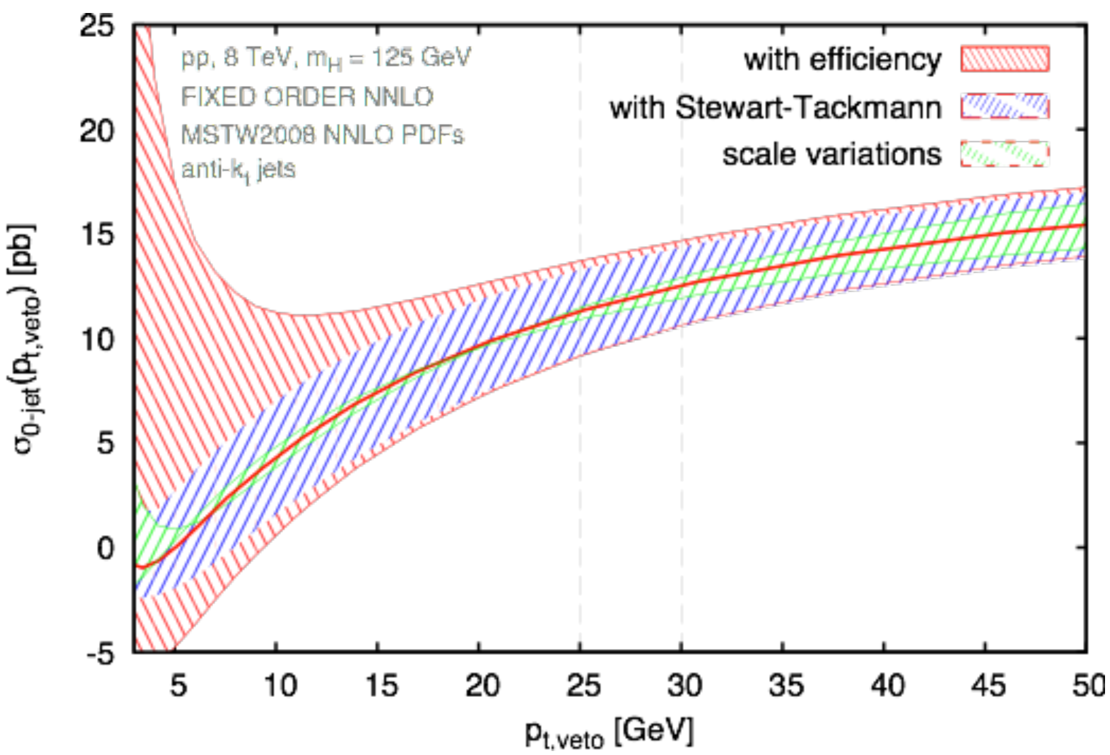


Becher Neubert Rothen



Resummation vs fixed order uncertainties

At fixed-order, due to infrared sensitivity, different methods to assess uncertainties, all compatible within perturbative accuracy, give very different results



