A general framework to combine NNLO calculations with a parton shower

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Parton Showers, Event Generators and Resummation
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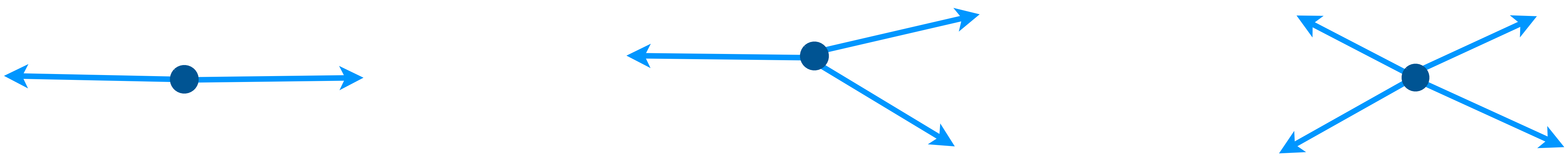
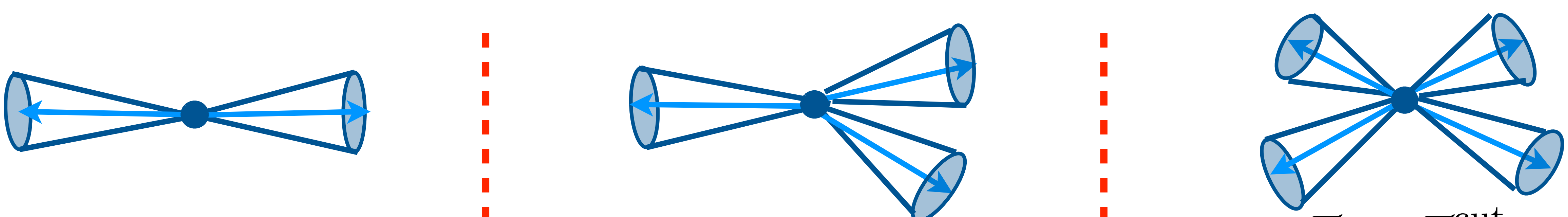
To cast perturbative calculations as event generators, separate the total hadronic event into different jet multiplicities

Φ_2 Φ_3 Φ_4





$\mathcal{T}_2 < \mathcal{T}_2^{\mathrm{cut}}$

$\mathcal{T}_2^{\mathrm{cut}}$

$\mathcal{T}_2 \geq \mathcal{T}_2^{\mathrm{cut}}$
$\mathcal{T}_3 < \mathcal{T}_3^{\mathrm{cut}}$

$\mathcal{T}_3^{\mathrm{cut}}$

$\mathcal{T}_2 \geq \mathcal{T}_2^{\mathrm{cut}}$
$\mathcal{T}_3 \geq \mathcal{T}_3^{\mathrm{cut}}$
$\mathcal{T}_4 < \mathcal{T}_4^{\mathrm{cut}}$

$$\frac{\mathrm{d}\sigma_2^{\mathrm{MC}}}{\mathrm{d}\Phi_2}(\mathcal{T}_2^{\mathrm{cut}}) \qquad \frac{\mathrm{d}\sigma_3^{\mathrm{MC}}}{\mathrm{d}\Phi_3}(\mathcal{T}_3^{\mathrm{cut}}) \qquad \frac{\mathrm{d}\sigma_4^{\mathrm{MC}}}{\mathrm{d}\Phi_4}(\mathcal{T}_4^{\mathrm{cut}})$$

With this step, the only question remains is what the differential cross-section is

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}?$$

At this point, this is a purely perturbative question

This leads to two conditions that each MC cross-section needs to satisfy

Condition 1:
Correct individual perturbative accuracy

	N-jet	(N+1)-jet	(N+2)-jet
Fixed order	NNLO	NLO	LO

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Condition 1:
Correct individual perturbative accuracy

	N-jet	(N+1)-jet	(N+2)-jet
Fixed order	NNLO	NLO	LO
Resummed order	LL	LL	LL

Resummation required to be able to take resolution variable small

This leads to two conditions that each MC cross-section needs to satisfy

Condition II: Correlation between different multiplicities

Jet resolution is artificial parameter. Inclusive cross-sections should not depend on it to the perturbative order one is working

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^c) = \frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\,\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\,\theta(\mathcal{T}_N < \mathcal{T}_N^c)$$

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This consistency can be written as

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{T}_N^{\mathrm{cut}}} \left[\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right]_{\mathcal{T}_N^{\mathrm{cut}}=\mathcal{T}_N} = \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\, \delta[\mathcal{T}_N - \mathcal{T}_N(\Phi_{N+1})]\, \frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