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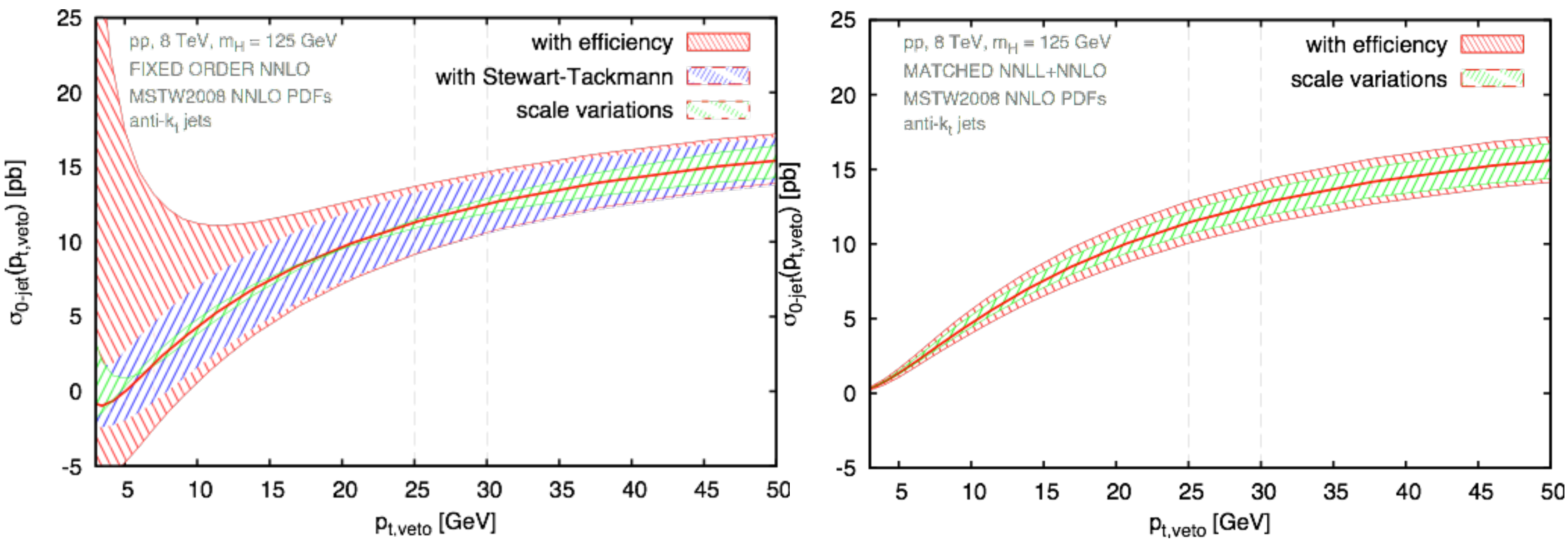


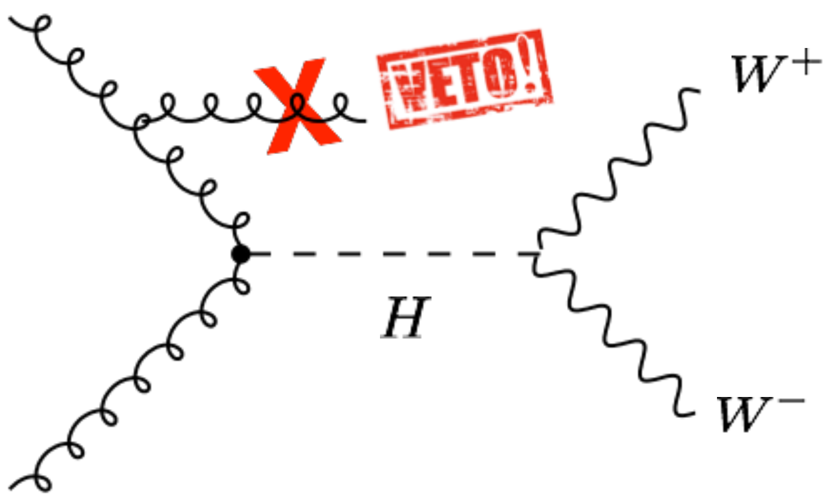
After resummation of large logarithms, also naive scale variations are a sensible way to estimate theoretical uncertainties, which at NNLL are around 10-12%

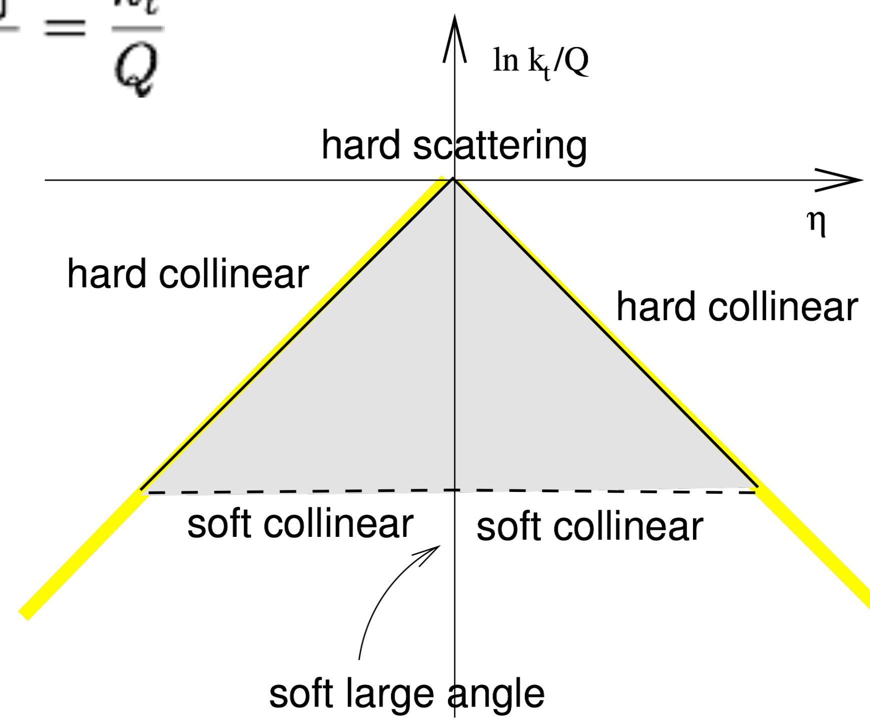
The main message is: if you feel you have to resum logs, just do it