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If this is enforced exactly, is unitarity talked about before

Explicit construction of the MC cross-sections

At fixed order in perturbation theory, the expressions are completely determined

Standard expression for an NNLO observable

$$\sigma^{\mathrm{NNLO}}(X) = \int \mathrm{d}\Phi_N \, (B_N + V_N + W_N)(\Phi_N) \, M_X(\Phi_N) + \int \mathrm{d}\Phi_{N+1} \, \big(B_{N+1} + V_{N+1}\big)(\Phi_{N+1}) \, M_X(\Phi_{N+1}) + \int \mathrm{d}\Phi_{N+2} \, B_{N+2}(\Phi_{N+2}) \, M_X(\Phi_{N+2}) \, ,$$

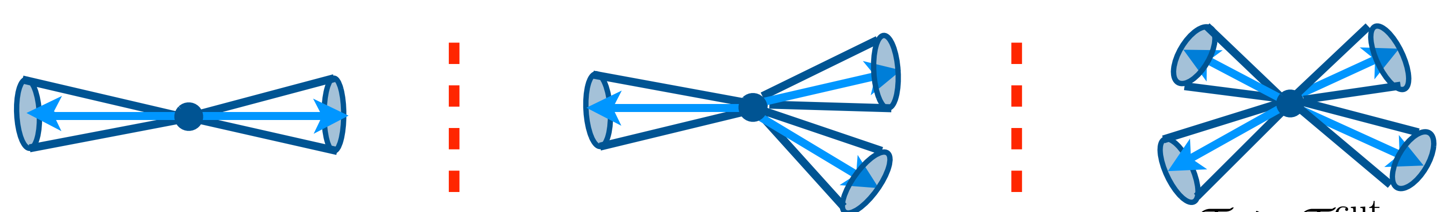
Want to write this in terms of MC cross sections

At fixed order in perturbation theory, the expressions are completely determined

Standard expression for an NNLO observable

$$
\begin{aligned}
\sigma^{\mathrm{NNLO}}(X) = &\int \mathrm{d}\Phi_N \,(B_N + V_N + W_N)(\Phi_N)\, M_X(\Phi_N) \\
&+ \int \mathrm{d}\Phi_{N+1} \,\big(B_{N+1} + V_{N+1}\big)(\Phi_{N+1})\, M_X(\Phi_{N+1}) \\
&+ \int \mathrm{d}\Phi_{N+2}\, B_{N+2}(\Phi_{N+2})\, M_X(\Phi_{N+2})\,,
\end{aligned}
$$

Want to write this in terms of MC cross sections



$\mathcal{T}_2 < \mathcal{T}_2^{\mathrm{cut}}$	$\mathcal{T}_2^{\mathrm{cut}}$	$\mathcal{T}_2 \geq \mathcal{T}_2^{\mathrm{cut}}$ $\mathcal{T}_3 < \mathcal{T}_3^{\mathrm{cut}}$	$\mathcal{T}_3^{\mathrm{cut}}$	$\mathcal{T}_2 \geq \mathcal{T}_2^{\mathrm{cut}}$ $\mathcal{T}_3 \geq \mathcal{T}_3^{\mathrm{cut}}$ $\mathcal{T}_4 < \mathcal{T}_4^{\mathrm{cut}}$
$\dfrac{\mathrm{d}\sigma_2^{\mathrm{MC}}}{\mathrm{d}\Phi_2}(\mathcal{T}_2^{\mathrm{cut}})$		$\dfrac{\mathrm{d}\sigma_3^{\mathrm{MC}}}{\mathrm{d}\Phi_3}(\mathcal{T}_3^{\mathrm{cut}})$		$\dfrac{\mathrm{d}\sigma_4^{\mathrm{MC}}}{\mathrm{d}\Phi_4}(\mathcal{T}_4^{\mathrm{cut}})$

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Standard expression for an NNLO observable

$$\sigma^{\mathrm{NNLO}}(X) = \int \mathrm{d}\Phi_N \, (B_N + V_N + W_N)(\Phi_N) \, M_X(\Phi_N) + \int \mathrm{d}\Phi_{N+1} \, \big(B_{N+1} + V_{N+1}\big)(\Phi_{N+1}) \, M_X(\Phi_{N+1}) + \int \mathrm{d}\Phi_{N+2} \, B_{N+2}(\Phi_{N+2}) \, M_X(\Phi_{N+2}) \,,$$

Want to write this in terms of MC cross sections

$$\sigma^{\mathrm{NNLO}}(X) = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \int \mathrm{d}\Phi_{N+1} \, \frac{\mathrm{d}\sigma_{N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}; \mathcal{T}_{N+1}^{\mathrm{cut}}) + \int \mathrm{d}\Phi_{N+2} \, \frac{\mathrm{d}\sigma_{\geq N+2}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+2}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\mathrm{cut}})$$

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$$+ \int \mathrm{d}\Phi_{N+2} \, \frac{\mathrm{d}\sigma_{\geq N+2}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+2}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\mathrm{cut}})$$

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$$+ \int \mathrm{d}\Phi_{N+2} \, \frac{\mathrm{d}\sigma_{\geq N+2}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+2}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\mathrm{cut}})$$

Required expression at fixed order are

$$\begin{aligned}
\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) &= (B_N + V_N + W_N)(\Phi_N) \\
&\quad + \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N} \, (B_{N+1} + V_{N+1})(\Phi_{N+1}) \, \theta[\mathcal{T}_N(\Phi_{N+1}) < \mathcal{T}_N^{\mathrm{cut}}] \\
&\quad + \int \frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_N} \, B_{N+2}(\Phi_{N+2}) \, \theta[\mathcal{T}_N(\Phi_{N+2}) < \mathcal{T}_N^{\mathrm{cut}}] \,,
\end{aligned}$$

At fixed order in perturbation theory, the expressions are completely determined

Want to write this in terms of MC cross sections

$$\sigma^{\mathrm{NNLO}}(X) = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \int \mathrm{d}\Phi_{N+1} \, \frac{\mathrm{d}\sigma_{N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}; \mathcal{T}_{N+1}^{\mathrm{cut}})$$

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Required expression at fixed order are

$$\frac{\mathrm{d}\sigma_{N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}; \mathcal{T}_{N+1}^{\mathrm{cut}})$$

$$= (B_{N+1} + V_{N+1})(\Phi_{N+1}) \, \theta[\mathcal{T}_N(\Phi_{N+1}) > \mathcal{T}_N^{\mathrm{cut}}]$$

$$+ \int \frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_{N+1}} \, B_{N+2}(\Phi_{N+2}) \, \theta[\mathcal{T}_N(\Phi_{N+2}) > \mathcal{T}_N^{\mathrm{cut}}] \, \theta[\mathcal{T}_{N+1}(\Phi_{N+2}) < \mathcal{T}_{N+1}^{\mathrm{cut}}]$$

At fixed order in perturbation theory, the expressions are completely determined

Want to write this in terms of MC cross sections

$$\sigma^{\rm NNLO}(X) = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma_N^{\rm MC}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\rm cut}) + \int \mathrm{d}\Phi_{N+1} \, \frac{\mathrm{d}\sigma_{N+1}^{\rm MC}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\rm cut}; \mathcal{T}_{N+1}^{\rm cut}) + \int \mathrm{d}\Phi_{N+2} \, \frac{\mathrm{d}\sigma_{\geq N+2}^{\rm MC}}{\mathrm{d}\Phi_{N+2}}(\mathcal{T}_N > \mathcal{T}_N^{\rm cut}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\rm cut})$$

Required expression at fixed order are

$$\frac{\mathrm{d}\sigma_{\geq N+2}^{\rm MC}}{\mathrm{d}\Phi_{N+2}}(\mathcal{T}_N > \mathcal{T}_N^{\rm cut}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\rm cut}) = B_{N+2}(\Phi_{N+2}) \, \theta[\mathcal{T}_N(\Phi_{N+2}) > \mathcal{T}_N^{\rm cut}] \, \theta[\mathcal{T}_{N+1}(\Phi_{N+2}) > \mathcal{T}_{N+1}^{\rm cut}]$$

How do I make these expression have the correct LL behavior?

There are many possible ways to make the expression correct to LL accuracy

Consider again the exclusive N-jet cross-section

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = {} & (B_N + V_N + W_N)(\Phi_N) \\
& + \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\,(C_{N+1} + VC_{N+1})(\Phi_{N+1})\,\theta[\mathcal{T}_N(\Phi_{N+1}) < \mathcal{T}_N^{\mathrm{cut}}] \\
& + \int \frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_N}\,C_{N+2}(\Phi_{N+2})\,\theta[\mathcal{T}_N(\Phi_{N+2}) < \mathcal{T}_N^{\mathrm{cut}}]\,,
\end{aligned}
$$

Split this into a singular piece and a non-singular piece

$$
= \frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})
$$

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$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) \;=\; \frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) \;+\; \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