Jet-veto efficiency at fixed order

The final-state observable corresponding to the jet-veto cross section is the transverse momentum of the hardest jet. For one soft and collinear gluon

$$V(k) = \frac{\max_j\{p_{t,j}\}}{m_H} = \frac{k_t}{Q}$$



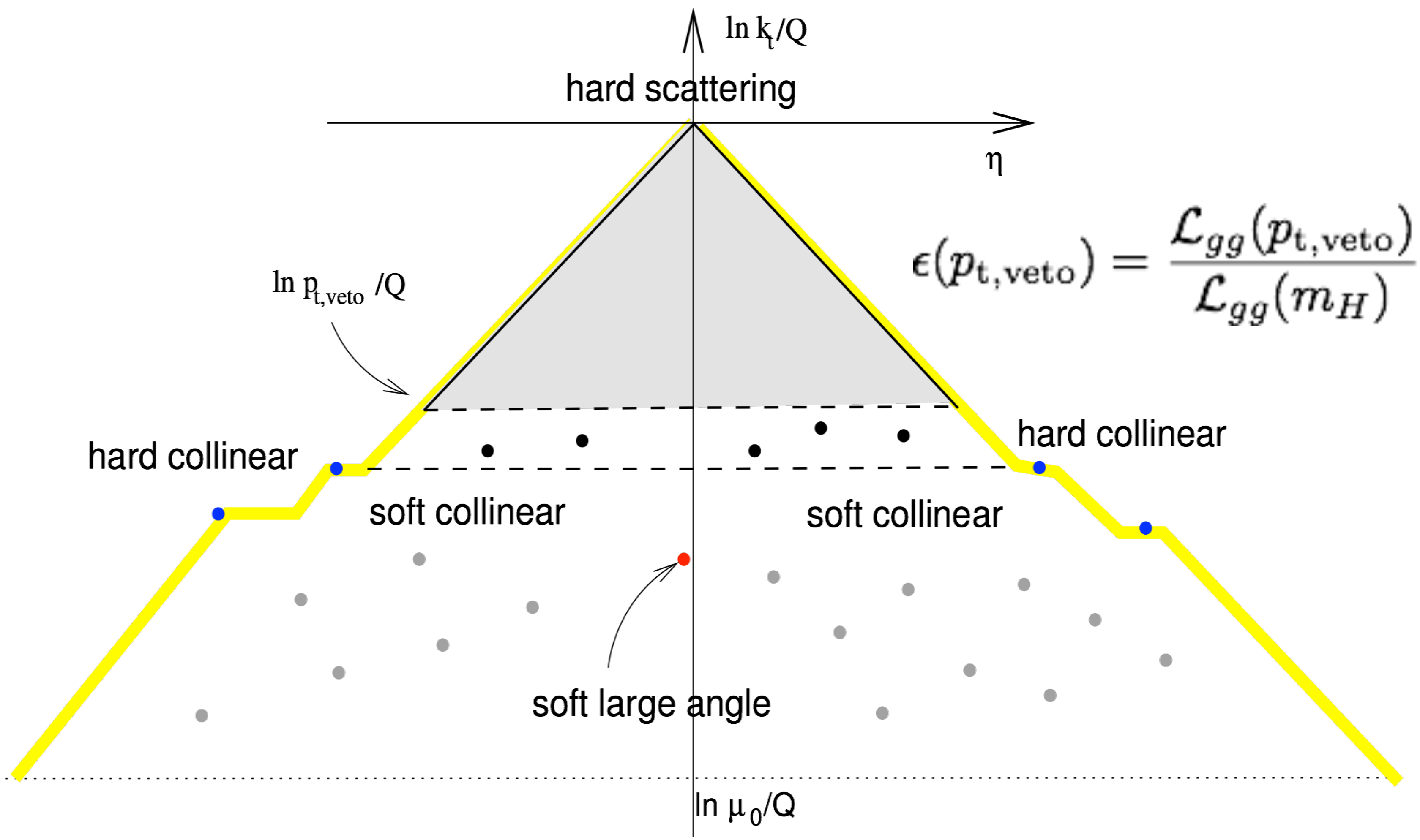


The cumulative distribution in the transverse momentum of the leading jet is called jet-veto efficiency $\epsilon(p_{t,\mathrm{veto}})$. For a single gluon this is obtained by computing the shaded area, and keeping in mind that emitting particles are gluons, with colour charge

$$\epsilon(p_{t,\mathrm{veto}}) = 1 - 2C_A\frac{\alpha_s}{\pi}\ln\left(\frac{m_H}{p_{t,\mathrm{veto}}}\right)^2$$

Single-logarithmic resummation

The transverse momentum of the leading jet scales like the transverse momentum of the hardest emissions, hence it is trivially rIRC safe.