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The singular piece contains all singular dependence on T_N^{cut}

$$\begin{aligned}\frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) &= (B_N + V_N + W_N)(\Phi_N) \\ &\quad + \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\,(C_{N+1} + VC_{N+1})(\Phi_{N+1})\,\theta[\mathcal{T}_N(\Phi_{N+1}) < \mathcal{T}_N^{\mathrm{cut}}] \\ &\quad + \int \frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_N}\,C_{N+2}(\Phi_{N+2})\,\theta[\mathcal{T}_N(\Phi_{N+2}) < \mathcal{T}_N^{\mathrm{cut}}]\,,\end{aligned}$$

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The singular piece contains all singular dependence on T_N^{cut}

$$\begin{aligned}\frac{d\sigma_N^{C}}{d\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = (B_N + V_N + W_N)(\Phi_N) \\ &+ \int \frac{d\Phi_{N+1}}{d\Phi_N}\,(C_{N+1} + VC_{N+1})(\Phi_{N+1})\,\theta[\mathcal{T}_N(\Phi_{N+1}) < \mathcal{T}_N^{\mathrm{cut}}] \\ &+ \int \frac{d\Phi_{N+2}}{d\Phi_N}\,C_{N+2}(\Phi_{N+2})\,\theta[\mathcal{T}_N(\Phi_{N+2}) < \mathcal{T}_N^{\mathrm{cut}}]\,,\end{aligned}$$

Non-singular piece has at most power dependence on T_N^{cut}

$$\begin{aligned}\frac{d\sigma_N^{B-C}}{d\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) &= \int \frac{d\Phi_{N+1}}{d\Phi_N}\,(B_{N+1} - C_{N+1} + V_{N+1} - VC_{N+1})(\Phi_{N+1})\,\theta[\mathcal{T}_N(\Phi_{N+1}) < \mathcal{T}_N^{\mathrm{cut}}] \\ &+ \int \frac{d\Phi_{N+2}}{d\Phi_N}\,(B_{N+2} - C_{N+2})(\Phi_{N+2})\,\theta[\mathcal{T}_N(\Phi_{N+2}) < \mathcal{T}_N^{\mathrm{cut}}]\,.\end{aligned}$$

There are many possible ways to make the expression correct to LL accuracy

Consider again the exclusive N-jet cross-section

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Now, resum singular dependence on T_N^{cut} to at least LL accuracy

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Now, resum singular dependence on T_N^{cut} to at least LL accuracy

$$\frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) \longrightarrow \left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right]\Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})$$

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Consider again the exclusive N-jet cross-section

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Now, resum singular dependence on T_N^{cut} to at least LL accuracy

$$\frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) \longrightarrow \left[\boxed{\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N}} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right]\Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})$$

Singular approximation to inclusive cross section

$$\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} = (B_N + V_N + W_N)(\Phi_N) + \int\frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\,(C_{N+1} + VC_{N+1})(\Phi_{N+1})$$
$$+ \int\frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_N}\,C_{N+2}(\Phi_{N+2})\,.$$

There are many possible ways to make the expression correct to LL accuracy

Consider again the exclusive N-jet cross-section

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Now, resum singular dependence on T_N^{cut} to at least LL accuracy

$$\frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) \longrightarrow \left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right] \boxed{\Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})}$$

Sudakov form factor

$$\Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}}) = \exp\left\{-\sum_m \left[\int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\,\frac{S_{N+1}(\Phi_{N+1})}{B_N(\Phi_N)}\,\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\right]_m\right\}$$

There are many possible ways to make the expression correct to LL accuracy

Consider again the exclusive N-jet cross-section

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Now, resum singular dependence on T_N^{cut} to at least LL accuracy

$$\frac{\mathrm{d}\sigma_N^{C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) \longrightarrow \left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \boxed{\frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})}\right] \Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})$$

Singular matching term

$$\begin{aligned}
\frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = & -\int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\,(C_{N+1} + VC_{N+1})(\Phi_{N+1})\,\theta[\mathcal{T}_N(\Phi_{N+1}) > \mathcal{T}_N^{\mathrm{cut}}] \\
& -\int \frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_N}\,C_{N+2}(\Phi_{N+2})\,\theta[\mathcal{T}_N(\Phi_{N+2}) > \mathcal{T}_N^{\mathrm{cut}}] \\
& - B_N(\Phi_N)\,\big[\Delta_N^{(1)}(\Phi_N;\mathcal{T}_N^{\mathrm{cut}}) + \Delta_N^{(2)}(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})\big] - V_N^C(\Phi_N)\,\Delta_N^{(1)}(\Phi_N;\mathcal{T}_N^{\mathrm{cut}}) \\
& + \Delta_N^{(1)}(\Phi_N;\mathcal{T}_N^{\mathrm{cut}}) \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\,(C_{N+1} - S_{N+1}^{(1)})(\Phi_{N+1})\,\theta[\mathcal{T}_N(\Phi_{N+1}) > \mathcal{T}_N^{\mathrm{cut}}]
\end{aligned}$$

There are many possible ways to make the expression correct to LL accuracy

Consider again the exclusive N-jet cross-section

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right]\Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Given this expression, and Condition II from before, the inclusive 1-jet cross section is determined by taking the derivative of the above

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \Bigg\{&\left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N)\right]_{\Phi_N=\hat{\Phi}_N}\frac{S_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\,\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\\ &+ \frac{\mathrm{d}\widetilde{\sigma}_{\geq N+1}^{C-S}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\Bigg\}\Delta_N(\hat{\Phi}_N;\mathcal{T}_N) + \frac{\mathrm{d}\sigma_{\geq N+1}^{B-C}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\end{aligned}$$

There are many possible ways to make the expression correct to LL accuracy

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \Bigg\{\Bigg[\frac{\mathrm{d}\sigma^{C}_{\geq N}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N)\Bigg]_{\Phi_N=\hat{\Phi}_N} \frac{S_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\,\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$
$$+ \frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\Bigg\}\Delta_N(\hat{\Phi}_N;\mathcal{T}_N) + \frac{\mathrm{d}\sigma^{B-C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

Each term uniquely determined

There are many possible ways to make the expression correct to LL accuracy

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \left\{ \boxed{\left[\frac{\mathrm{d}\sigma^{C}_{\geq N}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N)\right]_{\Phi_N=\hat{\Phi}_N} \frac{S_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\, \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})} \right.$$

$$\left. + \frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \right\} \Delta_N(\hat{\Phi}_N; \mathcal{T}_N) + \frac{\mathrm{d}\sigma^{B-C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

Each term uniquely determined

Terms shown before

There are many possible ways to make the expression correct to LL accuracy

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \Bigg\{ \bigg[\frac{\mathrm{d}\sigma^{C}_{\geq N}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N)\bigg]_{\Phi_N=\hat{\Phi}_N} \frac{S_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\,\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) + \boxed{\frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})}\Bigg\}\Delta_N(\hat{\Phi}_N;\mathcal{T}_N) + \frac{\mathrm{d}\sigma^{B-C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

Each term uniquely determined

Singular matching

$$\begin{aligned}
\frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) &= (C_{N+1} + VC_{N+1})(\Phi_{N+1})\,\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) + \int \frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_{N+1}}\, C_{N+2}(\Phi_{N+2})\,\theta[\mathcal{T}_N(\Phi_{N+2}) > \mathcal{T}_N^{\mathrm{cut}}] \\
&\quad - \bigg[1 + \frac{S^{(2)}_{N+1}(\Phi_{N+1})}{S^{(1)}_{N+1}(\Phi_{N+1})} + \frac{V^C_N(\hat{\Phi}_N)}{B_N(\hat{\Phi}_N)} + \Delta^{(1)}_N(\hat{\Phi}_N,\mathcal{T}_N)\bigg] S^{(1)}_{N+1}(\Phi_{N+1})\,\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \\
&\quad - \bigg\{\Delta^{(1)}_N(\hat{\Phi}_N;\mathcal{T}_N)\,(C_{N+1} - S^{(1)}_{N+1})(\Phi_{N+1}) \\
&\quad + \frac{S^{(1)}_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)} \int \frac{\mathrm{d}\Phi'_{N+1}}{\mathrm{d}\Phi_N}\,(C_{N+1} - S^{(1)}_{N+1})(\Phi'_{N+1})\,\theta[\mathcal{T}_N(\Phi'_{N+1}) > \mathcal{T}_N]\bigg\}\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})
\end{aligned}$$

There are many possible ways to make the expression correct to LL accuracy

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \left\{ \left[\frac{\mathrm{d}\sigma^{C}_{\geq N}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{N}}{\mathrm{d}\Phi_N}(\mathcal{T}_N) \right]_{\Phi_N = \hat{\Phi}_N} \frac{S_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\, \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\widetilde{\sigma}^{C-S}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \right\} \Delta_N(\hat{\Phi}_N; \mathcal{T}_N) + \boxed{\frac{\mathrm{d}\sigma^{B-C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})}$$