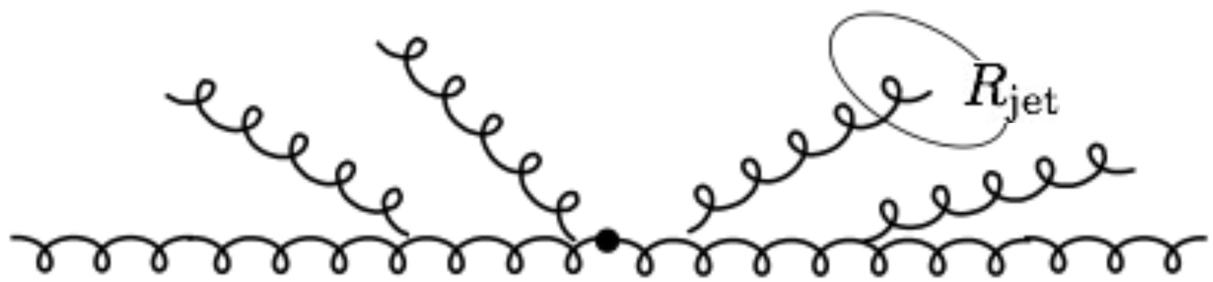


$$\epsilon(p_{t,\mathrm{veto}}) = \frac{\mathcal{L}_{gg}(p_{t,\mathrm{veto}})}{\mathcal{L}_{gg}(m_H)} e^{-R(p_{t,\mathrm{veto}})} \mathcal{F}(R')$$

$$R(p_{t,\mathrm{veto}}) = 4C_A \int_{p_{t,\mathrm{veto}}}^{m_H} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \left(\ln\left(\frac{m_H}{k_t}\right) - \frac{11C_A - 4T_R n_f}{12C_A} \right)$$

Multiple-emission effects

At NLL all relevant emissions are widely separated in rapidity



If $p_{\rm t,veto}$ is small enough, for a finite radius we always have $R_{\rm jet} \ll \ln(m_H/p_{\rm t,veto})$

This implies that for sufficiently small $p_{\rm t,veto}$, the jet algorithm will never be able to recombine two gluons widely separated in angle. This implies

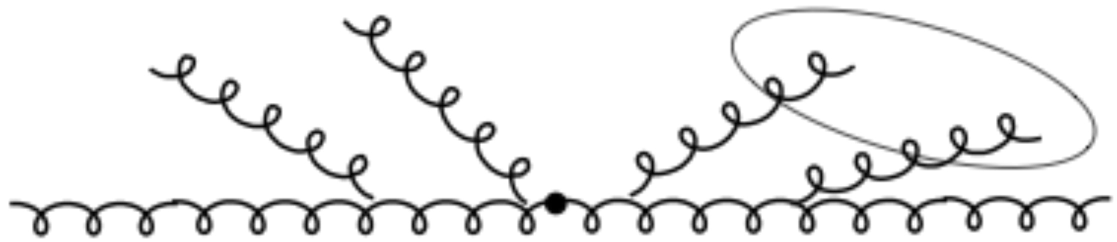
$$V(k_1,\dots,k_n) = \frac{\max_j\{p_{\rm t,jets}\}}{m_H} = \frac{\max_i\{k_{ti}\}}{Q}$$