Each term uniquely determined

Non-singular matching

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{B-C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) &\equiv \frac{\mathrm{d}\sigma^{\mathrm{NLO}}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) - \frac{\mathrm{d}\sigma^{C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \\
&= (B_{N+1} - C_{N+1} + V_{N+1} - VC_{N+1})(\Phi_{N+1})\, \theta[\mathcal{T}_N(\Phi_{N+1}) > \mathcal{T}_N^{\mathrm{cut}}] \\
&\quad + \int \frac{\mathrm{d}\Phi_{N+2}}{\mathrm{d}\Phi_{N+1}}\, (B_{N+2} - C_{N+2})(\Phi_{N+2})\, \theta[\mathcal{T}_N(\Phi_{N+2}) > \mathcal{T}_N^{\mathrm{cut}}]\,,
\end{aligned}$$

Given inclusive (N+1)-jet cross-section, can obtain the exclusive (N+1)-jet and inclusive (N+2)-jet rate in a similar manner

One can write the general expressions as…

$$
\begin{aligned}
&\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}; \mathcal{T}_{N+1}^{\mathrm{cut}}) \\
&\quad = \underbrace{\frac{\mathrm{d}\sigma'^{C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\, \Delta_{N+1}(\Phi_{N+1}; \mathcal{T}_{N+1}^{\mathrm{cut}})}_{\text{resummed}} + \Bigg(\underbrace{\frac{\mathrm{d}\sigma^{C-S}_{N+1}}{\mathrm{d}\Phi_{N+1}}}_{\substack{\text{FO singular}\\ \text{matching}}} + \underbrace{\frac{\mathrm{d}\sigma^{B-C}_{N+1}}{\mathrm{d}\Phi_{N+1}}}_{\substack{\text{FO nonsing.}\\ \text{matching}}}\Bigg)(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}; \mathcal{T}_{N+1}^{\mathrm{cut}})
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\geq N+2}}{\mathrm{d}\Phi_{N+2}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\mathrm{cut}}) \\
&\quad = \frac{\mathrm{d}\sigma'^{C}_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\bigg|_{\Phi_{N+1} = \hat{\Phi}_{N+1}} \frac{S_{N+2}(\Phi_{N+2})}{B_{N+1}(\hat{\Phi}_{N+1})}\, \Delta_{N+1}(\hat{\Phi}_{N+1}; \mathcal{T}_{N+1})\, \theta(\mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\mathrm{cut}}) \\
&\qquad + \Bigg(\frac{\mathrm{d}\sigma^{C-S}_{\geq N+2}}{\mathrm{d}\Phi_{N+2}} + \frac{\mathrm{d}\sigma^{B-C}_{\geq N+2}}{\mathrm{d}\Phi_{N+2}}\Bigg)(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}, \mathcal{T}_{N+1} > \mathcal{T}_{N+1}^{\mathrm{cut}})\,.
\end{aligned}
$$

…with explicit expressions for all terms in 1311.0286

Let’s take one step back
and do things at NLO…

Let’s recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right] \Delta_N(\Phi_N; \mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

Let's recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right] \Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

At NLO we find

$$\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} = (B_N + V_N)(\Phi_N) + \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\, C_{N+1}(\Phi_{N+1})$$

$$\frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \int \frac{\mathrm{d}\Phi_{N+1}}{\Phi_N}\, (B_{N+1} - C_{N+1})(\Phi_{N+1})\, \theta(\mathcal{T}_N < \mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = -\int \frac{\mathrm{d}\Phi_{N+1}}{\Phi_N}\, (C_{N+1} - S_{N+1})(\Phi_{N+1})\, \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

Let’s recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right] \Delta_N(\Phi_N; \mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_N^{B-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})$$

At NLO we find

$$\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} = (B_N + V_N)(\Phi_N) + \int \frac{\mathrm{d}\Phi_{N+1}}{\mathrm{d}\Phi_N}\, C_{N+1}(\Phi_{N+1})$$

$$\frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \int \frac{\mathrm{d}\Phi_{N+1}}{\Phi_N}\, (B_{N+1} - C_{N+1})(\Phi_{N+1})\, \theta(\mathcal{T}_N < \mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = -\int \frac{\mathrm{d}\Phi_{N+1}}{\Phi_N}\, (C_{N+1} - S_{N+1})(\Phi_{N+1})\, \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

Let’s make the choice $B_{N+1} = C_{N+1} = S_{N+1}$

$$\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N} \qquad \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_N^{B-C}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = 0$$