No jets implies all gluons have $k_{ti} < p_{\rm t,veto}$ $\Rightarrow$ No multiple emission corrections, the answer is just a Sudakov form factor

$$\epsilon(p_{\rm t,veto}) = \frac{\mathcal{L}_{gg}(p_{\rm t,veto})}{\mathcal{L}_{gg}(m_H)} e^{-R(p_{\rm t,veto})} \qquad \mathcal{F}(R') = 1$$

Beyond NLL: independent emission

If two emissions are not widely separated in rapidity, the jet algorithm will cluster them



This gives the following correction (calling $v = p_{t,\mathrm{veto}}/m_H$)

$$\delta\mathcal{F}_{\mathrm{indep}} = \overset{1}{\boxed{e^{-\int_{\epsilon v}^{v}[dk]M^2(k)}\left(\sum_{n=0}^{\infty}\frac{1}{n!}\prod_i \int_{\epsilon v}[dk_i]M^2(k_i)\prod_i \Theta(v - V(k_i))\right)}} \times$$
$$\times \frac{1}{2!}\int_{\epsilon v}[dk_a][dk_b]M^2(k_a)M^2(k_b)\left[\underbrace{\Theta(v - V(k_a,k_b))}_{\text{actual observable}} - \underbrace{\Theta(v - V(k_a))\Theta(v - V(k_b))}_{\text{NLL approximation}}\right]$$

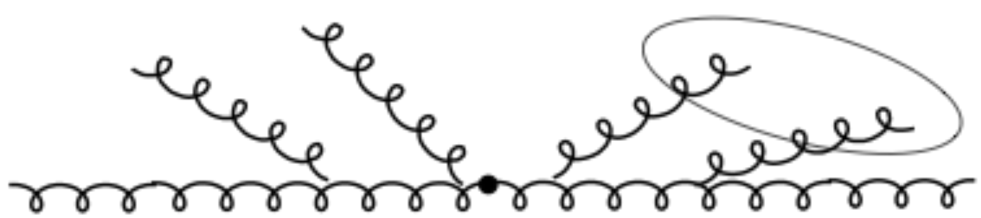
But the sum over all emissions that do not cluster and the Sudakov form factor give one

$$\delta\mathcal{F}_{\mathrm{indep}} = \frac{1}{2!}\int_{\epsilon v}[dk_a][dk_b]M^2(k_a)M^2(k_b)\left[\underbrace{\Theta(v - V(k_a,k_b))}_{\text{actual observable}} - \underbrace{\Theta(v - V(k_a))\Theta(v - V(k_b))}_{\text{NLL approximation}}\right]$$

Beyond NLL: independent emission

Let us define dimensionsless transverse vectors $\vec{v}_{a,b} = \vec{k}_{t;a,b}/m_H$

$$\delta\mathcal{F}_{\text{indep}} \simeq \left(\frac{2C_A}{\pi}\alpha_s(vm_H)\right)^2 \left(2\ln\frac{1}{v}\right)\frac{1}{2!}\int_{\epsilon v}^{\infty}\frac{dv_a}{v_a}\int_{\epsilon v}^{\infty}\frac{dv_b}{v_b}\left[\underbrace{\Theta(v-|\vec{v}_a+\vec{v}_b|)}_{\text{actual observable}} - \underbrace{\Theta(v-v_a)\Theta(v-v_b)}_{\text{NLL approximation}}\right]\times$$
$$\times\int_{\ln v}^{-\ln v} d\eta_a \int_{-\infty}^{\infty} d\eta_b \int_0^{2\pi}\frac{d\phi}{2\pi}\underbrace{\Theta(R_{\text{jet}}^2-(\eta_a-\eta_b)^2-\phi^2)}_{\text{clustering condition}}$$



Next step: we rescale $v_b = \zeta v_a$ and perform the integral over η_a, v_a and η_b