Let’s recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N} \Delta_N(\Phi_N; \mathcal{T}_N^{\mathrm{cut}})$$

Let's recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N}\ \Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \left\{\left[\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N} + \frac{\mathrm{d}\widetilde{\sigma}_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N)\right]\frac{S_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\ \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) + \frac{\mathrm{d}\widetilde{\sigma}_{\geq N+1}^{C-S}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\right\}\Delta_N(\hat{\hat{\Phi}}_N;\mathcal{T}_N) + \frac{\mathrm{d}\sigma_{\geq N+1}^{B-C}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

Let's recover a well know NLO matched result from our general setup

$$\frac{d\sigma_N^{\mathrm{MC}}}{d\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{d\sigma_{\geq N}^{\mathrm{NLO}}}{d\Phi_N} \Delta_N(\Phi_N; \mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{d\sigma_{\geq N+1}^{\mathrm{MC}}}{d\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \left\{\left[\frac{d\sigma_{\geq N}^{C}}{d\Phi_N} + \frac{d\widetilde{\sigma}_N^{C-S}}{d\Phi_N}(\mathcal{T}_N)\right] \frac{S_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)} \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \right.$$

$$\left. + \frac{d\widetilde{\sigma}_{\geq N+1}^{C-S}}{d\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\right\} \Delta_N(\hat{\Phi}_N; \mathcal{T}_N) + \frac{d\sigma_{\geq N+1}^{B-C}}{d\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

With the same choice $B_{N+1} = C_{N+1} = S_{N+1}$

$$\frac{d\sigma_{\geq N}^{C}}{d\Phi_N} = \frac{d\sigma_{\geq N}^{\mathrm{NLO}}}{d\Phi_N} \qquad S_{N+1}(\Phi_{N+1}) = B_{N+1}(\Phi_{N+1})$$

$$\frac{d\widetilde{\sigma}_{\geq N+1}^{C-S}}{d\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \frac{d\sigma_{\geq N+1}^{B-C}}{d\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \frac{d\widetilde{\sigma}_N^{C-S}}{d\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = 0$$

Let's recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N} \Delta_N(\Phi_N; \mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N} \frac{B_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)} \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) \Delta_N(\hat{\Phi}_N; \mathcal{T}_N)$$

Let's recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N}\ \Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N}\ \frac{B_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\ \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\ \ \Delta_N(\hat{\Phi}_N;\mathcal{T}_N)$$

This is the usual Powheg formula

Let's recover a well know NLO matched result from our general setup

$$\frac{\mathrm{d}\sigma_N^{\mathrm{MC}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N}\,\Delta_N(\Phi_N;\mathcal{T}_N^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq N+1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq N}^{\mathrm{NLO}}}{\mathrm{d}\Phi_N}\,\frac{B_{N+1}(\Phi_{N+1})}{B_N(\hat{\Phi}_N)}\,\theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})\,\Delta_N(\hat{\Phi}_N;\mathcal{T}_N)$$

This is the usual Powheg formula

One can reproduce the MC@NLO expressions with similar choices from our general framework

Comparison to other approaches

The MINLO reweighing approach appeared before our paper, so wanted to show how it fits into our general framework

Idea is to reweigh a variant of the Powheg approach (called MINLO)

$$\frac{d\sigma_{\geq 1}^{\text{ref. [38]}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = \widetilde{R}(\Phi_0; \mathcal{T}_0^{\text{cut}})\, \frac{d\sigma_{\geq 1}^{\text{HJ-MiNLO}}}{d\Phi_1}\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})$$

$$\frac{d\sigma_{\geq 1}^{\text{HJ-MiNLO}}}{d\Phi_1} = \left\{ B_1(\Phi_1)\big[1 - \widetilde{\Delta}_0^{(1)}(\hat{\Phi}_0; \mathcal{T}_0)\big] + V_1(\Phi_1) + \int \frac{d\Phi_2}{d\Phi_1}\, B_2(\Phi_2) \right\} \widetilde{\Delta}_0(\hat{\Phi}_0, \mathcal{T}_0)$$

$$\widetilde{R}(\Phi_0; \mathcal{T}_0^{\text{cut}}) = \frac{d\sigma_{\geq 0}^{\text{NNLO}}}{d\Phi_0} \Big/ \int \frac{d\Phi_1}{d\Phi_0}\, \frac{d\sigma_{\geq 1}^{\text{HJ-MiNLO}}}{d\Phi_1}\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})$$

The MINLO reweighing approach appeared before our paper, so wanted to show how it fits into our general framework

$$\frac{\mathrm{d}\sigma_{\geq 1}^{\text{ref. [38]}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = \widetilde{R}(\Phi_0; \mathcal{T}_0^{\text{cut}})\, \frac{\mathrm{d}\sigma_{\geq 1}^{\text{HJ-MiNLO}}}{\mathrm{d}\Phi_1}\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})$$

We can reproduce this result from our formalism by making the following choices

$$C_1(\Phi_1) = B_1(\Phi_1)$$

$$C_2(\Phi_2) = B_2(\Phi_2)$$

$$VC_1(\Phi_1) = V_1(\Phi_1)$$

$$S_1^{(1)}(\Phi_1) = B_1(\Phi_1)$$

$$S_1^{(2)}(\Phi_1) = V_1(\Phi_1) + \int \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_1}\, B_2(\Phi_2) - B_1(\Phi_1)\left[\frac{V_0^C(\hat{\Phi}_0)}{B_0(\hat{\Phi}_0)} + \Delta_0^{(1)}(\hat{\Phi}_0; \mathcal{T}_0)\right]$$

This gives

$$\frac{\mathrm{d}\sigma_0^{\text{MC}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{\mathrm{d}\sigma_{\geq 0}^{\text{NNLO}}}{\mathrm{d}\Phi_0}\, \Delta_0(\Phi_0; \mathcal{T}_0^{\text{cut}})$$

$$\frac{\mathrm{d}\sigma_{\geq 1}^{\text{MC}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = R(\hat{\Phi}_0)\left\{B_1(\Phi_1)\big[1 - \Delta_0^{(1)}(\hat{\Phi}_0; \mathcal{T}_0)\big] + V_1(\Phi_1) + \int \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_1}\, B_2(\Phi_2)\right\} \times \Delta_0(\hat{\Phi}_0; \mathcal{T}_0)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})\,,$$

The MINLO reweighing approach appeared before our paper, so wanted to show how it fits into our general framework

$$\frac{d\sigma_{\geq 1}^{\text{ref. [38]}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = \widetilde{R}(\Phi_0; \mathcal{T}_0^{\text{cut}}) \frac{d\sigma_{\geq 1}^{\text{HJ-MiNLO}}}{d\Phi_1} \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})$$

This gives

$$\frac{d\sigma_0^{\text{MC}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma_{\geq 0}^{\text{NNLO}}}{d\Phi_0} \Delta_0(\Phi_0; \mathcal{T}_0^{\text{cut}})$$

$$\frac{d\sigma_{\geq 1}^{\text{MC}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = R(\hat{\Phi}_0) \left\{ B_1(\Phi_1)\big[1 - \Delta_0^{(1)}(\hat{\Phi}_0; \mathcal{T}_0)\big] + V_1(\Phi_1) + \int \frac{d\Phi_2}{d\Phi_1} B_2(\Phi_2) \right\} \times \Delta_0(\hat{\Phi}_0; \mathcal{T}_0)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})\,,$$

With

$$R(\Phi_0) = \frac{d\sigma_{\geq 0}^{\text{NNLO}}}{d\Phi_0} \Big/ \left\{ \frac{d\sigma_{\geq 0}^{\text{NLO}}}{d\Phi_0} - \frac{V_0^C(\Phi_0)}{B_0(\Phi_0)} \int \frac{d\Phi_1}{d\Phi_0} S_1^{(2)}(\Phi_1)\, \Delta_0(\Phi_0, \mathcal{T}_0) \right\}$$

Thus, our general formalism encompasses the MINLO reweighing approach, but it could be implemented without reweighing

In Geneva, resum to high enough order that perturbative expressions and consistency conditions automatically satisfied

For any NNLL' resummed calculation one automatically gets

$$\frac{\mathrm{d}\sigma_N^{C-S}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) = 0$$

$$\begin{aligned}
\frac{\mathrm{d}\sigma_{\geq N}^{C}}{\mathrm{d}\Phi_N}\Delta_N(\mathcal{T}_N;\Phi_N) \quad &\to \quad \frac{\mathrm{d}\sigma_N^{\mathrm{resummed}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\,, \\
\sigma_N^{B-C}(\mathcal{T}_N^{\mathrm{cut}}) \quad &\to \quad \frac{\mathrm{d}\sigma_N^{\mathrm{nonsingular}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})
\end{aligned}$$

Thus, NNLO merging is automatically achieved, and the only thing missing is power suppressed non-singular pieces

These can be included with an explicit additional matching step

In conclusion, there is a very general approach to NNLO merging, and many possible implementations

Thank you!