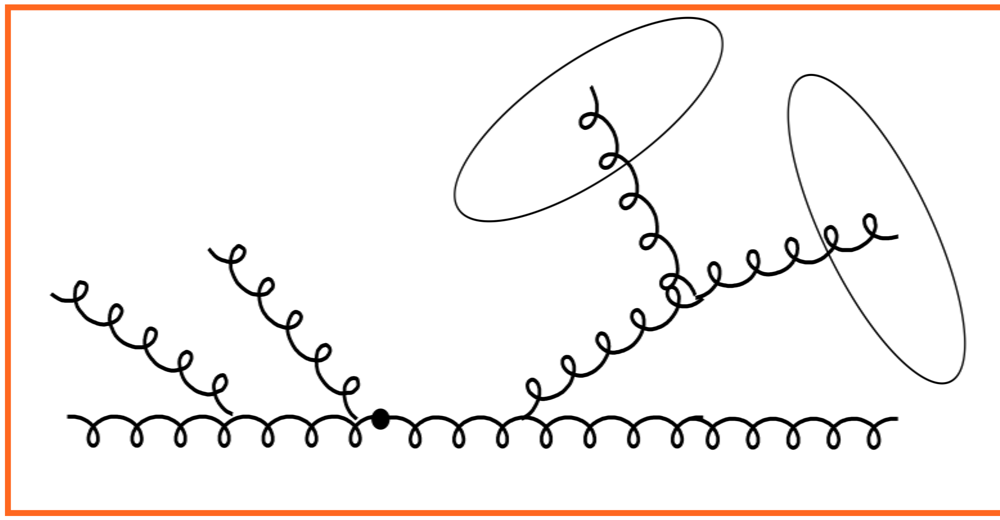
$$\delta\mathcal{F}_{\text{indep}} \simeq \left(\frac{2C_A}{\pi}\alpha_s(vm_H)\right)^2 \left(2\ln\frac{1}{v}\right)\frac{1}{2!}\int_{\epsilon}^{\infty}\frac{d\zeta}{\zeta}\int_{-R_{\text{jet}}}^{R_{\text{jet}}}\frac{d\phi}{2\pi}\ln\left(\frac{1+\zeta^2+2\zeta\cos\phi}{\max\{1,\zeta^2\}}\right)\sqrt{R_{\text{jet}}^2-\phi^2}$$

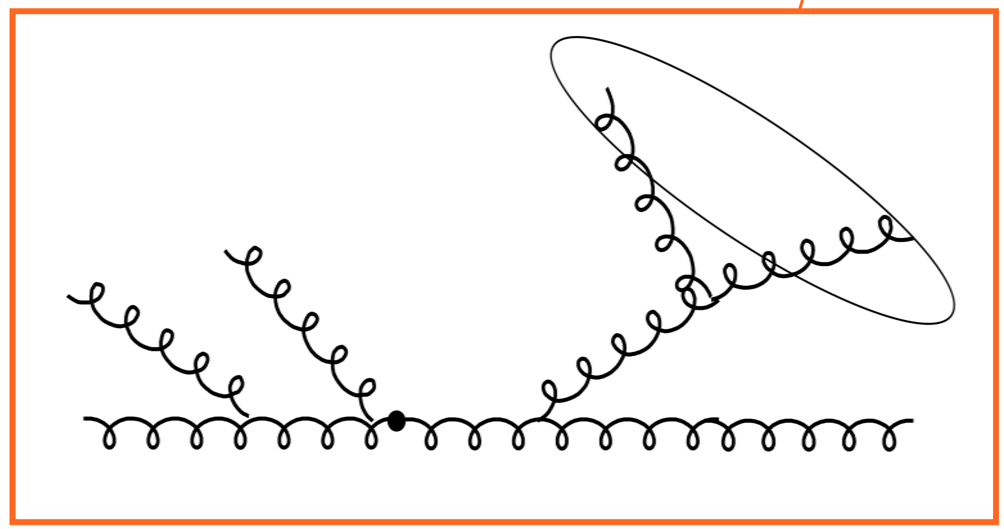
$$\boxed{\delta\mathcal{F}_{\text{indep}} \sim \alpha_s(p_{t,\text{veto}})R'(p_{t,\text{veto}})\,R_{\text{jet}}^2 \sim \alpha_s^2 L}$$ (small radius)

Beyond NLL: correlated emission

Similarly, one can compute the correction due to correlated emission

$$\delta\mathcal{F}_{\rm correl} = \int_{\epsilon p_{t,\rm veto}} [dk_a][dk_b]\tilde{M}^2(k_a,k_b)(1-\Theta_{\rm clust})\left[\underbrace{\Theta\left(p_{\rm t,veto}-\max(k_{ta},k_{tb})\right)}_{\text{actual observable}} - \underbrace{\Theta\left(p_{\rm t,veto}-|\vec{k}_{ta}+\vec{k}_{tb}|\right)}_{\text{NLL approximation}}\right]$$





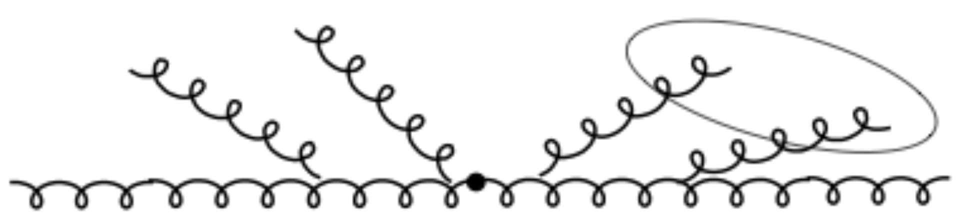
For small radius, rescaling $k_{ta} = zk_t$ and $k_{tb} = (1-z)k_t$ gives, for gluon-gluon splitting

$$\delta\mathcal{F}_{\rm correl} \sim \alpha_s(p_{\rm t,veto})R' \int_{R^2}^1 \frac{d\theta^2}{\theta^2}\int_0^1 dz\, p_{gg}(z)\ln\left[\frac{1}{\max(z,1-z)}\right] \sim \boxed{\alpha_s(p_{\rm t,veto})R' \ln\frac{1}{R_{\rm jet}}}$$

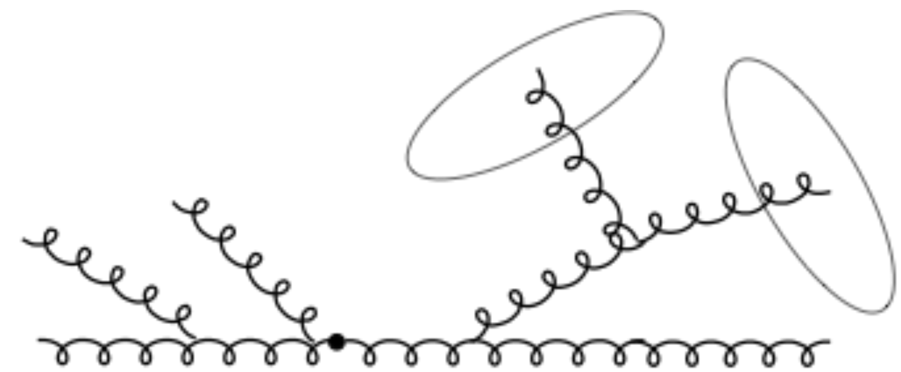
NNLL jet-veto distribution

Bringing all together one obtains a NNLL resummation formula, which contains for the first time a non-trivial dependence on the jet radius

Two nearby gluons clustered in one jet



One gluon giving two jets



$$\epsilon(p_{t,\mathrm{veto}}) = \frac{\mathcal{L}(p_{t,\mathrm{veto}})}{\mathcal{L}(m_H)} \times \left[1 + \underbrace{\alpha_s(p_{t,\mathrm{veto}})\, R'(p_{t,\mathrm{veto}}) f(R_{\mathrm{jet}})}_{\mathrm{NNLL}}\right] \times e^{-R(p_{t,\mathrm{veto}})}$$

The function $f(R_{\mathrm{jet}}) \sim \ln R_{\mathrm{jet}}$ due to a cutoff collinear singularity in gluon splitting. There is general interest in understanding the structure of these logarithms, and the term $\alpha_s^3 \ln^2(R_{\mathrm{jet}})$ has been recently computed by Alioli and Walsh

Summary

In this lecture we have learnt:

1. variation of renormalisation and factorisation scales is a theoretically sound procedure for scale uncertainties only for observables dominated by a single scale

2. for multi-scale observables, problems in scale variations might give an indication of the infrared sensitivity of cross sections

3. methods to assess theoretical uncertainties for resummed predictions

4. some principles of NNLL resummation in the specific case of the jet-veto efficiency